Curriculum Vitae

Alain Chenciner

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Alain Chenciner, born October 23 1943 at Villeneuve sur Lot (France)
French citizen
Languages spoken and written: french, english, spanish, russian, chinese (still studying).

1 EDUCATION

1963-1965 Student at Ecole Polytechnique,
1966-1971 CNRS (Attaché de recherche), founding member of the "Centre de Mathématiques de l'Ecole Polytechnique" created by Laurent Schwartz,
1971 Doctorat ès Sciences Mathématiques, University Paris XI-Orsay (director J. Cerf, jury: Henri Cartan, Jean Cerf, René Thom, Laurent Schwartz),

2 EMPLOYMENT

1971-1973 Maître de Conférences University Paris XI-Orsay,
1973-1981 Maître de Conférences University Paris VII-Denis Diderot (on leave at University of Nice 1975-78).
1981-1991 1st class Professor University Paris VII-Denis Diderot,
1991-2012 Professor in "classe exceptionnelle" University Paris VII-Denis Diderot; untill 2009, President of the thesis committee of the math department at University Paris VII-Denis Diderot.
From September 2012 on, Emeritus Professor University Paris VII-Denis Diderot

Until my retirement, codirector with Jacques Laskar of the research group "Astronomy and Dynamical Systems" which is part of IMCCE (Institute of Celestial Mechanics and Computation of Ephemeris) at Paris Observatory. We created

this research group in 1992 with both astronomers (Jacques Laskar and his students) and mathematicians (me and my students). To-day it is quite active in both the long term evolution of planetary systems and in celestial mechanics and this contributed a lot to maintain my enthusiasm for reearch.

3 AWARDS

A "Doistau-Blutel" prize from the french Academy of Science , december 1986,
A 3/4 h invited talk at the ICM Beijing 2002,
A 1h invited plenary talk at the ICMP Lisbon 2003,
Member of *Fellows of the AMS* since their creation.

4 EDITORIAL ACTIVITY

Grundlehren der Mathematischen Wissenschaften (Springer), Managing Editor,
Regular and Chaotic Dynamics member of the Editorial Board,
Nonlinearity member of the Editorial Board until 2017,
No DEA member of the Editorial Board until 2016,
In the past, also member of other Editorial Boards, like the ones of Astérisque and Discrete and Continuous Dynamical Systems.

Co-editor of :
Celestial Mechanics, Dedicated to Donald Saari for his 60th Birthday, Contemporary Mathematics, 292, AMS (2002),

Special issue of the *"Gazette des mathématiciens"* dedicated to René Thom, (february 2005).
Mathematical works of René Thom (Société Mathématique de France): the first volume appeared in 2017, the second one in 2019, the third and last one in 2022.

5 ORGANIZATION OF INTERNATIONAL CONFERENCES, SCIENTIFIC COMMITTEES

5.1 Organization (since 1999)

International Conference *The determination of homoclinic trajectories in Hamiltonian systems and Arnold's diffusion* September 6–17 , 1999 at IHES (with Giovanni Gallavotti and Jacques Laskar);
International Conference on Celestial Mechanics, Chicago (Northwestern University, celebrating Don Saari's 60th birthday) December 15–19, 1999 (with Clark Robinson and Jeff Xia);
International Conference in memory of Michel Herman, IHP Paris 4–8 novembre 2002;

AIM Conference AIM on Variational Methods, Palo Alto June 9–14, 2003 (with Richard Montgomery);
International Conference celebrating Marc Chaperon's 60th birthday, IHP January 2010;
Journée Poincaré organised at Paris Observatory July 9, 2012.

5.2 Scientific committees (since 1999)

International Conference *Géométrie au $XX^{ème}$ siècle, 1930-2000*, September 24–28, 2001, Institut Henri Poincaré (Paris);
International Conference *Modern Theory of Dynamical Systems and Celestial Mechanics* in honor of Alexeyev, Moscow, December 23–28, 2002;
International Conference in honor of Laurent Schwartz, Ecole Polytechnique, June 30–July 5, 2003;
Jubilé de Jean-Pierre Ramis, Toulouse, September 22–26, 2003;
Third international conference on Nonlinear Analysis : Hamiltonian Systems and Celestial Mechanics, Nankai University, Tianjin (Chine), June 2004;
Workshop on the mathematical aspects of celestial mechanics, IHP December 2007;
Conference to memory of Paulette Libermann, IHP December 2009;
International Conference in honor of Ken Meyer for his 75th birthday, RTNS 2012 ;
International Conference *Dynamical systems and classical mechanics*, in memory of V.I. Arnold, Edimburgh October 3-7, 2011;
Géométrie sans frontières, au contact de plusieurs disciplines, celebrating Daniel Bennequin's 60th birthday, March 12-17, 2012;
International Conference celebrating John Mather's 70th birthday, Lyon June 2012;
International Conference in memory of Henri Poincaré, Gigon September 2012;
New trends in Dynamical Systems, celegrating J. Llibre's 60th birthday, Salou, October 1–5, 2012;
Colloque international pour le centième anniversaire de la mort d'Henri Poincaré, IHP novembre 2012.
Journées Michel Hénon Paris Observatory, December 4–5, 2013
International Workshop in Astronomy and Dynamics celebrating Jacques Laskar's 60th birhday, Paris Obseratory April 28–30, 2015

6 THESES DIRECTED

6.1 Master theses

Jorge Billeke, *Formule d'Eisenbud-Levine et schéma de Hilbert*, Nice 1978 (direction en collaboration avec J. Briançon)

Thierry Debeneix, *Chocs dans les systèmes d'équations hyperboliques*, Paris VII 1979

Eric Benoit, *Equations de Van der Pol avec forcing*, Paris VII 1979

Angel Rodríguez Méndez, *Connexions homoclines en dimension 3*, Santiago de Compostela 1982

6.2 Doctorate theses

Marc Chaperon, *Germes hyperboliques d'actions de groupes commutatifs*, Univ. Paris VII, October 1980

Daniel Bennequin, *Entrelacements et équations de Pfaff*, Univ. Paris VII, November 1982

Eric Benoit, *Canards de* $\mathbb{R}^3$, Nice 1984

Jerome Los, *Phénomènes de petits diviseurs dans les dédoublements de courbes invariantes*, Univ. Paris VII, June 1986

Alain Albouy, *Variétés intégrales du problème des N corps*, Univ. Paris VII, April 1992

Philippe Levallois, *Non-intégrabilité des billiards définis par certaines perturbations algébriques d'une ellipse et du flot géodésique de certaines perturbations algébriques d'un ellipsoïde*, Univ. Paris VII, December 1993

David Sauzin, *Résurgence paramétrique et exponentielle petitesse de l'écart des séparatrices du pendule rapidement forcé*, Univ. Paris VII, February 1994

Emmanuelle Julliard-Tosel, *Non-intégrabilité algébrique et méromorphe de problèmes de N corps*, Univ. Paris VII, January 25 1999

Jacques Féjoz, *Dynamique séculaire globale dans le problème plan des trois corps et application à l'existence de mouvements quasipériodiques*, Univ. Paris XIII, Sept. 9 1999

Andrea Venturelli, *Action minimizing periodic orbits in the n-body problem*, September 2002.

Martin Celli, *Sur les mouvements homographiques de N corps associés à des masses de signe quelconque, le cas particulier où la somme des masses est nulle, et une application à la recherche de chorégraphies perverses.*, (codirection with Alain Albouy) September 2005.

Anne Robadey, *Différentes modalités de travail sur le général dans les recherches de Poincaré sur les systèmes dynamiques*, January 2006.

Qiaoling Wei, *Solutions de viscosité des équations de Hamilton-Jacobi et min-max itérés* (codirection with Marc Chaperon) Paris 7, May 2013.

Lei Zhao, *Solutions quasi-périodiques et solutions de quasi-collision du problème spatial des trois corps* (codirection with Jacques Féjoz) Paris 7, May 2013.

Jessica Massetti, *Quasi-périodicité et quasi-conservativité*, (codirection with Jacques Féjoz). Paris Observatory, October 2015.

7 RESEARCH

7.1 General overview

My first research works in *differential topology* werein the continuation of those of my advisor Jean Cerf, who had generalized to 1-parameter families the seminal works of Steve Smale on the h-cobordism theorem and the Poincaré oonjecture in higher dimensions. They led me to know René Thom, to develop an interest towards singularities of differentiable functions and, quite naturally, towards dynamical systems, that is the qualitative and geometric study of the asymptotic behavior of solutions of differential equations. More precisely, I was first interested in understanding the role of conservative systems (those of classical mechanics) as organizing centers of the bifurcations of dissipative systems (those which appear, for instance, in the theory of electrical oscillation). From there to the n-Body Problem – the model of conservative systems – which is my main theme to-day, there is the amazed discovery of the origin of qualitative dynamics in the works of Poincaré and Birkhoff while preparing a master course dedicated to Charles Conley's thesis. This course was indeed the occasion to meet Jacques Laskar with whom I founded the *Astronomy and Dynamical Systems* group (ASD) which we have co-directed until my retirement.

Starting with a brief panorama of my first publications, I describe more in detail those of my works that I consider the most important, from thre older ones:
– *bifurcations of elliptic fixed points* within the general theory of dynamical systems,
– *quasi-collisions in the Restricted Three-Body Problem*, in between the general theory of Dynamical Systems and Celestial Mechanics,
– *the role of symmetries in the n-Body Problem* in Celestial Mechanics and the importance of not limiting the study to an ambiant space of dimension 3 in order to understand the algebraic structure of the equations,

to the most recent ones:
– *cases of generic divergence of the normalization of elliptic fixed points* and introduction of the notion of *geometric normalization* which separates radial and angular behaviours.

7.2 Works in differential topology and first works in dynamics

7.2.1 1-parameter Smale theory and low dimensional singularities.

My first publications, [1] to [8], written in part with François Laudenbach, were dedicated to the geometry of some spaces of differentiable functions on a manifold; the question were aimed at generalizing Jean Cerf's Pseudo-Isotopy theorem to the non-simply connected case, this theorem being itself a generalization to 1-parameter families of Steve Smale's celebrated h-cobordism theorem. This

aimed was only attained later on by A. Hatcher and J. Wagoner on the one hand, Igusa on the other hand.

In [4], we had shown under mild hypotheses, that the space of differentiable functions defined on a trivial cobordism and possessing critical points of only two consecutive indices is connected. Soon after. the result of [3] on the géometry of "swallow tails" allowed me to simplify the proof and avoid heavy obstruction computations. In my thesis [6], I generalized the point of view of [3] by considering various problems of "returning to the organizing centre": given an unfolding F_1 of a function depending on parameters in a k-disc D, one considers a family f_2 of functions, parametrized by $S = \partial D$, whose critical values behave like the ones of the restriction $F_1|_{\partial D}$ to the boundary of D of the family F_1. The problem is to understand what are the obstructions to extend the family f_2 to a family F_2 parametrized by D, and whose critical values behave as those of the family F_1. This study led me to conjecture the existence of an exotic factor $\mathbb{Z}$ in the group $\pi_3(Diff D^n)$. The existence of such a factor was proved later on by Hsiang and his collaborators.

In [7], we had given a new definition of the group Wh_2, from which follows a much simpler proof of Cerf's Pseudo-Isotopy theorem in the simply connected case.

7.2.2 Singularities of différentiable functions.

In [5], a key lemma of the theory of singularities (perturbation by an element of the ideal $\mathcal{MJ}^2$) is used in the computation of the homotopy of the local isotropy group of a germ of function. This paper is the origin of my interest for the theory of singularities and of my active involvment in René Thom's seminar (see in particular [C4],[C5],[C6],[C11]).

7.2.3 Morse jets and h-principle

in a recent article [46] written in collaboration with François Laudenbach, we make a new incursion in the realm of Differential Topology : we characterize the sections of the space of 2-jets of Morse functions which are homotopic to a holonomic section.

7.2.4 Equations of Volterra's type

The two articles [9] et [11] are about quadratic differential equations of the type "species competition". They are the byproduct of dicussions with physicists in Nice. The criterion of non existence of periodic orbits given in [9] turned paradoxically useful to localize regions of the parameter space where Hopf bifurcations may occur.

7.2.5 Bifurcations of invariant tori

In [12] and [13], written in collaboration with Gérard Iooss during my stay in Nice, we generalize classical works by Sacker, Naimark, Ruelle-Takens. We deduce the existence of bifurcations of invariant tori of arbitrary dimension from hypotheses on the spectrum of the "graph transform" map. While natural, these hypotheses are of infinite codimension as soon as one is not in the classical cases (for the flows, bifurcations of periodic orbits from an equilibrium or of invariant 2-tori from a periodic solution). This of course is a strong limitation but we shall see in the next section that the phenomenon can nevertheless be observed in small codimension. An interesting feature of these articles is the attempt, probably optimal, to get rid of some diophantine hypotheses by solving only approximately some of the difference equations which are involved; the ony conditions of this kind which remain are directly related to the expression of the notion of "vague attractor".

7.3 Main works in dynamics

7.3.1 Bifurcations of elliptic fixed points.

The simplest non trivial (in the mathematical acception of the term) Mechanical Systems are those whose state condition may be defined by giving position and velocity of a point in the plane, that is 4 real numbers (the "initial conditions" at some time : 2 position coordinates, 2 velocity coordinates). Once the total energy of the system has been fixed, which takes off one dimension, the evolution is defined by a differential equation in dimension 3. A periodic solution is called"stable" if the solutions corresponding to nearby initial conditions remain indefinitely (or at least during extremely long spans of time) close to it. In order to understand the neighborhood of such a stable solution, it is sufficient to understand the repeated intersections of these nearby solutions with a small disc which cuts transversally transverse the periodic solution (which corresponds to a unique intersection point). This is the method of "Poincaré sections", which remplaces the qualitative study of a differential equation by the study of the orbits under iteration of a map P from a disc to a slightly larger disc, in the neighborhood of an "elliptic fixed point ". The conservative character of the mechanical problem becomes the preservation of areas by the map P (the area of the image by P of a small region equals the one of the region) or, more generally, the preservation of a measure with a smooth density with respect ot the Lebesgue measure. The *Restricted Problem of Three Bodies* belongs to this class and was studied in this way by Poincaré. The same is true of its caricature, the problem of geodesics on a deformed sphere. Following the seminal works by Poincaré and Birkhoff on periodic orbits, important works were dedicated to the study of such maps : the most famous are those by Kolmogorov, Arnold, Moser (K.A.M. theory of invariant closed curves) and those by Aubry and Mather (invariant Cantor sets).

In a series of articles (in particular [22, 23, 26]; see also their decription by J.C. Yoccoz in the Bourbaki), I have shown that all the phenomena present in the neighborhood of an elliptic fixed point of an area preserving diffeomorphism of a surface, still occur but "unfolded in the parameter space", in generic (i.e. without any special property) 2-parameter families of (non conservative) surface diffeomorphisms. In other words, the phenomena which occur simutaneously in a single conservative map are now distributed among an infinity of members of a generic family of dissipative maps close to one possessing an elliptic fixed point (i.e. a map which is conservative only at the level of its first derivative at this fixed point). To the Cantor set of invariant curves given by KAM theory corresponds a Cantor set of values of the parameters for which the corresponding diffeomorphism possesses a non normally hyperbolic invariant curve (i.e. an invariant curve such that the radial dynamics in its neighborhood is neither a contraction nor a dilatation, a behavior which, in the dissipative world, is as close as possible of being conservative). In the same way, Aubry-Mather sets, periodic orbits and their homoclinic orbits are also unfolded in the parameter space (figure 1).

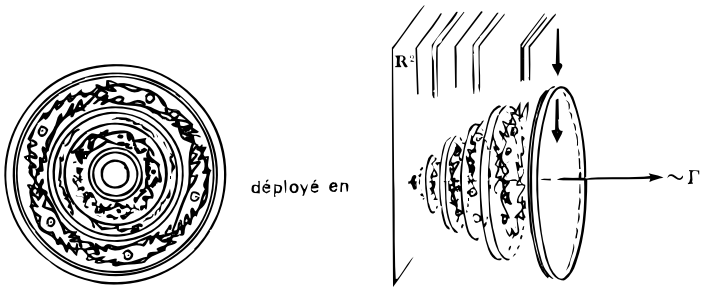


Figure 1 (from [22])

A consequence of these works is the discovery that, in the world of dissipative diffeomorphisms, the role held in the world of conservative diffeomorphisms by invariant closed curves zround a fixed point, is held by 1-parameter families of diffeomorphisms displaying the elimination ("saddle-node") of a couple of invariant closed curves. As in the conservative case and even in a more striking way, it is the *rotation numbers* of the orbits which play the dominant part when normal hyperbolicity becomes too weak. A surprizing result is, for instance, the contagious property of "well ordered" (i.e. ordered as the orbits of a rotation) orbits whose rotation number is *diophantine* (i.e. "far enough" from the rational in a precise sense coming from number theory). In the generic families which are considered, the existence of such an orbit is enough to exclude any "chaotic" behavior: the diffeomorphism "looks like" a normal form in a whole neighborhood of the fixed point containing the orbit and the closure of the orbitis a smooth invariant closed curve. This is a quite beautiful illustration of the way in which an angular distortion may replace a radial contraction or dilatation. Figure 2 shows the "bubles" of the parameter space in which chaotic behavior is confined.

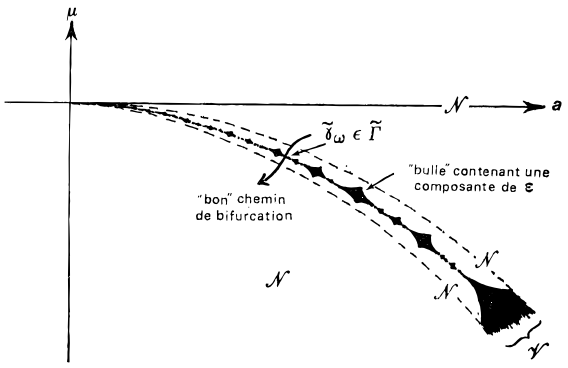


Figure 2 (d'après [22])

Figure 3 illustrates the "resonant elimination" of a couple of invariant curves, that is the complex pjenomena which occur in a generic family of diffeomorphisms corresponding to a path crossing one of these "bubles".

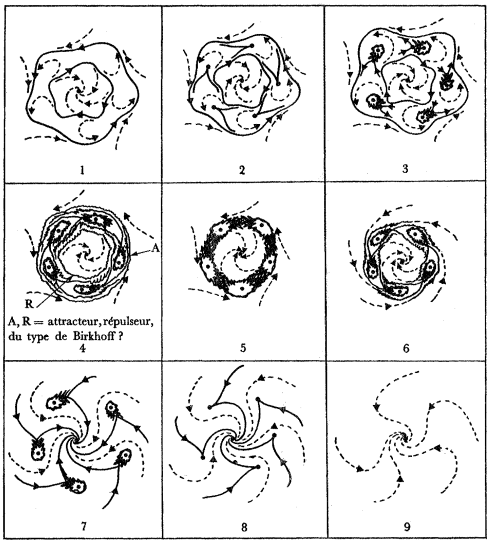


Figure 3 (from [26])

These works are described in a volume of the Russian Encyclopedia of Mathematics (Dynamical Systems V) and in the book "Dynamical Systems" by Arrowsmith and Place. The "bubles" of the parameter space are also present in other situations, for instance in the doubling of invariant curves studied in his thesis by my student Jérome Los.

It is during this period that I wrote in *Encyclopædia Universalis* the two long papers [C11, C14] which were well received bu both mathematicians and physicists.

7.3.2 The restricted problem of three bodies, secular systems.

My interest in Celestial Mechanics dates back to the eighties; it originates precisely from the preparation of a Master course on Dynamical Systems which I based on Charles Conley's thesis *On Some New Long Periodic Solutions of the Plane Restricted Three Body Problem*. Precisely, the course concerned the Plane Circular Restricted Three Body Problem in the lunar case, that is when the Jacobi constant is sufficiently negative to insure that the zero mass body (the Moon) remains confined in a compact component of the Hill region containing one of the primaries (the Earth), the other primary (the Sun) acting on this couple as a perturbation. It is natural in this, and this is what Conley does, to work in a rotating frame fixing the primaries and to regularize the Earth-Moon collisions. This regularization is accomplished following Levi-Civita by adding at infinity of the energy (=Jacobi constant) hypersurface a circle representing the collision directions, which leads to a compactified energy hypersurface with the topologuy of the rotation group $SO(3)$ (that is the projective space $P_3(R)$). At this occasion, I computed a good approximation of the Poincaré first return map in a global annulus of section chosen so as to contain the collision cercle (see [L4]).

In [27], written in collaboration with Jaume Llibre, we compare the McGehee regularization to the composition of the Levi-Civita regularization with a blowing up of the collision circle. This allows us to control with computations reduced to a minimum the intersection of this circle with its image under the first return map. We deduce the existence for the initial (non regularized) problem of *punctured invariant tori* (figure 4) corresponding to solutions in which the Moon comes arbitrarily to the Earth Terre without ever undergoing collision (of course, such *quasi-collisions* are only possible for point masses). This led me to ask in [C17] questions on conservative monotone distortions of the annulus. These questions are still unsolved to-day.

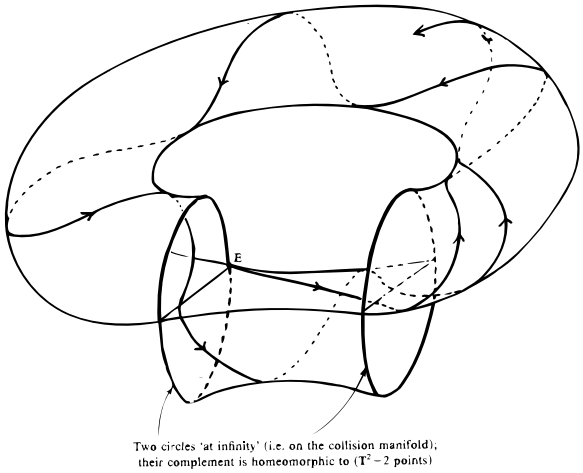


Figure 4 ("invariant punctured torus, from [27])

In the thesis of my student Jacques Féjoz, the existence of solutions undergoing such quasi-collisions was extended to the full Three Body Problem in the plane. This was a consequence of his global study of the associated "secular system" and in particular of the subtle relation between averaging and regularisation. I had proposed to him such a global study as a consequence of my attempt in [C16] to present geometrically the structure in the planetary case. Discussions with Alain Albouy had also been important.

The existence of solutions with quasi-collisions has been extended to the spatial Three Body Problem by Zhao Lei whose thesis I codirected with Jacques Féjoz.

7.3.3 Symmetries and the n Body Problem.

The researches on this topic were in particular at the origin of the discovery of a particularly fascinating family of periodic solutions of the n Body Problem, the *choreographies*. I shall describe them in more detail.

(i) Translation and rotation symmetries : the "shape" of n bodies. The article [28], which appeared in 1998 is the result of a long collaboration with my former student Alain Albouy. In it we generalize to an arbitrary number of bodies and an arbitrary dimension of ambient space the famous "Essai sur le problème des trois corps" (Essay on the the Three Body Problem) written by Lagrange in 1772. We introduce an algebraic coding (by a quadratic form) of the configuration up to isometry of a system of n points. The information contained in this quadratic form is the same as the one given by the set of mutual distances between the n points but it is coded in a way very suitable to computations; once the masses are introduced, it is interpreted as a kind of intrinsic inertia containing the information up to isometries of the inertia ellipsoid of the solid body which the n masses would form if rigidly bound by par massless bars.

(ii) Balanced configurations and central configurations. Using the coding of the shape described above one obtains equations of the "reduced" n-Body Problem (i.e. after quotient by translations and rotations) from which follows the most thorough understanding of *homographic motions*, that is of Kepler-like solutions such that the configuration remains at each time similar to a given one. These reduced equations played also an important part in Albouy's classification of the set of *central configurations* of four bodies with the same mass. These configurations are the only ones which admit homographic motions and their determination for more than three bodies is extraordinarily difficult. The basic observation in this work is that in a reduction "à la Lagrange" – i.e. one which uses mutual distances as variables, – ambient space dimension is just a constraint: in some sense, the Three Body Problem is structurally simpler in a four dimensional space. This leads to the generalization of central configurations that we have called *balanced configurations*: those are the configurations which admit a *relative equilibrium* motion (i.e. a rigid homothetic one) in a space with high enough dimension (that is $2(n-1)$ if there are n bodies). While central configurations are critical points of the potential function on the set of

configurations having a given moment of inertia with respect to their center of mass, the balanced configurations are the critical points of the potential function on the set of configurations whose whole inertia spectrum is given, that is those configurations whose inertia ellipsoid is given up to a rotation.

(iii) Homothety symmetry and analysis of collisions. In [29], also started in collaboration with Alain Albouy, I study the reduction of the "pseudo-symmetry" under homothety which originates from the homogeneity of the Newtonian potential . Already known to Elie Cartan, this symmetry accounts for the existence of tMcGehee's *collision manifold*. This point of view gives a unified conceptual understanding of the classical works of Sundman, Mc Gehee and Saari on collision solutions and (dually) on completely parabolic ones. The paper contains a mistake which, fortunately, is without consequence : an inverted inequality had led me to wrongly assert that Wintner's Tauberian inequalities were unnecessary. I used this way of understanding the collision manifold in my Bourbaki lecture [C17] of the famous result of Jeff Xia which asserts the existence of collision-free solutions of the spatial 5 Body Problem such that some bodies go to infinity in finite time.

(iv) Discrete symmetries and periodic solutions which minimize the Lagrangian action. What are the "simplest" solutions of the n Body Problem which satisfy some given symmetry constraints ? Here "simple" means "which minimize the Lagrangian action", which plays in this problem the part played by geodesics in Riemannian Geometry. My interest for this problem started in 1988. In the note [30], written in collaboration with Nicole Desolneux, we obtain a basic result which, curiously, was not known. Consider the following symmetry constraint among periodic loops of n-body configurations with a given period: after half of the period the configuration is the symmetric of the initial one with respect to the center of mass; in many cases, and in particular in all problems in the plane, the periodic solutions which minimize the Lagrangian action among all loops of a given period with this symmetry are always relative equilibria: the configuration behaves like a solid body, each body describing with constant angular velocity a cercle around the center of mass (see figure 5).

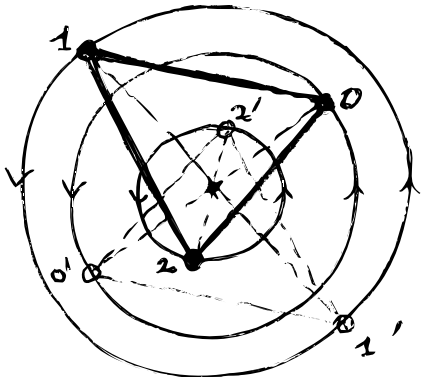


Figure 5 (Lagrange relative equilibrium (equilateral triangle)

(v) The Hip-Hop. Studying with my student Andrea Venturelli the first case not ruled by this theorem, namely 4 bodies in 3-dimensional ambient space, led to a new "simple" solution, that we named the Hip-Hop [31]: 4 equal masses whose configuration periodically hesitates between the regular tetrahedron, whose relative equilibrium would have realized the minimum of action in $\mathbb{R}^4$ and the square, whose relative equilibrium would have realized the minimum of action in $\mathbb{R}^2$ (figure 6). It appeared that such solutions had already been obtained numerically in 1983 by I.Davies,A.Truman and D.Williams and in 1999 by G.Hoynant.

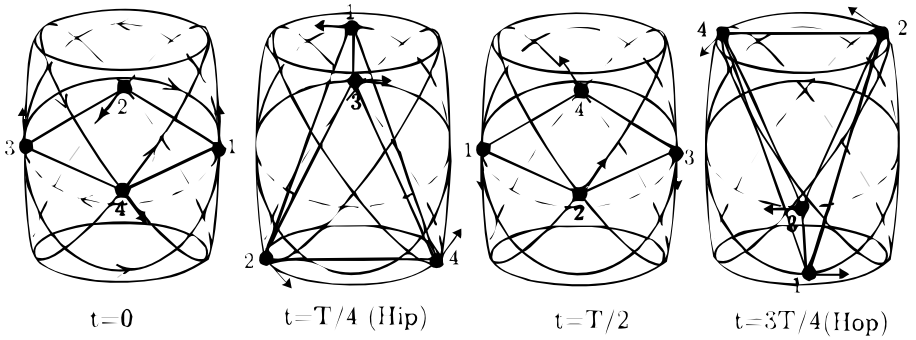


Figure 6 (from [33])

For a generalization to the n Body Problem with arbitrary masses see below the section "Generalized Les Hip-Hops".

(vi) The Eight. In [32], written in collaboration with Richard Montgomery, we prove the existence of a remarkable periodic solution of the 3 Body Problem with equal masses in $\mathbb{R}^2$: the three bodies indefinitely chase each other along one and the same eight-shaped closed curve. This reminds us of the Lagrange equilateral relative equilbrium: when the three masses are equal, the three bodies chase each other indefinitely along one and the same circle. Moreover, in both cases, the bodies are separated by equal time intervals. Nevertheless, while the Lagrange solution is dynamically unstable, Carles Simó proved numerically that the Eight is dynamically stable.

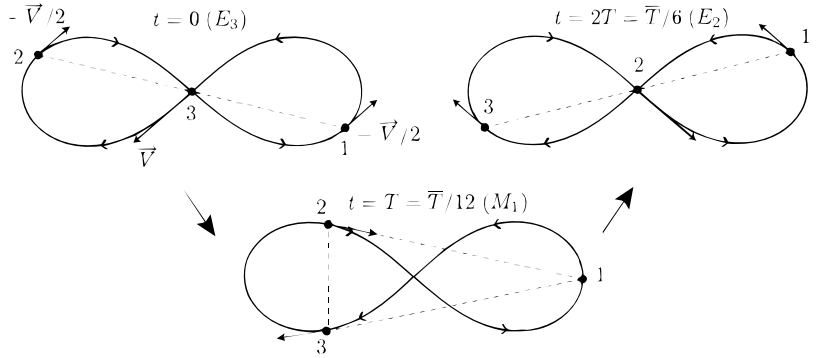
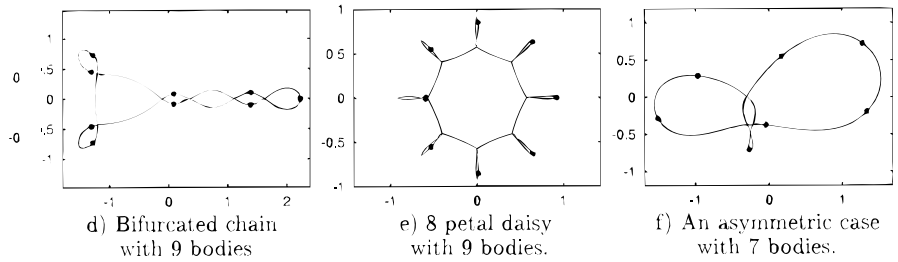


Figure 7 (d'après [32])

This discovery – actually a rediscovery because this solution had been numerically discovered in 1993 by Cris Moore – attracted much attention; papers about it appeared in various journals and even newspapers: *Science (March 17, 2000), Science News (April 14, 2000), Polityka (sic! July 15, 2000), Siam News (July/August 2000), Pour la Science (November 2000), cover of Notices of the AMS (may 2001), Le Monde (January 8, 2002).*

(vii) The Choreographies. Intense numerical (Joseph, Gerver, Carles Simo) and theoretical (Kuo Chang Chen) activity followed the discovery of the Eight. It led to the discovery of the *choreographies*, solutions of the n Body Problem in which n equal masses chase each other on one and the same closed curve and are separated by equal time intervals (see [34] for the state of knowledge in 2002). The corresponding symmetry principles are detailed in [33]. In many cases, such constraints allow to overcome the collision problem attached to weak forces (as is Newton's force) and hence to realize part of the program sketched by Poincaré in a short note dated 1896.



d) Bifurcated chain with 9 bodies

e) 8 petal daisy with 9 bodies.

f) An asymmetric case with 7 bodies.

Figure 8 (from [34])

Thanks to the works of many authors, the existence of many types of symmetric periodic solutions of the n Body Problem is now rigourosly established (see section (ix)).

(viii) Perverse solutions. From the question of whether n bodies admitting a choreographic solution must necessarily have equal masses, I introduced in [39], [40] the notion of *perverse solution* of the n Body Problem. These are solutions which exist for at least two sets of masses. In [P1], we construct examples with several hundred bodies of *really perverse solutions* where the same solution admits rwo systems of masses whose total mass and center of mass are identical. My student Martin Celli proved that in case of a logarithmic potential, choreograpy implies that the masses are all equal.

(ix) Dirichlet problem. A superb idea of Christian Marchal allowed to show that collisions never appear for the Dirichlet problem, that is when minimizing the action between two given configurations. In particular, *given two configurations in $\mathbb{R}^2$ or $\mathbb{R}^3$ of n arbitrary masses and $T > 0$, there always exist a solution of the n Body Problem wich in time T joins the first configuration to the second.* Marchal's proof treats the case of isolated collisions. In my invited lecture at Beijing ICM [35], using techniques from Montgomery, Terracini and Venturelli, I completed the proof and gave applications. A presentation for a wider mathematical audience is given in [C23]). A generalisation of this proof

to equivariant situations was given by Davide Ferrario and Susanna Terracini. This allowed to give existence proofs for a number of interesting periodic solutions which, after the discovery of the Eight, had been obtained numerically. A synthetic view of the state of the art is given in my plenary lecture at ICMP03 in Lisbon [42]. The absence of collisions in the solutions of the Dirichlet Problem explains the success of minimisation under symmetry constraints: the choice of a fundamental domain of the group action on the time interval brings us back to a Dirichlet Problem

(x) Generalized Hip-Hops. Among applications of the above result, one of the most interestings may be the existence for $n \geq 4$ arbitrary masses – of periodic solutions not lying in a plane generalizing the Hip-Hop [37]. These solutions may be viewed as spatial ersatz of the relative equilbria which necessarily remain in a fixed plane. This is the first meaningful application of the minimization method to the n Body Problem with arbitrary masses. Other interesting applications were given by K.C. Chen. Notice that the reason for the existence of these generalized Hip-Hop is the very one which implies the existence of spatial central configurations (Pacella-Moeckel theorem).

(xi) Rotating Eights. With Jacques Féjoz and Richard Montgomery [41], we investigated the various ways in which the Eight solution may bifurcate into a family of solutions of the 3 Body Problem which are periodic in a rotating frame (in a 3-dimensional space). We show the existence of three families, each of which correspond to a choreography in a frame rotating around one of the symmetry axes of the Eight.

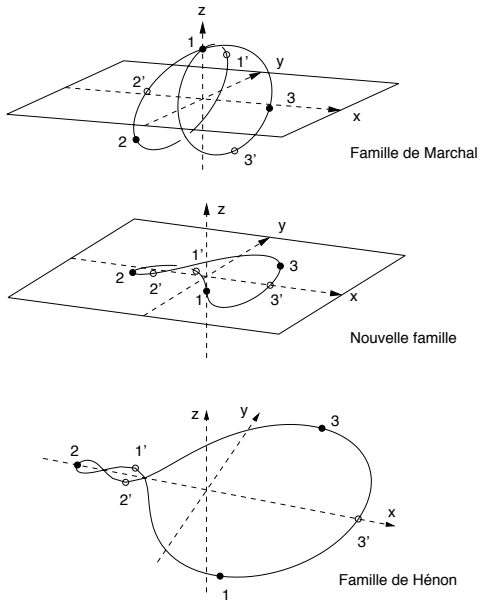


Figure 9 (d'après [41])

Formerly, only two of these families had been discovered respectively by Michel Hénon and Christian Marchal. Each family corresponds to a different breaking of the $D_6 \times \mathbb{Z}_2$ symmetry displayed by the Eight in $\mathbb{R}^3$ but the resulting subgroup is in each case isomorphic to the dihedral group D_6. Ins [44], we show that only Marchal's family P_{12} connects the Eight to lagrange equilateral relative equilibrium.

(xii) Bifurcations of quasi-periodic solutions from a very symmetric relative equilibrium. In [43],[45] we show that from a regular n-gon relative equilibrium of n equal masses bifurcate families of quasi-periodic solutions which terminate at some classes of spatial choreographies or Hip-Hops. Hence these two a priori very different classes of solutions may have a common origin. The general question is the global continuation of Lyapunov families of quasiperiodic solutions which bifurcate these relative equilibria are submitted to a perturbation in the direction orthogonal to the plane of motion (hereinafter named "vertical"). The symmetries of the l'*vertical variational equation* maybe analyzed and minimization under these symmetry constraints in a rotating frame allows the continuation. Nevertheless when the number of bodies exceeds 4, the existence of isomorphisms between these different symmetries (for instance the *chains* and the *general Eights* restrict the possibility of using minimisation to show the existence of some of these families (see fig. 10: the absolute minimum in this symmetry class is the Eight while the chain is merely a relative minimum).

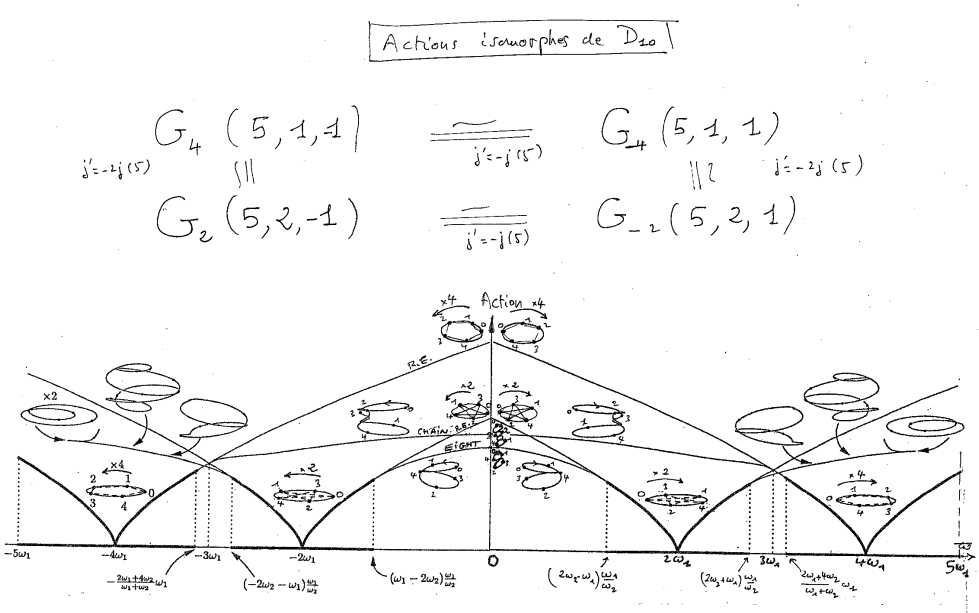


Figure 10 (from [45])

7.3.4 The N-Body Problem in high dimensions.

(i) Angular momentum in dimension greater than 3 and Horn problem. Il In [47], [48], I explain a surprizing relation between on the one hand the structure of the set of spectra of angular momenta of relative equilibria of a given central configuration of n point in a space $\mathbb{R}^{2p}$ when $p \geq 2$, on the other hand the problem of Horn, that is the description of the set of possible spectra of a sum of 2 hermitian (or real symmetric) whose spectra are prescribed.

The result may be stated in the following way : Relative equilibria necessarily take place in a space $\mathbb{R}^{2p}$ of even dimension (see [28] et [C36]) and they are in correspondence with hermitian structures on this space. The theorem is about the "frequency map" which, to such a structure associates the ordered frequencies of the angular momentum (turned into an endomorphism via the euclidean structure) of the corresponding relative equilibrium. It asserts that the image is the same as the one of the restriction of this map to a subset of structures compatible with the symmetries of the inertia ellipsoïde of the central configuration at stake. In [47], I had solved the case $p = 2$ by a direct computation of critical points and these had led me to formulate a conjectureon the general case. In [48], guided by the numerical experiments of my collaborator, Hugo Jiménez-Pérez, I prove this conjecture in full generality.

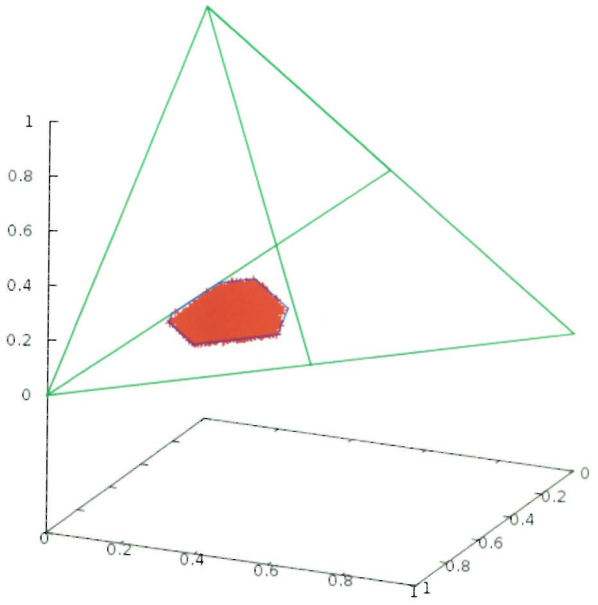


Figure 11 (from [48])

One of the motivations of this study was to understand the spectral structure of the angular momentum of a relative equilibrium in $\mathbb{R}^{2p}$ of a central configuration (hence periodic) when it bifurcates to a family of quasi-periodic relative equilibria of a balanced configuration (voir [28], [C36]). These spectra

correspond to a subskeleton of the image of the frequency map (see [50] where in particular the dimensions in which such relative equilibria may occur are studied).

These works suggested a new inequality between some Littlewood-Richardson coefficients; this inequality is proved in [49], written collaboration with Bernard Leclerc.

(ii) From one solution to another one through a fourth dimension In the same way as on goes from the equilateral relative equilibrium to the Eight threough a family of quasi-periodic solutions in $\mathbb{R}^3$ (see [45]), one can go from a HipHop with $\mathbb{Z}/4\mathbb{Z}$ symmetry to a Hip-Hop with $\mathbb{Z}/3\mathbb{Z}$ symmetry through a family of quasi-periodic solution in $\mathbb{R}^4$ (see [C36]).

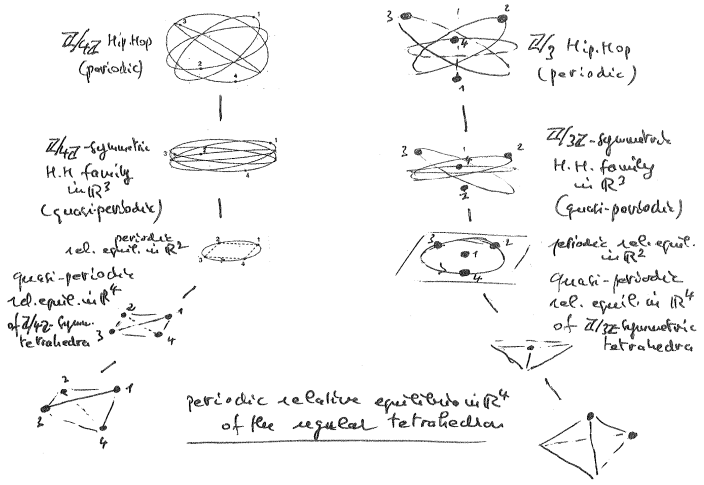


Figure 4: from a $\mathbb{Z}/4\mathbb{Z}$-symmetric Hip-Hop to a $\mathbb{Z}/3\mathbb{Z}$-symmetric one

Figure 11 (from [C36])

With Jacques Féjoz, we had during some time the project of studying how general is this phenomenon, and at first of finding enough significant examples in which going through higher dimension reveals a natural link between famillies of solutions without bovious relation.

7.3.5 Normalisation of analytic weakly attracting fixed points

Let $F : \mathbb{R}^2, 0 \to \mathbb{R}^2, 0$ be a local diffeomorphism of $\mathbb{R}^2$ fixing the origin and such that its dérivative $dF(0)$ be conjugate to a rotation. If F is a weak contraction, i.e. if no-linear terms make the origin attracting, F has no topological invariant: two such maps are topologically conjugate. This is not the case at the analytical level and arises the problem of a local conjugacy of F to a *normal form* N which preserves the foliation by circles centered at 0 and applies each leaf onto the image leaf by a rotation. If F is analytic and $dF(0)$ is a non-periodic rotation, such a conjugacy always exists formally but, in contrast to what happens in the

conservative case, the normal form is not uniquely determined and the existence of an analytic normalization is a delicate problem. This problem splits into
i) existence of a *geometric normalization* in which only the radial behaviour is prescribed,
ii) i) being granted, existence of a normalization in which the angular behavior is also prescribed.
In publication [52], which addresses part *ii)*, we give explicit examples of geometrically normalized analytic F's , related to sub-familles of the classical 2-parameter *Arnold family* $\theta \mapsto \theta + s + t \sin 2\pi\theta$ of circle diffeomorphisms, for which any normalization diverges.

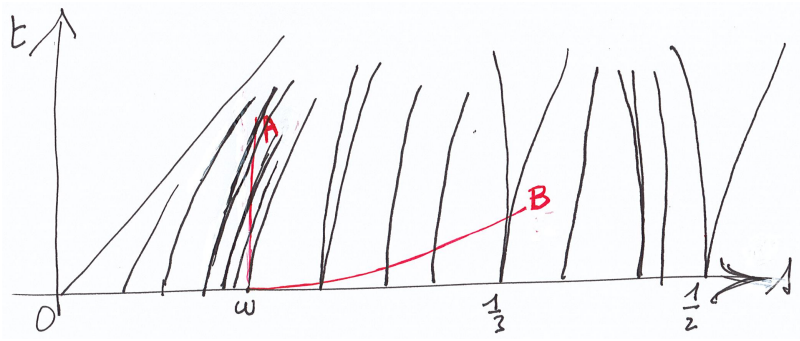


Figure 13 (d'après [52])

One shows moreover that if the derivative at the fixed point is a rotation by an angle $2\pi\omega$ such that ω is sufficiently badly approximated by rationals (technically, such that ω is not a Brjuno number), divergence of any normalization is generic.
Publication [53], in preparation, addresses to part *i)* ; there we study the genericity of divergence of all the geometric normalizations of F.

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This article was scandalously suppressed from the electronic version of E.U., the reason being difficulties encountered by the publisher to display some formulas (sic !) ; see the letter *Le niveau baisse* **in** *Gazette*

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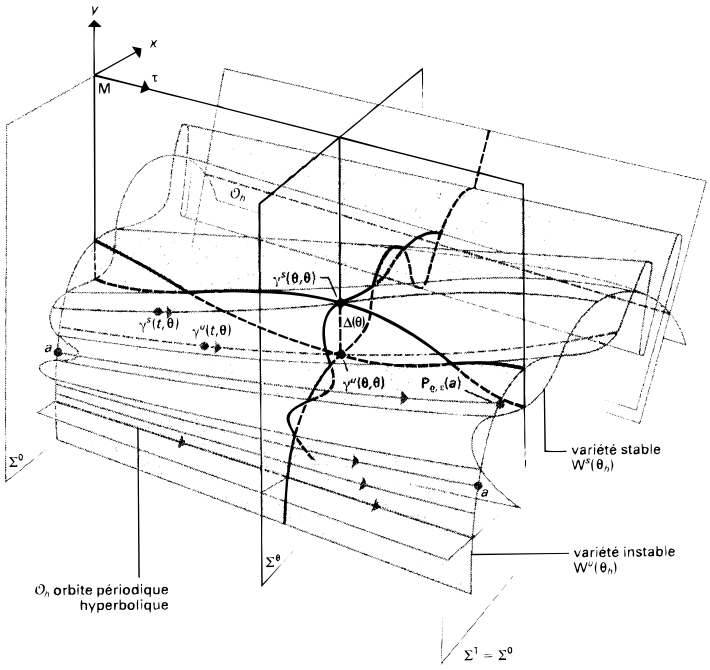
https://perso.imcce.fr/alain-chenciner/preprint.html



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[C30] Le problème des N corps et le système solaire, *numéro spécial de Sciences et Avenir ” la dixième planète”, 145 (2005)*
[C31] *Les polygones déchainés et le problème des n corps*, Images des mathématiques 2006 (CNRS)
http://images.math.cnrs.fr/Les-polygones-dechaines-et-le.html
[C32] *Conservative dynamics and the calculus of variations* (summer school ELAM, Montevideo 2005)
https://perso.imcce.fr/alain-chenciner/Montevideo.pdf
[C33] *Three Body Problem*, Scholarpedia (2007) 2[10]:2111
http://www.scholarpedia.org/article/Three_body_problem
[C34] *The “form” of a triangle*, Rendiconti di Matematica, Série VII, vol. 27, 1-16 (2007), Version anglaise de [C25] augmentée d'un appendice.
http://www1.mat.uniroma1.it/ricerca/rendiconti/200728129/1-16.pdf
[C35] *Four lectures on the N body problem*, in Hamiltonian Dynamical Systems and Applications, Edited by Walter Craig, Springer 2008
[C36] *The Lagrange reduction of the N-body problem: a survey*, Acta Mathematica Vietnamica (2013) 38: 165-186, preprint: http://arxiv.org/abs/1111.1334
[C37] *Une promenade dans les Méthodes nouvelles de la mécanique céleste*, Gazette des Mathématiciens $n^0$134, (octobre 2012) 37-47 & Quadratures http://smf4.emath.fr/Publications/Gazette/2012/134/smf_gazette_134_37-47.pdf
[C38] *Eloge de Poincaré*, prononcé le 9 juillet 2012 au cimetière du Montparnasse à l'occasion de la “journée”, Poincaré à l'Observatoire de Paris. Gazette des Mathématiciens $n^0$134, (octobre 2012) 116-121 http://smf4.emath.fr/en/Publications/Gazette/2012/134/smf_gazette_134_116-121.pdf

[C39] *Poincaré and the Three-Body Problem*, Séminaire Poincaré (Bourbaphy) XVI (2012) : Poincaré 1912–2012, pages 45-133
http://www.bourbaphy.fr/novembre2012.html

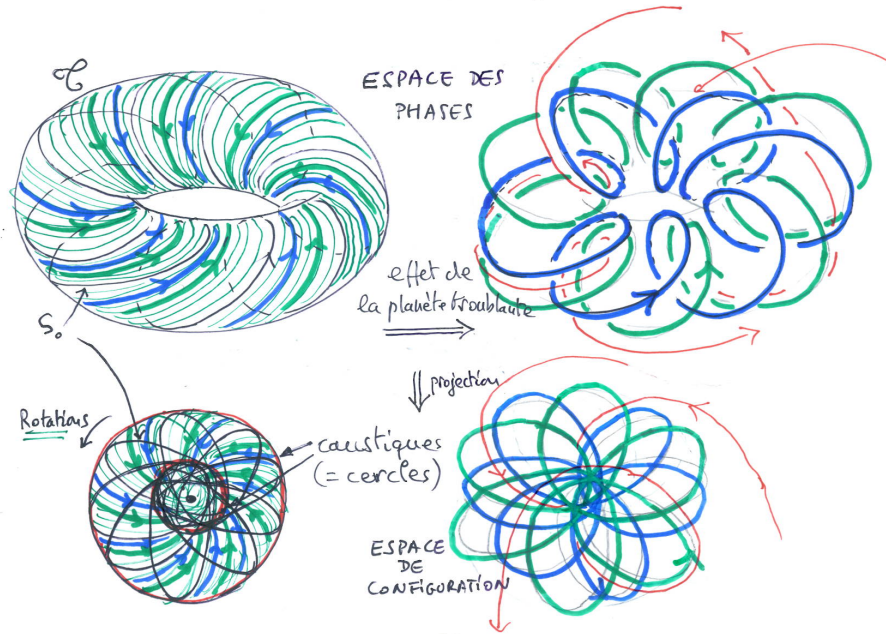


Figure 15 (Breaking of a resonant torus, from [C39])

[C40] *La force d'une idée simple : hommage à Claude SHANNON à l'occasion du centenaire de sa naissance*, Gazette des Mathématiciens $n^0$152, (avril 2017) 16-22 http://smf4.emath.fr/en/Publications/Gazette/2017/152/smf_gazette_152_16-22.pdf

[C41] *Ombres et traces*, chapitre du livre *René Thom : portrait mathématique et philosophique*, pp. 341-362, éditions du CNRS (2018)

[C42] *Perturbing a planar rotation: normal hyperbolicity and angular twist*, pp. 451-468 du livre Geometry in History (ed. S. G. Dani and A. Papadopoulos), Springer Verlaag (2019)

[C43] *Vous avez dit qualitatif ?*, in: Morel, JM., Teissier, B. (eds) *Mathematics Going Forward* . Lecture Notes in Mathematics, vol 2313. Springer, pp. 61-80 (2023)

On my page https://perso.imcce.fr/alain-chenciner/
one finds the notes of some courses as well as unpublished more historical or philosophical texts written at the occasion of seminar lectures, in particular:
[VFI] *Le vrai, le faux, l'insignifiant* (2015),

10 Books

[L1] *Courbes algébriques planes*, Editions de Paris VII (1979), re-published by Springer (2007)
[L2] *Illustrated glossary*, pages 138 to 171 of René Thom's book *Prédire n'est pas expliquer*, Eshel (1991)

Figure 16 (figure 25 p. 233 from [L2]: Black box)

[L3] 三体问题 *(Le problème des trois corps)* : livre en chinois constitué des transparents d'une conférence sur le problème des trois corps donnée le 20 septembre 2018 à Pékin à 首都师范大学 (Université Capital Normal), des notes préparatoires pour cette conférence et de quelques dessins. https://mp.weixin.qq.com/s/dtYobWkMt8xvUvvwMMUU2g

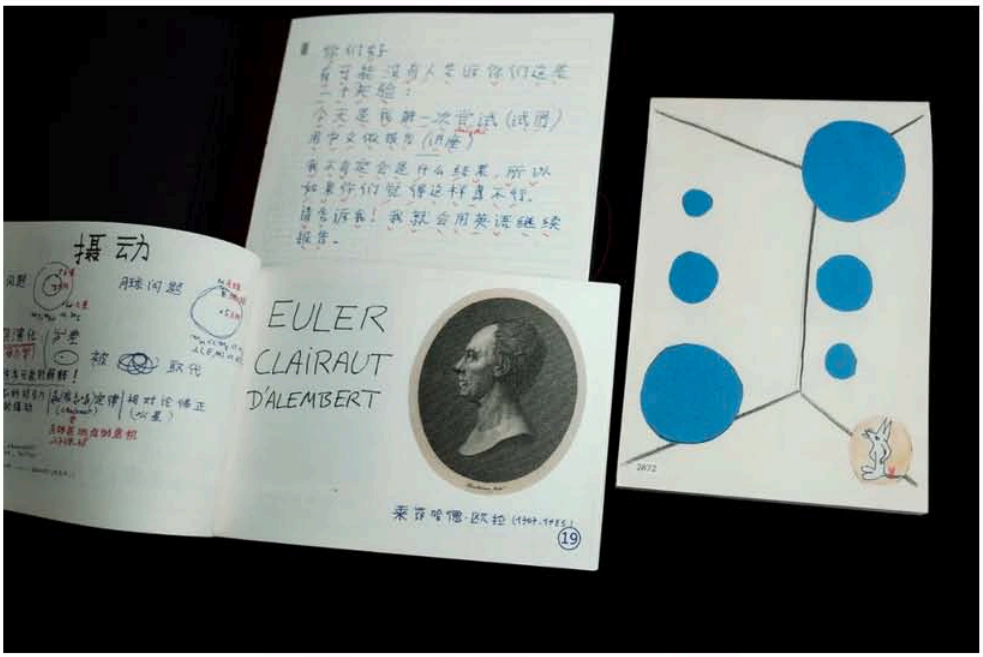


Figure 17 d'après [L3])

... books in preparation

[L3] *Extrémales et géométrie*, about 350 pages.

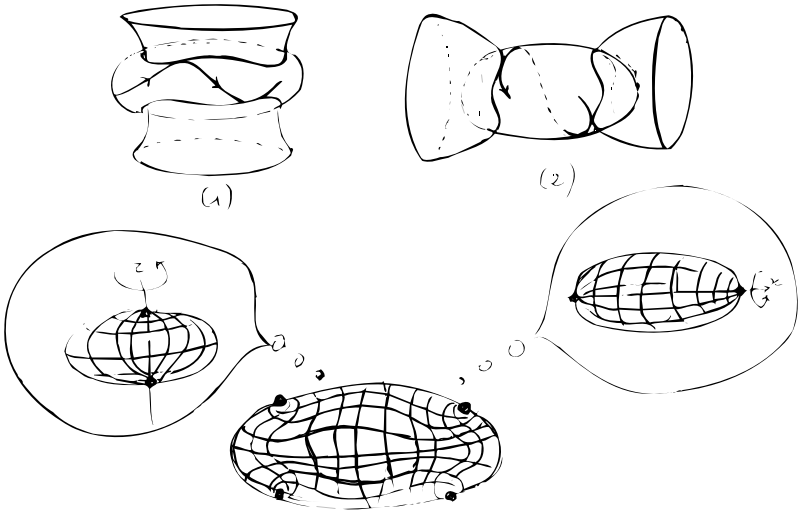


Figure 18 (geodesics on an ellipsoid hesitate between two types of revolution, from [L3])

[L4] *Le problème de la Lune et la théorie des systèmes dynamiques*, Paris VII (1985 et 1987). Could become part of a bok on the N-body problem.

[L5] *Géométrie*, manuscript (about 100 pages) given to students in a Master course.

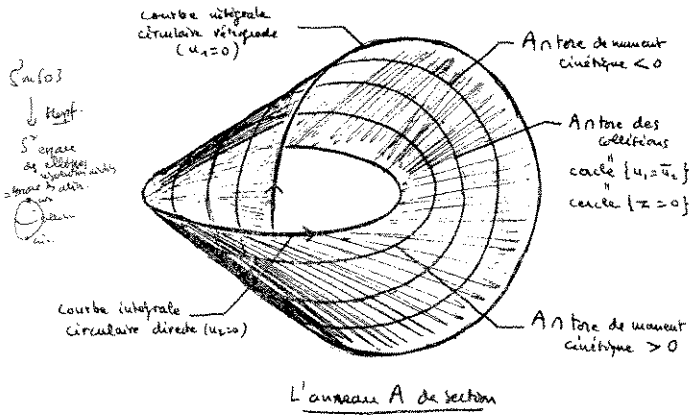


Figure 19 (from [L4])

[L6] *Théorie de l'information* notes from a course (54 pages).

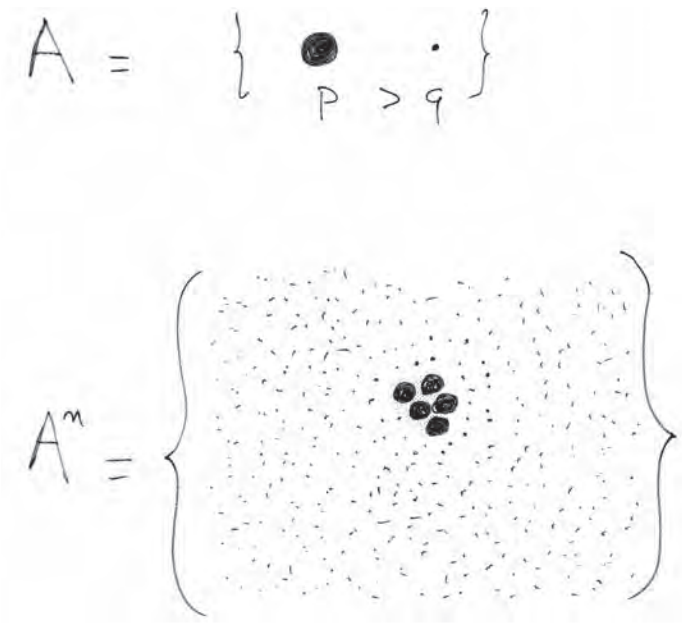
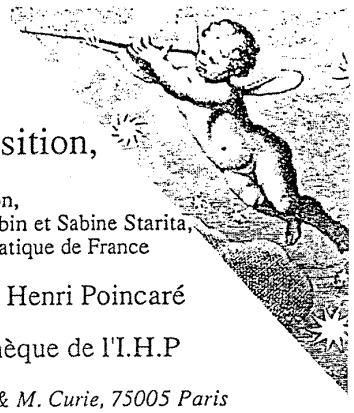


Figure 20 (Le 1er *Shannon's theorem, from [L6])*

... and an exhibition

[E] *La mécanique céleste... une exposition* : twelve billboards, four dedicated to the global evolution of the n-body problem, eight dedicated to the stability of the solar system. June 8–18, 1996 at the main hall of Institut Henri Poincaré (I.H.P.) the, till June December 1996 inside l'I.H.P.' library. This was done at the occasion of SMF's special day dedicated to Celestial Mechanics which I had organized with the help of IHP's staff (Hélène Nocton, Dominique Dartron, Antoine Gobin and Sabine Starita). The description of the exhibition is on my web page https://perso.imcce.fr/alain-chenciner/preprint.html

CETTE EXPOSITION EST
PRÉSENTÉE À NOUVEAU À
la BIBLIOTHÈQUE de l'I.H.P.
à partir du 16 octobre 2003



la mécanique céleste.... une exposition,

conçue par Alain Chenciner, et réalisée par Hélène Nocton,
avec la collaboration de Dominique Dartron, Antoine Gobin et Sabine Starita,
à l'occasion de la Journée Annuelle de la Société Mathématique de France

du 8 au 18 juin 1996 dans le hall de l'Institut Henri Poincaré

du 20 juin au 31 décembre 1996 à la bibliothèque de l'I.H.P

11, rue P. & M. Curie, 75005 Paris

L'exposition comprend douze panneaux. Les huit premiers, de format 70cm × 145cm et numérotés de 1 à 8, sont consacrés à l'évolution globale du Problème des n corps; les quatre autres, de format 90cm × 110cm et numérotés de I à IV, sont plus spécifiquement dédiés à la stabilité du Système Solaire. Le texte ci-dessous fournit les références de l'essentiel du contenu des panneaux, ainsi que les textes d'accompagnement. Cette exposition a été réalisée à l'occasion de l'Assemblée annuelle de la Société Mathématique de France du 8 juin 1996, dont le thème était la Mécanique Céleste. Les huit premiers panneaux concernent la conférence d'Alain Chenciner (publication de la S.M.F.), les quatre autres celle de Michel Herman.

Panneau 1. De l'attraction des globes homogènes ··· aux équations de Lagrange.

Portraits de Newton et Lagrange

Isaac Newton *Philosophiæ naturalis Principia Mathematica* Liber tertius Propositio VIII Theorema VIII : *"Si la matière de deux globes qui gravitent l'un vers l'autre est homogène à égales distences de leurs centres : le poids de l'un de ces globes vers l'autre sera réciproquement comme le quarré de la distance qui est entre leurs centres"* (traduction de la Marquise du Chastellet).

1

Figure 21 (from [E])

11 Conferences, seminar talks, etc

11.1 Invited lectures in international conferences since 1999

1999

Dynamical Systems, Oberwolfach, July 18-24, 1999 *(Action minimizing solutions of the 4-body problem in R^3)*

International workshop on separatrix splitting, IHES September 7-14, 1999 *(Minima de la fonctionnelle d'action pour le problème des 4 corps dans R^3)*

International conference in honor of D. Saari, Chicago, december 15-19, 1999 *(Action minimizing periodic orbits in the Newtonian n-body problem)*

2000

International conference on Dynamical Systems in honor of J. Palis, IMPA Rio de Janeiro, July 19-28, 2000 *(A new class of periodic solutions of the N body problem in the case of equal masses)*

International conference "Regular and unstable motions in Hamiltonian systems", Roma, September 5-8, 2000 *(Action minimizing periodic solutions in the Newtonian n-body problem: symmetry versus topology)*

2001 International conference on Hamiltonian Systems ans Celestial Mechanics (HAMSYS), Guanajuato, March 19-23, 2001 *Are the masses of a choreography necessarily equal ?)*

International conference in memory of Michael Herman, IMPA Rio de Janeiro, April 9-13, 2001 *Are the masses of a choreography necessarily equal ?)*

International conference in memory of Michael Herman and Jurgen Moser, ICMS Edinburgh, May 25, 2001 *(Action minimizing periodic solutions of the n-body problem)*

Global Analysis of Dynamical Systems, international workshop in honour of Floris Takens, Lorenz center, Leiden June 24-29, 2001 *(Action minimizing periodic solutions of the n-body problem)*

Conference to the memory of Gilles Chatelet, Ecole Normale Supérieure, Paris, jJune 27-29, 2001 *(La forme de n corps)*

Dynamical Systems, Oberwolfach, July 16-21, 2001 *(Action minimizing periodic solutions of the n-body problem)*

International conference on Dynamical Systems, ICTP Trieste, July 30-August 3, 2001 *(Action minimizing periodic solutions of the n-body problem)*

2002

Conference and course during the semester on Dynamical Systems, Pisa, February 5-8 and April 22-26, 2002 *(The role of symmetries in the n-body problem)*

Warwick symposium on Classical N-body Problems, Avril 15-19, 2002 *(From Lagrange to the Eight: Marchal's P12 family)*

45 minutes invited lecture at the International Congress of Mathematicians (ICM), Beijing, August 20-28, 2002 *(Action minimizing solutions of the Newtonian n-body problem: from homology to symmetry)*

Satellite ICM conference : "New Directions in Dynamical Systems", Kyoto, August 5-15, 2002 *(Simple non-planar periodic solutions of the n-body problem)*

Satellite ICM conference : "Nonlinear Functional Analysis", Taiyuan (Shanxi), August 14-18, 2002 *(Some facts and more questions on the "Eight")*
Mather's fest, Penn State University October 14-15, 2002 & Princeton University October 17-20, 2002, conference at Penn State : *(Continuing the "Eight" in 3-space: Marchal's P_{12} family)*
Invited lecture at Princeton University, October 21, 2002 *(Solutions of the n-body problem with prescribed initial and final configurations: existence and applications)*
International congress to the memory of V.M. Alekseyev "Modern Theory of Dynamical Systems and Celestial Mechanics", Moscow, December 23-28, 2002 *(Non-planar solutions of the n-body problem: from isosceles to generalized hip-hops)*

2003

AIM Conference, Palo Alto June 2003 *(Rotating Eights)*
Dynamical Systems, Oberwolfach July 2003 *(The fixed ends problem and symmetric periodic solutions of the N-body problem)*
Plenary lecture at the International Congress of Mathematical Physics (ICMP), Lisboa, Julyt 27-August 2, 2003 *(Symmetries and "simple" solutions of the n Body Problem)*

2004

International conference on Celestial Mechanics, B.I.R.S. Banff (Canada) April 2004
"Third international conference on Nonlinear Analysis : Hamiltonian Systems and Celestial Mechanics", Nankai University, Tianjin (Chine), June 2004] *(canceled because of teaching)*
Université polytechnique de Catalunya (End of a cycle of lectures on Poincaré), Barcelona June 30, 2004 *(Une note de Poincaré)*
International conference in honor of Ivar Ekeland, Paris July 6-8, 2004 *(The n-body problem and the principle of (really) least action)*
Final conference of he Conference Henri-Poincaré, IHP December 16-18, 2004 *(Une note de Poincaré)*

2005

International conference "De la Topologie à la Géométrie symplectique" Nantes, June 8-10, 2005 *(Unchained polygons and the n-body problem)*
Dynamical Systems, Oberwolfach July 10-16, 2005
"Taiwan-France Joint Conference on Nonlinear Partial Differential Equations and Related Topics", Taipeh July 18-22, 2005 *Unchained polygons in the equal-mass N-body problem*
CELMEC IV, San Martino el Cimino September 11-16, 2005 *Unchained polygons in the equal mass N-body problem*

2006

Colloque international en l'honneur de Carles Simó, 29 mai-13 juin, S'Agaró *The flow of the equal mass three-body problem in the neighborhood of the Lagrange equilateral relative equilibrium*
International conference on Hamiltonian systems, Nanjing June 18-23 *Unchained polygons and the n-body problem*

2007

International conference on variational methods, Nankai University, Tianjin May 20-26, 2007 *Action minimization and global continuation of Lyapunov families stemming from relative equilibria*

Dynamical Systems, Oberwolfach July 8-14, 2007 *Global continuation of relative equilibria and action minimization*

International conference in honor of S. Lopez de Medrano, Mexico, October 22-26, 2007 *Unchained polygons*

2008

Abel Symposium, Tromsö June 18-21, 2008 *Action minimization and global continuation of Lyapunov families stemming from relative equilibria*

International conference "Stability and instability in mechanical systems", Barcelona CRM September 22-26, 2008 *Unchained polygons and the N-body problem*

2009

2009 NCTS Workshop on Dynamical Systems Tsinghua University, Taiwan (May 2009)

Conference CelMech in Viterbo (September 2009)

Symposium Mathematics and Astronomy, a joint long journey, Madrid Novembee 2009 *Unchained Polygons and the N-body problem*

2010

Colloquium of the department of mathematics of the National Taiwan University, Taipei, May 10, 2010 : *Reduction of symmetries in the N-body problem*

Conference *From Dynamical Systems to Symplectic Topology* in honor of Eddy Zehnder, ETH Zürich, November 10, 2010: *The symmetries of the N-body problem*

HAMSYS 2010 in honor of Ernesto Lacomba, Mexico City, November 29 – December 3, 2010: *The angular momentum of relative equilibria in higher dimensions*

2011

International conference *Geometrical methods in Dynamics and Topology*, Hanoi April 18-22, 2011: *The angular momentum of a relative equilibrium*

Chern research center, Nankai Universiy (Tianjin), June 15, 2011: *The angular momentum of a relative equilibrium*

Dynamical Systems, Oberwolfach July 2011 *The angular momentum of a relative equilibrium*

International conference on Hamitonian dynamics, Nanjin August 21-27, 2011: *The angular momentum of a relative equilibrium*

2012

Inauguration of the Fibonacci center, Pisa, Scuola Normale Superiore, March 5-8, 2012 *Angular momentum and Horn problem*

International conference for the 60th birthday of Daniel Bennequin, Paris March 12-16, 2012: *Moment cinétique et problème de Horn*

Mathematical Institut Simon Stoilov in Bucarest , at the occasion of the hundredth anniversary of Poincaré's death, Bucarest March 23, 2012 : *Une note de Poincaré*

International conference on variational methods, Chern center, Nankai University, Tianjin, May 21-25, 2012: *Angular momentum and Horn problem*
Conference *Souriau's 90* to the memory of Jean-Marie Souriau, Aix en Provence June 25-29, 2012 : *Angular momentum and Horn problem*
Journée Henri Poincaré, Institut d'astrophysique de Paris, July 9, 2012 : *Une promenade dans les Méthodes nouvelles de la mécanique céleste*
Montevideo Dynamical Systems Conference 2012, Montevideo September 13-17, 2012 : *Angular momentum and Horn problem*
International conference to the memory of Poincaré, Gijon September 3-7, 2012: *A walk through the New Methods of Celestial Mechanics*
Conerence for the hundredth anniversary of Poincaré's death, IMPA, Rio de Janeiro, November 26-29, 2012 : *A walk through the New Methods of Celestial Mechanics*
Conference Poincaré Lorraine 2012, Metz & Nancy December 12-14, 2012 *Une promenade dans les Méthodes nouvelles de la mécanique céleste*

2013

Dynamical Systems, Oberwolfach July 7-13, 2013 *Angular momentum and Horn's problem*
Conference on Celestial Mechanics, Viterbo September 2-6, 2013 *Angular momentum in higher dimensions and Horn problem.*
Conference "Des dynamiques singulièrement perturbées aux dynamiques de population" (at the occasion of the retirement of Eric Benoît), La Rochelle, December 16-18, 2013 *Une note de Poincaré*

2014

Workshop "Symplectic techniques in topology and dynamics 2014", University of Cologne, September 22-26, 2014, *Angular momentum and Horn's problem*

2015

Workshop on Hamiltonian Dynamical Systems Fudan university (Shanghai), January 4-10, 2015, *Non-avoided crossings for n-body balanced configurations in* $\mathbb{R}^3$ *near a central configuration*
Astronomy and Dynamics, international workshop in honor of Jacques Laskar, Paris Observatory, April 28-30, 2015, *Non-avoided crossings or why asking Jacques Laskar to make a computation may lead to unexpected results*
Oviedo, Conference in honor of Angel Rodriguez-Mendez, June 3-5, 2015, *Non-avoided crossings for n-body balanced configurations in* $\mathbb{R}^3$ *near a central configuration*
Workshop "Symplectic Techniques in Topology and Dynamics 2015", University of Cologne, August 24-28, 2015, *Non-avoided crossings for n-body balanced configurations in* $\mathbb{R}^3$ *near a central configuration*

2016

Plenary lecture at the Congreso de Matemática Capricornio (Comca 2016) Catholic University of the North, Antofogasta (Chili) July 24 - AUgust 10, 2016 *Angular momentum and Horn's problem*
Conférence dans le colloque international en l'honneur d'Eldar Straume, *Angular momentum and Horn's problem*, Norvegian University of Science and Technology (NTNU) Trondheim, 3-4 novembre 2016

2017
Dynamical Systems, Oberwolfach July 9-15, 2017
Plenary lecture at the international conference "Contemporary Mathematics" for the 80th anniversary of the late Vladimir Arnold, Moscow December 18-23, 2017
2018
International conference "Geometry and Dynamics in interaction", Paris Observatory, January 15-17, 2018, *From periodic to quasi-periodic: bifurcations of n-body relative equilibria and Horn polytopes*
International Conference on Variational Methods (ICVAM-4), Chern Institute, Nankai University, Tianjin (China), May 14-18, 2018 *n-body relative equilibria in higher dimensions*
Topologie différentielle et mathématique d'aujourd'hui, en l'honneur de Jean Cerf, Orsay, June 11-13, 2018, *n-body relative equilibria in higher dimensions*
Perspective in Hamiltonian Dynamics, Venezia June 18-22, 2018, *n-body relative equilibria in higher dimensions*
2021
International conference on Hamiltonian Dynamics in honor of Jean-Pierre Marco *Local diffeomorphisms of the plane with a weakly attracting elliptic fixed point : some questions*, Paris, Université Paris-Dauphine, June 10 2021
International conference "Regular and Chaotic Dynamics" in memory of 'Alexey V. Borisov : *Elliptic fixed points with an invariant foliation: some facts and more questions*, Moscou (via Zoom because of Covid) November 30 2021 http://www.mathnet.ru/php/conference.phtml?confid=1952&option_lang=eng

11.2 Seminar talks, lecture notes, ... since 1999

1999
Cours de Mécanique Céleste à l'école d'été des Tourelles (Montpellier) 20-24 septembre 1999 *(Introduction au problème des n corps)*
2000
Séminaire *Aspects de la géométrie*, IHP, 13 janvier 2000 *(La forme de n corps)*
Séminaire de Sytèmes dynamiques de Paris VII, janvier 2000 *(Solutions périodiques du problème des n corps qui minimisent l'action)*
Séminaire à l'Université de Nice, 27 janvier 2000 *(Une solution périodique remarquable du problème des 3 corps dans le plan)*
Séminaire à l'Ecole Normale Supérieure (Paris), 1er mars 2000 *(Un monde nouveau de solutions périodiques du problème des n corps dans le cas de masseségales)*
Seminar in Londres (Board Nonlinearity meeting, University College), 14 mai 2000
Séminaire à l'Université Paris XIII, 17 mai 2000 *(Une nouvelle classe de solutions périodiques du problème des n corps dans le cas de masses égales)*
Séminaire à l'Université de Genève, 5 juin 2000 *(Une nouvelle classe de solutions périodiques du problème des n corps dans le cas de masses égales)*
Séminaire à l'Ecole Polytechnique, 9 juin 2000 *(De nouvelles solutions périodiques du problème des n corps : minima de l'action et symétrie)*

Journées Scientifiques de l'IMC, 7-9 juin 2000 *(Une nouvelle classe de solutions périodiques du problème des n corps dans le cas où les masses sont égales)*
Séminaire à l'Universidade Federal de Pernambuco (Recife, Brésil), 3 aout 2000 *Soluçoes periódicas do problema dos n corpos que minimizam a açao Lagrangianea*

2001

Séminaire à Marseille, 16 mai 2001 *(Solutions périodiques du problème des n corps minimisant l'action)*
Séminaire à l'Université de Barcelone, septembre 2001 *(Perverse solutions of the n-body problem)*
Séminaire à l'Université de Tours, 8 novembre 2001 *(Solutions périodiques du problème des n corps qui minimisent l'action)*

2002

Séminaire *Histoire de géométries*, Maison des sciences de l'homme, 11 mars 2002 *(Quelques aspects du problème des n corps)*
Séminaire de Sytèmes dynamiques de Paris VII, 2 avril 2002 *(Une belle idée de Christian Marchal et ce qui s'en suit)*
Laboratoire de Mécanique, 24 mai 2002 *(De nouvelles classes de solutions périodiques du problème des n corps)*
Ecole normale de Cachan, Colloquium du CMLA, 31 mai 2002 *(Symétries et solutions périodiques du problème des n corps)*
Université de Barcelone, 17 juillet 2002 *(The absence of collisions for a minimizer of the fixed-ends n-body problem : idea of the proof and some consequences)*
Journées IUF de Nice, 5 et 6 septembre 2002 *(Stables jongleries, instables Hip-Hops ou les surprises du problème des n corps)*
Journée de Physique, Chevalleret, 25 octobre 2002 *(Minimisation de l'action et solutions du problème des n corps)*

2003

Journées de l'IMCCE, 5 mai 2003 *(Solutions du problème des n corps joignant deux configurations : l'idée de Christian Marchal et ce qui s'en suit)*
Biséminaire Mathématique-Physique, 14 octobre 2003 *(Symmetries and simple solutions of the n-body problem)*
Journées Maths et Cerveau, Chevalleret 21 novembre 2003 *(Minimisation et symétries dans le problème des trois corps)*
Colloquium de l'Institut Fourier, Grenoble 27 novembre 2003 *(Symétries et solutions "simples" du problème classique des n corps)*
Séminaire à l'E.N.S. Lyon *(Symétries et solutions "simples" du problème classique des n corps)*

2004

Journée de Mathématiques de l'Académie d'Orléans-Tours, 19 mai 2004 *(De l'espace des triangles au problème des trois corps)*
Conférence dans le cycle Fisymat de l'Université de Grenade, Granada, vers mai 2004
Cours sur le *Calcul des variations et la théorie KAM faible* à l'Université de Barcelone (5 cours), 28 juin - 3 juillet 2004

Conférence lors des journées d'étude sur le rôle de l'analyse complexe organisées par F. Norguet et S. Ofman, IHP 28 septembre - 1er octobre 2004 *(De l'espace des triangles au problème des trois corps)*
Conférence dans le colloque "3 corps, classique, quantique, discret-continu" organisé à l'E.N.S. par Guiseppe Longo et Thierry Paul, 28-29 septembre 2004 *(Les surprises de la solution équilatérale de Lagrange)*
Conférence au séminaire ASD, 21 octobre 2004 *(Introduction à la théorie KAM faible, 1er exposé)*
Conférence au séminaire ASD, 4 novembre 2004 *(Introduction à la théorie KAM faible, 2ème exposé)*
Conférence au séminaire ASD, 2 décembre 2004 *(Introduction à la théorie KAM faible, 3ème exposé)*
2005
Conférence à Nancy (colloquium ADSG), 1er mars 2005 *Les surprises de la solution équilatérale de Lagrange*
Mini-cours (2 séances de 2 heures) dans le GDR Sud-Rhodanien, Avignon 10-12 mars 2005 *(Calcul des variations dans le cas convexe : une introduction au théorème KAM faible d'Albert Fathi et à la théorie de Mather des mesures minimales)*
Conférence à Avignon (GDR Sud-Rhodanien), 11 mars 2005 *Problème des trois corps : les surprises de la solution équilatérale de Lagrange*
Conférence à Lille, 18 mars 2005 *(Les surprises de la solution équilatérale de Lagrange)*
3 conférences at the University of Minnesota during my stay as Ordway Professor :
– at the mathématics colloquium, April 28, 2005 *Symmetries and "simple" solutions of the classical n-body problem*
– at the dynamical systems' seminar, April 29, 2005 *Unchained polygons in the equal-mass n-body problem*
– at the "junior colloquium", May ", 2005 *The shape of a triangle*
Mini-cours (5 séances d'1 heure) de *géométrie différentielle* dans l'Ecole d'été Maths & cerveau, Institut Henri Poincaré, 13-24 juin 2005
Participation à la rencontre CNRS – jeunes au futuroscope de Poitiers, le 22 octobre 2005
Mini-cours (4 séances d'1h30) *Conservative Dynamics and the Calculus of Variations* dans la 14ème Latin Amrican School of Mathematics (ELAM) à Montevideo du 1er au 9 décembre 2005
2006
Séminaire du Greco (Observatoire de Paris) 9 février 2006 *Symétries et solutions périodiques du problème des n corps*
Colloquium de l'Académie de Bruxelles, 17 mars 2006 *Unchained polygons and the n-body problem*
Colloquium ""Lezioni Guido Castelnuovo" Universita La Sapienza, Roma 5 mai 2006 *The "form" of a triangle*
Colloquium departement de Math. Universita Tor Vergata, Roma 8 mai 2006 *Symmetries and "simple" solutions of the classical n-body problem*

5 cours sur le problème des n corps à l'Université de Granada (en espagnol) 17 - 21 avril
Invited lecture at the Purple Mountain Observatory, Nanjing June 14, 2006 *The flow of the equal mass three-body problem in the neighborhood of the Lagrange equilateral relative equilibrium*
Séminaire de topologie Orsay, 19 octobre 2006 *Symétries et solutions périodiques du problème des n corps : chorégraphies, Hip-Hops et équilibres relatifs*
Maths club de Jussieu, 4 décembre 2006 *Des triangles dans le ciel*

2007

Colloquium département de Maths. Université d'Orléans, 18 janvier 2007 *Symétries et solutions périodiques du problème des n corps : chorégraphies, Hip-Hops et équilibres relatifs*
Colloquium département de Maths. Université d'Angers, 2 février 2007 *La "forme" de n corps*
Colloquium département de Maths. Université de Lille, 9 février 2007 *Symétries et solutions périodiques du problème des n corps : chorégraphies, Hip-Hops et équilibres relatifs*
Shoudu Shifan Daxue, Beijing, 2 invited lectures:
– May 30, 2007 *Symmetries and "simple" solutions of the classical n-body problem*
– May 31, 2007 *Rotating Eights*
4 lectures on the N-body problem, Nato school, Montreal June 18-29, 2007
5 cours à l'Universidad Catolica del Norte, Antofogasta, 3-14 décembre 2007 *Conservative dynamics and the calculus of variations*

2008

Conférence de Séminaire à l'Université Paul Sabatier, Toulouse 14 février 2008 *Polygones déchaînés dans le problème des n corps*
Conférence au séminaire d'histoire des sciences, Toulouse 15 février 2008 *Poincaré Ê: une promenade dans les Méthodes nouvelles de la mécanique céleste*
Conférence au séminaire de Systèmes dynamiques de l'IMJ, Paris 7 mars 2008 *Des polygones réguliers aux chorégraphies et aux Hip-hops*
Conférence au Séminaire d'histoire des géométries, Maison des sciences de l'homme, Paris 17 mars 2008 *Poincaré : une promenade dans les Méthodes nouvelles de la mécanique céleste* http://www.ahm.msh-paris.fr/Video.aspx?domain=84fa1a68-95c0-4c74-aed7-06055edaca16&language=fr&metaDescriptionId=f7631082-f5c2-4fff-af11-da2dc718d885&mediatype=VideoWithShots
5 courses at Tsing Hua University, *The N-body problem: 5 lectures on symmetries and periodic solutions*, Beijing May 25i-June 13, 2008
Conference at Capital Normal University (Beijing), June 3, 2008 *Poincaré: a trip through the New Methods of Celestial Mechanics*
Colloquium de Nantes, 9 octobre 2008 *Poincaré, une promenade dans les Méthodes nouvelles de la mécanique céleste*
Colloquium de Rennes, 20 octobre 2008 *Des polygones réguliers aux chorégraphies et aux Hip-hops*

Réunion de l'ANR KAM faible, Bordeaux, 23 octobre 2008 *Minimisation de l'action sous contraintes de symétrie et prolongement de familles de Lyapunov : un exemple.*

2009

Séminaire de géométrie complexe, IHP mars 2009 *Des polygones réguliers aux chorégraphies et aux Hip-Hops.*

Mini-course, Tsing Hua university, Hsinchu, Taiwan, May 2009 *On weak KAM theory*

Colloquium of Tsing Hua university, Hsinchu, Taiwan, May 25, 2009 *Continuous and discrete symmetries in the n-body problem.*

Conference at the differential equations seminar of the National Taiwan University, May 26, 2009 *Bifurcating from relative equilibria: surprises and limitations of the minimization method.*

Colloquium du département de mathématique de l'Université de Bâle, 15 octobre 2009 *Des polygones réguliers aux chorégraphies et aux Hip-Hops.*

2010

Colloquium du centre de mathématique de l'Ecole Polytechnique, 8 janvier 2010 *Des polygones réguliers aux chorégraphies et aux Hip-Hops.*

Cours de mécanique céleste à Segovia dans le cadre du réseau D.A.N.C.E., 25-30 janvier 2010

Rencontre sur l'observation en mathématique, organisée par Karine Chemla et Jean-Jacques Szczeciniarz, Université Paris 7, 19 février 2010

Colloque "La reconquête de la géométrie par la dynamique après Lagrange", IHES 24-26 mars 2010 *La réduction du problème des trois corps par Lagrange.*

Séminar on Mathematical General Relativity, March 31, 2010 *Institut d'Astrophysique Symétries continues et symétries discrètes dans le problème des N corps.*

Mini-cours *Reduction of symmetries in the N-body problem*, May 2010 TsingHua university, Hsinchu (Taiwan)

Journées de l'IMCCE, 22-24 novembre 2010 *(Le moment cinétique d'un équilibre relatif en grande dimension)*

2011

Séminaire de philosophie et mathématique, Ecole Normale Supérieure, Paris, 31 janvier 2011: *Minimisation de l'action lagrangienne et symétrie dans le problème des n corps*

Séminaire de l'équipe ASD, IMCCE, Paris 17 février 2011: *Le moment cinétique d'un équilibre relatif*

Minicourse (3 courses) at "Hanoi University of Education, Hanoi, April 18-22, 2011: *the Lagrange reduction of the N-body problem*

Course at "Capital Normal" university (Shoudu Shifan Daxue), Beijing, June 2011: *the Lagrange reduction of the N-body problem*

Lecture at the cultural center of the french ambassy at Beijing (Café des sciences), June 30, 2011 *La danse des planètes* http://www.ambafrance-cn.org/12eme-Cafe-des-sciences-La-danse-des-planetes

Minicours, Université Sergio Arboledo, Santa Marta, Colombie, 28 novembre au 2 décembre 2011: *An introduction to the n-body problem, with emphasis on symmetries and on periodic and quasi-periodic solutions*

2012

Séminaire de Systèmes dynamiques de l'I.M.J., Paris 13 janvier 2012: *Moment cinétique et problème de Horn*

Séminaire du département de mathématique de l'Université d'Avignon, Avignon 7 février 2012: *Moment cinétique et problème de Horn*

Minicourse at Tsing Hua university, Beijing May 7-19, 2012, *From Euler-Lagrange equations to twist maps*

Minicourse at Capital Normal university (Shoudu Shifan Daxue), Beijing May 28- June 16, 2012, *Weak KAM theory*

Conference at Xi Nan Jiaotong university, Chengdu, June 18-21, 2012, *The surprises of the symmetric relative equilibria of the N-body problem in* $\mathbb{R}^3$

Éloge de Poincaré, prononcé le 9 juillet 2012 au cimetière du Montparnasse à l'occasion de la "journée Poincaré" organisée à l'Observatoire de Paris.

Séminaire de Systèmes dynamiques, Universidad de la Republica, Montevideo, 20 aout 2012, *Un paseo por los nuevos métodos de la mecánica celeste*

Conférence à l'Alliance française de Montevideo, 20 aout 2012, *Le problème de trois corps*

Conférence au séminaire Poincaré (Bourbaphy) à l'I.H.P. Paris, 24 novembre 2012 *Poincaré et le problème des trois corps*

Conférence au séminaire de systèmes dynamiques, Itabayana, 3 décembre 2012 *Uma nota de Poincaré*

Conférence au séminaire de mécanique céleste, Aracaju 4 décembre 2012 *Momento angular e problema de Horn*

Conférence au colloquium, Aracaju 5 décembre 2012 *Uma caminhada por os Novos Métodos da Mecânica Celeste*

Conférence au séminaire, Recife 6 décembre 2012 *Angular momentum and Horn problem*

Conférence au colloquium, Recife 6 décembre 2012 *A walk through the New Methods of Celestial Mechanics*

2013

Minicourse Capital Normal university (Shoudu Shifan Daxue), Beijing, June 2–14, 2013 *Quasi-collision solutions of the three-body problem*

Conference at the Morningside center of mathematics, Academy of sciences Beijing, June 20, 2013 *Angular momentum and Horn problem*

Conference at Tsinghua university, Beijing, June 24 2013 *Poincaré and the three-body problem*

Conference at Tsinghua university, Beijing, June 24 2013 *A note by Poincaré*

Conférence "Sciences et Société" à Nancy, 26 septembre 2013 *Le problème des trois corps*

Conférence à l'Université d'Oviedo, 17 octobre 2013 *Newton tenia razón: El problema de los tres cuerpos es demasiado difícil*

Conférence aux étudiants de première année de l'Université d'Oviedo, 21 octobre 2013 *From the mean value theorem to Liouville numbers and Peano curves*

Mini-cours à l'Université d'Oviedo, 23, 30 et 31 octobre 2013 *1) El problema de Kepler, 2) Soluciones simétricas del problema de n cuerpos*

2014

Conférence à l'Université du Luxembourg dans le cadre des "jeudis des sciences", 27 février 2014, *Le problème des trois corps*
Conférence dans le "Séminaire Darboux", LPTHE, Jussieu 24 mars 2014, *Monments cinétiques en grandes dimensions et problème de Horn*
Minicourse at the Chern Institute of Nankai university (Tianjin), May 19-23, 2014 *Quasi-collision solutions of the three-body problem*
Conférence à l'Association des amis de Jean Cavaillès, ENS 29 novembre 2014, *Ombres et traces*

2015

Conference at theNormal University (Beijing Shifan daxue), *A note by Poincaré*, Beijing January 12, 2015
2 lectures at Capital Normal University (Shoudu shifan daxue) :
Non-avoided crossings for n-body balanced configurations in $\mathbb{R}^3$ *near a central configuration*, Beijing January 13, 2015
Shadows and traces, Beijing January 15, 2015
One month course at Tsinghua University (Beijing) : Discrete dynamical systems, March 2-29, 2015
Conference at Shandong University (Jinan), *Shadows and traces*, Jinan, March 6, 2015
Conference at the Chern Institute of Nankai University (Tianjin), *Non avoided crossings*, Tianjn March 20, 2015
Conference at Tsinghua University (Beijing) *Angular momentum and Horn problem*, Beijing March 26, 2015

2016

Mini-cours à Merida (Mexique), *The algebraic side of the N-body problem: reduction, singularities, relative equilibria*, 18-29 janvier 2016
Conférence au séminaire de philosophie et mathématique de l'E.N.S. *Vous avez dit ÒqualitatifÓ ?*, 15 février 2016 http://savoirs.ens.fr/expose.php?id=2477
Colloquium à l'Institut de Mathématique de l'Université de Münster *Angular momentum and Horn's problem*, 3 mai 2016
Colloquium à l'IRMAR, Université de Rennes *Angular momentum and Horn's problem*, 9 mai 2016
Conférence dans le séminaire ASD, Observatoire de Paris à l'occasion du centième anniversaire de la mort de Claude Shannon *Le grand théeorème de Shannon ou la force d'une idée simple*, 26 mai 2016
Conférence dans la Rencontre entre Mathématiciens et Physiciens Théoriciens dédiée à Poincaré *Une note de Poincaré*, Strasbourg, IRMA, 2-4 juin 2016
Conférences et minicours à l'Université Catholique du Nord, Antofogasta (Chili) 24 juillet - 10 aout 2016 :

2 conférences au colloquium du département de mathématique :
Poincaré and the three-body problem 27 juillet 2016
A note by Poincaré 28 juillet 2016
Minicours (4 séances) *Shannon's theorems: the strength of a simple idea*

Conférence dans la Rencontre entre Mathématiciens et Physiciens Théoriciens dédiée à Thom *The two ways of stability in dimension 2: normal hyperbolicity and tangential twist*, Strasbourg, IRMA, 1-3 septembre 2016
Minicourse at Capital Normal University (Shoudu Shifan Daxue) (Beijing) *An introduction to information theory*, Beijing, September 10 - October 2, 2016
Conference at the China Sciences and Technologies Museum (Beijing), *Claude Shannon et la théorie de l'information : la force d'une idée simple*, Beijing, September 24, 2016 http://v.qq.com/x/cover/7p9udmwg2eo5yil.html?vid=a0021hxauw1
http://www.ambafrance-cn.org/Conference-d-Alain-Chenciner
2 lectures at Xinan Jiaotong University (Chengdu), October 3-12 2016 :
Shannon's theorems: the strength of a simple idea October 8, 2016
The two ways of stability in dimension 2 : normal hyperbolicity and tangential twist October 10, 2016
Conferenceat the China University of Electronic Sciences and Technologies (UESTC, Chengdu) *Claude Shannon : the strength of a simple idea*, Chengdu, October 11, 2016
http://www.ambafrance-cn.org/Conference-d-Alain-Chenciner-a-l-UESTC
Conférence à l'ENS Lyon *Le grand théeorème de Shannon ou la force dÕune idée simple*, 8 décembre 2016

2017

One month course at Tsinghua University (Beijing) : *Discrete dynamical systems* Beijing February 20 - March 20, 2017
Conference at Tsinghua University (Beijing) : *Perturbing a planar rotation: normal hyperbolicity and anguar twist*, Beijing, February 23, 2017
Conference at Capital Normal Universiy (Shoudu Shifan Daxue) : *Perturbing a planar rotation: normal hyperbolicity and anguar twist*, Beijing, March 9, 2017
Minicourse at Xinan Jiaotong University (Chengdu), March 20 - April 2, 2017 *Introduction to the restricted three-body problem*
Conférence at China University of Electronic Sciences and Technologies (UESTC, Chengdu) : *A note by Poincaré*, Chengdu, March 28, 2017
Conférence au DMA (ENS Paris) : *La naissance de la théorie de l'information ou la force d'une idée simple*, 11 mai 2017
Séminaire de géométrie hamiltonienne, Paris 6, 24 novembre 2017, *Les coordonnées action-angle ? Une simple histoire d'action de tore (d'après Nguyen Tien Zung)*

2018

Colloquium Université d'Augsburg, 22 janvier 2018, *Perturbing a planar rotation: normal hyperbolicity and angular twist*
One month course at Tsinghua University (Beijing) : *From Euler-Lagrange equations to twist maps*, Beijng, March 25 - April 20, 2018
Conference at Capital Normal University (Beijing), *Action-angle coordinates ? A simple story of torus action (according to Nguyen Tien Zung)*, Beijing, April 3, 2018
2 weeks course at Xinan Jiaotong Univrsity (Chengdu) : *Introduction to Hamiltonian Dynamics*, Chengdu April 23 -May 7, 2018

Conference at the departement of compared litterature Tsinghua University (Beijing), *the Three-Body problem*, Beijing, May 10, 2018
Conference *in chinese* at Capital Normal University （首都师范大学） (Pékin) : 三体问题 = *Le problème des trois corps*, 20 septembre 2018
Colloquium du laboratoire J.A. Dieudonné, Université de Nice : *Equilibres relatifs de n corps en grandes dimensions*, 8 octobre 2018
Séminaire de l' équipe "Géometrie, analyse et dynamique" du laboratoire J.A. Dieudonné, Université de Nice : *Les coordonnées action-angle ? Une simple histoire d'action de tore*, 9 octobre 2018

2019

Cours d'un mois à l'Université Tsinghua （清华大学） (Pékin) : *From elliptic fixed points of 2d-diffeomorphisms to dynamics in the circle and the annulus*, 15 avril-17 mai 2019
Conférence à l'Université Beihang （北京航空航天大学） (Pékin) : *Angular momentum and Horn's problem*, 25 avril 2019
Conférence *en chinois* à l'Université Capital Normal (首都师范大学) (Pékin): *Equilibres relatifs du problème des N-corps en dimensions supérieures* (更高维度N体问题的相对平衡运动), 30 avril 2019
Conférence à l'Université Normale de Pékin (北京师范大学) : *Angular momentum and Horn's problem*, 17 mai 2019
Conférence à l'Académie des Sciences *Michel Herman, multiple et sans compromis*, Hommage à Michel Herman et Jean-Christophe Yoccoz, 1er octobre 2019

2021 (via Zoom because of Covid)

Conference dans le "Matemairacorana workshop", célébrant le 80 ème anniversaire d'Hildeberto Cabral : *ABC or N-body relative equilibria in higher dimensions*, Recife 23 février 2021
Conférence dans le séminaire GDM (Geometry, Dynamics and Mechanics) : *ABC or N-body relative equilibria in higher dimensions*, Padova, 11 mai 2021
https://mediaspace.unipd.it/media/GDM-2021-9.mp4/1_5g6poo27/165241241

2022

Conférence dans le "Working group on Hamiltonian and symplectic dynamics" : *ABC*, Université Paris-Dauphine 8 juin 2022

2023

Conférence à l'Université Capital Normal (首都师范大学) (Pékin): *Normal forms* 23 mai 2023
Conférence à l'Université Bei Hang (北航) (Pékin): *Normal forms* 25 mai 2023

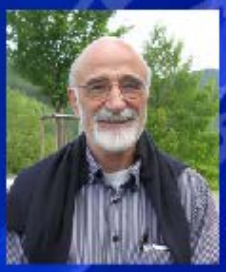
首都师范大学数学科学学院
School of Mathematical Sciences Capital Normal University

Distinguished Lecture Series in Mathematics 系列数学前沿学术讲座

Normal forms

主讲人：Alain Chenciner

法国巴黎天文台，巴黎七大



报告时间：2023年5月23日（周二）上午10:00–11:00

报告地点：教二楼827

摘要：

How to understand geometrically the main features of the action of a transformation and its iterates, or of the behavior of the solutions of a differential equation ?

At the end of 19th century, in his thesis, Poincaré introduces the notion of "normal form", that is the form taken by the map or the differential equation in a system of coordinates which reveals as much as possible of its qualitative behavior.

We shall consider this notion in one of its simplest occurrences, already very rich, the study of a local analytic diffeomophism of the plane around an elliptic fixed point.

报告人简介：

Alain CHENCINER是动力系统与三体问题的著名专家，法国巴黎第七大学特级数学教授，美国数学学会会士，2012年国际数学家大会邀请报告人。他和法国天文学家Jacques LASKAR院士一起创建巴黎天文台天体力学研究所天文学与动力系统研究小组。在中国作家刘慈欣的长篇小说《三体》的第146页，就提到了CHENCINER教授的研究工作。

邀 请 人：魏巧玲

主办单位：

School of Mathematical Sciences, Academy for Multidisciplinary Studies, CNU

Beijing National Center for Applied Mathematics

Beijing Advanced Innovation Center for Imaging Theory and Technology

Beijing Key Laboratory of Light-field Imaging and Digital Geometry

Musée des Sciences et Techniques, Pékin 24 septembre 2016

Figure 22 (illustration d'un texte de Pierre Duhem sur la "sensibilité aux données initiales", d'après [E])

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