Four Essays on Capital Markets and Asset Allocation

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Four Essays on Capital Markets and Asset Allocation

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Chapter 1

Introduction générale

Le risque est le thème prééminent de la théorie de la finance moderne. Comme Merton (1992, p. 1) résume, « le noyau de la théorie est l'étude de la meilleure façon d'allouer et de déployer des ressources à travers le temps dans un environnement incertain et du rôle des organisations économiques pour faciliter ces allocations ». Sans risque, le temps ne peut faire la différence que par une actualisation de l'allocation d'actifs.

En finance, la sélection de portefeuilles est le principal contexte pour étudier comment le risque affecte le choix d'allocation de l'investisseur. Pour les actifs risqués sur les marchés financiers, les investisseurs ne savent pas exactement comment ces actifs se comporteront pendant la période d'investissement. Sur la base des informations de retour disponibles sur les actifs disponibles, les investisseurs doivent décider de la proportion de leur patrimoine personnel à investir dans chaque actif. La décision d'investissement optimale devrait servir au mieux leur objectif, principalement la maximisation de leurs fonctions d'utilité attendues.

Levy (2016, p. 44) identifie deux étapes pour arriver à l'investissement optimal. La première étape est une décision objective de sélectionner des portefeuilles efficaces indépendamment de la préférence individuelle. En divisant l'ensemble du portefeuille réalisable en un ensemble de portefeuilles efficace et un ensemble de portefeuilles inefficace, tous les investisseurs rationnels s'accordent sur l'exclusion des portefeuilles inefficaces pour une évaluation plus approfondie, ce qui exclut les préférences individuelles. La deuxième étape est une décision subjective de mieux servir leurs préférences spécifiques. À l'intérieur de l'ensemble efficace, l'investisseur prend comme portefeuille optimal le portefeuille maximisant sa fonction d'utilité prévue. Étant donné que la préférence en matière de risque est assez personnelle, les portefeuilles optimaux peuvent être différents entre les investisseurs, mais ces portefeuilles sont nécessairement choisis parmi l'ensemble de portefeuilles efficace réalisé à la première étape.

Les deux étapes ne sont pas distinctement indépendantes. Au contraire, ils sont intrinsèquement liés d'une manière explicite ou implicite. Prenons l'optimisation de la variance moyenne comme exemple. En ce qui concerne la variance en tant qu'indice de risque, Markowitz (1952) développe la frontière efficace, qui est la collection de portefeuilles avec un risque minimum à un rendement attendu donné. Cette frontière correspond à la première étape de la décision objective, où tout portefeuille sous la frontière tombe dans l'ensemble inefficace. En fonction de son attitude particulière en matière de risque individuel, un investisseur peut choisir n'importe quel portefeuille sur la frontière efficiente. S'il est très averse au risque, le portefeuille de variance minimum sera l'investissement optimal; s'il est neutre au risque, le choix final sera le portefeuille de rendement maximum. En pratique, des choix intermédiaires entre ces deux situations sont faits. Cette sélection de préférence correspond à la deuxième étape de la décision subjective. Notez qu'en supposant généralement des rendements distribués normalement ou / et une fonction d'utilité quadratique, les deux étapes sont intégrées dans le sens que seuls les deux premiers moments comptent dans l'optimisation des investissements. Il est clair que si l'investisseur se soucie de plus que les deux premiers moments, alors la frontière efficace de la variance moyenne donne un ensemble efficace trompeur dès le premier pas. Étant donné que seule la moyenne et la variance sont prises en compte dans la détermination de la frontière efficace et que l'ensemble de portefeuilles réalisable est partiellement évalué, le portefeuille optimal global peut être filtré hors de l'ensemble efficace à la première étape. Ainsi, le portefeuille de l'ensemble efficace défectueux maximisant la fonction d'utilité de l'investisseur dans la deuxième étape est seulement localement optimal. En bref, les deux étapes de la construction efficace des frontières et de la maximisation de l'utilité sont soutenues par les mêmes considérations de décision, comme les moments de retour, et toute asymétrie entre elles crée une optimalité erronée.

La distribution normale n'est pas une description parfaite des rendements boursiers. Dans le Fama French Forum,[1] Les deux professeurs répondent à la question « Les retours de titres sont-ils normalement distribués? » avec la réponse suivante:

> « Les distributions de titres quotidiens et mensuels sont plutôt symétriques par rapport à leurs moyennes, mais les queues sont plus grosses (c'est-à-dire qu'il y a plus de valeurs aberrantes) que les distributions normales. (Ce sujet occupe la moitié de la thèse de doctorat de 1964 de Gene.) Dans l'ancienne littérature sur cette question, les alternatives populaires aux distributions normales étaient des distributions stables symétriques non normales (à queue grasse par rapport à la normale) et des distributions t avec de faibles degrés de liberté (qui sont également à queue grasse). Le message pour les investisseurs est: s'attendre à des rendements extrêmes, négatifs et positifs. »

De manière constante, les investisseurs devraient et devraient se préoccuper de moments plus élevés au-delà de la moyenne et de la variance, parmi lesquels l'asymétrie et l'aplatissement sont les plus prédominants pour saisir le risque de rendements extrêmes avec des interprétations économiques. Levy (1992) propose un exemple parfait sur ce point. Supposons qu'il y ait deux portefeuilles, x et y et que leurs profils de variance moyenne soient indiqués dans le tableau 1.1. Puisque Portfolio y a une moyenne plus élevée et une variance plus faible, il est sans doute supérieur à Portfolio x dans l'optimisation de la variance moyenne. Dans la première étape de la décision objective, Portfolio x sera classé comme inefficace et n'a aucune chance d'entrer dans l'examen de préférence de l'investisseur à la deuxième étape. Supposons que cet investisseur ait une fonction d'utilité logarithmique $u(W) = ln(W)$ où W est la richesse finale. Ensuite, l'utilité attendue pour Portfolio x, $E[u(x)]$, est de 2,3, alors que $E[u(y)]$ est de 0,9. Le surplus de $E[u(x)]$ sur $E[u(y)]$ est en contradiction avec la classification moyenne efficace du portefeuille, ce qui implique que l'optimisation de la variance moyenne conduit à un choix sous-optimal. Si nous examinons davantage l'asymétrie pour les deux portefeuilles, nous obtenons 9 413 841 pour Portfolio x et 93 149 pour Portfolio y.[2] Cette énorme différence légitime

[1] Ce forum est une section pour partager les observations et les idées d'Eugene Fama et de Kenneth French dans Dimensional Fund Advisors, le cabinet d'investissement dans lequel ils sont impliqués.

[2] Lorsque l'asymétrie et l'aplatissement accompagnent la variance dans le contexte, ils se réfèrent généralement au troisième moment et au quatrième moment pour plus de commodité. Cette dénotation est couramment utilisée dans les articles d'optimisation de portefeuille avec des moments d'ordre supérieur. Par exemple, dans Briec, Kerstens, and Jokung (2007, p. 138), l'asymétrie est explicitement définie comme le troisième moment.

le choix de l'investisseur Portfolio x sur Portfolio y selon une spécification d'utilité logarithmique.

TABLE 1.1: Exemple de mesure du risque dans Levy (1992, p. 567)

	x	Prob (x)	y	Prob (y)
State 1	10	0,99	1	0,8
State 2	1 000	0,01	100	0,2
Moyenne	19,9		20,8	
Variance	9 703		1 568	

Notez que nous corrigeons la variance initiale de 1 468 pour Portfolio y dans Levy (1992).

Les événements extrêmes, généralement peu fréquents, ont un impact important sur la distribution des rendements et sur le compromis rendement-risque. Par conséquent, l'optimisation devrait tenir compte de ces risques. Comme on peut le voir dans l'exemple du Table 1.1, l'énorme écart pour Portfolio x couvre sa perspective d'un rendement extrêmement positif de 1 000. La valeur des moments d'ordre supérieur à l'optimisation du portefeuille est qu'ils capturent mieux les risques d'événement. En raison de l'incompétence de la variance dans la spécification de l'impact des événements extrêmes, des informations précieuses sur les risques d'événements sont ignorées dans l'approche de la variance moyenne traditionnelle pour la sélection du portefeuille.

En bref, l'exemple précédent souligne l'avantage d'inclure des rendements extrêmes dans le processus d'optimisation des investissements, en particulier pour améliorer la première étape de la classification de l'efficience. Les moments d'ordre supérieur capturent l'impact de ces rendements extrêmes. De plus, les moments d'ordre supérieur aident à faire une distinction entre les caractéristiques de rendement des actifs des actions et des obligations. Les obligations sont généralement considérées comme un type particulier de titres à faible rendement et à faible volatilité. Cependant, une caractéristique de rendement clé pour les obligations est une combinaison de faible variance et de kurtosis élevé, ce qui n'est pas le cas pour les actions. Le Table 1.2 donne des statistiques descriptives sur les rendements annuels de l'indice S&P 500, des bons du Trésor à 3 mois et des obligations à 10 ans de 1928 à 2017. Comme prévu, les rendements moyens et les volatilités du court les obligations à terme et les obligations à long terme sont faibles, mais l'asymétrie et l'aplatissement sont beaucoup plus élevés que pour les actions. Spécifiquement, le kurtosis donne un aspect de risque différent de la volatilité, car on peut voir le tri de la volatilité et le kurtosis est inversé pour les obligations et les actions. Ces statistiques donnent deux implications. Premièrement, l'inclusion de moments d'ordre supérieur, en particulier l'aplatissement, permet de décrire un profil de risque de retour complet. Deuxièmement, la séparation des actifs en actions et des actifs obligataires est nécessaire dans la construction du portefeuille. En ce sens, le portefeuille d'actions est d'abord déterminé puis combiné avec l'investissement d'obligations pour constituer le portefeuille optimal global.

Le sujet principal de cette thèse est que nous essayons d'intégrer les risques d'événements (entendus au sens large) dans le processus d'allocation d'actifs. Plus précisément, nous développons deux approches pour l'amélioration des investissements qui tiennent compte des risques d'événement, où les informations

TABLE 1.2: Différentes caractéristiques de rendement entre les actions et les obligations

	S&P 500	3M T-Bill	10Y T-Bond
Moyenne	11,5%	3,4%	5,2%
Volatility	19,6%	3,1%	7,7%
Skewness	-42,1%	102,0%	100,8%
Kurtosis	11,5%	99,1%	169,5%

Les statistiques sont calculées avec les rendements annuels sur le marché boursier, les bons du Trésor à trois mois et les bons du Trésor à dix ans entre 1928 et 2017. Les données de rendement proviennent des données historiques d'Aswath Damodaran sur « les rendements historiques des actions, obligations et obligations - États Unis ».

d'ordre supérieur entrent explicitement dans les problèmes de construction du portefeuille. Une approche est une amélioration progressive dans le cadre classique de l'optimisation de la variance moyenne et du CAPM, en étendant ce cadre aux moments d'ordre supérieur. L'autre approche concerne le cadre alternatif de la dominance stochastique, pour lequel nous examinons l'efficience des investissements dans le sens de la dominance stochastique d'ordre supérieur. Avec ces adaptations, l'optimalité globale est améliorée pour la sélection de portefeuille en présence de risques d'événement. Au-delà du contexte de sélection des portefeuilles, nous étudions également les risques d'événements (aujourd'hui compris au sens classique) dans le contexte de la finance d'entreprise. Nous observons les changements de nom d'entreprise lors des fusions et acquisitions (M&As) et nous examinons comment les événements affectent la dynamique de rendement pour l'acquéreur et la cible. Cela permet d'avoir une vision panoramique de l'impact des risques événementiels en finance, pour lesquels les marchés financiers et la finance d'entreprise sont deux sphères intrinsèquement complémentaires.

Les études sur l'extension CAPM aux moments d'ordre supérieur, l'optimisation de la dominance stochastique d'ordre supérieur et les changements de nom d'entreprise au cours de M&A sont liées par une focalisation constante sur le portefeuille de marché. Le portefeuille de marché joue un rôle clé dans le modèle CAPM en tant que modèle général d'équilibre du marché qui se développe à partir de l'optimisation de la variance moyenne en tant qu'analyse d'optimisation individuelle. Le portefeuille de marché est utilisé dans tous les modèles d'évaluation des actifs en ce qui concerne le risque de marché. Comme le portefeuille de marché actuel est trop difficile à cerner, et pour des raisons de faisabilité pratique, les indices boursiers sont utilisés comme approximations.

Pour l'examen des risques événementiels dans cette thèse, nous mettons en œuvre les analyses sur les indices boursiers à différents niveaux de diversification des secteurs industriels et des composants individuels. Plus précisément, dans les deux analyses d'ordre supérieur, nous utilisons les secteurs de l'industrie comme actifs et le portefeuille est constitué de pondérations d'investissement dans ces secteurs. L'allocation optimale a alors des implications sur le bénéfice de la diversification, qui a été une règle d'or dans la théorie financière depuis l'optimisation de la variance moyenne. Nous avons également défini l'effet de changement de nom de M&A

au niveau des composants parmi l'indice S&P 500, l'indice boursier le plus important de la profession d'investissement indiciel. Généralement, les analyses de cette thèse donnent de fortes inférences sur l'efficacité des indices boursiers examinés.

1.1 Moyenne variance optimization et CAPM

Le cadre classique de l'optimisation de la variance moyenne et du CAPM est fondamental pour financer la théorie, et une partie importante de cette thèse est une extension de ce cadre ainsi qu'un effort pour remédier à son incapacité à gérer les risques d'événements. Donc, il est bénéfique d'avoir un examen rapide à ce sujet.

Supposons qu'il existe des actifs à risque n avec des profils de rendement et de variance réguliers dans un marché parfait, $\tilde{r}_i$ étant le rendement à période unique pour l'actif $i = 1, 2, ..., n$. Alors le vecteur de retour pour ces assets à risque est $\tilde{R} = [\tilde{r}_1, \tilde{r}_2, ..., \tilde{r}_n]'$, et le vecteur de retour attendu associé est $\mu = E[\tilde{R}]$, la matrice de covariance est $\Sigma = E[(\tilde{R} - \mu)(\tilde{R} - \mu)^{'}]$. Ensuite, pour un niveau de rendement attendu donné μ_P, le portefeuille désiré devrait avoir une variance minimale, qui est formulée comme

$$\begin{aligned} \min_{w} \quad & \frac{1}{2} w' \Sigma w \\ \text{sujet à} \quad & w' \mu = \mu_P \\ & w' \mathbf{1} = 1 \end{aligned}$$

où w est le vecteur poids du portefeuille de longueur n, et $\mathbf{1}$ est un vecteur unitaire de la même taille.

En résolvant le problème de la programmation quadratique, nous avons une hyperbole dans le plan de retour attendu et de volatilité. Le sommet de l'hyperbole est le portefeuille de la variance minimale et la partie située au-dessus du portefeuille de la variance minimale est la frontière efficace. Le long de cette courbe, les portefeuilles ont des volatilités minimales à des rendements attendus donnés ou, de manière équivalente, des rendements maximaux attendus à des volatilités données. Des expositions détaillées peuvent être trouvées, par exemple, dans Huang and Litzenberger (1988, Chapter 3) ou dans Back (2010, Chapter 5).

Sur la base de la frontière efficace de la variance moyenne, Sharpe (1964) développe le CAPM en passant des comportements de micro-optimisation aux équilibres de marché généraux. Avec les hypothèses principales des investisseurs homogènes et un marché des actifs sans risque parfait, tous les investisseurs ont le même choix de portefeuille pour les actifs risqués, qui est le portefeuille le long de la frontière efficace maximisant le ratio de Sharpe. Des différences existent seulement dans les proportions investies dans l'actif sans risque et dans ce portefeuille à risque. L'équilibre du marché implique que le portefeuille à risque doit être le portefeuille du marché. De cette intuition vient la formule CAPM, dont la dérivation directe, originaire de Sharpe (1964), est ci-dessous.

Supposons qu'un portefeuille P est composé du portefeuille de marché M et d'un actif i, tandis qu'une part de α est investie dans le portefeuille de marché M et le reste dans l'actif i, avec $\alpha \in [0, 1]$. Le retour du portefeuille est une combinaison linéaire des deux composants comme $\tilde{r}_P = \alpha \tilde{r}_M + (1 - \alpha) \tilde{r}_i$. La courbe de (μ_P, σ_P) par rapport à α touche la ligne du marché des capitaux (CML) à (μ_M, σ_M) où un investissement total est effectué dans le portefeuille de marché et $\alpha = 1$. Dans ce cas, la dérivée de la courbe au point de tangence est égale à la pente CML $\frac{\mu_M - r_f}{\sigma_M}$, comme le montre la

Figure 1.1. Donc,

$$\left.\frac{\partial \mu_P}{\partial \sigma_P}\right|_{\alpha=1} = \frac{\mu_M - r_f}{\sigma_M}.$$

Après quelques manipulations, on obtient

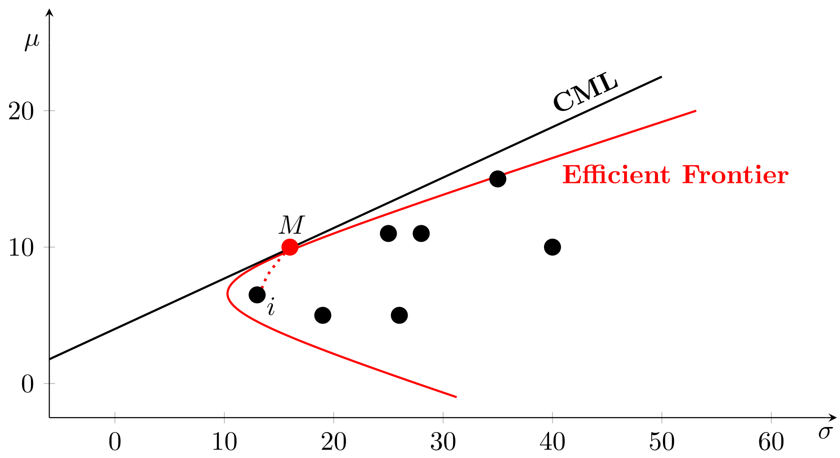
$$\frac{\mu_i - \mu_M}{(\sigma_{iM} - \sigma_M^2/\sigma_M)} = \frac{\mu_M - r_f}{\sigma_M},$$

qui donne facilement

$$\mu_i = r_f + \beta_i(\mu_i - r_f),$$

where $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$. C'est la formule commune du CAPM, qui relie le rendement attendu d'un actif à son exposition au risque de marché. Si l'actif est plus exposé au risque de marché, son rendement attendu est plus élevé. Cette formule indique une règle de tarification linéaire, selon laquelle l'excédent de rendement attendu de l'actif est positivement proportionnel à son exposition au risque de marché.

FIGURE 1.1: Ligne de marché de capitaux et dérivation de CAPM



1.1.1 Moments d'ordre supérieur

Par rapport à la classification dans Levy (2016) dans laquelle l'optimisation du portefeuille comporte deux étapes: l'identification efficace du portefeuille et la maximisation de l'utilité individuelle, Markowitz (1952) donne une spécification légèrement différente pour le processus d'investissement. Il maintient que

> « Le processus de sélection d'un portefeuille peut être divisé en deux étapes. La première étape commence par l'observation et l'expérience et se termine par des croyances sur les performances futures des titres disponibles. La deuxième étape commence par les croyances pertinentes sur les performances futures et se termine par le choix du portefeuille. »

Notez que cette classification ne contredit pas les deux étapes de la décision objective et subjective de Levy. Selon la première étape de Markowitz, la préparation est une partie essentielle du processus général d'optimisation, et elle implique des intrants clés pour la décision objective et la décision subjective. En particulier, « l'observation

» et « les performances futures des titres disponibles » sont importants pour la spécification de l'ensemble efficace, de même que « l'expérience » et les « croyances » pour la maximisation de l'utilité attendue.

Si l'observation et l'expérience aboutissent à une collecte d'intrants d'optimisation au-delà de la moyenne et de la variance, l'utilisation du cadre d'optimisation de la variance moyenne ne peut fournir qu'un choix sous-optimal. Cont (2001) offre un résumé des propriétés empiriques pour les retours d'actifs. Parmi les propriétés statistiques stylisées du rendement des actifs, les queues lourdes et l'asymétrie des pertes sont mises en évidence, montrant que l'asymétrie négative et l'excès de kurtosis sont des préoccupations substantielles au-delà de la moyenne et de la variance. De façon constante, les moments d'ordre supérieur sont également importants pour décrire les préférences des investisseurs. Par exemple, Kraus and Litzenberger (1976) réaffirme que les investisseurs ont une préférence pour l'asymétrie positive, et étend le CAPM à trois moments pour incorporer l'effet d'asymétrie dans l'évaluation. Les recherches ultérieures sur les moments d'ordre supérieur ont principalement porté sur l'incorporation de l'asymétrie, pour laquelle De Athayde and Flôres (2004) et Briec, Kerstens, and Jokung (2007) sont les tentatives les plus représentatives de la construction du portefeuille d'asymétrie de la variance moyenne.

Kurtosis est fréquemment utilisé pour capturer les risques extrêmes, qui ont été mis en lumière après la récente série de crises économiques et de catastrophes. Par exemple, Barro (2006) utilise kurtosis pour refléter le risque de catastrophe et montre que les désastres économiques du siècle dernier ont conduit à une baisse moyenne de 29% du GDP réel par habitant. Il trouve que le risque de catastrophe a des effets considérables sur le rendement des actifs et sur la prime d'équité. Dittmar (2002), Eraker, Johannes, and Polson (2003), Liu, Longstaff, and Pan (2003), Poon, Rockinger, and Tawn (2003), Bakshi and Madan (2006), Bates (2008), Todorov (2009), Benzoni, Collin-Dufresne, and Goldstein (2011), Bollerslev and Todorov (2011), Bates (2012), Gabaix (2012), Drechsler (2013), Wachter (2013), and Cremers, Halling, and Weinbaum (2015), parmi beaucoup, notez le rôle important des risques extrêmes dans la détermination des rendements. Plus intéressant encore, Malmendier and Nagel (2011) trouve que les faibles rendements boursiers contribuent à modeler les préférences individuelles vers une plus grande aversion au risque, et cet effet est plus fort pour les jeunes. Ainsi, l'effet des risques extrêmes n'est pas seulement contemporain mais aussi prospectif.

Par nature, l'aplatissement est un indicateur de risque complémentaire à la variance pour deux raisons. La première raison est que le risque englobe non seulement le risque baissier mais aussi le risque à la hausse, comme le préconise Damodaran (2003, Chapter 2). En ce sens, le kurtosis est plus approprié que l'asymétrie pour représenter les risques d'événements, car il assigne les mêmes pénalités aux événements extrêmes, qu'ils soient positifs ou négatifs. Pour illustrer comment l'incorporation de kurtosis influence le choix du portefeuille, nous revenons à l'exemple de Table 1.1. Nous voyons que Portfolio y domine Portfolio x dans le sens de la variance moyenne. Il est vrai que Portfolio x est fortement biaisé vers la valeur extrêmement positive de 1 000, mais son kurtosis est congruemment plus grand que celui de Portfolio y. En fait, le kurtosis de Portfolio x est de 9 227 456 454 (le kurtosis standard est de 98), alors que pour Portfolio y il est de 7 992 159 (le kurtosis standard est de 3,25). Supposons que l'expansion de Taylor de la fonction d'utilité se termine à l'ordre 4 et que l'utilité attendue puisse être exprimée comme $E[u(W)] = \gamma_1\mu - \gamma_2\sigma^2 + \gamma_3 s^3 - \gamma_4\kappa^4$, où les γs sont tous des charges utilitaires positives pour les moments de retour. Étant donné que Portfolio x est seulement meilleur en termes d'asymétrie, il ne peut être qu'un choix supérieur lorsque l'investisseur se soucie principalement de l'asymétrie bien au-delà de tout autre moment, ce qui est hautement improbable.

La deuxième raison est que les événements négatifs extrêmes sont plus importants que les événements positifs extrêmes, ce qui contribue à un effet d'asymétrie dégonflé et à une appréhension de la dispersion accrue. Cont (2001) obtient que les grands mouvements ascendants ne sont pas aussi égaux que les grands drawdowns. Cette perte d'asymétrie implique que toute valeur extrême positive potentielle est probablement compensée par des valeurs extrêmes négatives de taille similaire ou supérieure. Lorsque l'on compare deux distributions avec une asymétrie négative, l'aplatissement est plus informatif sur le niveau de dispersion général des distributions.

Toutes ces preuves suggèrent qu'il est essentiel d'inclure les risques extrêmes, en particulier l'aplatissement, dans l'optimisation des investissements.

1.1.2 Méthode d'amélioration de Pareto

Les portefeuilles pourraient bien partager des moments communs. Par exemple, supposons que nous examinions les profils de retour de deux portefeuilles A et B jusqu'au quatrième moment. Les deux portefeuilles ont les mêmes trois premiers moments, et Portfolio B a un quatrième moment inférieur. Ainsi, le portefeuille B présente une amélioration marginale du profil par rapport au portefeuille A au quatrième moment. Un inconvénient majeur des approches existantes concernant la construction efficace de portefeuilles plus performants est le manque de discernement parmi les portefeuilles montrant des améliorations marginales du profil de rendement comme dans l'exemple précédent. L'incapacité à saisir une amélioration marginale conduit à une mauvaise classification des portefeuilles inefficaces, ici le portefeuille A, aussi efficace. La méthode d'amélioration de Pareto surmonte une telle défaillance et améliore l'optimalité globale dans la construction de portefeuille d'ordre supérieur. Nous couvrons ce point ci-après.

Il y a deux méthodes principales qui étendent la frontière efficiente aux moments d'ordre supérieur. La première de ces méthodes est représentée par De Athayde and Flôres (2004), un ajustement simple de l'optimisation de la variance moyenne dans le contexte de la dimension d'asymétrie de la variance moyenne. En termes simples, cette méthode minimise la variance du portefeuille à des niveaux de retour et d'asymétrie attendus donnés en utilisant le programme suivant:

$$\begin{aligned} \min_{w} \quad & \sigma^2 \\ \text{sujet à} \quad & \mu \geq \mu_P \\ & s^3 \geq s_P^3 \end{aligned}$$

où le profil de retour de P est un niveau donné. Cette méthode implique l'actif sans risque dans la détermination du portefeuille de variance minimale, expliquée dans De Athayde and Flôres (2004, p. 1339) comme

$$\min_{w} \mathcal{L} = w'\Sigma w + \lambda_1(\mu_P - w'\mu - (1 - w'\mathbf{1})r_f) + \lambda_2(s_P^3 - w'S(w \otimes w)),$$

où $\otimes$ est le produit tensoriel. Comme le suggère le programme de minimisation, cette méthode ne peut pas produire un portefeuille purement efficace d'actifs risqués. Le Table 1.2 met en évidence les caractéristiques de retour de la faible / moyenne volatilité ainsi que l'asymétrie / kurtosis élevé pour les obligations, ce qui est assez différent des caractéristiques de retour des actions. En d'autres termes, cette méthode de variance minimale n'a aucun lien avec la séparation des deux fonds, et les portefeuilles optimaux selon cette méthode ne peuvent pas être caractérisés comme une partie d'investissement à risque commune plus une partie d'investissement sans risque.

La méthode de la variance minimale a également une implication étrange sur l'équilibre du marché des actifs sans risque. Dans l'hypothèse d'attentes homogènes du marché pour les investisseurs, la part de l'actif sans risque est fixée dans le choix optimal de l'investisseur selon la fonction de Lagrange, et les investisseurs ont la même proportion patrimoniale investie dans l'actif sans risque. Cette part est indépendante des préférences de risque individuelles, peu importe que l'agent soit à la recherche de risque ou d'aimer le risque. Ainsi, les investisseurs à la recherche de risques ne peuvent pas emprunter de l'argent pour investir davantage dans la partie risquée, et les investisseurs averses au risque ne peuvent pas réduire l'investissement risqué et prêter de l'argent. La préférence en matière de risque n'a aucun impact sur l'offre et la demande pour l'actif sans risque, et l'équilibre du marché sans risque est en cause.

Du point de vue de l'implémentation, le programme d'optimisation de la méthode de variance minimale doit rechercher toute la grille couvrant les rendements attendus μ_P et les valeurs de skewness s_P^3. Pour les portefeuilles ayant la même variance, le programme de minimisation de la variance ne peut pas détecter les portefeuilles inefficaces avec des valeurs de moyenne et d'asymétrie plus faibles, tout en gardant ces portefeuilles efficaces par erreur. Il apporte le risque de fausses erreurs négatives, illustré par l'exemple de Table 1.3. Supposons que nous ayons trois portefeuilles sur le marché et que nous souhaitons obtenir les portefeuilles efficaces parmi les portefeuilles x, y et z en utilisant la méthode de minimisation de la variance dans De Athayde and Flôres (2004). Maintenant, définissez $\mu_P = 1$ et $s_P^3 = 1.5$, et nous recherchons le portefeuille de variance minimum sous ces contraintes. Notez que les trois portefeuilles répondent aux exigences de moyenne et d'asymétrie. Comme ils ont la même variance de 2, le programme d'optimisation minimisant la variance n'aide pas du tout à identifier l'un d'entre eux comme étant les portefeuilles inefficaces. En dépit de la même variation que les autres portefeuilles, Portfolio z est le portefeuille efficient effectif car il présente une moyenne plus élevée et une asymétrie plus élevée. Cependant, l'information sur la moyenne et l'asymétrie n'est pas considérée dans le programme, ce qui conduit à une erreur dans la conservation des portefeuilles inefficaces. Du point de vue de l'écart de dualité, le programme dual est exact entre la maximisation du rendement et la minimisation de la variance dans l'optimisation de la variance moyenne due à la convexité. Lorsque le problème d'optimisation va à des moments d'ordre supérieur, l'écart de dualité n'est pas nécessairement nul. Dans ce cas, la variance de minimisation du portefeuille à des valeurs de rendement attendu et d'asymétrie donnée n'est pas nécessairement le rendement maximal attendu du portefeuille à des valeurs de variance et d'asymétrie données.

L'autre principale optimisation pour la construction de portefeuilles de moments d'ordre supérieur est la méthode de la fonction de pénurie, mise au point par Briec, Kerstens, and Jokung (2007). Cette méthode va plus loin que la première approche en égalisant les améliorations potentielles pour un portefeuille selon toutes les dimensions du profil de rendement. Le raisonnement est que si nous pouvons trouver un portefeuille avec un meilleur profil de rendement qui domine le portefeuille évalué, alors ce dernier est inefficace et le portefeuille obtenant l'amélioration de profil la plus importante est l'efficience correspondante. Si ce portefeuille n'atteint

TABLE 1.3: Profils de portefeuille hypothétiques et optimisation de l'efficacité

	x	y	z	z*
Moyenne	1	1	1,5	1,5
Variance	2	2	2	1,8
Skewness	1,5	2	2	2

Nous supposons que les portefeuilles x, y et z sont tous réalisables. La faisabilité de Portfolio z^* sera indiquée dans le texte.

pas un profil de rendement dominant par rapport au portefeuille évalué, le portefeuille évalué est efficace. L'ensemble efficace comprend les portefeuilles sans profil de rendement dominant.

Dans la méthode de la fonction de pénurie, l'objectif est de maximiser l'amélioration du profil

$$\begin{aligned} \max_{w} \quad & \delta \\ \text{sujet à} \quad & \mu \geq \mu_P + \delta g_1 \\ & \sigma^2 \leq \sigma_P^2 - \delta g_2 \\ & s^3 \geq s_P^3 + \delta g_3 \end{aligned}$$

où δ est la taille de l'amélioration, $g = [g_1 \ -g_2 \ g_3]'$ est un vecteur d'amélioration directionnelle prédéfini, et les paramètres de les δ, g_1, g_2 et g_3 sont positifs.

Pour voir comment fonctionne la méthode de la fonction de pénurie, nous revenons à l'exemple de Table 1.3. Supposons maintenant que nous ayons un portefeuille supplémentaire de z*, et x est le portefeuille évalué parmi les quatre choix. Pour une spécification de $g = [0,5 \ -0,2 \ 0,5]'$ avec le recul, la méthode de la fonction de pénurie renvoie Portfolio z* comme portefeuille efficace, puisqu'elle présente une amélioration moyenne de 0,5, une amélioration de la variance de 0,2, une asymétrie amélioration de 0,5, et il réalise la meilleure amélioration du profil de rendement, également $\delta = 1$, dans l'ensemble du portefeuille réalisable. Pour atteindre ce portefeuille efficace z*, la méthode de variance minimale doit exécuter le programme d'optimisation quatre fois, avec les contraintes de (μ_P, s_P^3) à (1, 1,5), (1, 2), (1,5, 1) et (1,5, 2) respectivement. La méthode de la fonction de pénurie est moins susceptible de souffrir de la recherche de grille redondante, augmente ainsi l'efficacité. Notez que pour obtenir un ensemble suffisant de portefeuilles efficaces, nous devons fournir un ensemble suffisamment large de portefeuilles évalués en tant qu'intrants pour obtenir les portefeuilles efficaces correspondants.

Bien qu'elle soit plus efficace que la méthode du portefeuille de variance minimale à la spécification des portefeuilles efficients, la méthode de la fonction de pénurie a aussi la faiblesse de ne pas pouvoir identifier l'amélioration du profil de rendement marginal. Remarque g_i doit être positif pour faciliter l'optimisation. Par conséquent, cette méthode ne parvient pas à capturer les améliorations marginales pour lesquelles g_i peut être 0. Dans l'exemple précédent, si Portfolio z est en cours d'évaluation, la fonction de pénurie la reconnaît également car la valeur maximisée de δ est toujours 0. En fait Portfolio z* est le seul portefeuille efficace, mais cette amélioration de profil par rapport à Portfolio z n'est pas identifiée par le programme du fait que les g_1 et g_3 correspondants sont tous les deux 0

Pour résoudre un tel problème, nous proposons d'utiliser un cadre plus général de vecteur d'amélioration dimensionnelle au-delà de la positivité stricte dans toutes les directions. Nous appelons cette méthode la méthode d'amélioration de Pareto. Cette méthode partage la même motivation que la méthode de la fonction de pénurie, car les portefeuilles efficients sont ceux sans aucune amélioration du profil de rendement. Pour toute paire de portefeuilles x et y, nous disons que portefeuille y a une amélioration de Pareto par rapport au portefeuille x, si et seulement si

$$\langle \mathcal{P}_y, \mathcal{P}_x \rangle \gneqq \mathbf{0},$$

où $\langle \mathcal{P}_y, \mathcal{P}_x \rangle = [\mu_y - \mu_x;\ \sigma_x^2 - \sigma_y^2;\ s_y^3 - s_x^3]'$ dans l'espace moyen d'asymétrie de la variance et $\gneqq$ signifie "pas moins de mais pas égal à". le vecteur arbitraire ε et un vecteur de taille identique à zéro $\mathbf{0}$, $\varepsilon \gneqq \mathbf{0}$ indique que chaque élément de ε n'est pas inférieur à 0 et qu'au moins un L'élément n'est pas 0.

Dans ce cadre général, nous pouvons facilement déceler Portefeuille z* comme le seul actif parmi les quatre portefeuilles, car il n'a pas d'amélioration de profil disponible parmi l'ensemble réalisable. Cela montre la supériorité de la méthode d'amélioration de Pareto par rapport à la méthode du portefeuille de variance minimale et la méthode de la fonction de pénurie pour saisir l'amélioration du profil de rendement marginal. Ainsi, la méthode d'amélioration de Pareto est robuste à l'erreur de mal classer un portefeuille inefficace comme efficace. En outre, il est flexible pour mener une optimisation d'investissement formulée avec n'importe quelle combinaison de moments de retour, comme des moments de saut. Plus précisément, lorsqu'un investisseur est opposé aux risques extrêmes mais neutre à l'asymétrie, la méthode d'amélioration de Pareto peut être utilisée pour optimiser l'espace moyen de kurtosis de la variance.

1.1.3 Approximations polynomiales de l'utilitaire

Dans l'optimisation des investissements, la première étape de la construction d'une frontière efficace se relie intrinsèquement à la deuxième étape de la maximisation de l'utilité. L'identification efficace du portefeuille dans les moments d'ordre supérieur concerne la première étape, et une extension comparable de la spécification de l'utilitaire aux moments d'ordre supérieur est nécessaire pour la deuxième étape. La méthode classique pour incorporer des moments d'ordre supérieur est l'approximation polynomiale de l'utilité attendue par l'expansion de Taylor, lancée par Pratt (1964) et Arrow (1965).

L'expansion générale de Taylor de l'utilité attendue du rendement des actifs est

$$E[u(\tilde{R})] = \sum_{i=0}^{\infty} \frac{u^i(E[\tilde{R}])}{i!} E[(R - E[\tilde{R}])^i].$$

Dans le cas le plus fondamental de la troncature à l'ordre 2, nous avons:

$$E[u(\tilde{R})] \approx u(E[\tilde{R}]) + \frac{1}{2} u''(E[\tilde{R}]) E[(R - E[\tilde{R}])^2].$$

Markowitz (1959) et Levy and Markowtiz (1979) lancent et rationalisent qu'une combinaison de la moyenne et de la variance fournit une bonne approximation de la fonction d'utilité attendue. L'utilitaire de variance moyenne est un benchmark de spécifications d'utilité concis et intuitif. Dans sa version la plus populaire, intégrée dans l'approche de Black Litterman Black and Litterman (1992), l'utilitaire de variance moyenne est écrit comme

$$\mu - \frac{1}{2}\lambda\sigma^2$$

où $\lambda = -\frac{u''(\tilde{R})}{u'(\tilde{R})}$ est le coefficient d'aversion au risque absolu de Arrow Pratt. Pour un investisseur avec une telle préférence, le programme d'optimisation devient

$$\begin{array}{ll} \max\limits_{w} & w'\mu - \frac{1}{2}\lambda w'\Sigma w \\ \text{sujet à} & w'\mathbf{1} = 1 \end{array}$$

tandis que les deux étapes de l'optimisation de l'investissement dégénèrent en une seule étape de la maximisation de l'utilité. Notez que cette dégénérescence de décision est basée sur une spécification exacte pour l'investisseur qui a alors le même coefficient d'aversion au risque que les autres investisseurs. Cette situation n'est pas susceptible d'être vraie. La fonction d'utilité de la variance moyenne révèle explicitement le compromis entre le rendement et le risque, où plus le coefficient d'aversion au risque est élevé, plus l'attrait des portefeuilles à forte variance est faible. Cette approche est assez simple et pratique, mais elle suppose que l'investisseur se concentre exclusivement sur les deux premiers moments de la distribution des rendements.

Comme mentionné précédemment, la combinaison de la moyenne et de la variance ne suffit pas pour caractériser complètement la distribution de rendement réelle. Les investisseurs se soucient des risques extrêmes, et une approximation plus générale de Taylor pour l'utilité attendue est nécessaire. Une troncature à l'ordre 3 conduit à l'utilité d'asymétrie de variance moyenne

$$E[u(\tilde{R})] = \gamma_1\mu + \gamma_2\sigma^2 + \gamma_3 s^3,$$

et une troncature à l'ordre 4 conduit à l'utilité de la variance skewness kurtosis de la variance

$$E[u(\tilde{R})] = \gamma_1\mu + \gamma_2\sigma^2 + \gamma_3 s^3 + \gamma_4\kappa^4,$$

où γ_i est le coefficient correspondant au moment de l'ordre $i \in [1, 2, 3, 4]$.

Ainsi, si l'investisseur utilise l'aplatissement comme un indicateur de risque extrême et est insensible à l'asymétrie, nous pouvons spécifier une fonction d'utilité moyenne de la variance de kurtosis pour cet ensemble de préférences de risque comme suit:

$$E[u(\tilde{R})] = \gamma_1\mu + \gamma_2\sigma^2 + \gamma_4\kappa^4.$$

Les estimations de γs reflètent l'attitude particulière de l'investisseur face au risque, γ_2 d'aversion au risque commune et γ_4 d'aversion au risque extrême.

Comme le démontre l'expansion de Taylor, une compilation consécutive d'un ordre consécutif jusqu'à l'ordre de n pour l'approximation de l'utilité est familière en recherche financière. Un moment sélectif de compilation jusqu'à n avec des accents spécifiques sur la préférence de risque, comme ici avec une aversion au risque extrême mais une indifférence à l'asymétrie, est nouveau. Cette spécification traitant d'une approximation de l'utilité du moment de saut est nouvelle pour la littérature financière existante, autant que nous le sachions.

L'expansion de Taylor peut être implémentée sur toutes les fonctions d'utilité spécifiques dans les classes de CARA, CRRA et HARA. Les paramètres de γs en cours d'étalonnage facilitent la convergence de l'utilité approximative espérée vers l'utilité attendue exacte. Par exemple, Garlappi and Skoulakis (2011) discute de l'approximation de Taylor pour les fonctions utilitaires de classe HARA basées sur une décomposition linéaire. C'est-à-dire que nous pouvons également approximer la fonction d'utilité du moment de saut en étalonnant les γs.

1.1.4 Monte Carlo

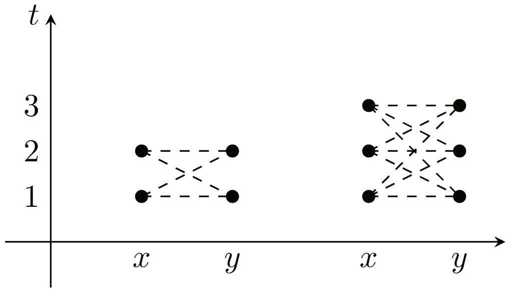
Au fur et à mesure que l'ordre du moment augmente, l'apport pour l'optimisation des investissements est plus exigeant. Dans l'optimisation de la variance moyenne, les niveaux de retour sont l'entrée; dans l'optimisation du portefeuille de variance minimale de De Athayde and Flôres (2004), une grille de niveaux de retour et d'asymétrie est l'entrée; dans la méthode de la fonction de pénurie de Briec, Kerstens, and Jokung (2007), un nombre suffisant de portefeuilles évalués comme entrée. Puisque l'optimisation de Pareto est plus générale que la méthode de la fonction de pénurie pour inclure les améliorations du profil de rendement marginal, sa contribution est une approximation suffisante de l'ensemble du portefeuille réalisable.

Une présentation parfaite de l'ensemble réalisable par dénombrement est impossible, car il existe des formations de portefeuille possibles indéfinies par diversification. Cependant, les frictions du marché telles que les coûts commerciaux rendent impossible la réalisation des choix de diversification indéfinis. Compte tenu de la présence de commissions de courtage, frais et taxes, un petit changement de portefeuille, disons, une augmentation de 0,1% du poids d'un actif compensée par une diminution du poids d'un autre actif n'est généralement pas pragmatique. Ce fait implique qu'il est probable d'obtenir une approximation raisonnable de l'ensemble du portefeuille réalisable.

La complexité d'approximation de l'ensemble réalisable explose lorsque le nombre d'actifs sous-jacents augmente. Pour illustrer ce point, nous introduisons un instrument simple appelé « densité d'investissement ». Supposons un actif, nous avons t possessions possibles sur l'actif, et la détention réelle i peut être un entier de $[1, t]$ Le plus élevé t, le plus proche entre deux exploitations adjacentes, ou l'écart de poids plus dense. Par exemple, supposons que nous ayons deux actifs x et y de 2 possessions possibles, chacun avec 1 et 2 comme densité de détention. Pour un investissement dans un seul actif, nous avons deux choix: investir en x ou en y. Pour les possibilités de cross assets de (x, y), on a (1, 1), (1, 2), (2, 1) et (2, 2). Le poids du portefeuille est ensuite transformé à partir de la paire de densité. Pour (1, 2), la densité totale est 3, donc (1, 2) désigne un portefeuille avec 1/3 investissement en x et 2/3 en y. Notez la première possibilité (1, 1) et la dernière possibilité (2, 2) correspondent à des investissements égaux. Par conséquent, nous devons moins un compte redondant, et nous avons cinq choix de portefeuille dans ce cas. Similaire est fait quand $t = 3$, comme montré dans la Figure 1.2.

Dans la même veine, supposons que pour chacun des actifs n, nous pouvons discrétiser la détention en possibilités t, alors un simple calcul donne une réponse de $n + t^n - t + 1$ choix de portefeuille. Notez que n est le nombre d'investissements de coin dans des actifs individuels, t^n est la combinaison de toutes les possibilités de détention entre les actifs, et $t - 1$ est un ajustement pour les investissements redondants égaux. La Figure 1.3 montre comment les possibilités d'investissement explosent en ce qui concerne n et t. Quand il y a 2 densités de détention pour chaque actif, nous avons 5 choix de portefeuille comme mentionné précédemment dans l'univers des 2 actifs, 1 033 choix dans l'univers des 10 actifs, 1,07 milliards dans l'univers des 30 actifs et $1,27 \times 10^{30}$ choix dans l'univers des 100 actifs. Quand il y a 30 densités de détention pour chaque actif, nous avons 873 choix de portefeuille dans l'univers

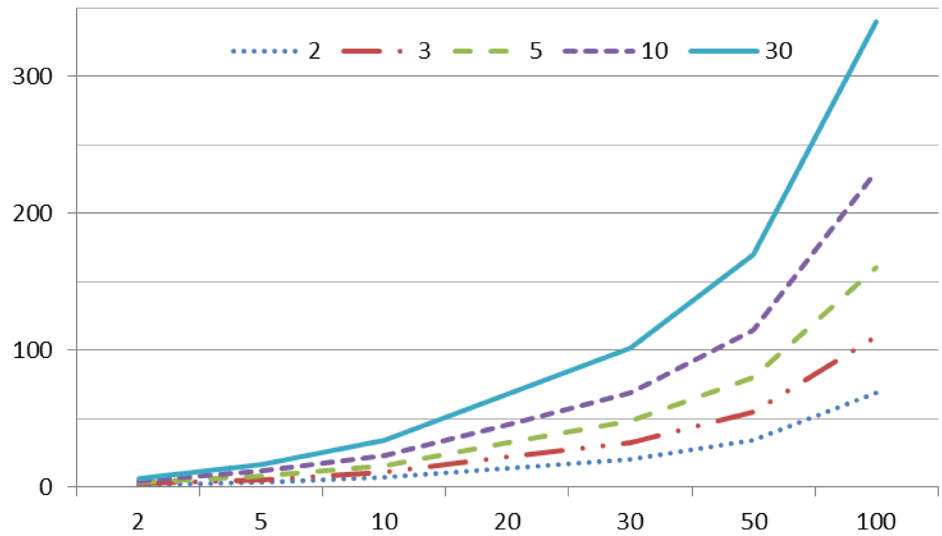
FIGURE 1.2: Densités d'investissement et choix de portefeuille



à 2 actifs, $5,9 \times 10^{14}$ choix dans l'univers 10-assets, $2,06 \times 10^{44}$ dans les 30 -asset universe, et $5,2 \times 10^{147}$ choix dans l'univers de 100 assets.

FIGURE 1.3: Choix de l'investissement, univers de l'actif et densité de détention

Cette figure montre comment les choix d'investissement total varient en fonction de la taille n de l'univers des actifs, ainsi que du nombre t de possibilités de détention. L'axe horizontal montre n, et l'axe vertical mesure le nombre logarithmique de choix d'investissement pour une paire correspondante de n et t.



L'explosion de la charge de calcul dans un univers d'actifs de grande taille implique le besoin d'une réduction de dimensionnalité. Une bonne façon est d'utiliser les sous-indices de l'industrie comme actifs sous-jacents plutôt que d'utiliser des actions individuelles. Par exemple, l'indice DJIA compte 30 actions qui peuvent être facilement classées en une dizaine de secteurs. Les avantages d'une réduction dimensionnelle sont évidents et plus importants lorsqu'on considère l'indice FTSE 100, l'indice NIKKEI 225 et l'indice S&P 500.

En outre, un algorithme de génération de poids approprié peut faciliter l'approximation de l'ensemble réalisable. Notez que lorsque la densité de maintien est excessivement raffinée, un changement de poids minimal se distingue de ses voisins. Lorsque la taille de l'univers des actifs n est d'environ 10, la différence marginale entre deux pondérations de portefeuille adjacentes est négligeable, et un changement de placement d'un portefeuille à l'autre est très probablement dissuadé par les coûts de transaction.

La distribution de Dirichlet est un schéma générateur de poids avec la caractéristique de somme régulière. Cette distribution a beaucoup d'applications en économie et en finance. Par exemple, Chotikapanich and Griffiths (2002) pionnier son utilisation dans l'échantillonnage des parts de revenu pour une estimation de la courbe de Lorenz. Leurs recherches suggèrent que la distribution de Dirichlet peut également être utilisée pour échantillonner les pondérations de portefeuille, dont les caractéristiques économiques et statistiques sont similaires aux parts de revenu.

La distribution de Dirichlet est la généralisation multivariée de la distribution bêta. Il a la fonction de densité de probabilité suivante:

$$f(w_1, w_2, ..., w_T; \alpha_1, \alpha_2, ..., \alpha_T) = \frac{\Gamma(\sum\limits_{i=1}^{T} \alpha_i)}{\prod\limits_{i=1}^{T} \Gamma(\alpha_i)} \prod_{i=1}^{T} w_i^{\alpha_i - 1}$$

où $\alpha_1, \alpha_2, ..., \alpha_T$ sont des paramètres de concentration et sont tous positifs. $\Gamma(\cdot)$ est la fonction Gamma telle que

$$\Gamma(\alpha_i) = \int_0^\infty x^{\alpha_i - 1} e^{-x} dx.$$

Pour la distribution de Dirichlet, $\Sigma_{i=1}^{T} w_i = 1$ et $0 \leqslant w_i$.

Notez que la somme régulière est inhérente à la distribution de Dirichlet, et la distribution uniforme est un cas particulier de la distribution de Dirichlet où tout $\alpha_i = 1$. De plus, les distributions marginales sont des distributions bêta et ne sont pas indépendantes. Une façon courante d'échantillonner les poids est de générer des nombres aléatoires positifs indépendamment, puis de les diviser par la somme de ces nombres aléatoires pour la normalisation. La distribution de Dirichlet est supérieure à une telle simulation de poids de portefeuille puisque les pondérations de portefeuille ne peuvent pas être indépendantes les unes des autres. L'autre façon courante d'échantillonner les poids est d'utiliser randfixedsum, qui produit des vecteurs de poids à partir de la distribution uniforme. Comme nous l'avons mentionné, randfixedsum n'est qu'un sous-ensemble de la distribution plus générale de Dirichlet. La propriété non négative de la distribution de Dirichlet est également conforme à la contrainte commune de la vente à découvert pour les actifs risqués dans l'équilibre du marché.

En somme, la distribution de Dirichlet nous équipe avec l'instrument pour approximer l'ensemble faisable et obtenir cette entrée essentielle pour la méthode d'amélioration de Pareto. Il favorise l'application de la méthode d'amélioration de Pareto dans le contexte de l'optimisation de portefeuille avec des moments d'ordre supérieur.

1.2 Dominance stochastique

Le cadre d'optimisation de la variance moyenne est fondamentalement paramétrique. Ce cadre se concentre sur la moyenne et la variance pour capturer des informations sur une distribution de probabilité. Cela fonctionne bien pour les retours distribués normalement, car dans ce cas la connaissance de ses deux premiers moments est suffisante pour encapsuler une distribution de probabilité.

Cependant, cela mène à un choix d'optimisation trompeur une fois que la vraie distribution dévie beaucoup de la normalité. Nous avons déjà discuté de l'inclusion de moments d'ordre supérieur pour mieux capturer l'information d'une distribution. C'est toujours une approche paramétrique générale. Alternativement, nous discutons maintenant l'utilisation d'une approche non paramétrique pour contourner la description incomplète d'une distribution de retour par un ensemble fini de moments.

L'approche non paramétrique alternative à l'optimisation de la variance moyenne basée sur le moment que nous utilisons est la dominance stochastique. Lancée par Mann and Whitney (1947) et Lehmann (1955), la dominance stochastique est apparue dans la théorie des probabilités et les statistiques comme un « ordre stochastique » pour comparer et ordonner deux variables aléatoires. Les fonctions de distribution cumulatives sont utilisées pour capturer l'ensemble des perspectives dans le processus de classement. Quirk and Saposnik (1962), Hadar and Russell (1969), Hanoch and Levy (1969) et Rothschild and Stiglitz (1970) introduisent et étendent l'application de la dominance stochastique dans un contexte économique pour comparer deux distributions de probabilité alternatives de la richesse future. Ces discussions sont basées sur les axiomes de rationalité de la théorie de l'utilité attendue. En effet, le premier ordre et la dominance stochastique du second ordre s'accompagnent respectivement de la présence de principes de non-satiété et d'aversion au risque.

Par la suite, l'approche de la dominance stochastique s'applique dans divers domaines à l'intérieur et au-delà de l'économie, comme en économie agricole pour trouver les meilleurs emplacements de plantation, et en médecine pour déterminer les traitements optimaux. Parce que le risque est le thème principal de la finance, la dominance stochastique trouve ses applications les plus importantes dans la recherche financière. Surtout dans le domaine de la sélection de portefeuilles, la dominance stochastique se développe pour analyser des stratégies de diversification efficaces à la recherche du portefeuille optimal. Par ce développement, la dominance stochastique va au-delà de la comparaison par paires pour la détermination de l'efficacité, ce qui élargit le champ d'application de l'approche de la dominance stochastique.

En conclusion, cette approche tire pleinement parti des informations de la distribution complète plutôt que des moments limités pour saisir le risque des perspectives d'investissement, et s'appuie sur les principes généraux de préférence des risques plutôt que sur une paramétrisation naïve d'une certaine spécification d'utilité. Il est également avantageusement plus conforme à la théorie de l'utilité attendue que le cadre d'optimisation de la variance moyenne et est plus cohérent sur le plan théorique. Néanmoins, pour son application plus large sur diverses classes d'utilité, sa réponse à la sélection optimale du portefeuille n'est pas aussi explicite que dans l'approche d'optimisation de la variance moyenne. Dans la première étape de la décision objective, il donne une identification claire des choix inefficaces pour tous les investisseurs dans la même classe d'utilité, indépendamment de leurs préférences spécifiques. Ainsi, l'ensemble inefficace peut être plus petit que celui du cadre d'optimisation de la variance moyenne. Il est donc moins probable que le portefeuille optimal mondial répondant le mieux aux préférences de l'investisseur soit éliminé. Par la suite, dans la deuxième étape de la décision subjective, la probabilité d'une fausse erreur négative pour le portefeuille optimal est minimisée. La dominance stochastique a de fortes implications sur l'optimisation des investissements avec l'hypothèse d'investisseurs hétérogènes, ce qui est beaucoup plus réaliste que son homologue homogène.

1.2.1 Dominance stochastique de premier ordre et de second ordre

Nous procédons maintenant à un examen rapide du premier ordre et de la dominance stochastique de second ordre pour montrer l'essence de l'approche de la dominance stochastique pour la sélection de portefeuille. Pour plus de détails, reportez-vous, par exemple, Kuosmanen (2004) et Levy (2016).

Soit G_x et G_y des fonctions de distribution cumulatives pour deux options d'investissement x et y, respectivement. Ensuite, Portfolio y domine Portfolio x par la dominance stochastique de premier ordre (FSD) si et seulement si la courbe de G_y n'est pas supérieure à G_x sur tout le domaine du niveau de retour r. Précisément,

$$y \succ^1 x \Leftrightarrow G_x(r) - G_y(r) \geq 0, \ \forall r \in \mathbb{R}.$$

De plus, Portfolio y domine Portfolio x par la dominance stochastique de second ordre (SSD) si et seulement si la zone en dessous de G_y n'est pas plus grande que la zone en dessous de G_x sur tout le domaine du niveau de retour r. Précisément,

$$y \succ^2 x \Leftrightarrow \int_{-\infty}^{r} [G_x(s) - G_y(s)]ds \geq 0, \ \forall r \in \mathbb{R}.$$

La dominance stochastique est congruente à la théorie de l'utilité attendue. Notons $\mathcal{U}_n$ la classe de fonction utilitaire qui inclut toutes les fonctions utilitaires u avec des dérivées impaires non négatives et des dérivées non positives jusqu'à l'ordre n, où n est un entier positif. Par exemple, $\mathcal{U}_1$ comprend toutes les préférences non-sonnées avec $u' \geq 0$, et $\mathcal{U}_2$ comprend toutes les préférences non-satisfaites ainsi que les aversion au risque avec $u' \geq 0$ plus $u'' \leq 0$. La dominance stochastique ne spécifie pas les fonctions d'utilité au-delà de ces hypothèses de préférence générale sur les dérivés d'utilité. Pour FSD, si $y \succ^1 x$, Portfolio a un utilitaire attendu plus élevé que Portfolio x pour toute spécification d'utilitaire de $\mathcal{U}_1$. C'est-à-dire,

$$y \succ^1 x \Leftrightarrow G_x(r) - G_y(r) \geq 0, \ \forall r \in \mathbb{R}.$$

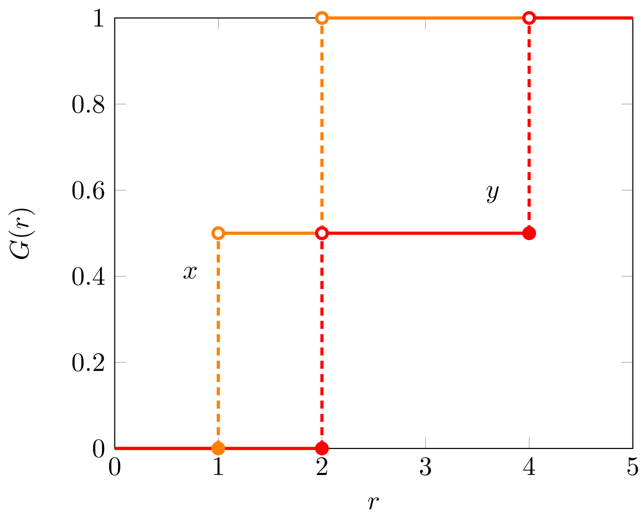
En conséquence, pour SSD, $y \succ^2 x$ signifie que tous les investisseurs ayant des fonctions d'utilité à partir de $\mathcal{U}_2$ choisiront Portfolio y sur Portfolio x. En d'autres termes,

$$y \succ^2 x \Leftrightarrow \int_{-\infty}^{r} [G_x(s) - G_y(s)]ds \geq 0, \ \forall r \in \mathbb{R}.$$

Un exemple simple dans Levy (2016, p. vii) présente le mécanisme de FSD. Supposons que Portfolio x a un rendement de 1 dans le mauvais état et 2 dans le bon état avec une probabilité égale, et Portfolio y double les rendements dans chaque état. Il est facile de savoir que Portfolio x a un rendement attendu de 1,5 et une variance de 0,25, et que Portfolio y a un rendement attendu de 3 et une variance de 1. Dans l'optimisation classique de la variance moyenne, aucun portefeuille inefficace n'est détecté. y a un rendement attendu plus élevé mais aussi une variance plus élevée. Cependant, Portfolio y a une supériorité évidente pour son rendement attrayant dans chaque état, bien que sa variance soit plus élevée. Cette supériorité est bien capturée par FSD, voir Figure 1.4. Puisque la distribution cumulative de Portfolio y est strictement inférieure à celle de son homologue, nous pouvons clairement obtenir $y \succ^1 x$. Par conséquent, tous les investisseurs qui préfèrent plus ou moins devraient choisir Portfolio y sur Portfolio x.

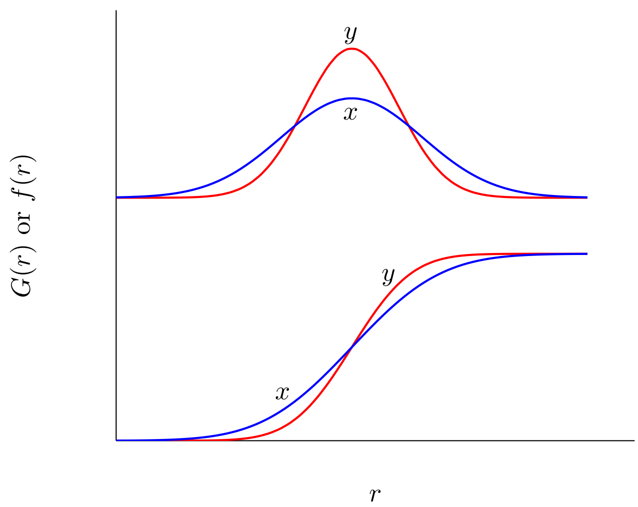
Nous illustrons ensuite SSD avec l'exemple de la Figure 1.5, où la distribution de retour de Portfolio x est un spread préservant moyen à celui de Portfolio y.

FIGURE 1.4: FSD pour le choix d'investissement



Supposons que les rendements des deux portefeuilles suivent les distributions normales avec la même moyenne et Portfolio x présente une variance plus importante. Comme nous le voyons, les di Nous illustrons ensuite SSD avec l'exemple de la Figure 1.5, où la distribution de retour de Portfolio x est un spread préservant moyen à celui de Portfolio y. Supposons que les rendements des deux portefeuilles suivent les distributions normales avec la même moyenne et Portfolio x présente une variance plus importante. Comme nous le voyons, les fonctions de distribution cumulative pour les deux portefeuilles se croisent, ce qui signifie qu'il n'existe aucune relation de FSD entre eux. Cependant, la zone en dessous de G_x est toujours plus grande que la zone en dessous de G_y. Par conséquent, la relation SSD existe avec $y \succ^2 x$, et tous les investisseurs averses au risque préfèrent le portefeuille y au portefeuille x. La preuve formelle d'équivalence entre les SSD et la propagation moyenne conservatrice peut être trouvée dans Rothschild and Stiglitz (1970).

FIGURE 1.5: SSD pour choix d'investissement



Comme indiqué précédemment, SSD correspond à la classe d'utilitaires de $\mathcal{U}_2$. Quand il s'agit d'une paire de portefeuilles avec la moyenne et la variance égaux, SSD est incompétent pour discerner un meilleur choix. Prenez l'exemple dans Table

1.4. Le portefeuille x et le portefeuille y ont le même rendement attendu de 1,5 et la même variance de 0,75. FSD et SSD ne sont pas détectés entre les deux portefeuilles. Cependant, Portfolio y a des rendements plus élevés dans chaque état que Portfolio x, et présente une plus grande asymétrie. Intuitivement, les investisseurs devraient choisir Portfolio y s'ils ont une préférence pour l'asymétrie, et Portfolio x devrait être classé comme inefficace. Cet exemple indique la nécessité d'étendre l'approche de dominance stochastique à un ordre supérieur, ce qui profite à la détermination d'ensemble efficace pour l'optimisation du portefeuille.

TABLE 1.4: Exemple de TSD dans Levy (1992, p. 93)

	x	Prob (x)	y	Prob (y)
Etat 1	0	0,25	1	0,75
Etat 2	2	0,75	3	0,25
Moyenne	1,5		1,5	
Variance	0,75		0,75	
Skewness	-0,75		0,75	

1.2.2 Dominance stochastique d'ordre supérieur

L'un des principaux avantages de l'approche de la dominance stochastique est l'intégration transparente de la première étape de la décision objective et de la deuxième étape de la décision subjective, qui minimise le risque d'exclure le portefeuille optimal global dans la première étape. Cet avantage se fait au prix d'un ensemble efficace relativement important ou d'un pouvoir moins discriminant. Un moyen efficace d'augmenter la compétence sélective de la première étape consiste à donner des hypothèses plus spécifiques pour les préférences. L'introduction d'une classe d'utilité restreinte facilite le processus de sélection des portefeuilles inefficaces.

Plus précisément, $\mathcal{U}_1$ inclut $\mathcal{U}_2$ et l'autre classe d'utilitaires avec $u' \geq 0$ plus $u'' \geq 0$. Notez que $y \succ^1 x$ signifie que Portfolio y est préféré par tout investisseur ayant une fonction d'utilité non rassasiée, peu importe s'il est averse au risque ou non. En revanche, $y \succ^2 x$ signifie que Portfolio y est préféré par tout investisseur ayant une fonction d'utilité non rassasiée et averse au risque. Entre les deux cas, la principale différence est que FSD doit également satisfaire les investisseurs aimant le risque. Pour s'assurer que Portfolio y est préférable pour eux, le rendement requis doit être supérieur au rendement pour les investisseurs averses au risque pour compenser leur désir de plus de risques. Comme dans la Figure 1.5, le portefeuille y est un moyen anti-propagation moyen pour Portfolio x, de sorte que tous les investisseurs averses au risque choisissent Portfolio y, mais les investisseurs aimant le risque préfèrent Portfolio x. Pour accroître l'attrait du Portefeuille pour ces investisseurs aimant le risque, le rendement attendu du Portefeuille y doit être plus élevé afin que les investisseurs averses au risque et ceux qui aiment le risque conviennent que Portfolio y est meilleur. Avec cette agrégation, Portfolio y est FSD over Portfolio x.

En d'autres termes, la classe d'utilité large $\mathcal{U}_1$ conduit inévitablement à un ensemble moins sélectif et efficace, car les portefeuilles inefficaces doivent être cohérents pour tous les investisseurs, peu importe qu'ils soient averses au risque ou à risque. Mais les investisseurs à la recherche de risques n'ont pas de compromis entre le risque et le rendement parce qu'ils préfèrent les deux. Cette caractéristique de préférence de risque n'est pas systématiquement stable selon les choix d'investissement de l'investisseur, ni cohérente entre les différents investisseurs sur le marché. Son omission dans l'optimisation de l'investissement n'altère pas matériellement l'intégrité de l'optimisation.

Ainsi, une extension de la dominance stochastique à l'ordre 3 et à l'ordre 4 est utile pour faciliter la détermination efficace de l'ensemble pour l'optimisation du portefeuille. Les classes d'utilitaires correspondantes sont $\mathcal{U}_3$ et $\mathcal{U}_4$. $\mathcal{U}_3$ inclut toutes les fonctions utilitaires u avec $u' \geq 0$, $u^{''} \leq 0$, et $u^{'''} \geq 0$. Ils sont ceux d'investisseurs non rassasiés, averses au risque et prudents. De même, $\mathcal{U}_4$ inclut toutes les fonctions utilitaires u avec $u' \geq 0$, $u^{''} \leq 0$, $u^{'''} \geq 0$, et $u^4 \leq 0$. Ils sont ceux d'investisseurs non rassasiés, averses au risque, prudents et tempérant. À mesure que la classe d'utilité devient plus limitée, l'ensemble inefficace devient plus grand et l'ensemble efficace comprend moins de portefeuilles pour accélérer la deuxième étape de la décision subjective. Cependant, une classe d'utilité surcontrainte est moins utile. La déduction de préférence doit être équilibrée par une explication économique, car la dérivée 100e d'une fonction d'utilité a une signification économique minimale.

Nous pouvons également confirmer ce point à partir de la définition de la dominance stochastique. Si $G_x(r) - G_y(r) \geq 0$, alors $\int_{-\infty}^{r}[G_x(s) - G_y(s)]ds \geq 0$. Donc, FSD implique SSD, mais pas textit vice versa. Il est possible qu'aucune relation FSD n'existe entre Portfolio x et Portfolio y, mais une relation SSD peut être détectée de sorte que $y \succ^2 x$ or $x \succ^2 y$. Cela modifiera la composition de l'ensemble efficace, car les deux portefeuilles sont dans l'ensemble efficace FSD mais un seul est dans l'ensemble efficace SSD.

1.2.3 Majorisation

Pour les DSE et les DSS, la détermination de la dominance est caractérisée intuitivement par une comparaison des hauteurs et des zones entre les deux fonctions de distribution cumulatives. Cependant, l'exposition n'est pas si simple pour le SD d'ordre supérieur, où les intégrales multiples apportent souvent une complexité de calcul.

Une manière pragmatique et pratique de surmonter ces difficultés consiste à établir un pont entre la théorie de la dominance stochastique et celle de la majorisation en utilisant la fonction de distribution empirique. Cette équivalence de SD et de théorisation de la majorité est une extension des arguments de Marshall and Olkin (1979) utilisés dans Levy (1992) et Kuosmanen (2004), montrant que la comparaison bidimensionnelle SD des distributions peut être réalisée à n'importe quel ordre en utilisant des sommes cumulatives. La dominance stochastique des fonctions de distribution empiriques à l'ordre n équivaut à la dominance dans le sens de la majorisation à l'ordre n, et va comme

$$y \succ^{[n]} x \quad \Leftrightarrow \quad \tilde{y}_t^{[n]} \geq \tilde{x}_t^{[n]} \text{ for all } t \leq T,$$

où $\tilde{x}^{[n]}$ est la somme cumulée du vecteur de retour ordonné de Portfolio x à l'ordre n, défini comme

$$\forall t \leq T \quad \tilde{x}_t^{[n]} = \sum_{j_{n-1}=1}^{t} \sum_{j_{n-2}=1}^{j_{n-1}} \cdots \sum_{j_1=1}^{j_2} \tilde{x}_{j_1}.$$

Les sommes cumulées correspondantes pour les quatre premiers ordres sont

$$\forall t \leq T \quad \tilde{x}_t^{[1]} = \tilde{x}_t,$$

et

$$\forall t \leq T \quad \tilde{x}_t^{[2]} = \sum_{j_1=1}^{t} \tilde{x}_{j_1},$$

et

$$\forall t \leq T \quad \tilde{x}_t^{[3]} = \sum_{j_2=1}^{t} \sum_{j_1=1}^{j_2} \tilde{x}_{j_1},$$

aussi bien que

$$\forall t \leq T \quad \tilde{x}_t^{[4]} = \sum_{j_3=1}^{t} \sum_{j_2=1}^{j_3} \sum_{j_1=1}^{j_2} \tilde{x}_{j_1}.$$

Considérons maintenant l'exemple simple de l'ensemble dominant pour un portefeuille avec deux observations de retour $(1,4)$. Nous voulons construire toutes les paires dominantes de retours (x_1, x_2) tels que $(x_1, x_2) \succ^{[n]} (1,4)$, pour $n = 1$ à 4. Pour simplifier, nous nous concentrons sur le cas $x_1 < x_2$ alors que le cas $x_1 > x_2$ est facilement obtenu par symétrie.

Avec les quatre équations de sommes cumulées mentionnées ci-dessus, nous voyons qu'un portefeuille (x_1, x_2) domine le portefeuille $(1,4)$ à la commande 1 dans le sens de la majoration lorsque $x_1 > 1$ et $x_2 > 4$ Alors, $(x_1, x_2) \succ^{[2]} (1,4)$ quand $x_1 > 1$ et $x_1 + x_2 > 1 + 4$, dont la bordure est un segment satisfaisant $x_2 = 5 - x_1$ du point $(1,4)$ à $x_2 = x_1$. En outre, $(x_1, x_2) \succ^{[3]} (1,4)$ lorsque $x_1 > 1$ et $x_1 + x_1 + x_2 > 1 + 1 + 4$, dont la bordure est un segment satisfaisant $x_2 = 6 - 2x_1$ du point $(1,4)$ à $x_2 = x_1$. Enfin, $(x_1, x_2) \succ^{[4]} (1,4)$ lorsque $x_1 > 1$ et $x_1 + x_1 + x_1 + x_2 > 1 + 1 + 1 + 4$, la bordure de qui est un segment satisfaisant $x_2 = 7 - 3x_1$ du point $(1,4)$ à $x_2 = x_1$.

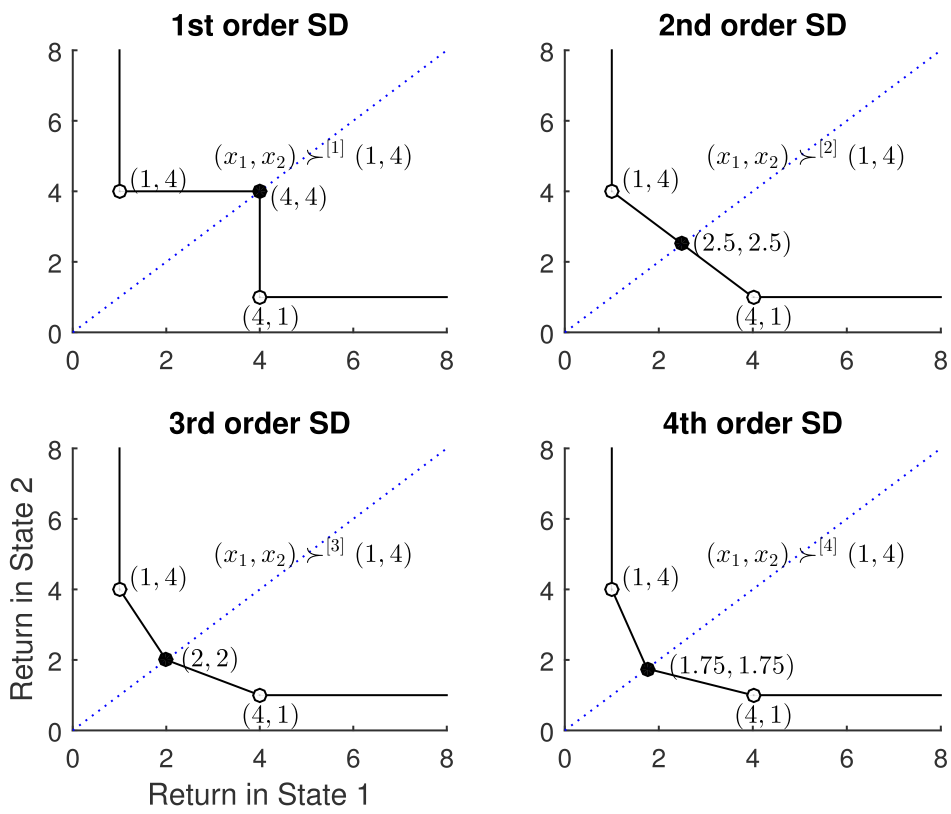
La Figure 1.6 offre un résumé de cette discussion. Les ensembles dominants de $(1,4)$ sont tous convexes sauf au premier ordre. La figure réaffirme une caractéristique de la dominance stochastique: les ensembles dominants augmentent par inclusion, et un ensemble dominant à un ordre élevé incorpore les ensembles dominants aux ordres inférieurs. Par exemple, si nous ne pouvons pas trouver de portefeuilles empiriques dans un ensemble dominant au second ordre, l'ensemble dominant au premier ordre est vide.

1.2.4 Diversification

La dominance stochastique classique est principalement un outil par comparaison par paires. Cependant, il existe de nombreuses possibilités d'investissement dans la finance, il est donc nécessaire d'effectuer des comparaisons larges et complexes au-delà des paires. Par exemple, si nous voulons connaître les investissements efficaces dans l'indice S&P 500, nous devons comparer directement chaque paire des 500 composants. [3] Cela signifie que nous obtenons $500 \times 499 = 249500$ paires. Bien qu'une dominance détectée pour une fois soit suffisante pour reléguer l'investissement dominé dans l'ensemble inefficace, les investissements complets

[3] En fait, il y a 505 actions dans l'indice, parce que 5 des 500 sociétés de composants ont plusieurs classes d'actions cotées. Il s'agit de Discovery Communications, de Google, de Comcast, de Twenty-First Century Fox et de News Corporation jusqu'en avril 2018.

FIGURE 1.6: Jeux dominants à l'ordre 1 à 4: une illustration



dans une seule composante ne représentent qu'une petite fraction de tous les investissements réalisables. En outre, l'ensemble efficace de l'investissement à un seul composant ne peut pas nécessairement garantir son efficacité générale sur l'ensemble réalisable.

Kuosmanen (2004) propose un test nécessaire pour l'efficacité du SSD, qui identifie comme optimal le portefeuille avec la plus grande amélioration du rendement moyen sur un portefeuille évalué, si ce dernier est détecté comme inefficace. Ce portefeuille optimal Kuosmanen SSD est obtenu en utilisant le programme suivant:

$$
\begin{aligned}
\max_{w,W} \quad & \Big(\sum_{t=1}^{T}\sum_{i=1}^{n} r_{it}w_i - \sum_{t=1}^{T} R_t\Big)/T \\
\text{sujet à} \quad & \sum_{i=1}^{n} r_{it}w_i \geq \sum_{j=1}^{T} W_{tj}R_t \\
& w \in \Lambda \\
& W \in \Xi
\end{aligned}
$$

où w est le vecteur de pondération et Λ l'ensemble des portefeuilles long uniquement, r_i est la série de rendements pour l'actif i et R la série pour le portefeuille évalué. W est une matrice doublement stochastique d'éléments non négatifs avec une somme unitaire pour chaque ligne ainsi que chaque colonne, et Ξ est l'ensemble des matrices doublement stochastiques. Par exemple,

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \text{ et } \begin{bmatrix}0.25 & 0.75\\0.75 & 0.25\end{bmatrix}$$

sont deux exemples de matrices doublement stochastiques de taille 2 ×2. Les matrices d'identité comme la première sont des cas particuliers de matrices doublement stochastiques.

Si la maximisation de l'objectif donne 0, le portefeuille évalué remplit la condition nécessaire pour l'efficacité du SSD. Si la maximisation donne un scalaire positif de δ, le portefeuille évalué est inefficace en SSD, et le portefeuille optimal du programme Kuosmanen a un excédent de retour moyen de δ sur le portefeuille évalué. Notez que le portefeuille évalué ne doit pas nécessairement faire partie des actifs risqués n ou une combinaison de ceux-ci, et la base de données de retour $r = [r_1\ r_2\ ...\ r_n]$ sur le marché peut être facilement ajustée pour augmenter le profil de rendement du portefeuille évalué comme $[R, r]$.

Nous développons une extension du test SSD Kuosmanen (2004) jusqu'au quatrième ordre pour améliorer l'optimisation des investissements de dominance stochastique. Le programme de portfolio optimal SD à la commande N est le suivant:

$$\begin{aligned} \max_{w,W} \quad & (\sum_{t=1}^{T}\sum_{i=1}^{n} r_{it}w_i - \sum_{t=1}^{T} R_t)/T \\ \text{sujet à} \quad & (rw)^{[N-1]} \geq WR^{[N-1]} \\ & w \in \Lambda \\ & W \in \Xi \end{aligned}$$

où $r = [r_1\ r_2\ ...\ r_n]$ est la base de données du marché.

1.3 Changement de nom et inefficacité du marché

Pour intégrer les risques de l'événement dans la construction du portefeuille, nous avons précédemment discuté de l'extension du cadre classique d'optimisation de la variance moyenne et du CAPM vers des moments d'ordre supérieur, ainsi que de l'extension de l'approche de dominance stochastique vers un ordre supérieur. Nous montrons que l'inclusion des risques d'événements dans la sélection du portefeuille améliore l'optimalité globale pour l'investissement. Nous passons maintenant les analyses des risques extrêmes au contexte de la finance d'entreprise. Certains événements d'entreprise affectent de manière significative la dynamique de rendement. Par exemple, l'effet de changement d'indice S&P 500 est abondamment documenté par diverses études. Les sociétés qui rejoignent l'indice S&P 500 connaissent une réaction positive significative aux prix suite à l'annonce d'un changement d'indice. Ce retour anormal implique des changements importants de caractéristiques économiques, par exemple, Barberis, Shleifer, and Wurgler (2005) remarquez le changement dans la comovement des entreprises ajoutées au marché. Par conséquent, la prise en compte de ces événements d'entreprise importants est également importante pour améliorer l'efficacité de l'allocation d'actifs.

Nous considérons un ensemble important d'événements d'entreprise: M&A et changements de nom d'entreprise. Pour accroître la comparabilité entre les sociétés de l'échantillon, nous concentrons notre couverture sur l'indice S&P 500. Après un M&A, la société combinée doit décider si elle doit conserver le nom de l'acquéreur, ou faire un changement et utiliser le nom de la cible, ou utiliser une combinaison de noms des deux sociétés ou introduire un nouveau nom. Le changement de nom contient des informations importantes sur les stratégies d'entreprise et les objectifs de gestion. Ainsi, lors de la révélation du changement de nom lors de l'annonce de M&A, les cours des actions devraient incorporer cette information rapidement dans des marchés efficaces. Comme il y a une période importante entre une annonce M&A et l'annonce de changement d'indice S&P 500 correspondante, nous nous attendons à ce que la réaction au changement de nom M&A et le changement de nom M&A soient indifférents. hypothèse de marché (EMH). Dans le cas contraire, une différence significative implique que les informations sur la valeur ne sont pas entièrement reflétées par le cours de l'action, un écart manifeste par rapport à l'efficacité du marché et une opportunité profitable d'améliorer les choix d'investissement.

Dans ce contexte, le changement de nom M&As et le changement de nom sans M&As impliquent tous les composants S&P 500, et la couverture médiatique sur les deux groupes est immédiate et généralement égale. Il n'y a donc pas d'asymétrie d'information substantielle entre eux, ce qui atténue la crainte que l'accessibilité de l'information des investisseurs n'aboutisse à des réactions différentes sur les prix. Notre objectif est d'étudier comment le changement de nom affecte la dynamique de rendement lors de l'annonce du changement d'indice S&P 500.

1.3.1 Méthodologie de l'étude d'événement

La méthodologie de l'étude paire est largement utilisée dans les études sur l'efficacité du marché, telles qu'elles ont été examinées dans Fama (1991). Il est décrit succinctement comme « l'analyse de l'existence d'une réaction statistiquement significative sur les marchés financiers à des occurrences passées d'un type donné d'événement supposé affecter la valeur marchande des entreprises ».

L'élément de base de cette analyse est la spécification d'un événement. L'événement d'entreprise doit être informatif et intuitif, comme un fractionnement d'actions, une annonce M&A et ainsi de suite. Comme l'événement-calendrier peut être différent d'un événement à l'autre, la première étape de la conception de l'étude d'événement consiste à organiser la chronologie de l'événement. La date de l'événement est définie sur le jour 0 et la fenêtre d'événement est déterminée autour de la date de l'événement afin d'examiner la réaction du marché au cours de cette période. La réaction du marché à l'événement est mesurée par rapport à un indice de référence, qui est l'indicateur de la série de rendements boursiers en l'absence d'un tel événement. Pour obtenir le retour attendu, une fenêtre d'estimation est spécifiée pour les paramètres d'ajustement du risque, et le retour réel sur le retour attendu pendant la fenêtre d'événement est la partie anormale, qui est considérée comme l'impact de l'événement. La signification du retour anormal est ensuite évaluée. Plus de détails peuvent être trouvés dans, par exemple, Kothari and Warner (2008) et Patel and Welch (2017).

Formellement, le rendement anormal du stock i à l'instant t, $AR_{i,t}$, est la différence entre le rendement réel $R_{i,t}$ et sa référence, le rendement attendu $E[R_{i,t}]$. Par conséquent, il est égal à

$$AR_{i,t} = R_{i,t} - E[R_{i,t}].$$

Le retour cumulatif anormal $CAR_{i,[t_1,t_2]}$ est la somme des rendements anormaux sur la fenêtre d'événements de $[t_1, t_2]$ pour stock i:

$$CAR_{i,[t_1,t_2]} = \sum_{t=t_1}^{t_2} AR_{i,t}.$$

Le retour anormal cumulé moyen $CAR_{[t_1,t_2]}$ est le rendement anormal moyen transversal sur l'échantillon d'événement, défini comme suit: Comme l'événement est couramment exposé à diverses actions du groupe G, le rendement anormal cumulé moyen est défini comme suit:

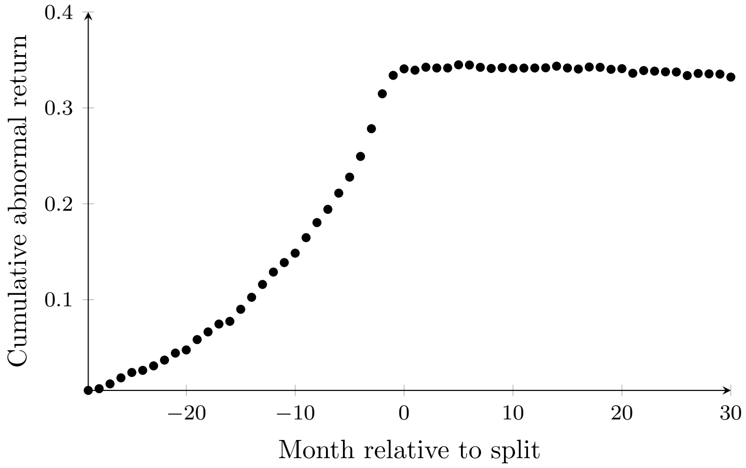
$$CAR_{[t_1,t_2]} = \frac{\sum CAR_{i,[t_1,t_2]}}{n},$$

où n est le nombre d'événements. La signification statistique associée est calculée avec le test standard t. Pour un résumé récent des méthodes d'étude des événements, voir Patel and Welch (2017).

Nous utilisons le travail pionnier de Fama, Fisher, Jensen, and Roll (1969) pour une exposition intuitive. Ces auteurs étudient comment les fractionnements boursiers affectent les ajustements de prix des actions, où les scissions avec des dividendes en actions de plus de 25% sont spécifiées en tant qu'événement. Ils collectent des stocks avec au moins deux années de données autour de la répartition de 1927 à 1959 pour un échantillon de 940 divisions. Le mois effectif divisé est défini comme étant le mois 0, et le rendement anormal cumulatif moyen est calculé de la même façon que le CAR susmentionné. La Figure 1.7 est le résultat principal de Fama, Fisher, Jensen, and Roll (1969). Il est évident que les rendements anormaux cumulés moyens augmentent significativement avant que les actions ne se divisent, mais pas après. Ils expliquent qu'un fractionnement d'actions est un signal positif pour les augmentations futures des dividendes, et le marché réagit efficacement à cette nouvelle information en la composant rapidement en prix de l'action au moins d'ici le mois fractionné.

FIGURE 1.7: Réaction de stock aux divisions

Cette figure correspond à Fama, Fisher, Jensen, and Roll (1969) Figure 2b, et les données cumulatives proviennent du Table 2 pour toutes les divisions.



Avec cette méthodologie d'étude d'événement, nous étudions les événements conjoints de M&A systématiques et les changements de nom.

1.3.2 L'indice S&P 500 et le portefeuille de marché

L'indice S&P 500 est le proxy indiciel le plus utilisé pour le portefeuille de marché, comme en témoigne sa prédominance dans les actifs indiciels. [4] Comme indiqué par les Indices S&P Dow Jones, 8,7 trillions d'actifs ont été comparés à l'indice S&P 500 à partir de 2016. Plus que cela, les ETF S&P 500 sont les plus importants en termes d'actifs sous gestion (AUM). Comme le montre le tableau 1.5, l'indice a une présence dominante dans la liste des plus gros ETF. Le ETF SPDR S&P 500, le ETF iShares Core S&P 500 et le ETF Vanguard S&P 500 ont un total d'environ 500 millions de dollars, tandis que le ETF Vanguard Total Stock Market et le ETF iShares MSCI EAFE ont beaucoup taille plus petite d'environ 90 millions de dollars. L'indice S&P 500 est l'indicateur de portefeuille de marché le plus approprié. Sa couverture sectorielle est beaucoup plus large que celle de l'indice Dow Jones Industrial Average, et elle est également plus flexible que l'indice CRSP US Total Market.

TABLE 1.5: Top 5 des ETF par actifs

1	SPY	SPDR S&P 500 ETF	$255,857,367.99	119,169,672
2	IVV	iShares Core S&P 500 ETF	$143,811,920.65	5,473,141
3	VTI	Vanguard Total Stock Market ETF	$94,019,144.07	3,307,988
4	VOO	Vanguard S&P 500 ETF	$87,959,472.81	3,562,306
5	EFA	iShares MSCI EAFE ETF	$78,349,083.63	29,966,174

Les données proviennent de la liste complète des « plus gros ETF: les 100 meilleurs ETF par actif » sur ETFdb.com, consulté le 30 mars 2018.

Prendre cet indice comme portefeuille de marché facilite notre analyse de l'impact des événements corporatifs importants. Tout d'abord, comme l'indice S&P 500 est l'indice de portefeuille de marché le plus suivi, toute composition change et les événements de composants sont immédiatement signalés par les médias tels que *Le Wall Street Journal*, *Le Washington Post* et *Le New York Times*. Divers acteurs du marché, tels que les gestionnaires de placements actifs et les arbitragistes, surveillent également de près les ajustements de l'indice pour leurs décisions d'investissement. Une exposition suffisante à l'information garantit que les informations sur la valeur des événements d'entreprise sont évidentes pour les investisseurs et devrait stimuler une réaction complète des prix à l'arrivée des informations dans le cadre de l'EMH. Deuxièmement, les M&A parmi l'indice S&P 500 sont systématiques car les événements modifient sensiblement la composition du portefeuille de marché et la corrélation au sein de l'industrie et entre les industries. Notez que seules les fusions entre composants provoquent un tel rééquilibrage de portefeuille de marché influent. Les fusions d'un acquéreur S&P 500 et d'une cible non S&P 500 ne peuvent guère le faire, comme on peut facilement imaginer le grand écart de marché pour la fusion systématique de Google et Motorola, et la fusion non systématique de Google et Songza. La fusion d'une cible S&P 500 et d'un acquéreur non S&P 500, bien que non impossible, est assez improbable et rare.

En bref, nous nous concentrons sur les M&A parmi l'indice S&P 500 pour assembler un échantillon de taille décente. Ces M&A sont systématiquement influents pour les ajustements du portefeuille de marché.

[4] Les indices S&P Dow Jones définissent que les actifs indiciels représentent des actifs dans des fonds institutionnels, des ETFs, des fonds communs de placement de détail et d'autres produits investissables qui cherchent à reproduire ou à égaler le rendement de l'indice respectif.

1.3.3 M&A systématiques et changement de nom

Nous désignons les M&A parmi les composants S&P 500 comme M&A systématiques. Notez qu'après la consolidation d'une entreprise, la société combinée doit désormais décider de l'identité de l'entreprise. Cette décision est très significative car tant l'acquéreur que la cible sont des leaders de l'industrie bien connus et des acteurs importants du marché, avec leurs marques de valeur, de renommée et d'histoire. Comme expliqué dans le rapport d'activité de The Economist (2014),

> « Aucun expert en gestion ne trouverait étrange que l'Impériale dépense la meilleure partie de $ 7 milliards sur quelque chose d'aussi éthéré que les marques. Ils sont la chose la plus précieuse que des entreprises aussi diverses que Apple et McDonald's, valent souvent beaucoup plus que des biens et des machines. Les marques représentent plus de 30% de la valeur boursière des sociétés de l'indice S&P 500. »

A cet égard, le choix du nom de l'entreprise combinée est d'un grand intérêt dans le cas d'un M&A systématique. La société peut conserver le nom de l'acquéreur, comme dans l'acquisition de Wyeth par Pfizer; il peut aussi changer son nom pour le nom de la cible, comme dans l'acquisition de AT&T par SBC; il peut faire une combinaison de noms, comme dans l'acquisition de Mobil par Exxon pour ExxonMobil, et dans l'acquisition de JP Morgan par JP Chase pour Chase Manhattan. Enfin, il peut aussi prendre un nouveau nom, comme dans l'acquisition de Gulf Oil par Chevron pour Standard Oil of California. Par rapport aux événements sans changement de nom, les événements de changement de nom sont un signal sur la réorientation de l'entreprise, la stratégie d'entreprise et l'attitude de la direction.

L'acquéreur systématique sur un M&A est enclin à garder sa propre identité, en particulier compte tenu des énormes coûts dissuasifs, directs et indirects, associés au changement de nom. Notez que le signal d'information de valeur d'un changement de nom est différent de celui d'un changement de nom dans des environnements d'entreprise sans prises de contrôle ou restructurations substantielles. Des études antérieures comme Cooper, Dimitrov, and Rau (2001) révèlent un effet significatif de changement de nom lors de la bulle Internet. À l'annonce de l'ajout de « dotcom » à son nom, l'entreprise qui change de nom reçoit une augmentation de valeur. Ce qui différencie un changement de nom pour un M&A systématique et un changement de nom dans un environnement commun est la caractéristique systématique. Dans le premier cas, au moins deux composantes du portefeuille de marché sont directement impliquées, alors que dans la dernière situation, au plus une composante est pertinente et les risques idiosyncratiques sont plus concernés.

La fusion de Westinghouse et de CBS montre comment le changement de nom marque la stratégie de l'entreprise. La composante S&P 500 Westinghouse a acquis son homologue CBS en 1995, puis a poursuivi sa transformation du conglomérat industriel à la force des médias. En 1996, il a procédé à la scission de l'activité industrielle, et s'est renommé CBS en 1997. Considérez, si Westinghouse a changé son nom à CBS sur cette fusion systématique plutôt que sur les retombées commerciales? L'ancien changement de nom hypothétique contient des informations prédictives précieuses, tandis que le dernier changement de nom n'est qu'une correction d'identité pour représenter son activité actuelle. L'impact sur le marché d'un changement de nom en 1995 serait plus important car il correspond à la taille de Westinghouse (conglomérat industriel) plus CBS (géant des médias), tandis que l'impact d'un changement de nom en 1997 implication unique des médias.

Les informations sur la valeur des changements de nom sur les M&A systématiques doivent être distinguées du reste des événements d'entreprise, en particulier des changements de nom à l'heure commune.

1.3.4 Rumination de l'information

L'EMH prédit que les prix reflètent pleinement toutes les informations disponibles. Comme les informations de changement de nom font partie de l'accord M&A entre deux sociétés, cette information est publique lors de l'annonce M&A. Par conséquent, nous prévoyons que les cours boursiers de l'acquéreur et de la cible s'ajusteront rapidement pour inclure cette information sur la valeur lors de l'annonce de l'opération. Lorsqu'un M&A systématique est finalisé, ce qui correspond généralement à l'annonce de changement d'indice S&P 500, il ne devrait pas y avoir de différences significatives de retour entre le groupe de changement de nom et le groupe de changement de nom, car l'information de valeur a déjà été incorporée dans le prix de l'action.

L'annonce M&A agit comme un filtre d'information, tandis que le retour anormal à l'annonce du changement d'indice S&P 500 est une mesure appropriée de l'efficacité du marché. Si le marché est efficace, la différence de rendement anormale entre les deux groupes à l'annonce de changement d'indice devrait être insignifiante. Cependant, si une différence significative est détectée, l'efficacité du marché est remise en question pour l'assimilation des informations de valeur de changement de nom. La différence de rendement implique également que le prix ne reflète que partiellement les informations disponibles pour le changement de nom, et l'annonce de changement d'indice déclenche une rumination de l'information qui entraîne une divergence de performance entre les deux groupes.

Le choix de l'annonce de changement d'indice en tant qu'événement d'étude atténue plusieurs préoccupations importantes comme l'effet M&A et l'effet de changement d'indice S &P 500, car ces effets sont principalement liés à l'annonce M&A. La première préoccupation est l'effet M&A, pour lequel des études antérieures comme Betton, Eckbo, and Thorburn (2008) indiquent que les rachats d'entreprises profitent aux actionnaires cibles et créent de la valeur pour les actionnaires généraux du soumissionnaire et la cible lors de l'annonce M&A. Ainsi, nous ne la considérons pas comme une contamination sérieuse de l'information à l'annonce de changement d'indice. La deuxième préoccupation concerne l'effet de changement d'indice S&P 500, pour lequel une littérature antérieure comme Lynch and Mendenhall (1997) rapporte des rendements anormaux positifs significatifs pour des ajouts d'index et des rendements anormaux négatifs pour des suppressions d'index lors de l'annonce de changement d'indice. Comme les M&A systématiques figurent parmi les composantes de l'indice S&P 500, la société combinée est toujours une composante et le stock cible sera échangé ou acheté à des conditions prédéfinies, puis radié, l'effet d'addition et l'effet de suppression ne sont pas défi conséquent. En définissant l'annonce de changement d'indice en tant qu'événement, nous supprimons avec succès les effets de confusion du M&A et du changement d'index S &P 500. Cette isolation aide à se concentrer sur les informations de changement de nom, qui sont propres au groupe de changement de nom. La comparaison des performances entre le groupe de changement de nom et le groupe de non-changement de nom reflète principalement cet effet.

Puis nous arrivons au mécanisme qui explique cette rumination de l'information. La rumination se réfère au fait de la réaction de stock significative à l'information périmée. Plus précisément, lors d'une annonce M&A systématique, les informations de changement de nom sont publiques mais éclipsées par d'autres types d'informations M&A plus courants, tels que la taille des transactions, les transactions, les industries impliquées, etc. Le changement de nom n'est pas une information nécessaire pour tous les M&A systématiques, donc l'attention de l'investisseur est moins attribuée à cette information de changement de nom spécifique que la ressource d'attention est limitée. Peng and Xiong (2006) documente que l'inattention des investisseurs extrait un comportement d'apprentissage par catégorie, dans lequel les informations sur le marché et l'ensemble du secteur sont traitées en priorité par rapport aux informations spécifiques à l'entreprise. Au fur et à mesure que les investisseurs assimilent les informations M&A plus générales sur l'annonce de l'opération, l'annonce de changement d'indice déclenche la rumination sur les informations de changement de nom. Cette rumination est en partie provoquée par le changement de ticker. Notez qu'un changement de nom provoque un changement de ticker, ce qui est crucial pour le trading de titres car le ticker est l'identité de stock primordiale pour les activités d'investissement. Par exemple, dans le changement de nom systématique M&A entre SBC et AT&T, l'acquéreur SBC a changé son nom en AT&T, ainsi que son ticker « SB » au classique « T » détenu par AT&T. Dans le cas de combinaison de noms, le ticker de Dow Chemical est passé de « DOW » à « DWDP » après son acquisition de Du Pont pour former DowDuPont en 2017. En résumé, les transactions M&A systématiques de changement de nom entraînent souvent des modifications d'identité. achèvement, incitant les investisseurs à reconnaître pleinement les informations périmées de changement de nom, ce qui entraîne l'effet de rumination. Cet effet de rumination entraîne le retour anormal du changement de nom M&As lors de l'annonce de changement d'indice.

1.4 Structure de la thèse

Cette thèse se déploie sur les trois aspects pour améliorer l'optimalité de l'investissement global en présence de risques extrêmes, tant dans le contexte de la sélection de portefeuilles que de la finance d'entreprise. Le Chapter **??** traite de l'extension des moments d'ordre supérieur de l'optimisation de la variance moyenne traditionnelle et du cadre CAPM. Nous considérons le choix d'investissement d'un investisseur avec une extrême aversion au risque, comme lors de la crise financière mondiale et de la crise de la dette souveraine européenne. Nous construisons une frontière efficace de portefeuille de variance moyenne avec la méthode d'amélioration de Pareto, validée par la simulation de Dirichlet pour approximer l'ensemble réalisable. La détermination de l'ensemble efficace de variance moyenne utilise la relation entre l'ensemble efficace de variance moyenne et l'ensemble efficace de variance moyenne efficace, pour lequel un portefeuille efficace de variance moyenne doit être un portefeuille efficace de variance moyenne mais pas textit vice versa. Nous proposons une étude empirique de l'indice S&P 500 pour mettre en œuvre l'optimisation moyenne de la variance de la kurtosis, où les sous-indices de l'industrie sont utilisés comme actifs individuels. Ce chapitre est basé sur l'article de Le Courtois and Xu (2017b), « Portfolio optimization when bond markets can crash ».

Chapter **??** étend le modèle de Kuosmanen (2004) et développe une approche opérationnelle pour tester l'efficacité de la dominance stochastique d'un portefeuille donné à n'importe quel ordre. En appliquant cette approche aux indices boursiers représentant dix-sept marchés développés et en développement à travers le monde, nous constatons que tous ces indices sont inefficaces, presque toujours à la troisième et très souvent à la deuxième, ce qui implique que toutes les prudentes et la plupart les investisseurs feraient mieux de ne pas investir dans ces indices de marché. Les indices sont souvent dominés par des sous-indices individuels de l'industrie, les biens de consommation, les services et les services publics étant particulièrement performants. Une règle de négociation simple basée sur des informations de dominance stochastique passées améliore le rendement hors échantillon moyen d'un portefeuille global de 2% par an tout en réduisant simultanément l'écart type de rendement de 3% par an. Il limite considérablement les pertes mondiales de portefeuille durant les crises financières de 2007-2008. Les portefeuilles de titres à faible volatilité et à faible volatilité dominent de manière stochastique les indices du marché et semblent être des alternatives plus souhaitables pour les investisseurs prudents et averses au risque. Ce chapitre est basé sur l'article de Kolokolova, Le Courtois, and Xu (2017), « Is it efficient to buy the index? A worldwide tour with stochastic dominance ».

Le rôle des indices boursiers est un fil conducteur inhérent à l'ensemble de la thèse, parcourant les chapitres et étayant la discussion de l'allocation d'actifs en présence de risques extrêmes. Pour simplifier, dans les deux premiers chapitres, nous nous concentrons sur l'optimisation du portefeuille au niveau des secteurs indiciels. Contrairement à l'indice S &P 500 ou à l'indice FTSE 100 avec des centaines de composants, le Dow Jones Industrial Average (DJIA) ne comprend que 30 composants et sa taille compacte facilite l'analyse de l'efficacité des investissements au niveau des composants. Le Chapter **??** se concentre sur la discussion de l'efficacité de DJIA. L'essor remarquable de DJIA en 2017 amène à critiquer largement le fait que le DJIA présente des inconvénients méthodologiques liés à la couverture limitée des entreprises et à la méthodologie des prix pondérés par rapport à son homologue de l'indice S&P 500, par exemple, Armstrong (2017) et Rosenbaum (2017). Pour examiner son efficacité en tant que benchmark de performance, nous construisons diverses variantes DJIA de pondération à partir des données de rendement total des composants entre 1988 et 2017. Nous trouvons que la variante pondérée par les prix surperforme la variante pondérée par la valeur marchande et le rendement total DJIA a le ratio Sharpe le plus élevé. après ajustement du coût du chiffre d'affaires. La comparaison par paire de dominance stochastique confirme l'efficacité de DJIA. Nous effectuons en outre une analyse optimale du portefeuille basée sur l'optimisation de la variance moyenne et l'optimisation de la dominance stochastique, en trouvant que l'optimalité dans l'échantillon n'est pas prédictive. Comparé au DJIA, l'augmentation du rendement du portefeuille et la réduction du risque au cours d'une année donnée n'ont pas de retour ou d'avantage de performance en termes de risque l'année suivante, ce qui renforce l'efficacité de DJIA. Nous montrons que le DJIA n'est pas seulement un premier indice dynamique conforme à la définition d'index moderne proposée par Lo (2016) mais aussi un indice de performance performant. Nos résultats remettent en question la croyance commune selon laquelle le DJIA est mal construit en tant qu'indice pondéré par les prix. Ce chapitre est basé sur l'article de Le Courtois and Xu (2018), « DJIA is more efficient than thought ».

Le Chapter **??** examine l'effet du changement de nom d'entreprise sur l'échantillon de M&A parmi les sociétés S&P 500 de 1979 à 2017. Cet échantillon systématique synthétise le développement des activités agrégées M&A et se distingue des autres activités non systématiques. traite des caractéristiques clés comme la taille économique énorme et la distribution symétrique de l'industrie. Le choix d'événement de l'annonce de changement d'indice S&P 500 aide à isoler l'effet de rumination de changement de nom d'autres effets de valeur confusionnels comme l'effet d'annonce M&A, l'effet de changement d'indice S&P 500 et l'effet de pression de prix. sont plus liés à l'annonce de l'accord M&A. Nous constatons que le changement de nom stimule des rendements anormaux économiquement et statistiquement significatifs pour les acquéreurs lors des annonces de changement d'indice. Le rendement anormal à court terme annualisé en une semaine est d'environ 60%, et le rendement anormal d'achat et de détention d'un an chez les pairs non nominatifs est d'environ 10%. Les résultats corroborent empiriquement notre hypothèse selon laquelle l'annonce du changement d'indice déclenche la rumination des investisseurs sur les informations de changement de nom. Ce chapitre est basé sur l'article de Le Courtois and Xu (2017a), « Corporate name changes of M&As among S&P 500 index ».

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Four Essays on Capital Markets and Asset Allocation

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Chapter 1

General introduction

Risk is the preeminent theme of modern finance theory. As Merton (1992, p. 1) summarizes, “the core of the theory is the study of how best to allocate and deploy resources across time in an uncertain environment and of the role of economic organizations in facilitating these allocations.” Without risk, time can only make a difference by no more than discounting in the asset allocation.

In finance, portfolio selection is the primary context to study how risk impacts the investor’s allocation choice. For risky assets in the financial markets, the investors do not know precisely how these assets will behave during the investment period. Based on available return information about the assets at hand, the investors have to decide on the proportions of their personal wealth to invest in each asset. The optimal investment decision should best serve their goal, principally the maximization of their expected utility functions.

Levy (2016, p. 44) identifies two steps to arrive at the optimal investment. The first step is an objective decision selecting efficient portfolios regardless of individual preference. By dividing the feasible portfolio set into an efficient portfolio set and an inefficient portfolio set, all rational investors agree on the exclusion of inefficient portfolios for further evaluation and this is free from individual preferences. The second step is a subjective decision to best serve their specific preferences. Inside the efficient set, the investor picks up as the optimal portfolio the portfolio maximizing his or her expected utility function. Since risk preference is quite personal, the optimal portfolios may be different between investors, but these portfolios are necessarily chosen from the efficient portfolio set conducted at the first step.

The two steps are not distinctively independent. Rather, they are intrinsically connected in an explicit or implicit manner. Take the mean variance optimization as an example. Regarding variance as a risk index, Markowitz (1952) develops the efficient frontier, which is the collection of portfolios with minimum risk at a given expected return. This frontier corresponds to the first step of the objective decision, where any portfolio below the frontier falls into the inefficient set. Depending on his or her particular individual risk attitude, an investor may choose any portfolio on the efficient frontier. If he or she is very risk averse, the minimum variance portfolio will be the optimal investment; if he or she happens to be risk neutral, the final choice will be the maximum return portfolio. In practice, intermediate choices between these two situations are made. This preference selection corresponds to the second step of the subjective decision. Note that usually assuming normally distributed returns or/and a quadratic utility function, the two steps are integrated in the sense that only the first two moments count in the investment optimization. Plainly, if the investor cares about more than the first two moments, then the mean variance efficient frontier gives a misleading efficient set from the first step on. Because only the mean and the variance are considered in the efficient frontier determination and the feasible portfolio set is partially evaluated, the global optimal portfolio can get filtered out of the efficient set at the first step. Thus, the portfolio from the flawed efficient set maximizing the investor's utility function in the second step is only locally optimal. In short, the two steps of efficient frontier construction and utility maximization are underpinned by the same decision considerations like return moments, and any asymmetry between them creates mistaken optimality.

Normal distribution is not a perfect description for stock returns. In the Fama French Forum,[1] the two professors respond to the question of "Are stock returns normally distributed?" with the following answer:

> "Distributions of daily and monthly stock returns are rather symmetric about their means, but the tails are fatter (i.e., there are more outliers) than would be expected with normal distributions. (This topic takes up half of Gene's 1964 PhD thesis.) In the old literature on this issue, the popular alternatives to the normal distributions were non-normal symmetric stable distributions (which are fat-tailed relative to the normal) and t-distributions with low degrees of freedom (which are also fat-tailed). The message for investors is: expect extreme returns, negative as well as positive."

Consistently, investors would and should care about higher moments beyond the mean and the variance, among which skewness and kurtosis are the most predominant for capturing the risk of extreme returns with economic interpretations. Levy (1992) proposes a perfect example on this point. Suppose there are two portfolios, x and y and their mean variance profiles are stated in Table 1.1. Since Portfolio y has a higher mean and a lower variance, undoubtedly it is superior to Portfolio x in the mean variance optimization. In the first step of the objective decision, Portfolio x will be classified as inefficient and has no chance to enter the investor's preference examination in the second step. Assume this investor has a logarithmic utility function $u(W) = ln(W)$ where W is the final wealth. Then, the expected utility for Portfolio x, $E[u(x)]$, is 2.3, while $E[u(y)]$ is 0.9. The surplus of $E[u(x)]$ over $E[u(y)]$ contradicts the mean variance efficient portfolio classification, implying that mean variance optimization leads to a suboptimal choice. If we further look at the skewness for both portfolios, we get 9,413,841 for Portfolio x and 93,149 for Portfolio y.[2] This huge difference legitimizes the investor's choice of Portfolio x over Portfolio y under a logarithmic utility specification.

Extreme events, usually not frequent, have a substantial impact on the return distribution and on the return-risk tradeoff. Therefore, optimization should account for such risks. As we see from the example in Table 1.1, the huge variance for Portfolio x covers its prospect of the extreme positive return of 1,000. The value of higher order moments to portfolio optimization is that they better capture the event risks. Due to the variance incompetence in specifying the impact of extreme events, valuable information about event risks is ignored in the traditional mean variance approach for the portfolio selection.

In short, the previous example highlights the benefit to include extreme returns into the investment optimization process, particularly to improve the first step of efficiency classification. Higher order moments capture the impact of these extreme

[1] This forum is a section to share observations and ideas from Eugene Fama and Kenneth French in Dimensional Fund Advisors, the investment firm they are involved in.

[2] When the skewness and the kurtosis go with the variance in the context, usually they refer to the third moment and the fourth moment for convenience. This denotation is commonly used in papers of portfolio optimization with higher order moments. For example, in Briec, Kerstens, and Jokung (2007, p. 138) the skewness is defined as the third moment explicitly.

TABLE 1.1: Risk measure example in Levy (1992, p. 567)

	x	Prob (x)	y	Prob (y)
State 1	10	0.99	1	0.8
State 2	1000	0.01	100	0.2
Mean	19.9		20.8	
Variance	9703		1568	

Note that we correct the initial variance error of 1468 for Portfolio y in Levy (1992).

returns. Moreover, higher order moments help to make a distinction between asset return characteristics of stocks and bonds. Bonds are commonly regarded as a special kind of stocks with low returns and low volatilities. However, a key return feature for the bonds is a combination of low variance and high kurtosis, which is not the case for the stocks. Table 1.2 gives descriptive statistics for the annual returns on the S&P 500 index, 3-month T-Bills and 10-year T-Bonds from 1928 to 2017. As expected, the mean returns and volatilities of the short term bills and long term bonds are low, but the skewness and the kurtosis are much higher than for the stocks. Specifically, the kurtosis gives a risk aspect different from the volatility, as we can see the sorting of the volatility and the kurtosis is reversed for the bonds and the stocks. These statistics give two implications. First, the inclusion of higher order moments, especially the kurtosis, helps to depict a complete return risk profile. Second, the separation of stock assets and bond assets is necessary in the portfolio construction. In this sense, the portfolio of stocks is first determined and then combined with the investment of bonds to constitute the global optimal portfolio.

TABLE 1.2: Different return characteristics between the stocks and the bonds

	S&P 500	3M T-Bill	10Y T-Bond
Mean	11.5%	3.4%	5.2%
Volatility	19.6%	3.1%	7.7%
Skewness	-42.1%	102.0%	100.8%
Kurtosis	11.5%	99.1%	169.5%

The statistics are calculated with annual returns on the stock market, the 3-month Treasury bills and the 10-year Treasury bonds since 1928 to 2017. The return data are from Aswath Damodaran's dataset of ''Historical Returns on Stocks, Bonds and Bills - United States.''

The primary topic in this thesis is that we try to incorporate event risks (understood in the broad sense) into the asset allocation process. Specifically, we develop two approaches for investment improvement that account for event risks, where the higher order information explicitly enter the portfolio construction problems. One approach is a progressive improvement within the classic framework of mean variance optimization and CAPM, extending this framework to higher order moments. The other approach is with the alternative framework of stochastic dominance, for which we examine investment efficiency in the sense of higher order stochastic dominance. With these adaptions, the global optimality is enhanced for portfolio selection in the presence of event risks. Beyond the context of portfolio selection, we also study event risks (now understood in the classic sense) in the context of corporate finance. We observe corporate name changes during mergers and acquisitions (M&As) and we examine how the events affect the return dynamics for the acquirer and the target. This provides for a panorama outlook of the impact of event risks in finance, for which financial markets and corporate finance are two intrinsically complementary spheres.

The studies on CAPM extension to higher order moments, higher order stochastic dominance optimization, and corporate name changes during M&As are related via a consistent focus on the market portfolio. The market portfolio plays a key role in the CAPM as a general market equilibrium model developing from the mean variance optimization as an individual optimization analysis. The market portfolio is used in every asset pricing models regarding market risk. As the actual market portfolio is too elusive, and for practical feasibility, stock market indices are used as proxies.

For the examination of event risks in this thesis, we implement the analyses on the stock indices at different diversification levels of industry sectors and individual components. Specifically, in the two higher order analyses we use the industry sectors as assets and the portfolio consists of investment weights in these sectors. The optimal allocation then has implications on the benefit of diversification, which has been a golden rule in finance theory since the mean variance optimization. We also set the name change effect of M&As at the component level among the S&P 500 index, the most important stock index in the index investment profession. Generally, the analyses in this thesis give strong inferences on efficiency of the stock indices examined.

1.1 Mean variance optimization and CAPM

The classic framework of mean variance optimization and CAPM is fundamental to finance theory, and a major part of this thesis is an extension to this framework as well as an effort to remedy its inability to deal with event risks. So, it is beneficial to have a quick review on it.

Suppose there are n risky assets with regular return and variance profiles in a perfect market, with $\tilde{r}_i$ being the single period return for asset $i = 1, 2, \dots, n$. Then the return vector for these risky assets is $\tilde{R} = [\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_n]'$, and the associated expected return vector is $\mu = E[\tilde{R}]$, the covariance matrix is $\Sigma = E[(\tilde{R} - \mu)(\tilde{R} - \mu)']$. Then, for a given expected return level μ_P, the desired portfolio should have minimum variance, which is formulated as

$$\begin{aligned} \min_{w} \quad & \frac{1}{2} w' \Sigma w \\ \text{subject to} \quad & w' \mu = \mu_P \\ & w' \mathbf{1} = 1 \end{aligned}$$

where w is the portfolio weight vector of length n, and $\mathbf{1}$ is a vector of ones of the same length.

Solving the quadratic programming problem, we have a hyperbola in the expected return and volatility plane. The vertex of the hyperbola is the minimum variance portfolio, and the part above the minimum variance portfolio is the efficient frontier. Along this curve, portfolios have minimum volatilities at given expected returns, or equivalently, have maximum expected returns at given volatilities. Detailed exhibitions can be found, for instance, in Huang and Litzenberger (1988, Chapter 3) or in Back (2010, Chapter 5).

Based on the mean variance efficient frontier, Sharpe (1964) develops the CAPM by going from micro optimization behaviors to general market equilibriums. With the principal assumptions of homogenous investors and a perfect riskfree asset market, all investors have the same portfolio choice for risky assets, which is the portfolio along the efficient frontier maximizing the Sharpe ratio. Differences only exist in the proportions invested in the riskfree asset and in this risky portfolio. Market equilibrium implies that the risky portfolio must be the market portfolio. From this intuition comes the CAPM formula, whose straightforward derivation, originally from Sharpe (1964), is below.

Suppose a portfolio P is composed of the market portfolio M and an asset i, while a share of α is invested in the market portfolio M and the rest in the asset i, with $\alpha \in [0,1]$. The portfolio return is a linear combination of the two components as $\tilde{r}_P = \alpha\tilde{r}_M + (1-\alpha)\tilde{r}_i$. The curve of (μ_P, σ_P) with respect to α touches the capital market line (CML) at (μ_M, σ_M) where a full investment is made in the market portfolio and $\alpha = 1$. In this case, the derivative of the curve at the tangency point is equal to the CML slope $\frac{\mu_M - r_f}{\sigma_M}$, as shown in Figure 1.1. Therefore,

$$\left.\frac{\partial \mu_P}{\partial \sigma_P}\right|_{\alpha=1} = \frac{\mu_M - r_f}{\sigma_M}.$$

After some manipulations, we obtain

$$\frac{\mu_i - \mu_M}{(\sigma_{iM} - \sigma_M^2/\sigma_M)} = \frac{\mu_M - r_f}{\sigma_M},$$

which readily yields

$$\mu_i = r_f + \beta_i(\mu_i - r_f),$$

where $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$. This is the common CAPM formula, which relates the expected return of an asset to its exposure to market risk. If the asset has a higher exposure in the market risk, its expected return is higher. This formula indicates a linear pricing rule, according to which the expected excess return for the asset is positively proportional to its market risk exposure.

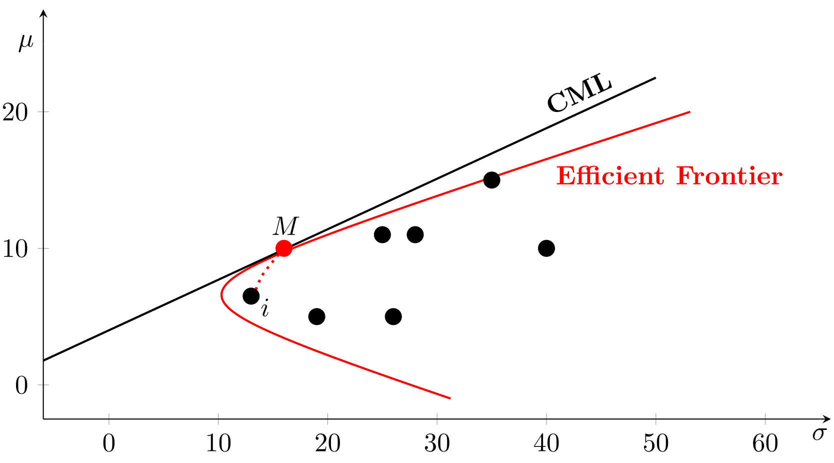
1.1.1 Higher order moments

Compared to the classification in Levy (2016) in which the portfolio optimization has two steps: efficient portfolio identification and individual utility maximization, Markowitz (1952) gives a slightly different specification for the investment process. He maintains that

> "The process of selecting a portfolio may be divided into two stages. The first stage starts with observation and experience and ends with beliefs about the future performances of available securities. The second stage starts with the relevant beliefs about future performances and ends with the choice of portfolio."

Note that this classification doesn't contradict Levy's two steps of objective decision and subjective decision. According to Markowitz's first stage, the preparation is

FIGURE 1.1: Capital market line and CAPM derivation



an essential part of the general optimization process, and it entails key inputs for the objective decision and the subjective decision. Specifically, "observation" and "the future performances of available securities" are important to the efficient set specification, so are "experience" and "beliefs" to the expected utility maximization.

If the observation and the experience lead to a collection of optimization inputs beyond the mean and the variance, the use of the mean variance optimization framework can only provide for a suboptimal choice. Cont (2001) offers a summary of empirical properties for asset returns. Among the stylized statistical properties of asset returns, heavy tails and loss asymmetry are highlighted, showing that the negative skewness and the excess kurtosis are substantial concerns beyond the mean and the variance. Consistently, higher order moments are also important to describe investor preferences. For example, Kraus and Litzenberger (1976) reaffirm that investors have a preference for positive skewness, and extend the CAPM to three moments to incorporate the skewness effect in valuation. Subsequent research on higher order moments was primarily conducted on the incorporation of skewness, for which De Athayde and Flôres (2004) and Briec, Kerstens, and Jokung (2007) are the most representative attempts on the mean variance skewness portfolio construction.

Kurtosis is frequently used to capture extreme risks, which have come into the spotlight after the recent series of economic crises and disasters. For example, Barro (2006) uses kurtosis to reflect disaster risk and shows that the economic disasters of the last century led to a 29% average decline in real per capita GDP. He finds that disaster risk has considerable effects on asset returns and on the equity premium. Dittmar (2002), Eraker, Johannes, and Polson (2003), Liu, Longstaff, and Pan (2003), Poon, Rockinger, and Tawn (2003), Bakshi and Madan (2006), Bates (2008), Todorov (2009), Benzoni, Collin-Dufresne, and Goldstein (2011), Bollerslev and Todorov (2011), Bates (2012), Gabaix (2012), Drechsler (2013), Wachter (2013), and Cremers, Halling, and Weinbaum (2015), among many, notice the significant role of extreme risks in the determination of returns. More interestingly, Malmendier and Nagel (2011) find that low stock returns contribute to shaping individual preferences towards greater risk aversion, and this effect is stronger for young people. Thus, the effect of extreme risks is not only contemporaneous but also prospective.

By nature, kurtosis is a complementary risk indicator to variance for two reasons. The first reason is that, risk encompasses not only downside risk but also upside risk, as Damodaran (2003, Chapter 2) advocates. In this sense, kurtosis is more appropriate than skewness at depicting event risks, because it assigns the same penalties to extreme events, no matter positive or negative. To illustrate how the incorporation of kurtosis impacts portfolio choice, we come back to the example in Table 1.1. We see that Portfolio y dominates Portfolio x in the mean variance sense. It is true that Portfolio x is highly skewed towards the extreme positive value of 1,000, but its kurtosis is congruently greater than that of Portfolio y. In fact, the kurtosis of Portfolio x is 9,227,456,454 (the standard kurtosis is 98), while for Portfolio y it is 7,992,159 (the standard kurtosis is 3.25). Assume that the Taylor expansion of the utility function ends at order 4 and that the expected utility can be expressed as $E[u(W)] = \gamma_1\mu - \gamma_2\sigma^2 + \gamma_3 s^3 - \gamma_4\kappa^4$, where the γs are all positive utility loadings for return moments. Since Portfolio x is only better in terms of skewness, it can only be a superior choice when the investor predominantly cares about skewness far beyond any other moments, which is highly improbable.

The second reason is that extreme negative events are more significant than extreme positive events, contributing to a deflated skewness benefit and an escalated dispersion apprehension. Cont (2001) obtains that the large upward movements are not as equally as the large drawdowns. This loss in asymmetry implies that any potential positive extreme values are likely offset by negative extreme values of similar or greater size. When comparing two distributions with negative skewness, kurtosis is more informative about the general dispersion level of the distributions.

All of these pieces of evidence suggest that it is vital to include extreme risks, especially kurtosis, into investment optimization.

1.1.2 Pareto improvement method

Portfolios may well share some common moments. For example, suppose we examine the return profiles of two portfolios A and B till the fourth moment. The two portfolios have the same first three moments, and Portfolio B has lower fourth moment. So, Portfolio B has a marginal profile improvement relative to Portfolio A at the fourth moment. A key drawback of the existing approaches about higher order efficient portfolio construction is the lack of discernment among portfolios showing marginal return profile improvements like in the previous example. The failure to capture a marginal improvement leads to a misclassification of inefficient portfolios, here Portfolio A, as efficient. The Pareto improvement method overcomes such a failure, and enhances the global optimality in the higher order portfolio construction. We cover this point hereafter.

There are two main methods extending the efficient frontier to higher order moments. The first of these methods is represented by De Athayde and Flôres (2004), a straightforward adjustment of mean variance optimization in the context of the mean variance skewness dimension. Simply put, this method minimizes the portfolio variance at given expected return and skewness levels using the following program:

$$\begin{aligned} \min_{w} \quad & \sigma^2 \\ \text{subject to} \quad & \mu \geq \mu_P \\ & s^3 \geq s_P^3 \end{aligned}$$

where the return profile of P is a given level. This method involves the riskfree asset in the minimum variance portfolio determination, explicated in De Athayde and

Flôres (2004, p. 1339) as

$$\min_{w} \mathcal{L} = w'\Sigma w + \lambda_1(\mu_P - w'\mu - (1 - w'\mathbf{1})r_f) + \lambda_2(s_P^3 - w'S(w \otimes w)),$$

where $\otimes$ is the tensor product. As the minimization program suggests, this method cannot produce a pure efficient portfolio of risky assets. Table 1.2 highlights the return features of low mean/volatility as well as high skewness/kurtosis for the bonds, which is quite different from the return features of the stocks. In other words, this minimum variance method has no link with the two fund separation, and the optimal portfolios by this method cannot be characterized as a common risky investment part plus a riskfree investment part.

The minimum variance method also has a strange implication on the riskfree asset market equilibrium. Under the assumption of homogenous market expectations for the investors, the share of riskfree asset is fixed in the investor's optimal choice according to the Lagrange function, and investors have the same wealth proportion invested in the riskfree asset. This share is independent of individual risk preferences, no matter the agent is risk seeking or risk loving. So risk seeking investors cannot borrow money for additional investment in the risky part, and risk averse investors cannot reduce the risky investment and lend money. The risk preference has no impact on the supply and demand sides for the riskfree asset at all, and the riskfree market equilibrium is in question.

From the implemental aspect, the optimization program for the minimum variance method has to search the entire grid spanning the expected returns μ_P and skewness values s_P^3. For portfolios with the same variance, the variance minimized program cannot detect the inefficient portfolios with lower mean and skewness values, still keeping these portfolios as efficient mistakenly. It brings the risk of false negative errors, illustrated by the example in Table 1.3. Assume we have three portfolios in the market and we want to get the efficient ones among Portfolios x, y and z using the variance minimization method in De Athayde and Flôres (2004). Now set $\mu_P = 1$ and $s_P^3 = 1.5$, and we search for the minimum variance portfolio under these constraints. Note that the three portfolios meet the mean and skewness requirements. As they have the same variance of 2, the optimization program minimizing the variance doesn't help at all to identify any of them as the inefficient portfolios. Despite having the same variance as the other portfolios, Portfolio z is the actual efficient portfolio because it has a higher mean and a higher skewness. However, the information of mean and skewness is not regarded in the program, leading to a mistake in retaining inefficient portfolios. From the perspective of duality gap, the dual program is exact between return maximization and variance minimization in the mean variance optimization due to convexity. When the optimization problem goes to higher order moments, the duality gap is not necessarily zero. In this case, the portfolio minimizing variance at given expected return and skewness values is not necessarily the portfolio maximizing expected return at given variance and skewness values.

The other main optimization for higher order moment portfolio construction is the shortage function method, pioneered by Briec, Kerstens, and Jokung (2007). This method steps further than the first approach by equalizing potential improvements for a portfolio along all of the return profile dimensions. The reasoning is that if we can find a portfolio with a better return profile that dominates the evaluated portfolio, then the latter is inefficient and the portfolio gaining the largest profile improvement is the corresponding efficient one. If there is no such portfolio achieving

TABLE 1.3: Hypothesized portfolio profiles and optimizing efficacy

	x	y	z	z*
Mean	1	1	1.5	1.5
Variance	2	2	2	1.8
Skewness	1.5	2	2	2

We suppose that Portfolios x, y and z are all feasible. The feasibility of Portfolio z^* will be indicated in the text.

a dominating return profile over the evaluated portfolio, then the evaluated portfolio is efficient. The efficient set includes those portfolios without any dominating return profile.

In the shortage function method, the objective is to maximize the profile improvement as

$$\begin{aligned} \max_{w} \quad & \delta \\ \text{subject to} \quad & \mu \geq \mu_P + \delta g_1 \\ & \sigma^2 \leq \sigma_P^2 - \delta g_2 \\ & s^3 \geq s_P^3 + \delta g_3 \end{aligned}$$

where δ is the size of the improvement, $g = [g_1 \ -g_2 \ g_3]'$ is a prespecified directional improvement vector, and the parameters of δ, g_1, g_2 and g_3 are positive.

To see how the shortage function method works, we come back to the example in Table 1.3. Now suppose we have an additional portfolio of z*, and x is the portfolio evaluated among the four choices. For a specification of $g = [0.5 \ -0.2 \ 0.5]'$ with hindsight, the shortage function method returns Portfolio z* as the efficient portfolio, since it has a mean improvement of 0.5, a variance improvement of 0.2, a skewness improvement of 0.5, and it achieves the best return profile improvement, equally $\delta = 1$, in the feasible portfolio set. To reach this efficient portfolio z*, the minimum variance method has to run the optimization program for four times, with the constraints of (μ_P, s_P^3) at (1, 1.5), (1, 2), (1.5, 1) and (1.5, 2) respectively. The shortage function method is less likely to suffer from the redundant grid searching, thus increases efficiency. Note that to get a sufficient set of efficient portfolios, we have to provide a sufficiently broad set of evaluated portfolios as inputs to obtain the corresponding efficient portfolios.

Despite being more efficient than the minimum variance portfolio method at the efficient portfolios specification, the shortage function method also has the weakness that it cannot identify marginal return profile improvement. Note g_i has to be positive to facilitate the optimization. Therefore, this method fails to capture marginal improvements for which a g_i can be 0. In the previous example, if Portfolio z is being evaluated, the shortage function also recognizes it as efficient because the maximized value of δ is still 0. Actually Portfolio z* is the only efficient portfolio, but this profile improvement over Portfolio z is not identified by the program due to the fact that the corresponding g_1 and g_3 are both 0.

To solve such a problem, we propose to use a more general setting of dimensional improvement vector beyond the strict positivity along all directions. We call this method as the Pareto improvement method. This method shares the same motivation as the shortage function method, because the efficient portfolios are those without any return profile improvements. For any pair of portfolios x and y, we say that Portfolio y has a Pareto improvement relative to Portfolio x, if and only if

$$\langle \mathcal{P}_y, \mathcal{P}_x \rangle \gneqq \mathbf{0},$$

where $\langle \mathcal{P}_y, \mathcal{P}_x \rangle = [\mu_y - \mu_x;\ \sigma_x^2 - \sigma_y^2;\ s_y^3 - s_x^3]'$ in the mean variance skewness space and $\gneqq$ means "no less than but not equal to." For an arbitrary vector ε and an identical size zero vector $\mathbf{0}$, $\varepsilon \gneqq \mathbf{0}$ indicates that each element of ε is no less than 0 and that at least one element is not 0.

Using this general framework, we can easily detect Portfolio z* as the only efficient one among the four portfolios, since it has no available profile improvement among the feasible set. This shows the superiority of the Pareto improvement method over the minimum variance portfolio method and the shortage function method at capturing the marginal return profile improvement. Thus, the Pareto improvement method is robust to the error of misclassifying an inefficient portfolio as efficient. Also, it is flexible for conducting an investment optimization formulated with any combination of return moments, like jumping moments. Specifically, when an investor is averse to extreme risks but neutral to skewness, the Pareto improvement method can be used for optimizing in the mean variance kurtosis space.

1.1.3 Polynomial approximations to utility

In the investment optimization, the first step of efficient frontier construction intrinsically connects to the second step of utility maximization. The efficient portfolio identification in higher order moments is about the first step, and a comparable extension of utility specification to higher order moments is needed for the second step. The classic method to incorporate higher order moments is the polynomial approximation to the expected utility by Taylor expansion, which is pioneered by Pratt (1964) and Arrow (1965).

The general Taylor expansion of the expected utility of asset return is

$$E[u(\tilde{R})] = \sum_{i=0}^{\infty} \frac{u^i(E[\tilde{R}])}{i!} E[(R - E[\tilde{R}])^i].$$

In the most fundamental case of the truncation at order 2, we have:

$$E[u(\tilde{R})] \approx u(E[\tilde{R}]) + \frac{1}{2} u''(E[\tilde{R}]) E[(R - E[\tilde{R}])^2].$$

Markowitz (1959) and Levy and Markowtiz (1979) initiate and rationalize that a combination of the mean and the variance provides a good approximation to the expected utility function. The mean variance utility is a concise and intuitive utility specification benchmark. In its most popular version, embedded in the Black Litterman approach Black and Litterman (1992), the mean variance utility is written as

$$\mu - \frac{1}{2}\lambda\sigma^2$$

where $\lambda = -\frac{u''(\tilde{R})}{u'(\tilde{R})}$ is the Arrow Pratt absolute risk aversion coefficient. For an investor with such a preference, the optimization program becomes

$$\begin{array}{ll} \max_{w} & w'\mu - \frac{1}{2}\lambda w'\Sigma w \\ \text{subject to} & w'\mathbf{1} = 1 \end{array}$$

while the two steps of the investment optimization degenerate into only one step of the utility maximization. Note that this decision degeneration is based on an exact specification for the investor who then has the same risk aversion coefficient as other investors. This situation is not likely to be true. The mean variance utility function explicitly reveals the tradeoff between return and risk, where the higher the risk aversion coefficient, the lower the attraction of portfolios with a high variance. This approach is quite straightforward and practical, but it assumes that the investor focuses exclusively on the first two moments of the return distribution.

As aforementioned, the combination of the mean and the variance is not enough to fully characterize the actual return distribution. Investors do care about extreme risks, and a more general Taylor approximation for the expected utility is needed. A truncation at order 3 leads to mean variance skewness utility as

$$E[u(\tilde{R})] = \gamma_1\mu + \gamma_2\sigma^2 + \gamma_3 s^3,$$

and a truncation at order 4 leads to mean variance skewness kurtosis utility as

$$E[u(\tilde{R})] = \gamma_1\mu + \gamma_2\sigma^2 + \gamma_3 s^3 + \gamma_4\kappa^4,$$

where γ_i is the corresponding coefficient for the moment of order $i \in [1, 2, 3, 4]$.

Thus, if the investor uses kurtosis as an extreme risk indicator and is insensitive to skewness, we can specify a mean variance kurtosis utility function for this set of risk preferences as follows:

$$E[u(\tilde{R})] = \gamma_1\mu + \gamma_2\sigma^2 + \gamma_4\kappa^4.$$

The estimations of γs reflect the investor's particular risk attitude to risks, γ_2 of common risk aversion and γ_4 of extreme risk aversion.

A consecutive moment compilation up to order n for utility approximation, as the Taylor expansion shows, is familiar in finance research. A selective moment compilation up to order n with specific emphases on risk preference, like here with an aversion to extreme risk but an indifference to skewness, is new. This specification dealing with a jumping moment utility approximation is novel to the existing finance literature, as far as we know.

The Taylor expansion can be implemented on any specific utility functions in the classes of CARA, CRRA, and HARA. The parameters of γs under calibration facilitates the convergence of the approximate expected utility to the exact expected utility. For example, Garlappi and Skoulakis (2011) discuss the Taylor approximation for the HARA class utility functions based on a linear decomposition. That's to say, we can also approximate the jumping moment utility function by calibrating the γs.

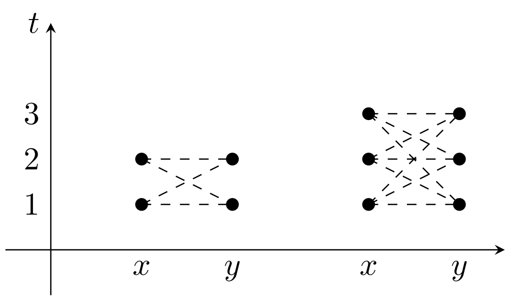
1.1.4 Monte Carlo

As the moment order goes higher, the input for the investment optimization is more demanding. In the mean variance optimization, return levels are the input; in the minimum variance portfolio optimization of De Athayde and Flôres (2004), a grid of return and skewness levels is the input; in the shortage function method of Briec, Kerstens, and Jokung (2007), a sufficient number of evaluated portfolios as the input. Since the Pareto optimization is more general than the shortage function method for including the marginal return profile improvements, its input is a sufficient approximation to the feasible portfolio set.

A perfect presentation of the feasible set by enumeration is impossible, because there are indefinite possible portfolio formations by diversification. However, market frictions such as trading costs make it impossible to realize the indefinite diversification choices. Given the presence of brokerage commissions, fees and taxes, a small portfolio change, say, a 0.1% increase in an asset's weight compensated by a decrease in another asset's weight is usually not pragmatic. This fact implies that it is probable to get a reasonable approximation to the feasible portfolio set.

The approximation complexity of the feasible set explodes as the number of underlying assets increases. To illustrate this point, we introduce a simple instrument called "investment density." Suppose for an asset, we have t possible holdings on the asset, and the actual holding i can be any integer of $[1, t]$. The higher t, the closer between two adjacent holdings, or the denser weight spacing. For example, suppose that we have two assets x and y of 2 possible holdings, each with 1 and 2 as holding density. For an investment in a single asset, we have two choices: investment in x or in y. For cross assets possibilities of (x, y), we have (1, 1), (1, 2), (2, 1) and (2, 2). The portfolio weight is then transformed from the density pair. For (1, 2), the total density is 3, so (1, 2) denotes a portfolio with 1/3 investment in x and 2/3 in y. Note the first possibility (1, 1) and last one possibility (2, 2) correspond to equal investments. Therefore, we have to minus one redundant count, and we have five portfolio choices in this case. Similar is done when $t = 3$, as shown in Figure 1.2.

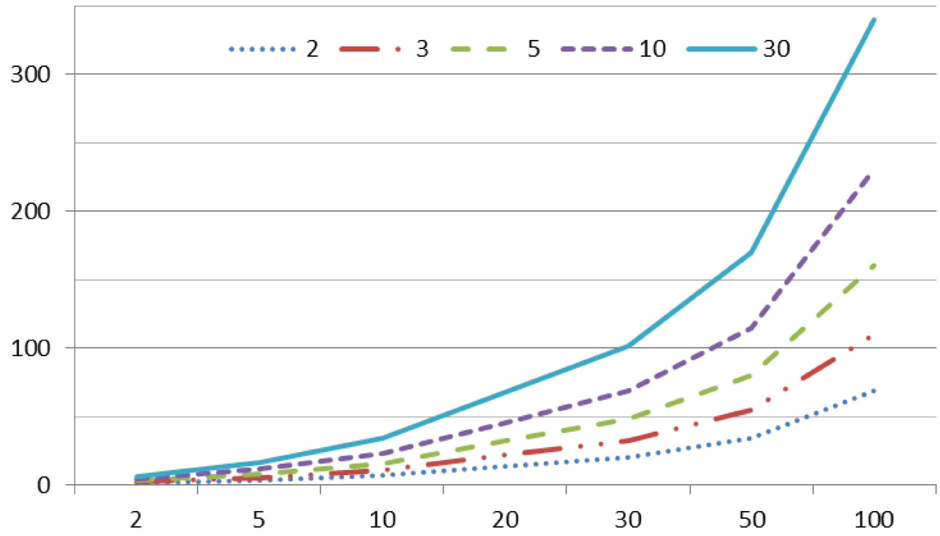
FIGURE 1.2: Investment densities and portfolio choices



In the same vein, suppose that for each of the n assets we can discretize the holding into t possibilities, then a simple calculation gives an answer of $n + t^n - t + 1$ portfolio choice. Note that n is the number of corner investments in single assets, t^n is the combination of all holding possibilities across assets, and $t - 1$ is an adjustment for redundant equal investments. Figure 1.3 exhibits how the investment possibilities explode with regard to n and t. When there are 2 holding densities for each asset, we have 5 portfolio choices as aforementioned in the 2-asset universe, 1,033 choices in the 10-asset universe, 1.07 billion choices in the 30-asset universe, and 1.27×10^{30} choices in the 100-asset universe. When there are 30 holding densities for each asset, we have 873 portfolio choices in the 2-asset universe, 5.9×10^{14} choices in the 10-asset universe, 2.06×10^{44} choices in the 30-asset universe, and 5.2×10^{147} choices in the 100-asset universe.

FIGURE 1.3: Investment choice, asset universe, and holding density

This figure shows how the total investment choices vary with respect to the size n of the asset universe, as well as the number t of holding possibilities. The horizontal axis shows n, and the vertical axis measures the logarithmic number of investment choices for a corresponding pair of n and t.



The explosion of the computational burden in a large size asset universe entails a need of a dimensionality reduction. A proper way is to use industry subindices as underlying assets rather than use individual stocks. For example, the DJIA index has 30 stocks that can be readily classified into about 10 sectors. The benefits of a dimensional reduction are obvious and more important when we consider the FTSE 100 index, the NIKKEI 225 index, and the S&P 500 index.

Furthermore, a proper weight generating algorithm can facilitate the feasible set approximation. Note that when the holding density is excessively refined, a minimal weight change is distinguished from its neighbors. When the asset universe size n is about 10, the marginal difference between two adjacent portfolio weights is negligible, and an investment change from one portfolio to the other is most probably deterred by the trading costs.

Dirichlet distribution is a weight generating scheme with the feature of regular sum. This distribution has lots of application in economics and finance. For example, Chotikapanich and Griffiths (2002) pioneer its use in sampling the income shares for a Lorenz curve estimation. Their research provides a hint that the Dirichlet distribution can also be used for sampling the portfolio weights, whose economic and statistical characteristics are similar to the income shares.

The Dirichlet distribution is the multivariate generalization of the beta distribution. It has the following probability density function:

$$f(w_1, w_2, ..., w_T; \alpha_1, \alpha_2, ..., \alpha_T) = \frac{\Gamma(\sum_{i=1}^{T} \alpha_i)}{\prod_{i=1}^{T} \Gamma(\alpha_i)} \prod_{i=1}^{T} w_i^{\alpha_{i-1}}$$

where $\alpha_1, \alpha_2, ..., \alpha_T$ are concentration parameters and are all positive. $\Gamma(\cdot)$ is the Gamma function such that

$$\Gamma(\alpha_i) = \int_0^{\infty} x^{\alpha_i - 1} e^{-x} dx.$$

For the Dirichlet distribution, $\Sigma_{i=1}^{T} w_i = 1$ and $0 \leqslant w_i$.

Note that the regular sum is inherent in the Dirichlet distribution, and the uniform distribution is a special case of the Dirichlet distribution where all $\alpha_i = 1$. Moreover, the marginal distributions are beta distributions, and are not independent. A common way to sample weights is to generate positive random numbers independently and then get divided by the sum of these random numbers for normalization. The Dirichlet distribution is superior to such a portfolio weight simulation since portfolio weights cannot be independent from each other. The other common way to sample weights is to use randfixedsum, which produces weight vectors from the uniform distribution. As we mentioned, randfixedsum is just a subcase for the more general Dirichlet distribution. The non-negative property of the Dirichlet distribution also conforms to the common constraint of non-short selling for risky assets in market equilibrium.

In sum, the Dirichlet distribution equips us with the instrument to approximate the feasible set and get this essential input for the Pareto improvement method. It promotes the application of the Pareto improvement method in the context of portfolio optimization with higher order moments.

1.2 Stochastic dominance

The mean variance optimization framework is fundamentally parametric. This framework focuses on the mean and the variance to capture information about a probability distribution. It works well for normally distributed returns, because in this case the knowledge of its first two moments is sufficient to encapsulate a probability distribution. However, it leads to misleading optimization choice once the true distribution deviates a lot from normality. We previously discussed the inclusion of higher order moments to better capture a distribution's information. This is still a general parametric approach. Alternately, we now discuss the use of a nonparametric approach to circumvent the incomplete description of a return distribution by a finite set of moments.

The nonparametric approach alternative to the moment-based mean variance optimization that we use is stochastic dominance. Pioneered by Mann and Whitney (1947) and Lehmann (1955), stochastic dominance appeared in probability theory and statistics as "stochastic ordering" to compare and order two random variables. Cumulative distribution functions are employed to capture the full spectrum of prospects in the ranking process. Quirk and Saposnik (1962), Hadar and Russell (1969), Hanoch and Levy (1969) and Rothschild and Stiglitz (1970) introduce and expand the application of stochastic dominance in an economic setting to compare two alternative probability distributions of the future wealth. These discussions are based on the rationality axioms of the expected utility theory. Indeed, the first order and the second order stochastic dominance are accompanied with the presence of non-satiation and risk aversion principles, respectively.

Subsequently, the stochastic dominance approach applies in various fields within and beyond economics, such as in agricultural economics to find best planting locations, and in medicine to determine optimal treatments. Because risk is the main theme of finance, stochastic dominance finds its most applications in finance research. Especially in the field of portfolio selection, stochastic dominance develops to analyze efficient diversification strategies in search of the optimal portfolio. By this development, stochastic dominance goes beyond pairwise comparison for efficiency determination, broadening the applicability scope for stochastic dominance approach.

In conclusion, this approach takes full advantage of the information from the complete distribution rather than from limited moments to capture the risk of investment prospects, and relies on general risk preference principles rather than on a naïve parameterization of a certain utility specification. It also advantageously conforms to the expected utility theory better than the mean variance optimization framework and is more theoretically consistent. Nonetheless, for its broader application over various utility classes, its answer to the optimal portfolio selection is not as explicit as in the mean variance optimization approach. In the first step of objective decision, it gives a clear identification of inefficient choices for all the investors in the same utility class, regardless of their specific preferences. Thus, the inefficient set can be smaller than that in the mean variance optimization framework. So, it is less likely that the global optimal portfolio best serving the investor's preferences gets screened out. Subsequently, in the second step of subjective decision, the probability of a false negative error for optimal portfolio is minimized. Stochastic dominance has strong implications on investment optimization with the assumption of heterogeneous investors, which is much more realistic than its homogenous counterpart.

1.2.1 First order and second order stochastic dominance

We now conduct a quick review on the first order and the second order stochastic dominance to exhibit the essence of the stochastic dominance approach for portfolio selection. For details, refer to, for instance, Kuosmanen (2004) and Levy (2016).

Let G_x and G_y be cumulative distribution functions for two investment options x and y, respectively. Then, Portfolio y dominates Portfolio x by the first order stochastic dominance (FSD) if and only if the curve of G_y is not above G_x over the whole domain of return level r. Precisely,

$$y \succ^1 x \Leftrightarrow G_x(r) - G_y(r) \geq 0, \ \forall r \in \mathbb{R}.$$

Further, Portfolio y dominates Portfolio x by the second order stochastic dominance (SSD) if and only if the area below G_y is no more than the area below G_x over the whole domain of return level r. Precisely,

$$y \succ^2 x \Leftrightarrow \int_{-\infty}^{r} [G_x(s) - G_y(s)]ds \geq 0, \ \forall r \in \mathbb{R}.$$

Stochastic dominance is congruent to the expected utility theory. Denote as $\mathcal{U}_n$ the utility function class that includes all the utility functions u with nonnegative odd derivatives and nonpositive even derivatives up to the order n, where n is a positive integer. For example, $\mathcal{U}_1$ comprises all nonsatiated preferences with $u' \geq 0$, and $\mathcal{U}_2$ comprises all nonsatiated as well as risk averse preferences with $u' \geq 0$ plus $u'' \leq 0$. Stochastic dominance doesn't specify utility functions beyond these general preference assumptions on utility derivatives. For FSD, if $y \succ^1 x$, then Portfolio y has a higher expected utility than Portfolio x for any utility specification from $\mathcal{U}_1$. That is to say,

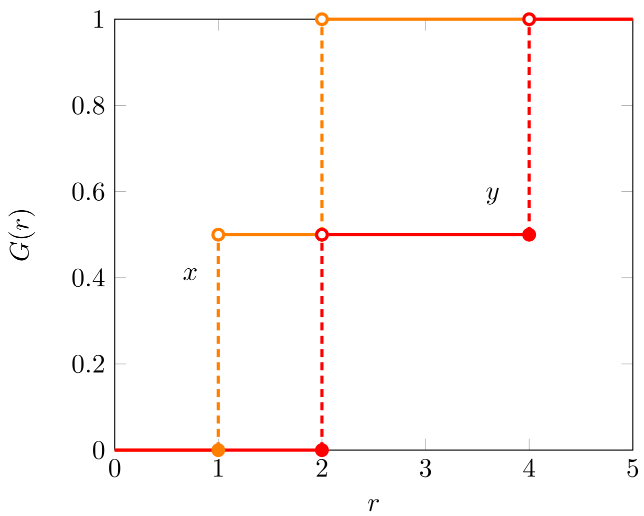
$$y \succ^1 x \Leftrightarrow E_y[u(r)] \geq E_x[u(r)],\ \forall u \in \mathcal{U}_1.$$

Correspondingly, for SSD, $y \succ^2 x$ means that all investors with utility functions from $\mathcal{U}_2$ will choose Portfolio y over Portfolio x. In other words,

$$y \succ^2 x \Leftrightarrow E_y[u(r)] \geq E_x[u(r)],\ \forall u \in \mathcal{U}_2.$$

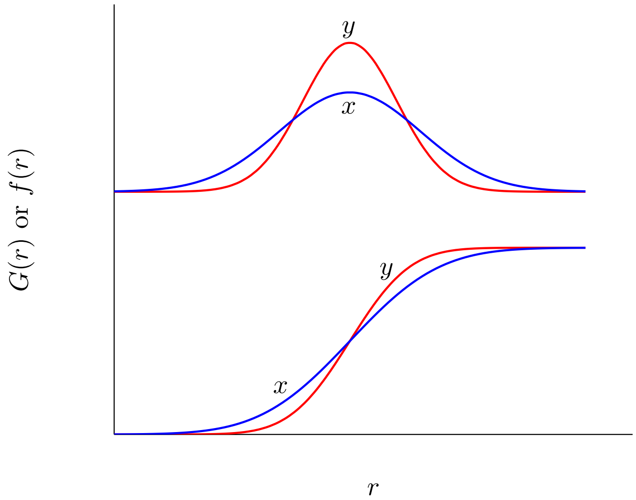
A simple example in Levy (2016, p. vii) presents the mechanism of FSD. Suppose Portfolio x has a return of 1 in the bad state and 2 in the good state with equal probability, and Portfolio y doubles the returns in each state. It is easy to know that Portfolio x has an expected return of 1.5 and a variance of 0.25, and that Portfolio y has an expected return of 3 and a variance of 1. In the classic mean variance optimization, no inefficient portfolio is detected as Portfolio y has a higher expected return but also a higher variance. However, Portfolio y has obvious superiority for its attractive return in each state, although its variance is higher. This superiority is well captured by FSD, see Figure 1.4. Since the cumulative distribution of Portfolio y is strictly below that of its peer, we can clearly get $y \succ^1 x$. Therefore, all investors who prefer more to less should choose Portfolio y over Portfolio x.

FIGURE 1.4: FSD for investment choice



We then illustrate SSD with the example in Figure 1.5, where the return distribution of Portfolio x is a mean preserving spread to that of Portfolio y. Suppose both portfolios' returns follow normal distributions with the same mean, and Portfolio x has a bigger variance. As we see, the cumulative distribution functions for the two portfolios intersect, which means no FSD relationship exists between them. However, the area below G_x is always bigger than the area below G_y. Therefore, SSD relationship exists with $y \succ^2 x$, and all risk averse investors prefer Portfolio y over Portfolio x. The formal equivalence proof between SSD and the mean preserving spread can be found in Rothschild and Stiglitz (1970).

FIGURE 1.5: SSD for investment choice



As discussed earlier, SSD corresponds to the utility class of $\mathcal{U}_2$. When it comes to a pair of portfolios with equal mean and variance, SSD is incompetent to discern a better choice. Take the example in Table 1.4. Portfolio x and Portfolio y has the same expected return of 1.5, and the same variance of 0.75. FSD and SSD are not detected between the two portfolios. However, Portfolio y has higher returns in each state than Portfolio x, and has higher skewness. Intuitively, investors should choose Portfolio y if they have preference for skewness, and Portfolio x should be classified as inefficient. This example indicates the necessity to extend stochastic dominance approach to higher order, which benefits the efficient set determination for portfolio optimization.

TABLE 1.4: TSD example in Levy (1992, p. 93)

	x	Prob (x)	y	Prob (y)
State 1	0	0.25	1	0.75
State 2	2	0.75	3	0.25
Mean	1.5		1.5	
Variance	0.75		0.75	
Skewness	-0.75		0.75	

1.2.2 Higher order stochastic dominance

A main advantage of stochastic dominance approach is the seamless integration of the first step of objective decision and the second step of subjective decision, which minimizes the risk of screening out the global optimal portfolio in the first step. This benefit comes at the cost of a relatively large efficient set, or equivalently of less discriminating power. An effective way to increase the selective competence of the first step is to give more specific assumptions for the preferences. The introduction of a restricted utility class facilitates the screening process for inefficient portfolios.

Specifically, $\mathcal{U}_1$ includes $\mathcal{U}_2$ and the other utility class with $u' \geq 0$ plus $u'' \geq 0$. Note that $y \succ^1 x$ means that Portfolio y is preferred by any investor with a non-satiated utility function, no matter if he or she is risk averse or not. By contrast, $y \succ^2 x$ means that Portfolio y is preferred by any investor with a non-satiated and risk averse utility function. Between the two cases, the key difference is that FSD has to satisfy risk loving investors as well. To make sure that Portfolio y is preferable for them, the required return has to be higher than the return for risk averse investors to compensate their desire for more risks. As in Figure 1.5, Portfolio y is a mean preserving anti-spread to Portfolio x, so all risk averse investors choose Portfolio y, but risk loving investors prefer Portfolio x. To increase the attraction of Portfolio y to these risk loving investors, the expected return for Portfolio y has to be higher so that both risk averse and risk loving investors agree that Portfolio y is better. With this agrrement, Portfolio y is FSD over Portfolio x.

In other words, the broad utility class $\mathcal{U}_1$ inevitably leads to a less selective efficient set, because the inefficient portfolios have to be consistent for all investors no matter if they are risk averse or risk seeking. But risk seeking investors have no tradeoff between risk and return because they prefer both. This risk preference characteristic is not systematically stable along the investor's investment choices, neither consistent across various investors in the market. Its omission in the investment optimization doesn't materially impair the optimization integrity.

So, an extension of stochastic dominance to order 3 and order 4 is worthwhile to facilitate the efficient set determination for portfolio optimization. The corresponding utility classes are $\mathcal{U}_3$ and $\mathcal{U}_4$. $\mathcal{U}_3$ includes all the utility functions u with $u' \geq 0$, $u'' \leq 0$, and $u''' \geq 0$. They are those of non-satiated, risk averse and prudent investors. Similarly, $\mathcal{U}_4$ includes all utility functions u with $u' \geq 0$, $u'' \leq 0$, $u''' \geq 0$, and $u^4 \leq 0$. They are those of non-satiated, risk averse, prudent and temperant investors. As the utility class becomes more constrained, the inefficient set becomes larger, and the efficient set includes fewer portfolios to accelerate the second step of subjective decision. However, an over-constrained utility class is less helpful. The preference deduction has to be balanced by an economic explanation, because the 100th derivative of a utility function has minimal economic meaning.

We can also confirm this point from the definition of stochastic dominance. If $G_x(r) - G_y(r) \geq 0$, then $\int_{-\infty}^{r}[G_x(s) - G_y(s)]ds \geq 0$. So FSD implies SSD, but not *vice versa*. It is possible that no FSD relationship exists between Portfolio x and Portfolio y, but SSD relationship can be detected so that $y \succ^2 x$ or $x \succ^2 y$. This will change the efficient set composition, as both portfolios are in the FSD efficient set but only one is in the SSD efficient set.

1.2.3 Majorization

For FSD and SSD, the dominance determination is intuitively characterized by a comparison of heights and areas between the two cumulative distribution functions. However, the exhibition is not so straightforward for higher order SD, where the multiple integrals often bring computational complexity.

A pragmatic and convenient way to overcome these difficulties is to bridge the stochastic dominance and majorization theory by using empirical distribution function. This equivalence of SD and majorization theorem is an extension of the arguments of Marshall and Olkin (1979) used in Levy (1992) and Kuosmanen (2004), showing that the pairwise SD comparison of distributions can be achieved at any order by using cumulative sums. Stochastic dominance of empirical distribution functions at order n is equivalent to dominance in the majorization sense at order n, and goes as

$$y \succ^{[n]} x \quad \Leftrightarrow \quad \tilde{y}_t^{[n]} \geq \tilde{x}_t^{[n]} \text{ for all } t \leq T,$$

where $\tilde{x}^{[n]}$ is the cumulative sum of the ordered return vector of Portfolio x at order n, defined as

$$\forall t \leq T \quad \tilde{x}_t^{[n]} = \sum_{j_{n-1}=1}^{t} \sum_{j_{n-2}=1}^{j_{n-1}} \cdots \sum_{j_1=1}^{j_2} \tilde{x}_{j_1}.$$

The corresponding cumulative sums for the first four orders are

$$\forall t \leq T \quad \tilde{x}_t^{[1]} = \tilde{x}_t,$$

and

$$\forall t \leq T \quad \tilde{x}_t^{[2]} = \sum_{j_1=1}^{t} \tilde{x}_{j_1},$$

and

$$\forall t \leq T \quad \tilde{x}_t^{[3]} = \sum_{j_2=1}^{t} \sum_{j_1=1}^{j_2} \tilde{x}_{j_1},$$

as well as

$$\forall t \leq T \quad \tilde{x}_t^{[4]} = \sum_{j_3=1}^{t} \sum_{j_2=1}^{j_3} \sum_{j_1=1}^{j_2} \tilde{x}_{j_1}.$$

Consider now the simple example of the dominating set for a portfolio with two return observations $(1, 4)$. We want to construct all the dominating pairs of returns (x_1, x_2) such that $(x_1, x_2) \succ^{[n]} (1, 4)$, for $n = 1$ to 4. For simplicity, we focus on the case $x_1 < x_2$ while the case $x_1 > x_2$ is readily obtained by symmetry.

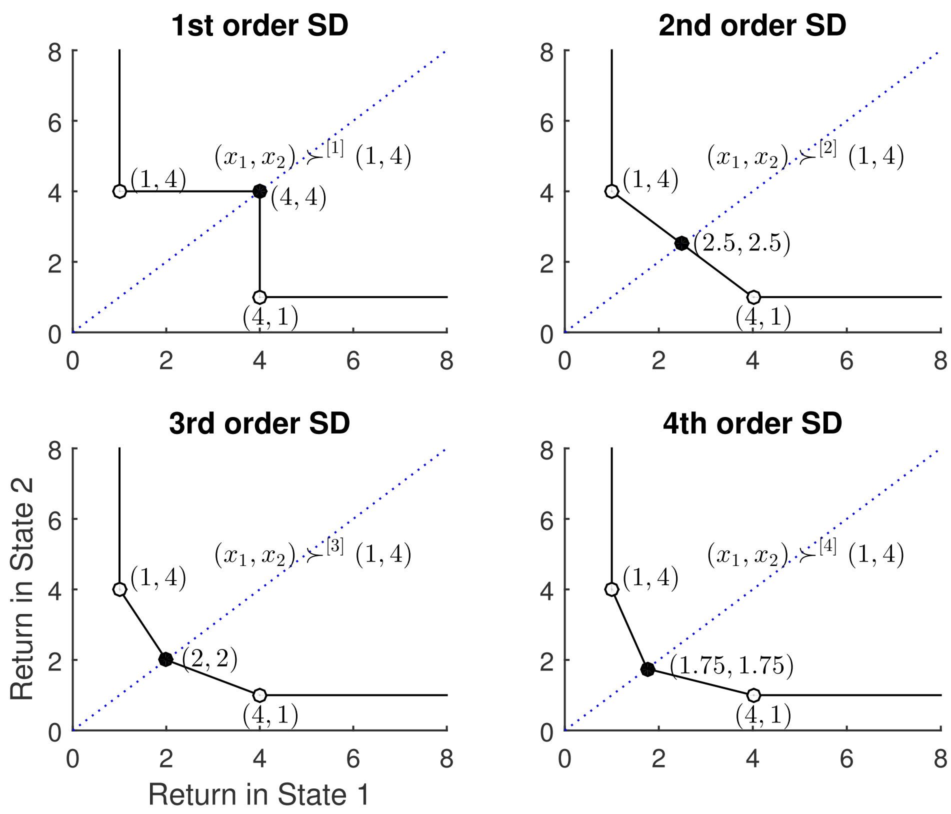
With the aforementioned four equations of cumulative sums, we see that a portfolio (x_1, x_2) dominates the portfolio $(1, 4)$ at order 1 in the majorization sense when $x_1 > 1$ and $x_2 > 4$. Then, $(x_1, x_2) \succ^{[2]} (1, 4)$ when $x_1 > 1$ and $x_1 + x_2 > 1 + 4$, the border of which is a segment satisfying $x_2 = 5 - x_1$ from the point $(1, 4)$ to $x_2 = x_1$. Also, $(x_1, x_2) \succ^{[3]} (1, 4)$ when $x_1 > 1$ and $x_1 + x_1 + x_2 > 1 + 1 + 4$, the border of which is a segment satisfying $x_2 = 6 - 2x_1$ from the point $(1, 4)$ to $x_2 = x_1$. Finally, $(x_1, x_2) \succ^{[4]} (1, 4)$ when $x_1 > 1$ and $x_1 + x_1 + x_1 + x_2 > 1 + 1 + 1 + 4$, the border of which is a segment satisying $x_2 = 7 - 3x_1$ from the point $(1, 4)$ to $x_2 = x_1$.

Figure 1.6 offers a summary of this discussion. The dominating sets of $(1, 4)$ are all convex except at the first order. The figure reaffirms a feature of stochastic dominance: the dominating sets are increasing by inclusion, and a dominating set at a high order incorporates the dominating sets at lower orders. For instance, if we cannot find empirical portfolios in a dominating set at the second order, the dominating set at the first order is empty.

1.2.4 Diversification

Classic stochastic dominance is primarily a tool via pairwise comparison. However, there are numerous investment possibilities in finance, so it is necessary to conduct complex broad comparisons beyond pairwise. For example, if we want to

FIGURE 1.6: Dominating sets at order one to four: an illustration



know the efficient investments in the S&P 500 index, most directly we have to compare each pair of the 500 components.[3] This means we get $500 \times 499 = 249,500$ pairs. Although a detected dominance for once is enough to relegate the dominated investment into the inefficient set, full investments into a single component only account for a small fraction of all the feasible investments. Also, the efficient set from single component investment cannot necessarily guarantee its general efficiency over the whole feasible set.

Kuosmanen (2004) proposes a necessary test for SSD efficiency, which identifies as optimal the portfolio with the greatest mean return improvement over an evaluated portfolio, if the latter one is detected as inefficient. This Kuosmanen SSD optimal portfolio is obtained using the following program:

$$\begin{aligned} \max_{w,W} \quad & \Big(\sum_{t=1}^{T}\sum_{i=1}^{n} r_{it}w_i - \sum_{t=1}^{T} R_t\Big)/T \\ \text{subject to} \quad & \sum_{i=1}^{n} r_{it}w_i \geq \sum_{j=1}^{T} W_{tj}R_t \\ & w \in \Lambda \\ & W \in \Xi \end{aligned}$$

[3]In fact, there are 505 stocks in the index, because 5 of the 500 component companies have several listed share classes. They are Discovery Communications, Google, Comcast, Twenty-First Century Fox, and News Corporation till April 2018.

where w is the weight vector and Λ the set of long only portfolios, r_i is the return series for asset i and R the series for the evaluated portfolio. W is a doubly stochastic matrix of nonnegative elements with unit sum for each row as well as each column, and Ξ is the set of doubly stochastic matrices. For example,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 0.25 & 0.75 \\ 0.75 & 0.25 \end{bmatrix}$$

are two examples of doubly stochastic matrices of size 2×2. The identity matrices like the first one are special cases of doubly stochastic matrices.

If the objective maximization gives 0, the evaluated portfolio meets the necessary condition for SSD efficiency. If the maximization gives a positive scalar of δ, the evaluated portfolio is SSD inefficient, and the optimal portfolio by the Kuosmanen program has a mean return surplus of δ over the evaluated portfolio. Note that the evaluated portfolio doesn't have to be part of or a combination of the n risky assets, and the return database $r = [r_1 \; r_2 \; \ldots \; r_n]$ in the market can be easily adjusted to augment the evaluated portfolio return profile as $[R, r]$.

We develop an extension of Kuosmanen (2004) SSD test up to the fourth order to improve the stochastic dominance investment optimization. The program for SD optimal portfolio at order N is the following:

$$\begin{aligned} \max_{w,W} \quad & \Big(\sum_{t=1}^{T}\sum_{i=1}^{n} r_{it} w_i - \sum_{t=1}^{T} R_t\Big)/T \\ \text{subject to} \quad & (rw)^{[N-1]} \geq WR^{[N-1]} \\ & w \in \Lambda \\ & W \in \Xi \end{aligned}$$

where $r = [r_1 \; r_2 \ldots r_n]$ is the market return database.

1.3 Name change and market inefficiency

To incorporate the event risks into portfolio construction, we previously discussed the extension for the classic framework of mean variance optimization and CAPM to higher order moments, as well as the extension for the stochastic dominance approach to higher order. We show that the inclusion of event risks into portfolio selection improves the global optimality for investment. Now we turn the extreme risks analyses to the context of corporate finance. Some corporate events significantly affect the return dynamics. For example, the S&P 500 index change effect is richly documented by various studies. The companies to be joining the S&P 500 index experience significant positive price reaction upon index change announcement. This abnormal return implies important economic characteristics change, for example, Barberis, Shleifer, and Wurgler (2005) notice the change in the comovement of the added companies with the market. Therefore, the consideration of these significant corporate events is also important to improve the asset allocation efficiency.

We consider an important set of corporate events: M&As and corporate name changes. To increase the comparability across sample companies, we focus our corporate coverage on the S&P 500 index. After an M&A, the combined company has to decide whether to keep the acquirer's name, or make a change and use the target's name, or to use a name combination of both companies, or to introduce a new name. The name change contains important value information about corporate strategies and management objectives. Thus, upon name change revelation at the M&A announcement, stock prices should incorporate this information quickly in efficient markets. Since there is a substantial time span between an M&A announcement and the corresponding S&P 500 index change announcement, we expect the price reaction to the name change M&A and the non name change M&A to be indifferent under the efficient market hypothesis (EMH). Otherwise, a significant difference implies that the value information is not fully reflected by the stock price, a clear deviation from market efficiency and a profitable chance to improve investment choices.

In this setting, the name change M&As and the non name change M&As all involve the S&P 500 components, and the media coverage about the two groups is immediate and generally equal. So, there is no substantial information asymmetry between them, which alleviates the concern that the investors' information accessibility leads to different price reactions. Our goal is to study how name change affects the return dynamics upon the S&P 500 index change announcement.

1.3.1 Event study methodology

The even study methodology is widely used in the studies on market efficiency, as reviewed in Fama (1991). It is succinctly characterized as the "analysis of whether there was a statistically significant reaction in financial markets to past occurrences of a given type of event that is hypothesized to affect firms' market values."

The basic element of this analysis is the specification of an event. The corporate event should be informative and intuitive, like a stock split, an M&A announcement, and so on. As the calendar event-time can be different across the events, the first step of the event study design is to arrange the event timeline. The event date is set to be Day 0, and the event window is ascertained around the event date to examine the market reaction over this period. The market reaction to the event is measured against a benchmark, which is the proxy for the stock return series in the absence of such an event. To get the expected return, an estimation window is specified for risk adjustment parameters, and the actual return over the expected return during the event window is the abnormal part, which is regarded as the event impact. The significance of the abnormal return is then evaluated. More details can be found in, for example, Kothari and Warner (2008) and Patel and Welch (2017).

Formally, the abnormal return of stock i at time t, $AR_{i,t}$, is the difference between the actual return $R_{i,t}$ and its reference, the expected return $E[R_{i,t}]$. Therefore, it is equal to

$$AR_{i,t} = R_{i,t} - E[R_{i,t}].$$

The cumulative abnormal return $CAR_{i,[t_1,t_2]}$ is the sum of abnormal returns over the event window of $[t_1, t_2]$ for stock i:

$$CAR_{i,[t_1,t_2]} = \sum_{t=t_1}^{t_2} AR_{i,t}.$$

The mean cumulative abnormal return $CAR_{[t_1,t_2]}$ is the cross-sectional average abnormal return over the event sample, defined as

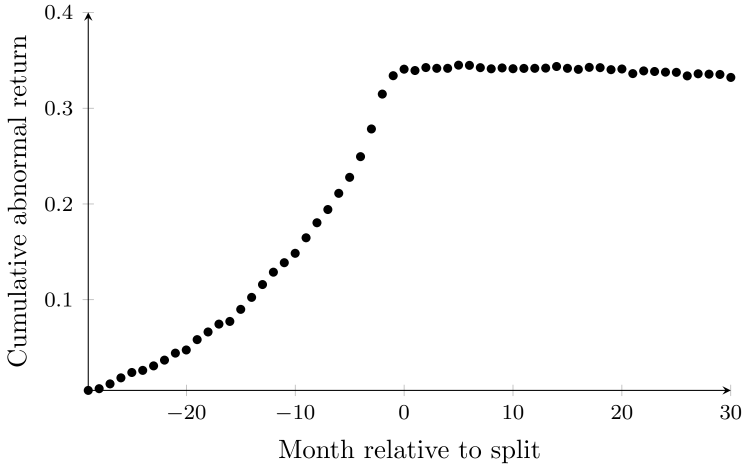
$$CAR_{[t_1,t_2]} = \frac{\sum CAR_{i,[t_1,t_2]}}{n},$$

where n is the number of events. The associated statistical significance is computed with the standard t test. For a recent summary of event study methods, see Patel and Welch (2017).

We use the pioneering work of Fama, Fisher, Jensen, and Roll (1969) for an intuitive exhibition. These authors study how stock splits affect stock price adjustments, where splits with stock dividends of more than 25% are specified as the event. They collect stocks with at least two years of data around the split from 1927 to 1959 for a sample of 940 splits. The split effective month is defined as Month 0, and the mean cumulative abnormal return is calculated similar to the aforementioned CAR. Figure 1.7 is the main result in Fama, Fisher, Jensen, and Roll (1969). It is straightforward that the mean cumulative abnormal returns increase significantly before stock splits, but not hence after. They explain that a stock split is a positive signal for future dividend increases, and the market reacts to this new information efficiently by compounding it into stock price rapidly at least by the split month.

FIGURE 1.7: Stock reaction to splits

This figure corresponds to Fama, Fisher, Jensen, and Roll (1969) Figure 2b, and the cumulative data are from Table 2 for all splits.



With this event study methodology, we study the joint events of systematic M&As and name changes.

1.3.2 The S&P 500 index and the market portfolio

The S&P 500 index is the most used index proxy for the market portfolio, as evidenced by its predominance in indexed assets.[4] As reported by the S&P Dow Jones Indices, $8.7 trillion of assets were benchmarked to the S&P 500 index as of 2016. More than that, the S&P 500 index ETFs are the largest in terms of asset under management (AUM). As shown in Table 1.5, the index has a dominating presence in the list of the largest ETFs. The SPDR S&P 500 ETF, the iShares Core S&P 500 ETF and the Vanguard S&P 500 ETF have a total AUM of about $500 million, while the Vanguard Total Stock Market ETF and the iShares MSCI EAFE ETF have a much smaller AMU size of about $90 million. The S&P 500 index is the most appropriate market portfolio proxy. It has a much broader industry coverage than the Dow Jones Industrial Average index, and it is also more tractable than the CRSP US Total Market Index.

[4]S&P Dow Jones Indices defines that indexed assets represent assets in institutional funds, ETFs, retail mutual funds, and other investable products that seek to replicate or match the performance of the respective index.

TABLE 1.5: Top 5 ETFs by assets

	Symbol	Name	AUM	Avg Volume
1	SPY	SPDR S&P 500 ETF	$255,857,367.99	119,169,672
2	IVV	iShares Core S&P 500 ETF	$143,811,920.65	5,473,141
3	VTI	Vanguard Total Stock Market ETF	$94,019,144.07	3,307,988
4	VOO	Vanguard S&P 500 ETF	$87,959,472.81	3,562,306
5	EFA	iShares MSCI EAFE ETF	$78,349,083.63	29,966,174

Data are from the complete list of "Largest ETFs: Top 100 ETFs By Assets" at ETFdb.com, accessed in Mar 30, 2018.

Taking this index as the market portfolio facilitates our analysis of the impact of significant corporate events. First, because the S&P 500 index is the most followed market portfolio index, any composition changes and component events are immediately reported by the mass media such as *The Wall Street Journal*, *The Washington Post*, and *The New York Times*. Various market participants like active investment managers and arbitrageurs also closely watch the index adjustments for their investment decisions. Enough information exposure guarantees that the value information of the corporate events is evident to the investors and should stimulate a full price reaction upon the information arrival under the EMH. Second, M&As among the S&P 500 index are systematic as the events substantially change the composition of the market portfolio and the correlation within industry and across industries. Note that only mergers between components prompt such influential market portfolio rebalancing. Mergers of a S&P 500 acquirer and a non S&P 500 target can hardly do that, as one can easily imagine the great discrepancy in market impact for the systematic merger of Google and Motorola, and the nonsystematic merger of Google and Songza. Merger of a S&P 500 target and a non S&P 500 acquirer, though not impossible, are quite unlikely and rare.

In short, we focus on the M&As among the S&P 500 index to assemble a sample of decent size. These M&As are systematically influential for the market portfolio adjustments.

1.3.3 Systematic M&A and name change

We denote the M&As among the S&P 500 components as systematic M&As. Note that after a business consolidation, the combined company has to decide on the business identity henceforth. This decision is quite significant because both the acquirer and the target are well-known industry leaders and important market players, with their brands carrying value, fame and history. As explained in the business report of The Economist (2014),

> "No management expert would think it strange that Imperial would spend the best part of $7 billion on something as ethereal as brands. They are the most valuable thing that companies as diverse as Apple and McDonald's own, often worth much more than property and machinery. Brands account for more than 30% of the stock market value of companies in the S&P 500 index."

In this respect, the combined company's name choice is of great interest in the case of a systematic M&A. The company can keep the acquirer's name, as in Pfizer's acquisition of Wyeth; it can also change its name to the target's name, as in SBC's acquisition of AT&T; it can make a name combination, as in Exxon's acquisition of Mobil for ExxonMobil, and in Chase Manhattan's acquisition of JP Morgan for JP Morgan Chase. At last, it can also take a new name, as in Standard Oil of California's acquisition of Gulf Oil for Chevron. Compared to non name change events, name change events are a signal about the business reorientation, corporate strategy and management attitude.

The systematic acquirer upon an M&A is inclined to keep its own identity, especially given the deterring huge costs, direct and indirect, associated with the name change. Note that here the value information signal of a name change is different from that of a name change in corporate environments without takeovers or substantial restructurings. Prior studies like Cooper, Dimitrov, and Rau (2001) reveal a significant name change effect during the Internet bubble. Upon the announcement of adding "dotcom" to its name, the name changing company receives a valuation boost. What differentiates a name change for a systematic M&A and a name change in a common environment is the systematic characteristic. In the former situation, at least two market portfolio components are directly involved, while in the latter situation at most one component is relevant and idiosyncratic risks are more concerned.

The merger of Westinghouse and CBS shows how name change signals the corporate strategy. The S&P 500 component Westinghouse acquired its index peer CBS in 1995, and then continued its transformation from industry conglomerate to media powerhouse. In 1996, it proceeded to spin off the industrial business, and renamed itself as CBS in 1997. Consider, what if Westinghouse changed its name to CBS upon this systematic merger rather than upon the business spinoff? The former hypothesized name change contains valuable predictive information, while the latter name change is only an identity correction to represent its current business. The market impact of a name change in 1995 would be greater because it corresponds to the size of Westinghouse (industry conglomerate) plus CBS (media giant), while the impact of a name change in 1997 after the industrial business spinoff is smaller due to the single involvement of media business.

The value information of name changes upon systematic M&As should be distinguished from the rest of corporate events, especially from name changes at common time.

1.3.4 Information rumination

The EMH predicts that prices fully reflect all the available information. As name change information is part of the M&A agreement between two companies, this information is public at the M&A announcement. Therefore, we expect the stock prices of both the acquirer and the target to adjust quickly to include this value information upon the deal announcement. When a systematic M&A finalizes, which is usually along with the S&P 500 index change announcement, there should be no significant abnormal return differences between the name change group and the non name change group, because the value information has already been incorporated into the stock price.

The M&A announcement acts as an information filter, while the abnormal return at the S&P 500 index change announcement is a proper measure of market efficiency. If the market is efficient, the abnormal return difference between the two groups at the index change announcement should be insignificant. However, if a significant difference is detected, the market efficiency is in question for the assimilation of name change value information. The return difference also implies that the price only partially reflects the available information for the name change, and the index change announcement triggers information rumination which results in a performance divergence between the two groups.

The choice of index change announcement as study event mitigates several important concerns like the M&A effect and the S&P 500 index change effect, because these effects are mainly related to the M&A announcement. The first concern is the M&A effect, for which prior studies like Betton, Eckbo, and Thorburn (2008) register that corporate takeovers benefit target firm shareholders and create value for the general shareholders of the bidder and the target upon the M&A announcement. Thus we don't regard it as a serious information contamination to the index change announcement. The second concern is about the S&P 500 index change effect, for which prior literature like Lynch and Mendenhall (1997) report significant positive abnormal returns for index additions and negative abnormal returns for index deletions upon the index change announcement. Because the systematic M&As are among the S&P 500 index components, the combined company is still a component, and the target stock would be exchanged or purchased at prespecified terms and then delisted, the addition effect and the deletion effect is not a consequential challenge. By setting the index change announcement as the event, we successfully remove the confounding value effects of the M&A and the S&P 500 index change. This isolation helps to concentrate on the name change information, which is unique to the name change group. The performance comparison between the name change group and the non name change group primarily reflects this effect.

Then we come to the mechanism that explains this information rumination. The rumination refers to the fact of significant stock reaction to stale information. Specifically, upon a systematic M&A announcement, the name change information is public but eclipsed by other more common M&A information types, such as deal size, deal parties, involved industries and so on. The name change is not necessary information to all systematic M&As, so the investor's attention is less attributed to this specific name change information as attention resource is limited. Peng and Xiong (2006) document that investor inattention extracts a category learning behavior, in which is market and sector-wide information is processed in priority than firm-specific information. As investors assimilate the more general M&A information upon the deal announcement, the index change announcement triggers rumination on the name change information. This rumination is partly provoked by the ticker change. Note that a name change causes a ticker change, which is crucial for security trading as the ticker is the paramount stock identity for investing activities. For example, in the name change systematic M&A among SBC and AT&T, the acquirer SBC changed its name to AT&T, as well as its ticker "SB" to the classic "T" owned by AT&T. In the name combination case, the ticker of Dow Chemical changed from "DOW" to "DWDP" after its acquisition of du Pont to form DowDuPont in 2017. In short, name change systematic M&A deals often lead to trading identity modifications upon deal completion, prompting investors to fully recognize the stale information of name change, which results in the rumination effect. This rumination effect brings about the abnormal return for name change M&As upon the index change announcement.

1.4 Thesis structure

This thesis unfolds over the three aspects to improve the global investment optimality in the presence of extreme risks, at the context of both portfolio selection and corporate finance. Chapter 2 deals with the higher order moments extension of the traditional mean variance optimization and CAPM framework. We consider an investor's investment choice with extreme risk aversion, as during the global financial crisis and the European sovereign debt crisis. We construct a mean variance kurtosis efficient portfolio frontier with the Pareto improvement method, empowered by the Dirichlet simulation to approximate the feasible set. The determination of the mean variance kurtosis efficient set uses the relationship between the mean variance efficient set and the mean variance kurtosis efficient set, for which a mean variance efficient portfolio must be a mean variance kurtosis efficient portfolio but not *vice versa*. We propose an empirical study of the S&P 500 index to implement the mean variance kurtosis optimization, where the industry sub-indices are used as individual assets. This chapter is based on the paper of Le Courtois and Xu (2017b), "Portfolio optimization when bond markets can crash."

Chapter 3 extends the model of Kuosmanen (2004) and develops an operational approach to test for the stochastic dominance efficiency of a given portfolio at any orders. Applying this approach to equity indices representing seventeen developed and developing markets across the globe, we find that all of these indices are inefficient, nearly always at order three and very often at order two, implying that all of the prudent and most of the risk averse investors would be better off not investing in these market indices. The indices are often dominated by individual industry sub-indices, with consumer goods, services, and utilities performing especially well. A simple trading rule based on past stochastic dominance information improves the average out-of-sample return of a global portfolio by 2% per year while simultaneously reducing the return standard deviation by 3% per year. It substantially limits global portfolio losses during the financial crises of 2007–2008. Portfolios of low-beta and low-volatility stocks consistently stochastically dominate the market indices and seem to be more desirable alternatives for prudent and risk-averse investors. This chapter is based on the paper of Kolokolova, Le Courtois, and Xu (2017), "Is it efficient to buy the index? A worldwide tour with stochastic dominance."

The role of stock market indices is an inherent thread across the whole thesis, running through the chapters and underpinning the discussion of asset allocation in the presence of extreme risks. For simplicity, in the first two chapters we focus on the portfolio optimization at the level of index sectors. Unlike the S&P 500 index or the FTSE 100 index with hundreds of components, the Dow Jones Industrial Average (DJIA) as a major stock index only includes 30 components and its compact size facilitates investment efficiency analysis at component level. Chapter 4 concentrates on the discussion of DJIA efficiency. The remarkable rise of DJIA in 2017 brings about extensive criticism that the DJIA has methodological drawbacks of limited corporate coverage and price weighted methodology compared to its peer of S&P 500 index, as reported in, for example, Armstrong (2017) and Rosenbaum (2017). To examine its efficiency as a performance benchmark, we construct various weighting DJIA variants from components' total return data between 1988 and 2017. We find that the price weighted variant outperforms the market value weighted variant, and the total return DJIA has the highest Sharpe ratio after turnover cost adjustment. Stochastic dominance pairwise comparison confirms the DJIA efficiency. We further perform an optimal portfolio analysis based on the mean variance optimization and the stochastic dominance optimization, finding that the in-sample optimality is not predictive. Compared to the DJIA, the portfolio weight gaining return enhancement and risk reduction in a certain year has no return or risk performance advantage in the next year, adding evidence to the DJIA efficiency. We show that the DJIA is not only a first dynamic index conforming to the modern index definition proposed by Lo (2016) but also an efficient performance benchmark. Our results challenge the common belief that the DJIA is badly constructed as a price weighted index. This chapter is based on the paper of Le Courtois and Xu (2018), "DJIA is more efficient than thought."

Chapter 5 examines the corporate name change effect on the sample of M&As among the S&P 500 companies from 1979 to 2017. This systematic sample synthesizes the development of aggregate M&A activities and is distinct from the other nonsystematic deals with key characteristics like huge economic size and symmetric industry distribution. The event choice of the S&P 500 index change announcement helps to isolate name change rumination effect from other confounding value effects as the M&A announcement effect, the S&P 500 index change effect and price pressure effect, because the three confounding effects are more related to the M&A deal announcement. We find that name change stimulates economically and statistically significant abnormal returns for the acquirers upon the index change announcements. The annualized short-term abnormal return within a week is about 60%, and the 1-year buy and hold abnormal return over non name change peers is about 10%. The results empirically corroborate our hypothesis that the index change announcement triggers investor rumination on name change information. This chapter is based on the paper of Le Courtois and Xu (2017a), "Corporate name changes of M&As among S&P 500 index."

Chapter 2

A high order CAPM when bond markets can crash

2.1 Introduction

The last decades have witnessed several economic disasters, among which the global financial crisis and the European sovereign debt crisis are the two most dramatic ones in recent memory. These events impact macro economy and financial markets. Asset prices become more volatile. The change is similar for the yields of government bonds, which assets are commonly considered as risk free. The phenomena of these extreme events call for better explanations. The traditional finance theory especially the classic benchmark of Capital Asset Pricing Model (CAPM) is based on the mean variance optimization without specific attention to extreme risk. This concern of event risk can be consequential to investor's behavior and portfolio construction. Some research try to account for this concern by incorporating liquidity risk as an additional systematic risk factor as in Acharya and Pedersen (2005), so economic crises are interpreted as liquidity crises. The key drawback for this additional systematic risk factor approach is that it has no suggestion on the portfolio construction. Moreover, there is no plausible utility function adjustment corresponding to the additional risk factor. Generally, expected utility approximation still follows the polynomial approximation pioneered by Pratt (1964). It's obvious that the additional risk factor cannot be independently identified in the expected utility approximation, thus this method cannot give an intuitive answer for the efficient portfolio identification and optimal portfolio selection according to individual utility optimization. While other research, for example, De Athayde and Flôres (2004), and Briec, Kerstens, and Jokung (2007) develop higher order moment efficient frontier to at order of three and four. However, their efficient portfolio selection methods of constrained variance minimization and shortage function can misclassify inefficient portfolios as efficient. Neither the additional risk factor approach nor the higher moment CAPM approach is specifically dedicated to extreme risk as well as a hike in risk free rate in the analysis.

In this paper, we capture the extreme risk by use of kurtosis, and consider a novel portfolio selection problem based on the expected return as well as variance and kurtosis of the asset return probability distribution.[1] Efficient portfolios and efficient frontier are achieved by the Pareto improvement method, which recognizes portfolios with inferior moment profile to another portfolio as inefficient. The Pareto

[1] When the terms of skewness and kurtosis go with variance in the context, usually they refer to the third moment and the fourth moment for convenience. This denotation is commonly used in papers of portfolio optimization with higher order moments. For example, in Briec, Kerstens, and Jokung (2007, p. 138) the skewness is defined as the third moment explicitly. In Jurczenko, Maillet, and Merlin (2006) and other papers, the same denotation is used.

improvement method, in the spirit of the Pareto efficiency of neoclassical economics, is a generalized and simple algorithm for inefficient portfolios detection and is free from the aforementioned problem of misclassification by constrained optimization and shortage function method. This intuitive Pareto improvement method is fueled by Dirichlet sampling for feasible portfolio set approximation, which is another innovation of this paper. The Dirichlet distribution generates multivariate vectors with non-negative values that sum to 1, perfectly consistent with the non-short selling portfolio formation. Deducing the relationship between mean variance efficient portfolios and mean variance kurtosis efficient portfolios, we vary Dirichlet concentration parameters and simulate 5.1 million portfolios as the input to Pareto improvement method. After achieving the mean variance kurtosis efficient frontier, we correspondingly implement an expected utility approximation with these three moments and obtain an original linear approximation for the utility specification with extreme risk aversion.

In modern finance, risk is primarily defined as variance. Remarkably, Markowitz (1952) interprets risk as return variance and investor's goal is to minimize portfolio variance given a certain level of expected portfolio return. The foundational Sharpe (1964), Lintner (1965) and Mossin (1966) specify the ratio of an asset's covariance with the market to market variance as systematic risk, and CAPM instructs that only systematic risk gets compensated by the market. However, the effect of higher order moments on asset pricing and return determination gets more attention as investors' and researchers' increasing interest in extreme events and associated impact on asset prices, especially after the 2008 global financial crisis. Barro (2006) explores the effect of three rare economic disasters, namely two World Wars and the Great Depression, on asset markets, and calibrates disaster probability to account for asset pricing puzzles like high equity premium. Cvitanić, Polimenis, and Zapatero (2008) warn that ignoring the effect of higher moments leads to wealth loss by overinvesting in risky assets. Aït-Sahalia, Cacho-Diaz, and Hurd (2009) advise that investors should concentrate on the management of the downward jump risk exposure in their portfolio choice decision. Aït-Sahalia and Matthys (2017) further derive the optimal weights showing that the investor's robust investment in the risky asset is lower when there is model uncertainty of jump or drift. Bates (2012) inspects US stock market crash risk since 1926 to 2010, and features the importance of fat tail property in the explanation of market crash, which can be captured by a time-changed Lévy process. Santa-Clara and Yan (2010), Benzoni, Collin-Dufresne, and Goldstein (2011), Bollerslev and Todorov (2011), Gabaix (2012), Drechsler (2013), Wachter (2013) and others also study general equilibrium models subject to tail events, or rare jump shocks, in attempt to explain market anomalies like the equity premium puzzle. Extreme risk becomes an important concern for asset pricing models.

There are also researchers trying to augment the CAPM by incorporating the effect of higher moments in the belief that the asset profile including higher order moments is a better description of asset's risk characteristics than variance. Kraus and Litzenberger (1976) derive a three moment CAPM and specify a quadratic characteristic line, which is the traditional security market line plus a quadratic term of market return volatility on the right hand side. They find that the coefficient of gamma capturing the third moment effect is statistically significant, and show that the preference for skewness is systematic in determining asset's return. Similarly, Fang and Lai (1997) study the effect of kurtosis on the determination of asset's risk premium. Their derivation of a four-moment CAPM is based on the understanding that increasing portfolio size cannot diversify skewness and kurtosis. They get empirical evidence that investors receive compensation for bearing higher kurtosis. Notably, studies for efficient frontier extension to higher dimension are not as numerous as for CAPM developments with additional risk factors. The most representative expansion of the efficient frontier to higher moments uses shortage function to find available improvement to portfolio moment profile as a way for portfolio efficiency evaluation. Typically, Briec, Kerstens, and Jokung (2007) give an instructive description of how to detect mean variance skewness efficient portfolios using shortage function method. If no directional improvement exists for the moment profile of evaluated portfolio within the feasible portfolio set, the evaluated portfolio can be labeled as efficient; otherwise, the detected portfolio achieving the largest directional improvement is efficient. Based on this method, Briec and Kerstens (2010) make an extension to non-convex mean variance skewness space, and Kerstens, Mounir, and Woestyne (2011) give an illustration of mean variance skewness efficient frontier. Jurczenko, Maillet, and Merlin (2006) step further to the mean variance skewness kurtosis space, where they identify the mean variance skewness kurtosis efficient portfolios in the same methodology of Briec, Kerstens, and Jokung (2007). They also make an attempt to display the efficient frontiers in the mean variance skewness space and the mean variance kurtosis space. Some other methods include multiple-objective approach by Lai (1991), and Bayesian approach by Harvey, Liechty, Liechty, and Müller (2010). For these methods, how to compromise several objectives and to disentangle the effect of higher moments from estimation error in the coherent treamtment of Baysian framework is still worth further investigation.

However, the shortage function method, as well as the closely related constrained variance minimization program, has the exposure to the mis-identification of efficient portfolios. Constrained optimization, as used in De Athayde and Flôres (2004) and Mencía and Sentana (2009), may misclassify inefficient portfolio as efficient by misaligning the objective and the constraints. On the other hand, the shortage function utilizes positive profile improvement along all directions, thus it can misstate the efficiency of portfolios with marginal profile improvement

Kurtosis is appropriate to measure the tail thickness of the return distribution. It's good at reflecting extreme risk, but there is hardly research properly focusing on kurtosis in portfolio selection without divergent consideration such as skewness. This paper proposes a first portfolio selection method with explicit emphasis on kurtosis, taking advantage of Pareto improvement method which is a straightforward algorithm of portfolio selection to minimize the misclassification error.

We contribute to the portfolio simulation literature as well. Most discussions on sampling methods are about alleviating the problem of estimation errors and associated high sensitivity of output to input. In the mean variance framework, the improvements by use of simulation includes "resampled efficient frontier" as illustrated by Michaud and Michaud (2008). The sampling method can also be used for random portfolio formation. There are many papers on random portfolio construction from a certain security pool at a predetermined size. For example, Canina, Michaely, Thaler, and Womack (1998) show that the compounded CRSP equal-weighted return can be misleading by randomly forming five kinds of equal-weighted portfolios at sizes of 10, 100, 300, 600 and 900 from the CRSP security pool. Sunder (1980), Jobson and Korkie (1982), So (2013), Bekaert, Harvey, Lundblad, and Siegel (2011), Grullon, Michenaud, and Weston (2015), Affleck-Graves and McDonald (1990), and Eun, Huang, and Lai (2008) adopt similar portfolio creation methods for their empirical studies. This random portfolio construction is also used when enumeration is impossible. For example, Porter and Bey (1974) generate random equal-weighted portfolios at various sizes to form second order stochastic dominance efficient sets and compare the sets with mean variance efficient sets. Note that they take on equal-weighted scheme at the randomly chosen securities from the feasible stocks pool. As the equal-weighted scheme is not representative enough for general portfolio development, we innovatively use the Dirichlet distribution to generate portfolio weights for the innate fitness of Dirichlet distribution. The inference that mean variance efficient portfolios are still mean variance kurtosis efficient facilitates the efficient portfolio identification and increases the efficacy of sample efficient frontier with Dirichlet simulations.[2] More than that, Dirichlet simulations make the Pareto improvement method implementable, and the combination of the two programs offers a simple and straightforward approach for the construction of higher moments efficient frontier.

The rest of this paper is arranged as follows. Section 2.2 presents the Dirichlet approach for efficient frontier generation in the presence of extreme risks. Section 2.3 discusses the linear approximation of expected utility when extreme risks are featured and considered. Section 2.4 proposes an empirical study of 10 economic sector sub-indices of the S&P 500 index for the tangency of Dirichlet efficient frontier and extreme risk utility surface. Section 2.5 concludes the paper.

2.2 Dirichlet approach for high dimensional efficient frontier generation

2.2.1 Extreme risk and kurtosis

Extreme events refer to those ostensibly rare yet bringing consequential aftermath, such as economic depressions, financial crises and market crashes. These events lie at the tail of the asset return probability distribution.

Suppose the financial market has N risky assets, and an investor constructs portfolio by deciding on the weight vector $w = [w_1\ w_2\ \dots\ w_N]'$ where w_i is the portfolio weight allocated to asset i with $i \in [1, 2, \dots, N]$. For a regular portfolio, it's guaranteed that $\Sigma_{i=1}^{N} w_i = 1$, plus $w_i \geqslant 0$ in the case of no short selling. Each asset has its return observation series, as well as its expected return. For example, $r_i = [r_{i1}\ r_{i2}\ \dots\ r_{iT}]'$ is the return series of asset i, r_{it} its return observation at time t, $E[r_i]$ its expected return. Then the portfolio return series and its expected return can be characterized as $R = [r_1\ r_2\ \dots\ r_N]w$ and $E[R] = [E[r_1]\ E[r_2]\ \dots\ E[r_N]]w$.

We calculate the central moment of order k as

$$m_k = E[R - E[R]]^k.$$

Ranging k from 2 to 4, we get M2 (variance), M3 (skewness), and M4 (kurtosis). From the definition of kth order moment, it's apparent that the deviation of extreme R observations from the expected value have more significant effect on higher order moments. This demonstrates the advantage of higher order moment in capturing the impact of extreme returns, since the compounding index is bigger.

Kurtosis is frequently used to capture extreme risks, which have come into the spotlight after the recent series of economic crises and disasters. For example, Barro (2006) uses kurtosis to reflect disaster risk and shows that the economic disasters of the last century led to a 29% average decline in real per capita GDP. He finds

[2] Some researches have already shown that Monte Carlo simulation can be used to compute optimal portfolios. See, for example, Cvitanić, Goukasian, and Zapatero (2003) and Detemple, Garcia, and Rindisbacher (2003)

that disaster risk has considerable effects on asset returns and on the equity premium. Dittmar (2002), Eraker, Johannes, and Polson (2003), Liu, Longstaff, and Pan (2003), Poon, Rockinger, and Tawn (2003), Bakshi and Madan (2006), Bates (2008), Todorov (2009), Benzoni, Collin-Dufresne, and Goldstein (2011), Bollerslev and Todorov (2011), Bates (2012), Gabaix (2012), Drechsler (2013), Wachter (2013), and Cremers, Halling, and Weinbaum (2015), among many, notice the significant role of extreme risks in the determination of returns. More interestingly, Malmendier and Nagel (2011) find that low stock returns contribute to shaping individual preferences towards greater risk aversion, and this effect is stronger for young people. Thus, the effect of extreme risks is not only contemporaneous but also prospective.

By nature, kurtosis is a complementary risk indicator to variance for two reasons. The first reason is that, risk encompasses not only downside risk but also upside risk, as Damodaran (2003, Chapter 2) advocates. In this sense, kurtosis is more appropriate than skewness at depicting event risks, because it assigns the same penalties to extreme events, no matter positive or negative. The second reason is that extreme negative events are more significant than extreme positive events, contributing to a deflated skewness benefit and an escalated dispersion apprehension. Cont (2001) obtains that the large upward movements are not as equally as the large drawdowns. This loss in asymmetry implies that any potential positive extreme values are likely offset by negative extreme values of similar or greater size. When comparing two distributions with negative skewness, kurtosis is more informative about the general dispersion level of the distributions.

We further examine the loss asymmetry from the perspective of investment value preservation. The return gains needed to recover from the return losses are disproportionate. For instance, a return loss of 10% in previous trading day asks for a return gain of 11% in this trading day for value recovery, while a loss of 50% needs a large gain of 100% just to keep the value. By common sense, a 50% loss is much more possible than a 100% gain. Therefore, extreme negative returns still leave investment ruins behind, even subsequently a similar size of gain partly recoups the value. What is worse, empirical record reminds that extreme negative returns are more frequent than extreme positive returns in a tense way. Since 1950s, the largest daily gain for S&P 500 index is 11.58% in Oct 13, 2008, only about half of the largest daily loss of 20.47% in Oct 19, 1987. Even the 20th largest daily loss of 5.28% is still greater than its gain counterpart of 4.76% in size.[3] If we take a longer horizon back to 1835 for the S&P 500 total return index, the extreme risk is more pronounced although the general growth trend for the index is quite clear. The maximum market drawdown is in the Great Depression with a log drawdown over 1.8, or about 85% value shrinkage equivalently. In the 180 years from 1835 to 2015, the probability of a 20% drawdown is about 10%. 23.1% of the period since 1927 to 2016 for the S&P 500 index has a drawdown of 20% or even worse.[4] The value preservation deduction and these empirical statistics mitigate the aforementioned concern.

It's also well discussed about investor's preference for odd moments and aversion to even moments **p**[e.g.][]jondeau2006optimal, and kurtosis better depicts investor's psychology of risk as disutility. This is also consistent with the traditional treatment of variance as a risk indicator, therefore we extend the risk definition beyond variance, taking kurtosis as the complementary risk measure to variance for its better characterization of extreme risks. That's why we primarily focus on kurtosis

[3] More details see https://en.wikipedia.org/wiki/List_of_largest_daily_changes_in_the_S%26P_500_Index.

[4] Statistics are from "180 Years of Market Drawdowns" by Robert Frey and the comment by Ben Carlson.

for extreme risks rather than skewness, despite that to some extent skewness also magnifies the effect of tail events with an exponent of 3.

We highlight the necessity of taking kurtosis into portfolio selection consideration for an extreme risk averse investor with a simple example as shown by Table 2.1.

TABLE 2.1: Tail return and moments: a portfolio example

	R	$R-E[R]$	$(R-E[R])^2$	$(R-E[R])^4$
		Portfolio A		
1	0	0.036	0.001296	2.82E-12
2	-0.01	0.026	0.000676	2.09E-13
3	-0.02	0.016	0.000256	4.29E-15
4	-0.12	-0.084	0.007056	2.48E-09
5	-0.03	0.006	0.000036	1.68E-18
Average	-0.036	0	0.001864	4.96E-10
		Portfolio B		
1	0.00303	0.040424	0.001634	7.13E-12
2	0	0.037394	0.001398	3.82E-12
3	-0.09	-0.05261	0.002767	5.87E-11
4	-0.09	-0.05261	0.002767	5.87E-11
5	-0.01	0.027394	0.00075	3.17E-13
Average	-0.03739	0	0.001864	2.57E-11
		Portfolio C		
1	0.002	0.038	0.001444	4.35E-12
2	-0.004	0.032	0.001024	1.10E-12
3	-0.088	-0.052	0.002704	5.35E-11
4	-0.09	-0.054	0.002916	7.23E-11
5	0	0.036	0.001296	2.82E-12
Average	-0.036	0	0.001877	2.68E-11

Suppose we have 3 portfolios, A, B and C, and each of them has 5 realizations for return R. Return scenarios are with equal probabilities, and average gives mean values for corresponding items. For simplicity, we refer respectively variance and kurtosis as

$$E(R-E[R])^2 = \Sigma(R-E[R])^2/5,$$

$$E(R-E[R])^4 = \Sigma(R-E[R])^4/5.$$

Note that there is a tail return of -12% in Portfolio A, while Portfolios B and C have a relatively modest minimum return of -9%. As a result, kurtosis of Portfolio A is about 20 times greater than the kurtosis of Portfolios B and C. If an investor is extreme risk averse, he or she trades off variance and kurtosis, so C is more preferred than A at the same mean return of -3.6%, even the variance of C is still slightly higher than A. Also he or she may well choose B over A at the same variance of 0.1864% if he or she trades off mean and kurtosis, even the mean return of B is yet slightly smaller than A. This kind of choice is also consistent with Tversky and Kahneman (1986) where loss aversion is an important property of value function and extreme negative losses are overweighed.

There are several reasons why an investor is repugnant to extreme returns, especially extreme negative returns. The following reasons are some explanations to the extreme risk aversion.

First, the financing burden transmitted by mark-to-market system. Individual investors especially those using futures and other derivatives to trade the underlying asset have to meet the margin call, which is often triggered by an extreme return. If the return is not so extreme, the investors probably don't have to make adjustments since the margin surplus may offset the loss. Also, institutional investors especially mutual funds and exchange-traded funds are more sensitive to such big price changes, since notable fund outflows and share redemptions may force them to realize losses and restrains institutional investors' maneuver for subsequent trading and value recovery. Even for hedge funds, extreme returns also have great impact to managers due to the requirements of hurdle rate and high water mark. The failure of LTCM is an exhibition that how wild asset price movements can be destructive.

Second, the danger of evolving into fire sales. When an asset is experiencing extreme negative return, it has prospective downward price pressure as investors holding this asset can go into panic and compete for sales. Therefore, the divergence of market price and intrinsic value can be wider and finally the competition leads to fire sales for this asset. Not only individual investors can get involved in this vicious circle, but also intuitional investors like mutual funds and hedge funds for their distressed selling, as explained by Coval and Stafford (2007) and Stein (2009).

Third, financial contagion across assets. The effect of extreme returns is not restricted to one specific asset, and it can spillover to other assets, asset classes, and financial markets. Canaries in the coal mine, the sharp price decline excites the investor psychology, which can be contagious since the rising across-sectional correlation and financial globalization. This systematic correlation is enhanced in bear markets other than bull markets for extreme events, see Longin and Solnik (2001) and Bae, Karolyi, and Stulz (2003).

This example reaffirms our contention that variance doesn't capture extreme risk and kurtosis is its excellent companion. We see for the three portfolios, they share similar expected return and variance. In traditional mean variance framework, Portfolio A is optimal since it has higher expected return at the same variance level compared to Portfolio B, and has smaller variance at the same expected return level compared to Portfolio C. However, it is this diagnosed optimal portfolio that carries the highest extreme risk with much greater kurtosis. As stated above, a loss of 12% is more disastrous than a loss of 9% for the portfolio development and performance assessment, while variance only gives a hint on the extreme risk rather than an alarm as kurtosis gives. For Portfolio A, the quadratic term for -12% scenario is about 6 times greater than that for the 0% scenario, but its quartic term is about 1,000 times greater than its counterpart. This comparison highlights the advantage of kurtosis in capturing the impact of extreme returns. This fact reaffirms the necessity to include kurtosis in the portfolio selection procedure.

2.2.2 Pareto improvement method

We first review constrained optimization and shortage function, the two most representative methods in higher moment efficient portfolio identification in literature. We show the incompetency of the two methods in detecting marginal profile improvements. We introduce the more general and simple Pareto improvement method. Comparing the Pareto improvement method with the other two methods with a simple example, we affirm the merit of Pareto improvement method as well as the potential pitfall of the two methods in specifying efficient portfolio with higher moments. This example also reveals the subtle relationship of efficient sets in mean variance space and in higher order moment space, which gives a clue about the composition for the set of higher moment efficient portfolios.

For each portfolio weight vector w, we compute its corresponding M1, M2, and M4, stacking them as the moment profile

$$\mathcal{P} = [M1\ M2\ M4]'.$$

The classic optimization problem for the mean variance analysis is

$$\begin{aligned} \text{minimize} \quad & M2 \\ \text{subject to} \quad & M1 \geqslant \mu \end{aligned}, \tag{P1}$$

where M1 and M2 are elements in the portfolio's moment profile, and μ a specified expected return level. The optimal portfolio by P1 has the minimum M2 under the condition that its M1 is no less than the prespecified μ. This is a constrained optimization problem in the two dimensional space of M1 and M2, and its natural extension to the three dimensional space of M1, M2, and M4 is

$$\begin{aligned} \text{minimize} \quad & M4 \\ \text{subject to} \quad & M1 \geqslant \mu \\ & M2 \leqslant \sigma^2 \end{aligned}, \tag{P2}$$

where the size of portfolio's moment profile is 3, and the added σ^2 a specified variance level. The optimal solution of P2 has the minimum M4 with the constraints that M1 is no less than μ and M2 is no bigger than σ^2.

The constrained optimization method for higher moment pioneered by De Athayde and Flôres (2004) is essentially the same as P2. Since De Athayde and Flôres (2004) discuss efficient portfolios in the M1-M2-M3 space, the minimization objective is M2 with constraints that M1 is no less than μ and M3 no less than a specified third moment level s^3. They state the problem in the method of Lagrange multipliers as follows[5]

$$\min_w \mathscr{L} = M2 + \lambda_1(\mu - M1) + \lambda_2(s^3 - M3)$$

where λ_1 and λ_2 are two associated Lagrange multipliers. As the minimization program suggests, this method cannot produce a pure efficient portfolio of risky assets. In other words, this constrained optimization method has no link with the two fund separation, and the optimal portfolios by this method cannot be characterized as a common risky investment part plus a riskfree investment part.

Shortage function, though in nature still a constrained optimization problem and introduced into portfolio optimization with higher moment by Briec, Kerstens, and Jokung (2007), is a little more complex. Its intuition is clear: for a portfolio, if we can find another portfolio with better moment profile, then this portfolio is inefficient and the portfolio with the largest profile improvement is efficient. Assume we evaluate portfolio i with its moment profile $\mathcal{P}^i = [M1^i\ M2^i\ M4^i]'$. If we can find another

[5]This method involves the riskfree asset in the minimum variance portfolio determination, explicated in De Athayde and Flôres (2004, p. 1339) as

$$\min_w \mathcal{L} = w'\Sigma w + \lambda_1(\mu_P - w'\mu - (1 - w'\mathbf{1})r_f) + \lambda_2(s_P^3 - w'S(w \otimes w))$$

where $\otimes$ is the tensor product.

portfolio j for which $M1^j > M1^i$, $M2^j < M2^i$, and $M4^j < M4^i$, then we say portfolio i is inefficient since $\mathcal{P}^j \succ \mathcal{P}^i$. If there is no such portfolio achieving dominating moment profile over it, then portfolio i is efficient. Efficient portfolios are those without any dominating moment profiles.

More formally, a directional improvement is

$$g = [g_1 \ \ -g_2 \ \ -g_4]^{'},$$

with all positive elements of g_1, g_2, and g_4. For an evaluated portfolio i with moment profile $\mathcal{P}^i$, its shortage function is[6]

$$\begin{aligned} \text{maximize} \quad & \delta \\ \text{subject to} \quad & M1^i + \delta g_1 \leqslant \ M1 \\ & M2^i - \delta g_2 \geqslant \ M2 \\ & M4^i - \delta g_4 \geqslant \ M4 \end{aligned} \quad , \tag{P3}$$

where δ, M1, M2, and M4 are all correspond to the portfolios in the feasible set. A positive δ means that the evaluated portfolio i is inefficient, and the portfolio with δ unit of the directional improvement g is efficient relative to it. If δ is 0, then portfolio i is efficient along the direction of g. The case of negative δ makes no sense, because it is equivalent to positive δ accompanied by a negative directional improvement $-g$ which is in fact a moment profile deterioration. It implies that the evaluated portfolio is not attainable within the feasible set.

We can rewrite P3 in a concise way:

$$\begin{aligned} \text{maximize} \quad & \delta \\ \text{subject to} \quad & \mathcal{P}^i + \delta g \preceq \ \mathcal{P} \end{aligned} \quad . \tag{P4}$$

P4 indicates that the shortage function method is a variant of the constrained optimization method. Now we make the moment profile explicitly relative to the portfolio weight vector

$$\begin{aligned} \underset{w,\delta}{\text{minimize}} \quad & (\mathbf{0}_{1\times N} \ \ -1) \begin{pmatrix} w \\ \delta \end{pmatrix} \\ \text{subject to} \quad & \mathcal{P}^i + \delta g \preceq \ \mathcal{P}(w) \end{aligned} \quad , \tag{P5}$$

where $\mathbf{0}_{1\times N}$ is a 1 by N vector full of 0.

The shortage function method has an evident limitation: the choice of the directional improvement is arbitrary and this vector cannot coincide with its projections in the space. That is to say, g cannot include 0 in the default optimization problem otherwise the computational complexity rockets. This vulnerable characteristic prompts us for a more general method taking into consideration of marginal improvement including 0 element in directional vector and free from the arbitrary choice.

The motivation of Pareto improvement method is the same with that of the shortage function method: only the portfolios without any moment profile improvement can be labeled as efficient. Given a well-defined directional improvement g and a set of evaluated portfolios, the shortage function method produces the efficient portfolio set. For each evaluated portfolio, P4 is implemented to spot one corresponding

[6] We rephrase the problem in the context of mean variance kurtosis space. For details refere to the third section of Briec, Kerstens, and Jokung (2007).

efficient portfolio. On the contrary, the Pareto improvement method screens out inefficient portfolios in the feasible set, and this deletion does not depend on the directional improvement g specification. Portfolios whose moment profiles are found to be dominated by any other portfolio in the pairwise comparison are removed immediately from the candidate pool for portfolio efficient set. The surviving portfolios are efficient and they constitute the efficient frontier.

We make formal exhibition for this method. Assume we have a feasible portfolio set

$$\Omega = \{w \in \mathbb{R}^N; \Sigma_{i=1}^N w_i = 1, w \geqslant \mathbf{0}\}.$$

Corresponding moment calculations are defined in Section 2.2.1. The moment profile $\mathcal{P}$ is a function of w

$$\mathcal{P}(w) = [M1(w)\ M2(w)\ M4(w)]^{'}.$$

The set for portfolios' moment profiles is defined as

$$\mathscr{P} = \{\mathcal{P}(w); w \in \Omega\}.$$

A Pareto improvement is an available dominance surplus over a portfolio's moment profile.

Definition 2.2.1. *Portfolio i has a Pareto improvement relative to Portfolio j, if and only if*

$$\langle \mathcal{P}^j, \mathcal{P}^i \rangle \gneqq \mathbf{0},$$

where $\langle \mathcal{P}^j, \mathcal{P}^i \rangle = [M1^j - M1^i\ M2^i - M2^j\ M4^i - M4^j]^{'}$ *and* $\gneqq$ *means "no less than but not equal to." For an arbitrary vector* ε *and the zero vector* $\mathbf{0}$ *in same size,* $\varepsilon \gneqq \mathbf{0}$ *indicates that each element of* ε *is no less than 0 but at least one is not 0.*

Thus, the set for efficient portfolios' moment profiles, $\mathscr{E}\mathscr{P}$, can be restated as a subset of $\mathscr{P}$,

$$\mathscr{E}\mathscr{P} = \{\mathcal{P}(w); \langle \mathcal{P}(w^{'}), \mathcal{P}(w) \rangle \gneqq \mathbf{0} \Rightarrow w^{'} \notin \Omega\}.$$

Similarly, the set of efficient portfolios is a subset of the feasible portfolio set Ω,

$$\Phi = \{w \in \Omega; \mathcal{P}(w) \in \mathscr{E}\mathscr{P}\}.$$

Compared with the shortage function method, the Pareto improvement method has two important advantages of generality and simplicity. It is more general, because the definition of the Pareto improvement is more relaxed than that of the directional improvement. The Pareto improvement is not pre-specified, so it is free from the potential optimization bias due to the choice of directional improvement, which is embedded in P4 to perform on all evaluated portfolios for corresponding efficient portfolios. And the Pareto improvement can be any linear combination of the standard bases with non-negative coefficients, while the pre-specified directional improvement in the shortage function method has to be a linear combination of the standard bases with all positive coefficients. In other words, the shortage function method demands improvement to be along each dimension of the moment profile, while the Pareto improvement method only requires improvement to be along at least one dimension. This method epitomizes the very essence of Pareto efficiency analysis in neoclassical economics, applying such spirit in the context of portfolio selection.

Moreover, this method is simpler than the shortage function method in terms of computational complexity. By the shortage function method, a large set of evaluated portfolios is necessary for a commensurate set of efficient portfolios and the optimization is executed for each evaluated portfolio. For example, if we want 1,000 efficient portfolios, we have to prepare at least 1,000 evaluated portfolios and compute P4 every time. However, by the Pareto improvement method only pairwise comparisons among portfolio moment profiles are needed, thus it is much less computationally demanding. It's effective since the confirmation of only one Pareto improvement relative to a portfolio is enough to classify the portfolios as inefficient.

Some may argue that the efficacy of Pareto improvement method relies on the defining of the feasible portfolio set Ω, and its related set of moment profiles $\mathscr{P}$. Actually, the shortage function method also depends on $\mathscr{P}$, which is shown by the right hand side of the constraint in P4. Briec, Kerstens, and Jokung (2007) explicitly admit that the optimization has to search over "all possible combinations of returns, risk, and skewness of the portfolios" for efficient portfolios, which clearly refers to $\mathscr{P}$. The claim that the Pareto improvement method is not as theoretically sound as the shortage function with respect to $\mathscr{P}$ is untenable. The key is to sample as representatively as possible for $\mathscr{P}$, which will be discussed in Section 2.2.3.

We use a simple example to illustrate the superiority of the Pareto improvement method for high dimensional efficient frontier construction, as well as the reason why the classical constrained optimization method may fail for that.

TABLE 2.2: Optimization and efficient portfolios

	M1	M2	M4
1	8%	12%	2.1%/1.9%
2	8%	10%	2.1%
3	6%	11%	2%
4	6%	9%	2%

Table 2.2 gives 4 portfolios with their corresponding moment profiles. Now we come to see the nuance of method implementations in 2-dimensional space and 3-dimensional space. The mean variance optimization P1 easily spots Portfolio 2 as efficient if $\mu = 8\%$ since it gives lower variance at 10% than Portfolio 1, and spots Portfolio 4 as efficient if $\mu = 6\%$ since its variance of 9% is lower than any other portfolio. By the Pareto improvement method, Portfolio 1 has a Pareto improvement towards Portfolio 2 along the variance dimension. The situation is similar for the comparison of Portfolios 3 and 4. Based on the Pareto improvements identified, this method classifies Portfolios 1 and 3 as inefficient. The constrained optimization method yields the same solution as the Pareto improvement method does in the mean variance space, since the duality connection of mean and variance is direct.

In the mean variance kurtosis space, P2 is the constrained optimization scheme. Now set $\mu = 6\%$ and $\sigma^2 = 11\%$, the solutions are Portfolios 3 and 4, both giving the same fourth moment of 2%. But we can clearly identify the dominance of Portfolio 4 over Portfolio 3 by its lower variance, which is suggested by the Pareto improvement method. The pair of Portfolios 1 and 2 is a similar case, where P2 spots them all as efficient at $\mu = 8\%$ and $\sigma^2 = 12\%$ for the same M4 of 2.1%, while the Pareto improvement method gives a different answer which makes more sense. Thus, in high dimensional space the constrained optimization method may misclassify inefficient portfolio as efficient.

The nub of this error lies on the fact that the three dimensional connection of M1, M2, and M4 is intertwined, and the constrained optimization has to search all the data grid of M1 and M2 taking them as pure constraints but ignoring the innate portfolio quality revealed by the M1 and M2 information. Yet, the Pareto improvement method circumvents such an error by equalizing constraints and objective. Therefore, this method is better than the constrained optimization method and its variant of the shortage function method in high dimensional portfolio optimization. The Pareto improvement method also obviates the technical difficulty of reaching an explicit solution beyond the mean variance space with the traditional optimization, as indicated by the attempt of De Athayde and Flôres (2004).

Another important point demonstrated in the above example is that Portfolios 2 and 4, which are efficient in the mean variance space, are also efficient in the mean variance kurtosis space. Due to their advantage in the $[M1\ M2]$ part, they cannot be inferior in each dimension no matter if their M4 are superior or not. Only portfolios with all inferior moments will be classified as inefficient by the Pareto improvement method. So, we can infer a key relationship between the set of mean variance efficient portfolios and the set of mean variance kurtosis efficient portfolios: the former set is a subset of the latter one. This inference plays an important role at increasing the simulation accuracy in the later step.

We have another relationship inference about the sets of efficient portfolios. If Portfolio 1 has a fourth moment of 1.9%, then it is also efficient in the mean variance kurtosis space since it has the overall lowest kurtosis. This suggests that an inefficient portfolio in the mean-variance space can be efficient in the mean variance kurtosis space. The two inferences give us a hint on the composition of the set of mean variance kurtosis efficient portfolios: the mean variance efficient portfolios are all mean variance kurtosis efficient, and the mean variance inefficient portfolios with kurtosis edge can be mean variance kurtosis efficient.

2.2.3 Dirichlet simulations

As previously stated, it's pivotal to sample representative feasible portfolio set Ω in order to take full advantage of the Pareto improvement method. The representative sampling for portfolios is both practically and theoretically feasible. In practice, market frictions deter minimal portfolio adjustments. Because of trading costs like brokerage commissions, fees, and taxes, a small portfolio change, say, with a 0.1% increase in one asset's weight at the cost of another asset's is not profitable, or even incurs losses. Several papers have documented the large economic losses by investors' overtrading costs, like Barber, Lee, Liu, and Odean (2008).

Chotikapanich and Griffiths (2002) initiate the use of Dirichlet distribution for sampling income shares to estimate Lorenz curve. This offers a theoretical inspiration that the Dirichlet distribution can be also used for sampling portfolio weights, whose characteristics are similar to income shares. Next we introduce the Dirichlet distribution and its application in the context of portfolio selection.

The Dirichlet distribution is the multivariate generalization of the beta distribution. It has the following probability density function:

$$f(x_1, x_2, ..., x_K; \alpha_1, \alpha_2, ..., \alpha_K) = \frac{\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \prod_{i=1}^{K} w_i^{\alpha_{i-1}}$$

where $\alpha_1, \alpha_2, ..., \alpha_K$ are concentration parameters and are all positive. $\Gamma(\cdot)$ is the Gamma function such that

$$\Gamma(\alpha_i) = \int_0^\infty x^{\alpha_i - 1} e^{-x} dx.$$

For the Dirichlet distribution, $\sum_{i=1}^{K} x_i = 1$ and $0 \leqslant x_i$.

The property of regular sum makes Dirichlet distribution a naturally appropriate distribution scheme for simulating portfolio weights. Its non-negativity, consistent with the constraint of non-short selling, also includes extreme cases where individual assets are fully invested.

If we define $\alpha_0 = \sum_{i=1}^{K} \alpha_i$, then

$$E[X_i] = \frac{\alpha_i}{\alpha_0},$$

$$\sigma_i^2 = \frac{\alpha_i(\alpha_0 - \alpha_i)}{\alpha_0^2(\alpha_0 + 1)},$$

showing that the concentration parameters affect the marginal distributions.

We first consider the Dirichlet distribution with equal concentration parameters to simulate the weights of three-asset portfolios as an exhibition. For a simulated portfolio scheme, x_1 is the weight for Asset 1, x_2 the weight for Asset 2, and x_3 the weight for Asset 3, and $\alpha = \alpha_1 = \alpha_2 = \alpha_3$. As stated above, the weight schemes generated by Drichlet distribution here form a standard 2 simplex in the 3 dimensional space, as shown by Figure 2.1. The impact of concentration parameters on simulated outcomes is straightforward. We see that the simulated points are evenly scattered when α is 1, and in this case, the Dirichlet distribution is equal to the uniform distribution. This special setting is called the flat Dirichlet distribution. As α gets smaller, points are less sparse toward the boundaries. That's to say, extreme silutaions including 0 values are more frequent. If α is bigger than 1, points are more condensed around the simplex center point. That's to say, extreme simulates including similar values are more frequent. For symmetric Dirichlet distribution, we can also get a clue about the impact of concentration parameter from the expected value and variance equations. In the case of 3 assets, the expected values are equally to be $1/3$, and the variance is $\frac{2}{9(3\alpha+1)}$. So, the four sub-cases in Figure 2.1 with varying α share the same expected value, but the variance is negatively related with α.

Then we come to see how unequal parameters lead to asymmetric concentration, as shown in Figure 2.2. It's obvious that the higher concentration parameter relative to the others, the more clustering toward high values for this asset's weight. In the first sub-case of Figure 2.2, α_3 is way bigger than the others, thus more portfolios are close to the individual investment in Asset 3. On the contrary, if α_3 is only half of the others in the second sub-case, more portfolios are simulated as major investment in Asset 1 and Asset 2 minimal investment in Asset 3. The asymmetric feature can also be inferred from the expected value and variance equations. In the first subcase, $E[X_1] = E[X_2] = 0.2$, and $E[X_3] = 0.6$. So the simulated portfolios have more weight in Asset 3, which is about 60%. In the second subcase, $E[X_1] = E[X_2] = 0.4$, and $E[X_3] = 0.2$. So the simulated portfolios have less weight in Asset 3, which is about 20%.

Now we have an intuitive illustration of Dirichlet distribution and an understanding of how the concentration parameters impact the Dirichlet simulation. Next

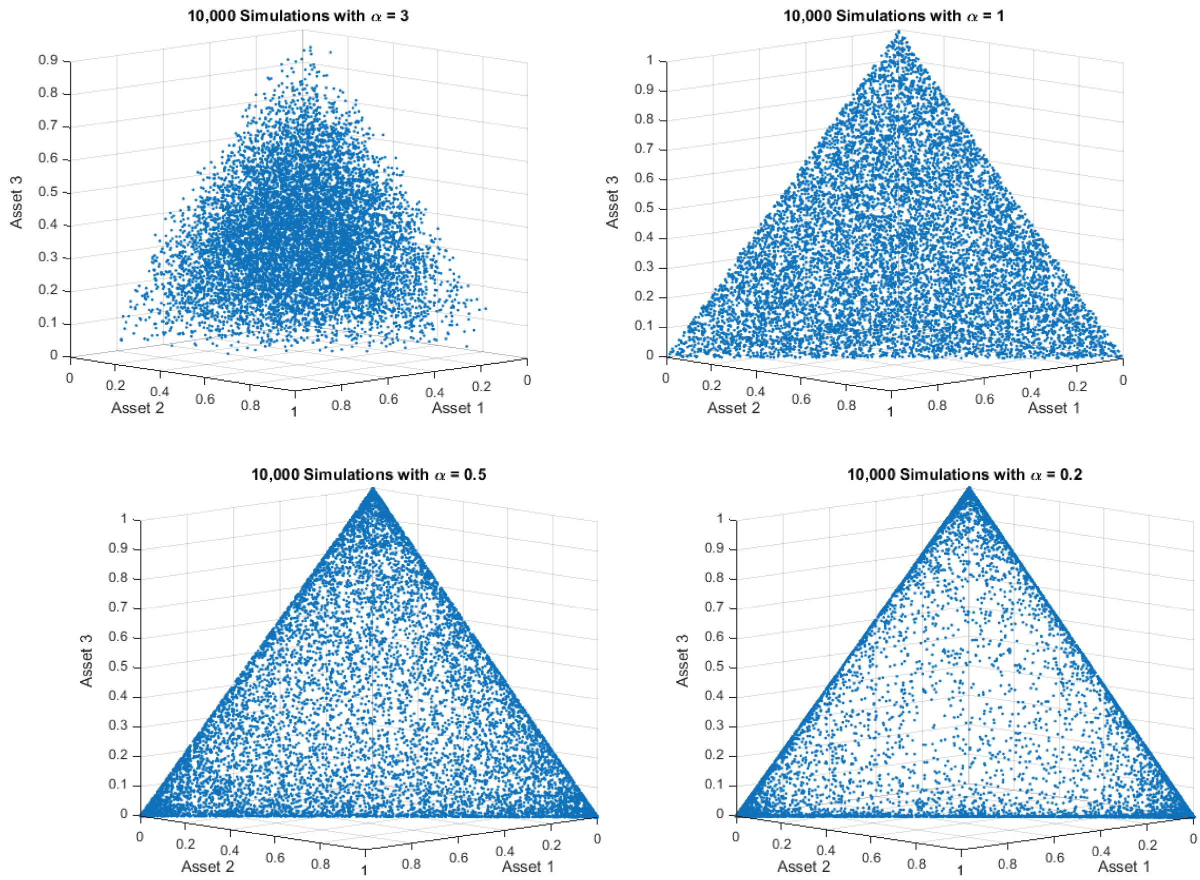


FIGURE 2.1: Dirichlet simulations with equal α

we use Dirichlet weight simulations as input to the Pareto improvement method to construct the mean variance kurtosis efficient frontier.

2.2.4 Mean variance kurtosis efficient frontier

The core of our innovative way for mean variance kurtosis efficient frontier construction is the combination of the Pareto improvement method and the Dirichlet distribution. The weight simulations by Dirichlet distribution to approximate a representative feasible portfolio set are fed into the Pareto improvement method. The weight simulations are the input of the Pareto improvement comparisons between moment profiles, the process of which directly eliminates the inefficient portfolios and their associated moment profiles from the efficiency pool. The leftover ones are efficient and they constitute the sample efficient frontier.

Formally, we sample a representative set of feasible portfolios by Dirichlet distribution for Ω

$$\Omega^* = \{w \mid w \sim \mathcal{D}ir(\boldsymbol{\alpha}), \boldsymbol{\alpha} > \mathbf{0}\},$$

where $\boldsymbol{\alpha}$ is the vector of the concentration parameters defined in Section 2.2.3 It is in the same size as of the asset universe size N. Then the corresponding sample set for the simulated portfolios' moment profiles is

$$\mathscr{P}^* = \{\mathcal{P}(w); w \in \Omega^*\},$$

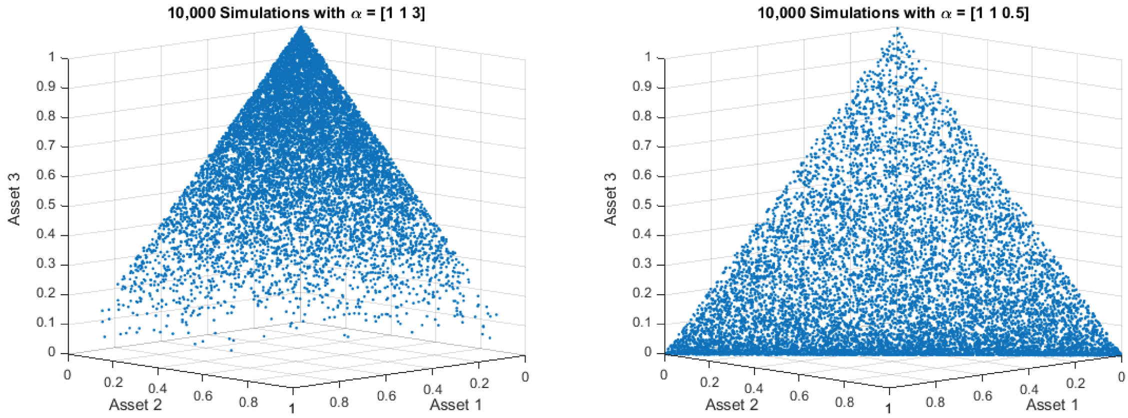


FIGURE 2.2: Dirichlet simulations with unequal α

After the Pareto improvement comparisons among $\mathscr{P}^*$ and the identification of inefficient portfolios, the sample set for the efficient portfolios' moment profiles is

$$\mathscr{E}\mathscr{P}^* = \{\mathcal{P}(w); \langle \mathcal{P}(w^{'}), \mathcal{P}(w) \rangle \gneqq \mathbf{0} \Rightarrow w^{'} \notin \Omega^*\},$$

and the set of efficient portfolios in simulation is

$$\Phi^* = \{w \in \Omega^*; \mathcal{P}(w) \in \mathscr{E}\mathscr{P}^*\}.$$

Note that the efficient frontier is a direct exposition of the set of the efficient portfolios' moment profiles other than the set of the efficient portfolios, since the latter set is just a collection of the efficient portfolio weights. Therefore, the simulated efficient frontier of mean variance kurtosis is a display of $\mathscr{E}\mathscr{P}^*$.

Regarding the relationship of the simulated efficient frontier $\mathscr{E}\mathscr{P}^*$ and the theoretical efficient frontier $\mathscr{E}\mathscr{P}$, we have 3 propositions as following.

Proposition 2.2.1. *The simulated efficient frontier is the lower bound for the theoretical efficient frontier.*

$\forall\ \mathcal{P}(w) \in \mathscr{E}\mathscr{P}^*, \exists\ \mathcal{P}(w^{'}) \in \mathscr{E}\mathscr{P}$ such that $\mathcal{P}(w^{'}) \succeq \mathcal{P}(w)$, which mean the theoretical efficient frontier is at least as good as the simulated efficient frontier.

Proposition 2.2.2. *At certain number of simulations, the efficacy of the simulated efficient frontier is negatively associated with the size of asset universe N.*

This proposition stems from the property of decreasing coverage ratio for a fixed number of simulations. Theoretically, the feasible portfolio set should span over the standard $N-1$ simplex. For example, the coverage ratio of 1,000 simulated portfolios is considerably higher in a 3 assets universe with a standard 2 simplex, a triangle with vertices of (1,0,0), (0,1,0), and (0,0,1), than in a 4 assets universe with a standard 3 simplex, a tetrahedron with vertices of (1,0,0,0), (0,1,0,0), (0,0,1,0), and (0,0,0,1). It means the simulated feasible portfolio set Ω^* is less representative as the size of asset universe increases, making the simulated efficient frontier $\mathscr{E}\mathscr{P}^*$ biased.

We experiment on Proposition 2 for the size effect on the simulation efficacy. We fetch the Dow Jones Industrial Average (DJIA) components' adjusted daily close prices from Sep 1, 2009 to Aug 31, 2015. Setting concentration parameters uniformly to 0.5, we simulate 100,000 portfolio schemes for the first 5 assets of DJIA and for the total 30 assets of DJIA respectively and plot these simulated portfolio profiles as

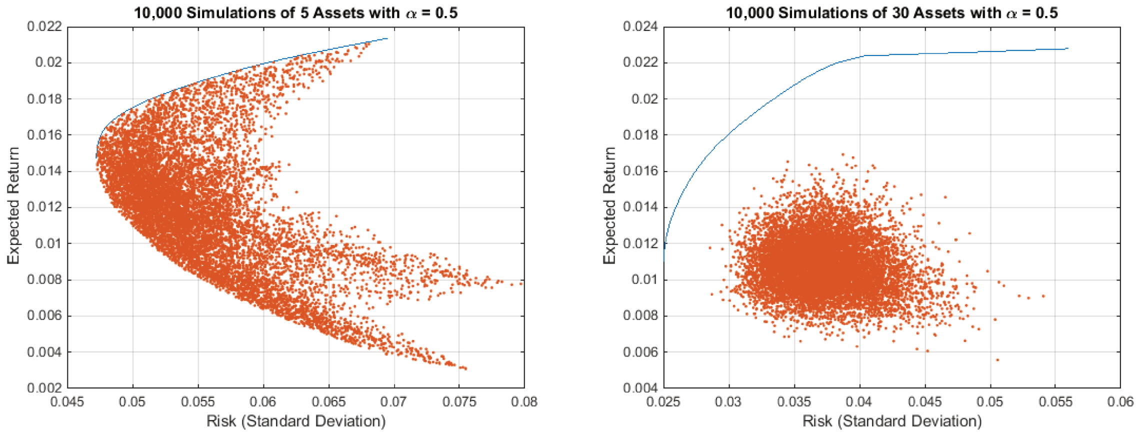


FIGURE 2.3: Dirichlet simulation efficacy and asset universe

red points. For each case, we also plot the corresponding real efficient frontiers by blue curve in Figure 2.3 for better comparison. Apparently the efficient portfolios of the 5 assets simulations closely approach the real efficient frontier, which is occupied by many red points. However, in the 30 assets simulation case, the red points cluster strictly below the real efficient frontier for. It suggests that the simulated efficient frontier has a substantial downward bias for the real efficient frontier. This observation lends evidence for Proposition 2 on the size effect.

Proposition 2.2.3. *At an appropriate size of asset universe N, the simulated efficient frontier is asymptotically consistent with the real efficient frontier. That's to say,*

$$\lim_{X\to\infty} d(\mathcal{P}(w), \mathcal{P}(w^{'})) = 0,$$

where $\mathcal{P}(w)$ is any moment profile from $\mathscr{E}\mathscr{P}^*$ and $\mathcal{P}(w^{'})$ its counterpart from $\mathscr{E}\mathscr{P}$, and $d(\cdot,\cdot)$ is the Euclidean distance.

In the mean variance space, this proposition states that at an expected return level of r, σ is the corresponding volatility along the real efficient frontier, $\tilde{\sigma}$ the corresponding volatility along the simulated efficient frontier. We argue that as the number of simulations X goes to infinity, the difference of $\tilde{\sigma}$ over σ goes to 0, which means

$$\lim_{X\to\infty} \tilde{\sigma}(r) - \sigma(r) = 0,$$

for all $r_{\text{minimum variance portfolio}} \leqslant r \leqslant r_{\text{maximum return portfolio}}$.

Figure 2.4 exhibits Proposition 3 from the perspective of coverage ratio for the simplex. In this 3 assets example, the concentration parameters are uniformly equal to 1. We see that the 1,000 portfolio simulations only scatter loosely over the standard 2 simplex, while the 100,000 portfolio simulations cover the simplex extensively. Naturally, the feasible portfolio set approximated by the latter simulations is more consistent with the true feasible set. In other words, the second simulated efficient frontier is more consistent with the true efficient frontier.

Recall that we have two important inferences on the relationship of the mean variance efficient portfolio set and the mean variance kurtosis efficient portfolio set. These inferences are useful in the construction of mean variance kurtosis efficient frontier. Because the mean variance efficient portfolio set is a necessary subset of the mean variance kurtosis efficient portfolio set, this fact calls for a specification of

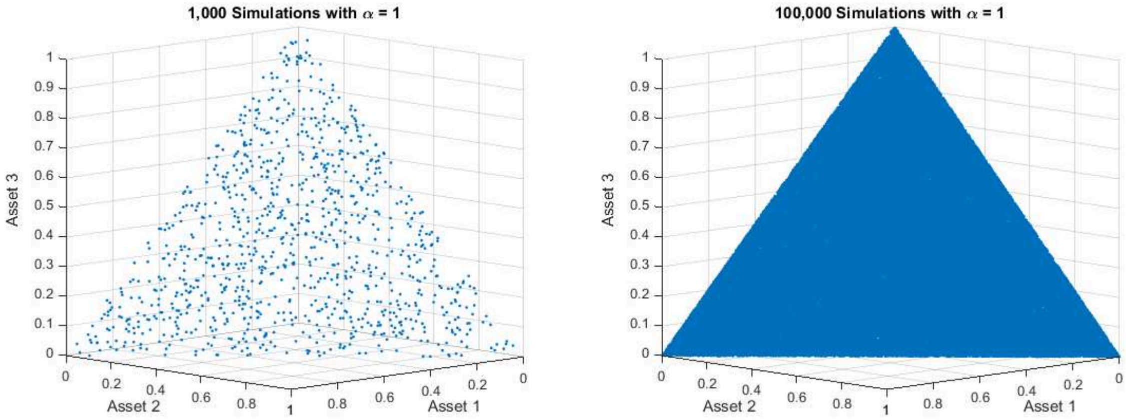


FIGURE 2.4: Number of Dirichlet simulations and simplex coverage

the Dirichlet concentration parameters which produce the most representative simulated mean-variance efficient portfolios first. We vary the concentration parameters so that the simulated mean variance efficient frontier best mimics the real efficient frontier. This set of parameters is used to simulate portfolios to construct the efficient subset by mapping mean variance efficiency to mean variance kurtosis efficiency. The other inference that mean-variance inefficient portfolios can be mean variance kurtosis efficient requires a broad coverage for simulated portfolios. Therefore we put the set of parameters in a wide range to generate portfolios as diverse as possible. These two inferences, combined with a careful treatment of the size effect and the asymptotic effect by the preceding propositions, facilitate the combination of the Pareto improvement method and the Dirichlet distribution in developing the mean variance kurtosis efficient frontier with favorable efficacy.

2.3 Linear approximation to utility functions

2.3.1 Mean variance utility and Taylor expansion

Markowitz (1959) and Levy and Markowtiz (1979) initiate and rationalize the use of mean variance utility, a concise and intuitive utility specification benchmark. They show that the function of mean and variance provides good approximation to the expected utility

$$E[U(R)] \approx U(E[R]) + \frac{1}{2}U''(E[R])E(R - E[R])^2.$$

Its most popular version, as embedded in Black Litterman approach Black and Litterman (1992), is

$$\mathcal{U} = E[R] - \frac{1}{2}\lambda\sigma^2,$$

where λ is the Arrow Pratt absolute risk aversion coefficient.

This mean variance preference reveals a key intuition of the tradeoff between return and risk. With such utility, an investor focuses exclusively on the first two moments of return distribution, and achieves the personal utility maximization by deciding portfolios with the highest expected return at certain variance, or with the lowest variance at given expected return. Furthermore, the higher the risk aversion coefficient is, the greater punishment each unit of variance receives. This utility function is straightforward and practical, specifying that personal utility has positive relationship with expected return, negative relationship with variance, and that risk aversion controls the tradeoff proportion. Due to these advantages, it has gained considerable dominance in application.

Although this utility specification is succinct, it is inadequate to subscribe to the empirical evidence for return characterizations, because high order moment information is absent. Correspondingly, the more general Taylor expansion approach for expected utility gets more and more popular, as the importance of return characteristics beyond mean and variance is better recognized. For example, Briec, Kerstens, and Jokung (2007), Jurczenko, Maillet, and Merlin (2006), Jondeau and Rockinger (2006), and Harvey, Liechty, Liechty, and Müller (2010) adopt Taylor expansion as utility approximation in their analyses of portfolio optimization with higher moments. Specifically, Briec, Kerstens, and Jokung (2007) and Harvey, Liechty, Liechty, and Müller (2010) concentrate on utility approximation by expanding to third moment while Jurczenko, Maillet, and Merlin (2006) and Jondeau and Rockinger (2006) expanding to fourth moment. The general Taylor expansion of the expected utility over return is

$$E[U(R)] = \sum_{i=0}^{\infty} \frac{U^i(E[R])}{i!} E[(R - E[R])^i].$$

More explicitly, we have the mean-variance-skewness utility

$$\mathcal{U}^3(R) = \gamma_1 E[R] - \gamma_2 \sigma^2 + \gamma_3 s^3,$$

and the mean-variance-skewness-kurtosis utility

$$\mathcal{U}^4(R) = \gamma_1 E[R] - \gamma_2 \sigma^2 + \gamma_3 s^3 - \gamma_4 \kappa^4,$$

where γ_i is the corresponding positive coefficient for the ith order moment, $i \in [1, 2, 3, 4]$.

Apparently, the more moments included, the better approximation to the expected utility, but we have to balance the model brevity and explanatory power. As emphasized in Section 2.2.1, kurtosis is a proper complementary measure for extreme risk; it also displays the essence of risk, conforming to the general rationality axioms of the investors' aversion to even moments. Therefore, in the spirit of model parsimony, we specify a mean variance kurtosis utility exhibiting the psychology of extreme risk averse investors. The mean variance kurtosis utility is expressed as

$$\mathcal{U}(R) = \gamma_1 E[R] - \gamma_2 \sigma^2 - \gamma_4 \kappa^4.$$

To our best knowledge, existing literature on higher moment utility approximation are about consecutive order expansion, with little work dealing with utility approximation with jumping moment like our paper. Next we explain our novel treatment for the mean variance kurtosis utility.

2.3.2 Matching of CRRA and mean variance kurtosis utility functions

Suppose the simple return Y follow a lognormal distribution. That is to say,

$$Y = e^X,$$

where $X \to \mathcal{N}(m, \sigma^2)$.

Corresponding to Section 2.2.1, we have the expressions for the key moments of Y in use. The expected value is

$$E(Y) = e^{m+\frac{\sigma^2}{2}},$$

and the variance is

$$Var(Y) = E(Y^2) - E(Y)^2 = e^{2m+2\sigma^2} - e^{2m+\sigma^2} = e^{2m}\left(e^{2\sigma^2} - e^{\sigma^2}\right).$$

The kurtosis can be expressed as

$$m_4(Y) = E(Y^4) - 4E(Y)E(Y^3) + 6E(Y)^2E(Y^2) - 3E(Y)^4,$$

which we reformulate as

$$m_4(Y) = e^{4m+8\sigma^2} - 4e^{m+\frac{\sigma^2}{2}}e^{3m+\frac{9\sigma^2}{2}} + 6e^{2m+\sigma^2}e^{2m+2\sigma^2} - 3e^{4m+2\sigma^2}.$$

In the compact form,

$$m_4(Y) = e^{4m}\left(e^{8\sigma^2} - 4e^{5\sigma^2} + 6e^{3\sigma^2} - 3e^{2\sigma^2}\right).$$

The Constant Relative Risk Aversion (CRRA) utility function of Y is

$$u_\theta(Y) = \begin{cases} \frac{Y^{1-\theta}}{1-\theta}, & \text{if } \theta > 0, \theta \neq 1 \\ ln(Y), & \text{if } \theta = 1 \end{cases},$$

where θ is the relative risk aversion coefficient. Assume that θ is positive but not 1. Because $(1-\theta)X \to \mathcal{N}((1-\theta)m, (1-\theta)^2\sigma^2)$, we have the expected utility as

$$E(u_\theta(Y)) = E\left(\frac{Y^{1-\theta}}{1-\theta}\right) = \frac{e^{(1-\theta)m+\frac{(1-\theta)^2\sigma^2}{2}}}{1-\theta}.$$

To match the CRRA preference with the mean variance kurtosis utility function, we have to calibrate the parameters so that the sum of squared utility difference between the two utilities is minimized. That is to say, our goal is to solve the following optimization program:

$$\min_{\alpha,\beta,\gamma} \sum_{i=0}^{N_m}\sum_{j=0}^{N_\sigma}\left[E(u_\theta(Y_{i,j})) - \alpha E(Y_{i,j}) - \beta Var(Y_{i,j}) - \gamma m_4(Y_{i,j})\right]^2,$$

where

$$Y_{i,j} = e^{X_{i,j}}$$

and

$$X_{i,j} \to \mathcal{N}(m_i, \sigma_j^2).$$

We want m_i to span $[-M, M]$ in $N_m + 1$ points, so we set:

$$m_i = -M + i\frac{2M}{N_m}.$$

Similarly, we want σ_j to span $[0, \Sigma]$ in $N_\sigma + 1$ points, so we set:

$$\sigma_j = j\frac{\Sigma}{N_\sigma}.$$

For illustration, we take $M = 20\%$, $\Sigma = 80\%$, $N_m = 50$, $N_\sigma = 50$, and $\theta = 5$ as a starting point. After optimization, we get $\hat{\gamma}_1 = 1.13$, $gam\hat{m}a_2 = 15.19$ and $\hat{\gamma}_4 = 0.02$.

2.3.3 Parameter sensitivity

As stated, the initial setting is

$$M = 20\%; \Sigma = 80\%; N_m = 50; N_\sigma = 50; \theta = 5.$$

We first experiment on the first four parameters, as shown by Figure 2.5.

We then look at the impact of relative risk aversion on the linear approximation with Figure 2.6. As θ goes bigger, parameters estimated especially for beta and gamma are crazily negative. Therefore we zoom on $[2, 9]$ which is relatively reasonable by the second sub-case.

2.4 Empirical implementation on the S&P 500 index

2.4.1 Reverse optimization for equilibrium returns

In this section of empirical implementation, we choose the S&P 500 index constituents as our underlying assets. As the most widely followed stock market index, it covers almost all influential blue-chips over diverse industries, and accounts for about 80% of the total capitalization of US stock markets. This index is also commonly used as a proxy for the market portfolio, so it is reasonable to assume that its components span the practical asset universe.

Rather than taking the individual stocks as risky assets for investment, we use a sector approach wrapping them into sub-indices due to the aforementioned size effect of simulation. If we invest directly in the constituents, the size of asset universe N is 500.[7] As explained, the weight vectors are in a standard 499 simplex, and the simulation results in this case can be unrepresentative at certain number of simulations. Moreover, the adjustments of index membership can affect the asset universe as additions and deletions make some assets newly tradable, or untradeable anymore. To keep up with the index dynamics, investment universe has to change accordingly. This problem is serious when a long sample period is needed for better return estimates. However, the investment in sub-indices circumvents all these drawbacks. Despite minor differences across various industry classification methods, the number of economic sectors are about 10, a great reduction from the number of constituents, 500. Therefore, this method enhances the simulation efficacy. Besides, since the sub-indices are synthesized from the underlying assets, the sub-index composition has already adjusted for the constituent changes, the concern of which is resolved in the sub-indices dynamics. All in all, the sub-indices investment has obvious advantages compared with the direct investment in components, thus we use sector investment in the empirical study.

[7] In fact, there are 505 stocks in the index, since some component companies have several listed share classes, like Discovery Communications, Google, Comcast, Twenty-First Century Fox, and News Corporation in April 2018.

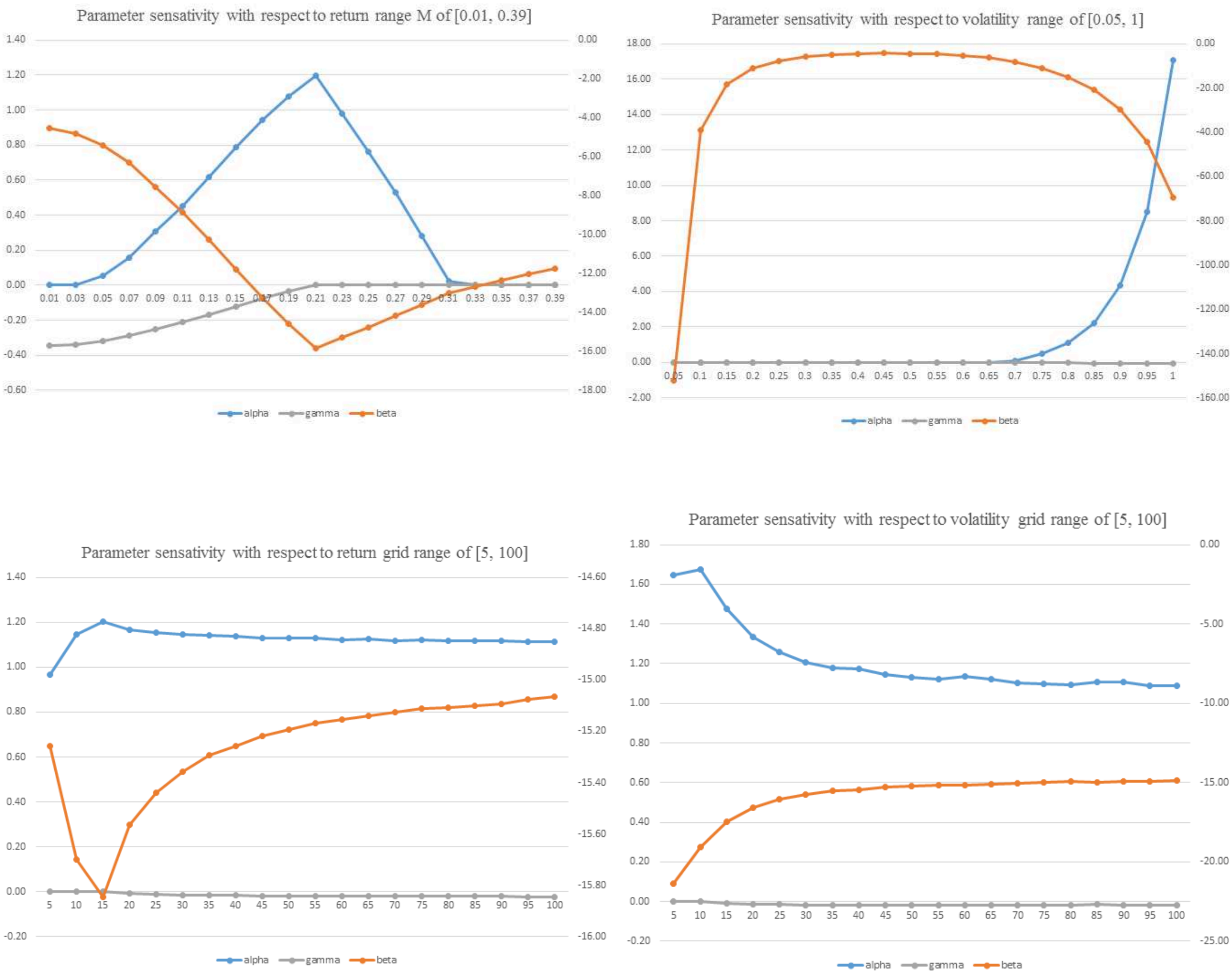
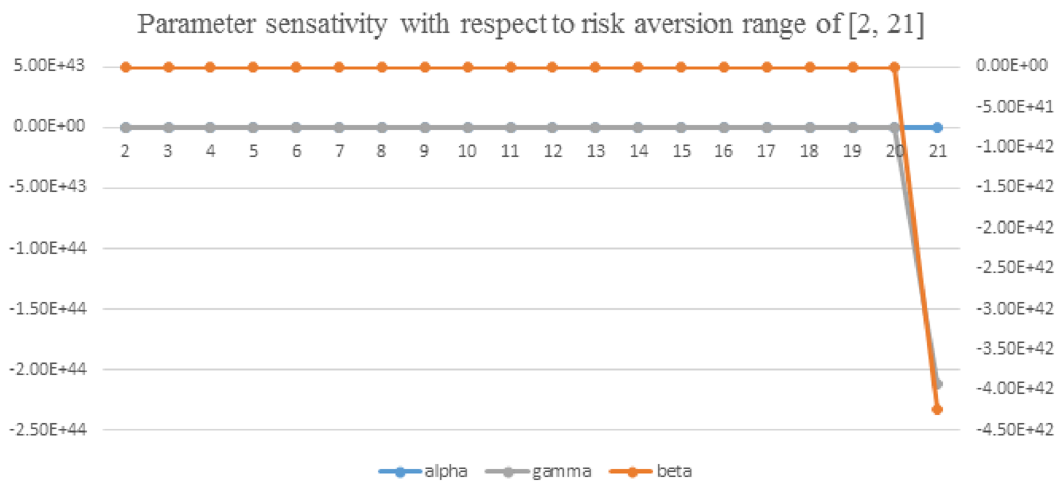


FIGURE 2.5: Experiments on parameters

Note: alpha (γ_1) and gamma ($-\gamma_4$) are on the left axis, while beta ($-\gamma_2$) the right.



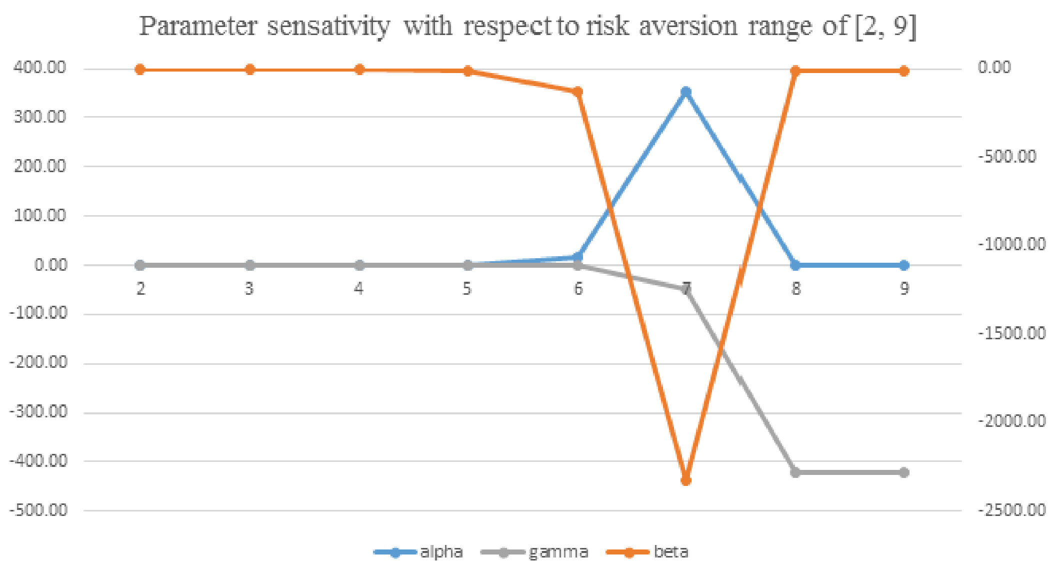


FIGURE 2.6: Experiments on risk aversions

Note: alpha and gamma are on the left axis, while beta the right.

Datastream offers data for 10 economic sectors of the S&P 500 index. These economic sectors are classified according to Global Industry Classification Standard (GICS) which was proposed by MSCI and Standard and Poor's in 1999. The level of Economic sector is the highest classification hierarchy, and these 10 sectors are Consumer Discretionary (CDis), Consumer Staples (CSta), Energy (Ene), Financials (Fin), Health Care (HCare), Industrials (Ind), Information Technology (Info), Materials (Mat), Telecommunication Services (Tel), and Utilities (Utl) respectively. A major change about the GICS structure is that Real Estate would be the 11th sector since September 1st, 2016 to capture investment evolvement. However, Datastream doesn't cover this new sector immediately. Judging from the capitalization difference between S&P 500 index and the summation of 10 sectors, not until September 19th, 2016 did Datastream, also perhaps the financial market, seriously take Real Estate as an independent economic sector.

We want to construct a reverse optimization as in the Black Litterman model for equilibrium returns, see Black and Litterman (1992) and Idzorek (2011). We take the equilibrium returns as a neutral input for the higher order optimization. Here the implied excess equilibrium return Π is calculated as

$$\Pi = \lambda \Sigma w_{mkt},$$

where λ is risk aversion coefficient, Σ the covariance matrix of excess return, and w_{mkt} the vector of market capitalization weights. The market implied risk aversion coefficient is defined as

$$\lambda = \frac{E[r_{mkt}] - r_f}{\sigma^2} = \frac{E[r_{mkt}] - r_f}{w'_{mkt} \Sigma w_{mkt}},$$

where r_{mkt} is market portfolio return, σ^2 its variance, and r_f risk free rate. See Idzorek (2011) for details.

Therefore, we have to collect data for market portfolio return (S&P 500 index), risk free rate (T-bill treasury secondary market for 3-month), covariance matrix (10 sub-indices return), and market weight (10 sub-indices capitalization). We use total return (RI) data for better return characterization. Note that in Datastream, RI data for 10 sub-indices uniformly start from September 11th, 1989 and market capitalization (MV) data mostly at least from January 23rd, 1995. S&P 500 index data and 3M T-bill data have better period coverage. Thus we set sample period as from 1995 to 2015, for which the ending year is just before the sector restructuring. For each year, we get daily data for both RI and MV. Covariance matrix is calculated with daily return for these 10 sub-indices, and market cap weight is characterized as the capitalization fraction of each sub-index to S&P 500 index at the last day of that year. Excess return is set as daily return difference between market portfolio and 3M T-bill. At the end, we have market implied excess equilibrium return for each industry in each sample year.

With this reverse optimization technique, we get annualized equilibrium excess returns for these sub-indices and market implied risk aversion coefficient for each year, shown in Table 2.3. We see that the commonly believed high beta sectors, Financials and Information Technology, for example, have the highest average annual excess return as well as the highest standard deviation. The commonly believed low beta sectors, Utilities and Consumer Staples, for example, have the lowest average annual excess return as well as the lowest standard deviation. This is quite reasonable and conforms to the tradeoff between risk and return in finance theory.

TABLE 2.3: Market implied risk aversion and equilibrium returns

	λ	Ene	Mat	Ind	CDis	CSta	HCare	Fin	Info	Tel	Utl
1995	44	0.18	0.25	0.31	0.28	0.25	0.22	0.4	0.63	0.33	0.2
1996	11.34	0.13	0.11	0.17	0.17	0.16	0.18	0.22	0.25	0.17	0.11
1997	7.72	0.26	0.22	0.27	0.22	0.29	0.34	0.34	0.39	0.23	0.13
1998	5.3	0.16	0.19	0.24	0.27	0.19	0.25	0.33	0.34	0.18	0.06
1999	4.37	0.07	0.06	0.12	0.16	0.09	0.15	0.19	0.27	0.16	0.05
2000	-2.98	-0.04	-0.08	-0.12	-0.12	-0.06	-0.08	-0.15	-0.19	-0.11	-0.05
2001	-3.03	-0.05	-0.11	-0.15	-0.15	-0.03	-0.05	-0.13	-0.25	-0.11	-0.05
2002	-3.41	-0.18	-0.19	-0.21	-0.21	-0.1	-0.17	-0.23	-0.29	-0.23	-0.16
2003	8.75	0.16	0.28	0.28	0.33	0.18	0.24	0.32	0.43	0.33	0.18
2004	7.74	0.08	0.12	0.11	0.11	0.07	0.09	0.1	0.14	0.09	0.06
2005	2.02	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
2006	10.27	0.13	0.15	0.11	0.11	0.07	0.08	0.11	0.15	0.09	0.07
2007	0.87	0.03	0.03	0.02	0.02	0.01	0.02	0.03	0.02	0.02	0.02
2008	-2.37	-0.38	-0.36	-0.31	-0.34	-0.21	-0.25	-0.46	-0.31	-0.31	-0.27
2009	3.25	0.31	0.33	0.33	0.32	0.13	0.15	0.77	0.27	0.19	0.16
2010	4.68	0.19	0.21	0.2	0.18	0.09	0.11	0.23	0.17	0.1	0.11
2011	0.88	0.06	0.06	0.06	0.05	0.03	0.04	0.07	0.05	0.03	0.03
2012	9.56	0.21	0.21	0.19	0.17	0.09	0.12	0.22	0.19	0.1	0.06
2013	23.18	0.35	0.38	0.35	0.35	0.26	0.31	0.41	0.3	0.23	0.23
2014	10.37	0.16	0.15	0.16	0.15	0.09	0.16	0.15	0.16	0.09	0.07
2015	1.05	0.03	0.03	0.02	0.03	0.02	0.03	0.03	0.03	0.02	0.02
Mean	6.84	0.09	0.1	0.1	0.1	0.08	0.09	0.14	0.13	0.08	0.05
StD	10.59	0.16	0.18	0.18	0.18	0.12	0.15	0.26	0.24	0.16	0.12
Min	-3.41	-0.38	-0.36	-0.31	-0.34	-0.21	-0.25	-0.46	-0.31	-0.31	-0.27
Max	44.00	0.35	0.38	0.35	0.35	0.29	0.34	0.77	0.63	0.33	0.23
Skew	2.39	-1.14	-0.85	-0.72	-0.79	-0.35	-0.53	-0.03	-0.26	-0.69	-1.06
Kurt	7.35	2.33	0.89	-0.08	0.20	0.38	0.13	1.32	-0.13	0.52	1.96

Most importantly, we see the average risk aversion coefficient is 6.84. Also the risk aversion varies a lot from 44 to -3, suggesting that there are periods when investors, or the whole market, shall be very risk averse. In these periods, investors are quite sensitive to extreme risk, revealed by the remarkably high risk aversion coefficient. This is an evidence of extreme risk in affecting investor's portfolio decision.

By Taylor series, the expected utility is expanded in terms of central moments. As the difference of higher order compounded return is quite small (like values in Table 2.1), the importance of kurtosis in utility approximation is underweighted. Since risk aversion is primarily related with variance which is much greater than kurtosis in size by compounding less times, it is not strange that the role of kurtosis, as well as its coefficient, is not as significant as its variance counterparts. However, faced with extreme events such as financial crisis and sovereign debt crisis, the investor's aversion to extreme returns should substantially increase in the utility loading specification, even if the risk aversion loading stands still. By this inference, the utility plane should tile downward kurtosis, meaning that the investor is willing to trade some positive return for lower fourth moment, *ceteris paribus*.

Proposition: Suppose the utility linear approximation is $U(x) = aM_1 - bM_2 - cM_4$. Faced with extreme events, investor's aversion to extreme risk rises dramatically and the loading of fourth moment in utility specification rockets.

2.4.2 Dirichlet parameterization

Now we turn to weight simulation. Previously maintained, Dirichlet distribution is a natural choice for this purpose since it produces simulate standard vector with non-negative values summed to 1, which is perfect for non-short portfolio weights. The distribution parameter α controls the concentration for its corresponding variable's marginal density distribution. Generally, the higher concentration parameter, the more condensed weight assignment for this variable. For simplicity we consider Dirichlet with equal distribution parameters. This symmetric Dirichlet distribution is very useful when there is no concrete evidence or knowledge of one variable's privilege over others.

Our goal is to take advantage of Dirichlet distribution to simulate abundant weights for the construction of efficient frontier in three-dimensional space of first, second, and fourth moments. So, a good simulation scheme should give simulated efficient frontier as close as the real one. Also, a good simulation scheme offering fine solution in high dimensional space shall give at least same or better performance in lower dimensional space, while the *vice versa* is not necessarily true. In our case, a good simulation for 3 dimensions should also be a good one for 2 dimensions.

Moreover, due to the unique mapping of weight and moment profiles, we reach that the efficient portfolios in 2D are also efficient in 3D. In short, 2D efficient set is a distinguished subset of 3D efficient set.

With this reasoning, we can primarily focus on those Dirichlet distributions best matching traditional mean-variance efficient frontier. We experiment 10,000 weight vector simulations with different parameters in Figure 2.7. The blue curve is the true efficient frontier constructed by mean and covariance matrix of the implied equilibrium excess returns. Red points are coordinates of mean and volatility for each simulated portfolio.

We want to use the parameter settings to enhance simulation efficacy. As the parameters decreases, more and more simulated points lie on the true efficient frontier. When the parameter is bigger than 1, simulated points are more concentrated

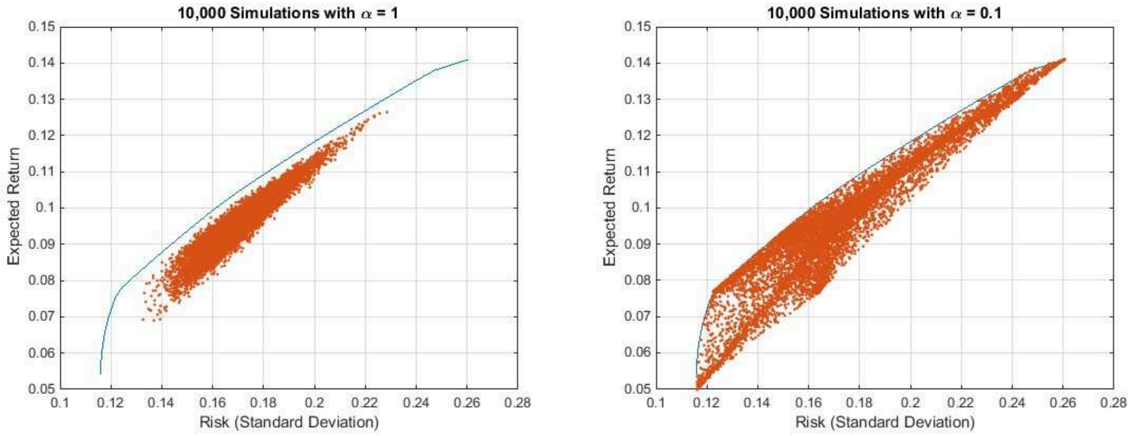


FIGURE 2.7: Efficient frontier relative to α

and the fitted frontier is strictly inferior to the real one. We notice that when the concentration parameter goes below 1, simulations are more dispersed and more points approach the real efficient counterpart. The fitted frontier is closer to the real one and the efficacy of simulation is better. Especially when it's down to 0.1 or even 0.01, simulations include most of the real efficient portfolios on different return ranges. Almost all of the blue efficient frontier is occupied with red points of simulated portfolios, suggesting that the fitted efficient frontier greatly resembles, if not perfectly coincide with, the real efficient frontier. Therefore we have convincing justification in choosing this simulation parameter for efficient frontier construction with dimensionality accession.

In short, the 2D efficient portfolio set is a prominent subset of the 3D efficient portfolio set. So, we have to increase the accuracy of simulation covering 2D efficient portfolios. This part is done by setting suitable Dirichlet parameter to values such as 0.01. Then, for those portfolios outside of the 2D efficient portfolio set but still within the 3D efficient portfolio set, we take more simulation scenarios into consideration in case of 3D efficient portfolio omission. This part is done by varying the Dirichlet parameter in a wide range additional to the exact identification of 2D efficient portfolios. For both parts, enlarging the simulation size can lead to better results, especially for the second part.

For the next simulation phase, we simulate 100,000 Dirichlet portfolio weights for each concentration parameter ranging from 0.1 to 5 with a step of 0.1, in addition to 0.01 which serves the best of the 2D efficient frontier. Therefore, we have 5,100,000 weight simulations in total. For each parameter, we record the portfolios without any Pareto improvements and at the end we conduct Pareto comparisons once again among these 51 simulated efficient sets for the final simulated efficient set. Over 6,000 portfolios are left in record and we plot these points in 3D as in Figure 2.9.

As shown in the marginal figures plotting the efficient frontier to the mean variance plane and the mean kurtosis plane, we can easily see a tradeoff between return and risk. The higher expected return, the higher variance; also, the higer expected return, the higher kurtosis. This 3D efficient frontier is similar to the 2D efficient frontier exhibition especially at the two ends, while in the middle section several points stroll around the trend at the same expected return level.

This shape is consistent to the upper edge of the higher moments efficient frontier by shortage function, which is demonstrated by Jurczenko, Maillet, and Merlin (2006) and shown besides the efficient frontier by the Pareto improvement method.

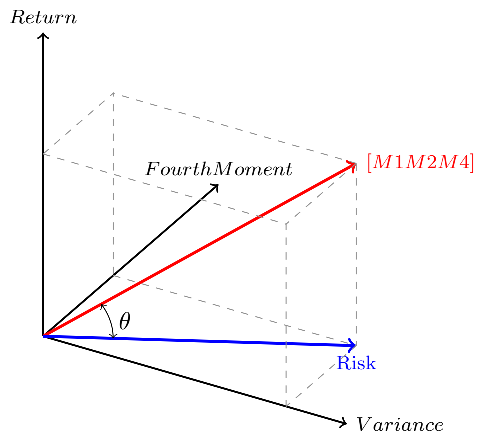


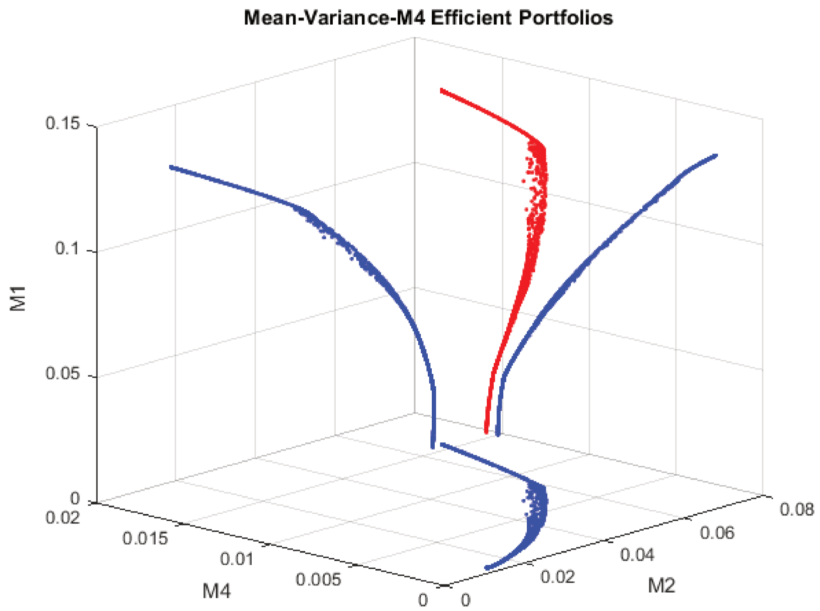
FIGURE 2.8: Generalized Sharpe ratio

It is still a question to choose among the efficient portfolios in the mean variance kurtosis space. In the mean variance space, the optimal choice is the efficient portfolio with the highest Sharpe ratio. There the tradeoff between return and risk is straightforward: the expected return and volatility. In the mean variance kurtosis space, we need a generalized Sharpe ratio to characterize such a tradeoff, as illustrated in Figure 2.8. As variance and kurtosis are two risk indicators, we can make a synthesized measure to balance the risk aspects revealed by the two indicator. In the risk plane, i.e., the variance and kurtosis plane, the vector addition is a convenient indicator for the risk synthesis. Among all the 3D efficient portfolios, the one achieving the highest angle θ should be the optimal one, as it has the highest return per unit of synthesized risk.

We then make a simple exhibition for the tangency of efficient portfolios and utility surface as well. From Table 2.3, we see that the average market implied risk aversion coefficient is 6.84, so we take 7 rather than 5 as the θ value in the utility parameter calibration setting. Then, we get the estimates that $\hat{\gamma}_1 = 353.9$, $ga\hat{m}ma_2 = 2327.1$ and $\hat{\gamma}_4 = 48.5$. For the specification of $u = 353.9M1 - 2327.1M2 - 48.5M4$, we manipulate the utility surface to examine the tangency of the utility surface and the efficient frontier. We use *gridfit* for 3D efficient surface, and this function interpolates some points as additional surface support. If we excise apparently redundant extrapolated points, we get a reduced one and its tangency to the utility surface is similar to that by efficient portfolios. Following figures show the tangency.

2.5 Conclusion

In this paper, we conduct the higher order moment extension of the traditional mean variance optimization and CAPM framework. Kurtosis captures the impact of extreme returns, thus we use it as a measure of extreme risks. It complements variance for a comprehensive depiction of investment risk, as investment with relatively low variance may well have relatively large kurtosis. We then consider the portfolio construction problem for those investors with extreme risk aversion. The preference of extreme risk aversion is instructive and useful given the presence of extreme financial events, especially during the global financial crisis and the European sovereign debt crisis recently. Therefore, we consider the portfolio development in the mean variance kurtosis space, to highlight the impact of extreme risks on portfolio selection.



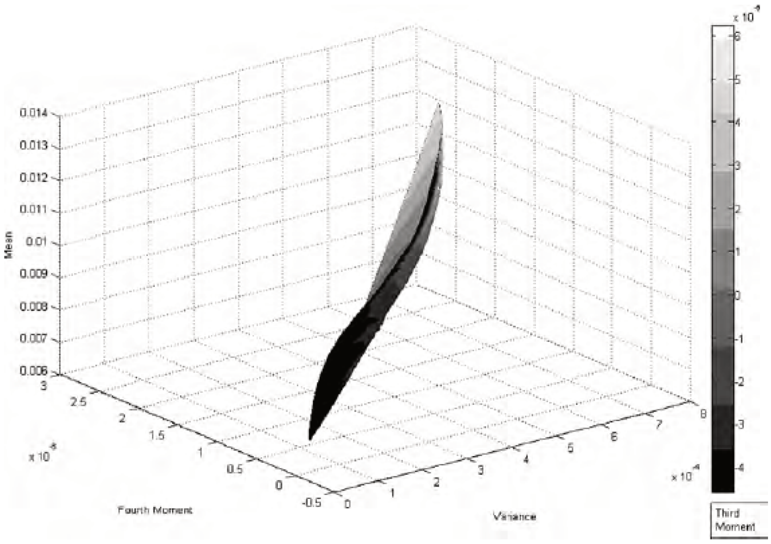


Figure 3.2 Mean–variance–skewness–*kurtosis* constrained efficient frontier in the mean–variance–fourth moment space. *Source*: HFR, monthly net asset values (1995–2005), computations by the authors. The constrained efficient frontier is obtained after optimisation of 20 hedge funds in 1296 directions. Grey shading represents the level of the third noncentral moment.

FIGURE 2.9: Mean variance kurtosis efficient portfolios

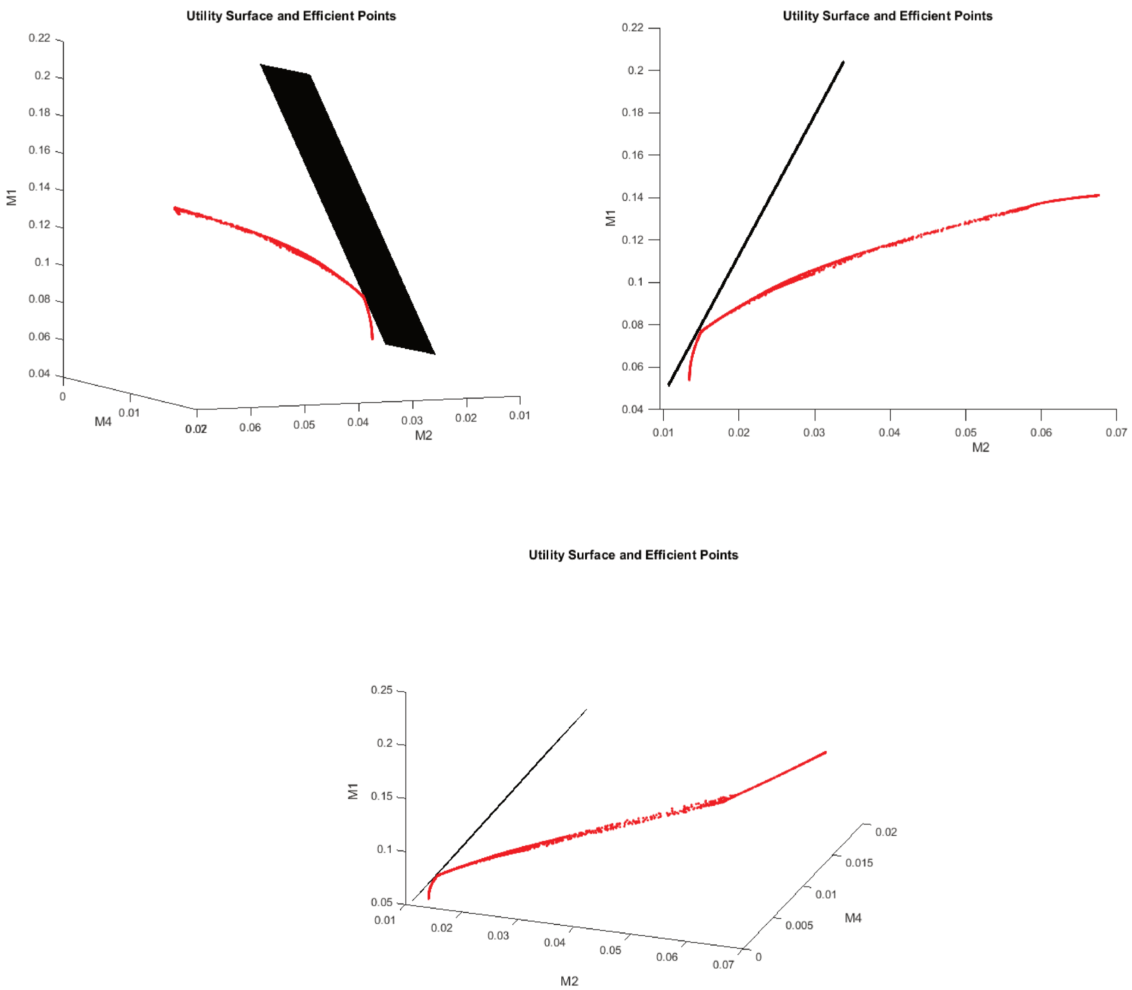


FIGURE 2.10: Mean variance kurtosis efficient portfolios and utility surface

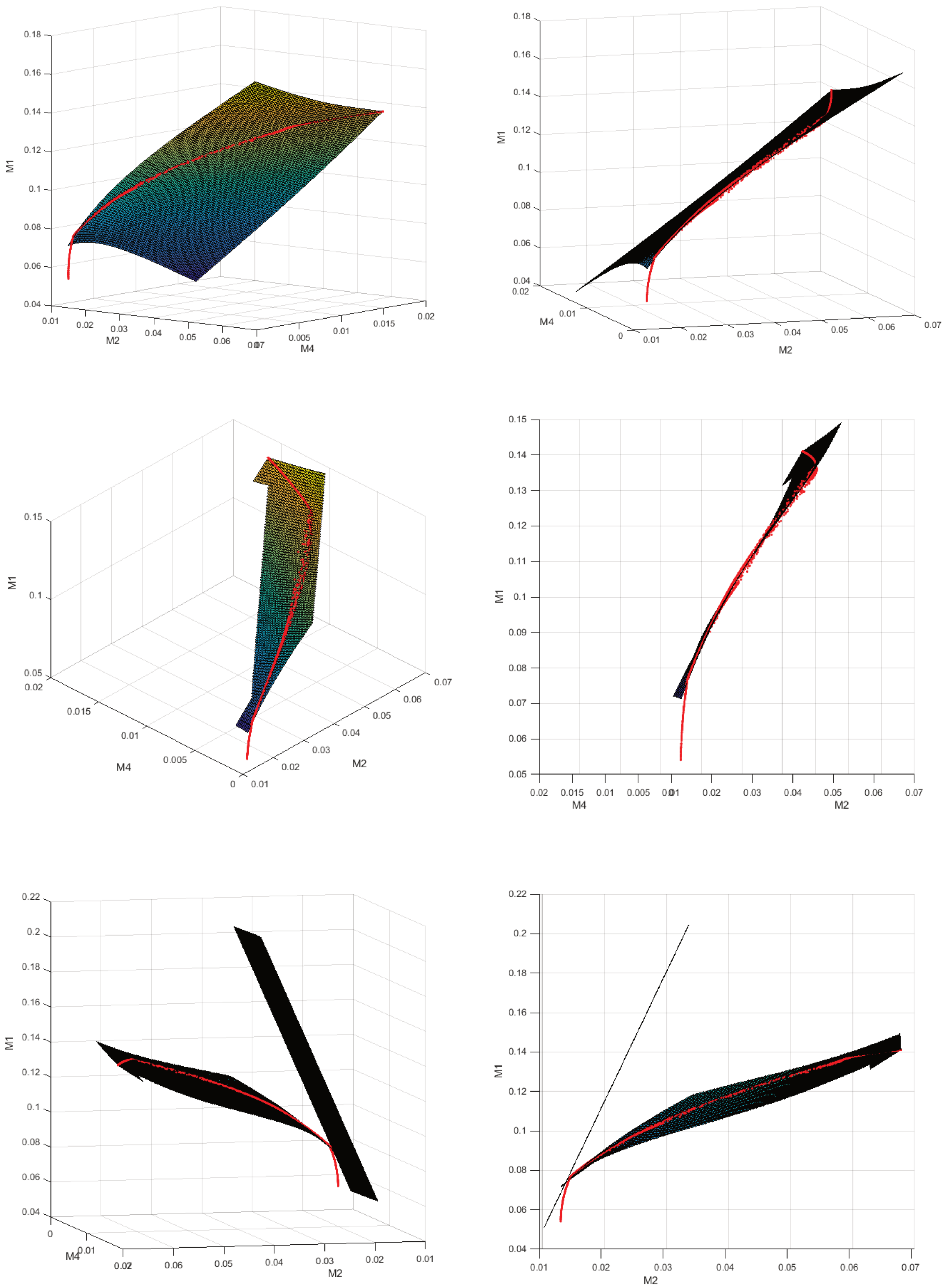


FIGURE 2.11: Mean variance kurtosis efficient frontier and utility surface

We propose the Pareto improvement method for the mean variance kurtosis efficient portfolio specification. Compared to current methods for higher order moment efficient frontier construction, i.e., the constrained variance minimization program by De Athayde and Flôres (2004) and the shortage function method by Briec, Kerstens, and Jokung (2007), the Pareto improvement method has a key advantage that it is able to detect marginal improvements for portfolio profiles. This advantage is important because the failure of the two current methods in specifying marginal improvements leads to a serious potential mistake: the misclassification of inefficient portfolios as efficient. Moreover, the Pareto improvement method is simple and efficient. By pairwise comparison among portfolio profiles, this method classifies the portfolios with inferior profiles as inefficient and only those portfolios without any profile improvement are labeled as efficient.

To implement the Pareto improvement method, a proper approximation of feasible portfolio set is necessary. We approximate this set by using the Dirichlet distribution. The Dirichlet distribution produces vectors of nonnegative elements in sum of 1. This feature makes the Dirichlet distribution a proper program for portfolio weight simulation. We illustrate the use of the Pareto improvement method and Dirichlet simulation with an empirical implementation of the S&P 500 index sectors with total return data from 1995 to 2015. The market implied risk aversion parameters and equilibrium sector returns are obtained from the Black Litterman model. The equilibrium sector returns are combined with Dirichlet weight simulations for Pareto improvement comparisons. The Dirichlet parameterization utilizes two properties of the mean variance efficient set and the mean variance kurtosis efficient set: mean variance efficient portfolios are mean variance kurtosis efficient portfolios, and mean variance inefficient portfolios can be mean variance kurtosis efficient portfolios. With regard to these two properties, we have 5.1 million weight vectors by Dirichlet simulation. We then produce the mean variance kurtosis efficient frontier, which behaves like two line segments at the two ends, and a band spread in the middle.

Further work will be on the mean variance kurtosis utility specification, the generalized Sharpe ratio for return and risk tradeoff, and the tangency of utility surface and efficient frontier in the mean variance kurtosis space. The correlation between the bond market extreme risk and the stock market extreme risk is also an issue to investigate.

Chapter 3

High order stochastic dominance and portfolio efficiency

3.1 Introduction

One of the most dynamic markets over the last decade has been the ETF market. By the end of June 2017, the total size of the assets under management (AUM) by ETFs reached USD 4 trillion, overcoming that of hedge funds.[1] ETFs provide an easy and cheap way for investors to track various market indices. The most heavily traded single ETF is SPDR S&P 500 ETF with the AUM being over USD 240 billion as of November 14, 2017.[2] The popularity of this ETF is not surprising, given that it tracks a well diversified stock portfolio, which often serves as a proxy for the "market" portfolio.

Starting from the seminal fork of Markowitz (1952), the diversification idea has been playing a decisive role in portfolio allocation theory and practice. Such concepts as the market portfolio and the security market line has become a natural benchmark for portfolio managers and academics. Despite its obvious merits, a problem with this approach is that it works only for jointly normally distributed asset returns or for mean-variance investors with quadratic utility function.

The optimality of the market portfolio has been challenged ever since it was introduced. For example, preference for skewness, behavioral biases, ambiguity with respect to underlying distribution and investor heterogeneity lead to optimal deviation from the market portfolio Conine and Tamarkin (1981), Shefrin and Statman (2000), Uppal and Wang (2003), and Mitton and Vorkink (2007).

Taking into account investor preference for skewness is an important step forward in portfolio theory. However, looking at a limited number of moments of return distribution is still restrictive, especially if a decision maker has a complex utility function, or an optimal portfolio should be constructed to satisfy preferences of multiple investors with heterogeneous utility functions (as, e.g., is the case of delegated portfolio management, including mutual and pension funds). A concept of stochastic dominance overcomes these limitations and provides an efficient tool for comparing complete distributions between each other, instead of focusing on a limited number of moments. First developed as a statistical tool Markowitz (1952) and Lehmann (1955), the concept soon found its way into economics and finance Hanoch and Levy (1969), Porter and Gaumnitz (1972), Tehranian (1980), Post (2003), Kuosmanen (2004), De Giorgi and Post (2008), Annaert, Van Osselaer, and Verstraete (2009), Constantinides, Czerwonko, Carsten Jackwerth, and Perrakis (2011), Hodder, Jackwerth, and Kolokolova (2014), Longarela (2015), and Post, Karabati, and

[1] Financial Times, September 10, 2017, "Regulators descend on booming ETF market".
[2] http://etfdb.com/etf/SPY/.

Arvanitis (2018), to name a few. One of the appealing features of the concept of stochastic dominance (and related stochastic dominance efficiency) is that it can be easily linked to the expected utility preference framework, and then applied to ranking potential portfolio return distributions. For example, if the return distribution X second-order stochastically dominates the return distribution Y, then all risk averse investors, regardless of the exact shape of their utility functions and the levels of risk aversion, would prefer X over Y. Similarly, third-order stochastic dominance leads to all risk averse and prudent investors choosing the dominating distribution.

Stochastic dominance efficiency is an even broader concept, which is applied to portfolios of assets. If a portfolio is stochastically efficient at a given order with respect to a set of underlying assets, then it is not possible to construct any other portfolio using the underlying set of assets that would dominate the portfolio in question at this order. Put differently, if a portfolio is not efficient, for example, at the second order, it is possible to construct a different portfolio using the same underlying assets that would be preferred by all risk averse investors.

To this end, the approach of evaluating portfolios from the stochastic dominance perspective is extremely appealing for delegated portfolio management industry, in which portfolio managers should cater for interests of multiple heterogeneous investors. Similarly, the rising in popularity ETFs should be assessed in terms of the stochastic dominance efficiency of their returns, or that of the indices they track. Previous empirical evidence worryingly suggest, however, that the market portfolio is inefficient relative even to the Fama and French benchmark size and book-to-market portfoliosPost (2003).

If the benchmark (market) portfolio is found not to be efficient, the follow up question naturally arises of whether it is possible to construct a dominating efficient portfolio. Kuosmanen (2004) has developed an operational test that, using standard linear programming algorithms, not only allows testing for the first and second order efficiency of a given portfolio relative to the underlying set of assets, but also provides optimal weights for an efficient portfolio. Applying this approach to twenty five Fama and French industry portfolios, Hodder, Jackwerth, and Kolokolova (2014) further show that efficient portfolios chosen in such a way perform reasonably well out of sample.

Post and Versijp (2007) develop a multivariate tests for second and third order stochastic dominance and show that the CRSP all-share index is not mean-variance efficient relative to the 10 beta-sorted portfolios, but the second order stochastic dominance efficiency cannot be rejected. Kopa and Post (2009) show that the U.S. market portfolio is not first-order stochastic dominance efficient in their sample relative to portfolios formed on book-to-market and size. Post (2016) develop a bootstrap empirical likelihood ratio test for stochastic dominance optimality, which jointly compares a given distribution with multiple possible alternatives, and show that the Fama and French small growth stock portfolio is not optimal for risk-averse investors. Post and Levy (2005) apply a wider range of stochastic dominance criteria including prospect stochastic dominance (that assumes an S-shape utility function) and Markowitz stochastic dominance (that assumes a reverse S-shaped utility function) to the market portfolio and show that the market portfolio is clearly inefficient using second order or prospect stochastic dominance criteria, but Markowitz stochastic dominance efficiency cannot be rejected. Post, Karabati, and Arvanitis (2018) combined the stochastic dominance decision criterion and the empirical likelihood optimization technique to improve the out-of-sample performance of portfolios relative to a set of benchmarks.

The majority of the existing empirical studies focus largely on second order stochastic dominance[3] and the U.S. market. There exists, however, growing evidence of substantial cross-county differences that result in different economic decisions of agents and pricing of assets. The important differences include, for example, observable legal rules that can impact ownership concentration La Porta, Silanes, Shleifer, and Vishny (1998), perception of risk that leads to variations in option pricing Weber and Hsee (1998), cultural differences influencing trade agreements Guiso, Sapienza, and Zingales (2009), investor portfolio choice Grinblatt and Keloharju (2001), and takeover activities Frijns, Gilbert, Lehnert, and Tourani-Rad (2013). Cross-county differences also manifest themselves through a better performance of country-specific Fama-French three-factor model compared to its global version in explaining time-series variation in international stock returns Griffin (2002). A related issue raised in Jorion and Goetzmann (1999) is that the U.S. market is one of the most successful markets in the world, and, consecutively, the estimates of the expected return on equity derived from this market are subject to a survivorship bias. The authors show substantial differences in the expected real return on assets across 39 countries, with the U.S. equity having the highest real return.

Our paper makes several contributions to the literature. First, on the theoretical front, we extend the operational approach of Kuosmanen (2004) to allow us to test for higher order stochastic dominance efficiency in majorization sense and derive efficient portfolios of orders higher than two.

Second, we use this methodology to test for the efficiency of well diversified stock indices across seventeen countries, spanning both developed and developing markets. We show that these indices are usually not efficient at order two, which also implies inefficiency at order three and beyond. In the cases where inefficiency at order two is not shown, then, in the vast majority of circumstances, stock indices are inefficient at order three and beyond. That is, all of the prudent investors and most of the risk averse investors would be better off not investing in those well-diversified indices but should instead hold more concentrated portfolios, focusing on several industries. The average potential improvement of a portfolio Sharpe ratio is 0.73 per year when an inefficient market index is substituted by an efficient portfolio.

Next, we perform a comparison of each well-diversified market index with its industry components and find striking difference across countries, which cannot be attributed only to the fact that a country is an emerging or developed economy. For example, the Japanese Nikkei 225 index is dominated by some of its sub-indices in 13 of 14 years in our sample (93% of years), whereas the Indian BSE SENSEX index is dominated only in 3 of 11 years (27%). Some counter-cyclical industries, such as consumer goods and services, health care, and utilities, are more likely to dominate the respective market indices in all countries in our sample.

Looking further into determinants of dominating industries, we find that the key factor is the relative volatility of the industry compared to the market, as well as past dominance of a sector index over the market index. Macroeconomic variables also help to forecast the years during which market indices are dominated and are not suitable for risk-averse and prudent investors. The information content of the aggregate indicators, however, is very distinct for developed and developing markets. For example, a higher GDP growth in developed markets indicates a relative homogeneous improvement in all sectors and, thus, predicts a lower likelihood of the market index to be dominated. A higher GDP growth in developing economies, that are usually tilted towards one or two main industries, signals a disproportional

[3] Notable exceptions are Post (2016) and Fang and Post (2017).

growth of one industries that makes the market index a relatively less desirable investment for a risk-averse investor as compared to the GDP-driving industry.

Motivated by the prediction results, we propose a simple trading rule based on past information on stochastic dominance. The rule allows improving the out-of-sample performance relative to a benchmark global portfolio with the mean return increasing on average by 1–2% annualized and return standard deviation declining by 2–3%. The improvement is consistent across time.

Last but not least, we contribute to the discussion on the exceptional performance of low-beta low-risk stocks. We show that the portfolios of low beta and low volatility stocks stochastically dominate the market indices in majority of years at order 3 and often at order 2. Thus, these portfolios are more suitable for risk-averse and prudent investors than the market indices across most of world economies considered in this paper.

3.2 Key concepts and theoretical results

In this section, we extend the arguments of Marshall and Olkin (1979) used in Kuosmanen (2004) and show that pairwise SD comparison of distributions can be achieved at any order of stochastic dominance.[4] We also construct necessary and sufficient conditions for a portfolio to be SD efficient with respect to the whole market at any order.

3.2.1 Pairwise comparisons of distribution

Let $F = F^{[1]}$ be a cumulative distribution function defined on real numbers and further define $F^{[n]}$ recursively by

$$F^{[n]}(r) = \int_{-\infty}^{r} F^{[n-1]}(s)\, ds. \tag{3.1}$$

Then, we define the stochastic dominance of continuous distributions as follows: a distribution F is said to dominate distribution G at order n in the stochastic sense[6] when $F^{[n]}(r) \leq G^{[n]}(r)$ for all $r \in \mathbb{R}$.

To compare distributions F and G in terms of stochastic dominance, we consider T return observations $x_{t=1,\ldots,T}$ associated with F, and T return observations $y_{t=1,\ldots,T}$

[4]Other authors devise tests for higher order SD. For example, Post and Kopa (2017) use a super-convex TSD (third order stochastic dominance) formulation – a more restrictive sufficient condition for TSD – to construct portfolios with enhanced out-of-sample performance relative to a benchmark. Fang and Post (2017) derive systems of equations that can exactly characterize portfolio SD efficiency up to the order 5. Davidson (2009) proposes a test of restricted stochastic dominance at any order that applies to theoretical cumulative distribution functions. This test considers stochastic dominance of distributions on a restrictive support, and it does not incorporate the most extreme information in the tails of the distributions. Accounting for tail events, however, is crucial for the analysis of financial returns, as has become apparent after the financial crisis of 2007-2008. The (extended) approach of Kuosmanen (2004) uses empirical cumulative distribution functions and accounts for all of the available information on the stock performance up to date. We believe it is the most appropriate for application to financial data.[5] Hodder, Jackwerth, and Kolokolova (2014) also show that the optimal second-order SD portfolios constructed using the Kuosmanen (2004) approach exhibit good out-of-sample performance.

[6]Note that stronger forms of stochastic dominance exist. For example, the existence of an $r_0 \in \mathbb{R}$ such that $F^{[n]}(r_0) < G^{[n]}(r_0)$ is sometimes also imposed. In decision theory, it is also necessary to add conditions on the moments of probability distributions from order three on to derive equivalences between stochastic order dominance results and preferences of probability distributions by groups of agents - such as prudent agents at order 3 and temperant agents at order 4.

associated with G. We denote the corresponding observations ranked in increasing order by $\tilde{x}_{t=1,\dots,T}$ and $\tilde{y}_{t=1,\dots,T}$.

Because we only work with discrete data, we do not use the cumulative distribution functions defined in Equation (3.1) for the continuous case. Rather, we construct and use cumulative distribution functions denoted as $\hat{F}^{[n]}$ and $\hat{G}^{[n]}$ that are multiply cumulative weights at each data point, with horizontal plateaus between data points. So, we extend the definition of an empirical cumulative distribution function to any order n as follows:

Definition 1 (Empirical Cumulative Distribution and Survival Functions at Order n)**.** *We classically define an empirical cumulative distribution function $\hat{F}$ as a function that is worth $\frac{k}{n}$ for any x that pertains to $[\tilde{x}_k, \tilde{x}_{k+1})$. The cumulative contribution at point $\tilde{x}_k$ of this empirical cumulative distribution function $\hat{F}$ is $\sum_{i=1}^{k} \frac{i}{n} = \frac{k(k+1)}{2n}$. So, we define $\hat{F}^{[2]}$ as follows: $\hat{F}^{[2]}(x) = \frac{k(k+1)}{2n}$ for all $x \in [\tilde{x}_k, \tilde{x}_{k+1})$. Similarly, the cumulative contribution at point $\tilde{x}_k$ of the cumulative empirical cumulative distribution function $\hat{F}^{[2]}$ is $\sum_{i=1}^{k} \frac{i(i+1)}{2n} = \frac{k(k+1)(k+2)}{6n}$. Therefore, we define $\hat{F}^{[3]}$ as follows: $\hat{F}^{[3]}(x) = \frac{k(k+1)(k+2)}{6n}$ for all $x \in [\tilde{x}_k, \tilde{x}_{k+1})$. The definition of $\hat{F}^{[n]}$ at any order n follows by repeating multiple times this process. We have:*

$$\hat{F}^{[n]}(x) = \sum_{j_n=1}^{k} \cdots \sum_{j_1=1}^{j_2} \frac{j_1}{n}, \qquad \forall x \in [\tilde{x}_k, \tilde{x}_{k+1}). \tag{3.2}$$

At any order n, we also define an empirical survival function $\bar{\hat{F}}^{[n]}$. This function is constructed as follows: $\bar{\hat{F}}^{[n]}(x) = \hat{F}^{[n]}(\tilde{x}_n) - \hat{F}^{[n]}(x)$ for all $x \in (\tilde{x}_{k-1}, \tilde{x}_k]$. So, $\bar{\hat{F}}(x) = 1 - \hat{F}(x)$ for all $x \in (\tilde{x}_{k-1}, \tilde{x}_k]$, $\bar{\hat{F}}^{[2]}(x) = \frac{n+1}{2} - \hat{F}^{[2]}(x)$ for all $x \in (\tilde{x}_{k-1}, \tilde{x}_k]$, $\bar{\hat{F}}^{[3]}(x) = \frac{(n+1)(n+2)}{6} - \hat{F}^{[3]}(x)$ for all $x \in (\tilde{x}_{k-1}, \tilde{x}_k]$, and so on. We have:

$$\bar{\hat{F}}^{[n]}(x) = \sum_{j_n=1}^{n} \cdots \sum_{j_1=1}^{j_2} \frac{j_1}{n} - \sum_{j_n=1}^{k} \cdots \sum_{j_1=1}^{j_2} \frac{j_1}{n}, \qquad \forall x \in [\tilde{x}_k, \tilde{x}_{k+1}). \tag{3.3}$$

Definition 2 (Stochastic Dominance of Empirical Distribution Functions)**.** *An empirical distribution $\hat{F}$ dominates an empirical distribution $\hat{G}$ at order n in the stochastic sense when $\hat{F}^{[n]}(r) \leq \hat{G}^{[n]}(r)$ for all $r \in \mathbb{R}$ and provided there exists $r_0 \in \mathbb{R}$ such that $\hat{F}^{[n]}(r_0) < \hat{G}^{[n]}(r_0)$.*

This article constructs a method that is equivalent to the comparison of empirical distribution functions. For this purpose, we need to introduce the following definition.

Definition 3 (Cumulative sums)**.** *The cumulative sums of $\tilde{x}$ at order n are given by*

$$\forall t \leq T \quad \tilde{x}_t^{[n]} = \sum_{j_{n-1}=1}^{t} \sum_{j_{n-2}=1}^{j_{n-1}} \cdots \sum_{j_1=1}^{j_2} \tilde{x}_{j_1}. \tag{3.4}$$

These cumulative sums are discrete equivalents to the integrals of a cumulative distribution function provided in Equation (3.1). We use this formula up to order 4 for most financial applications[7]. Using this definition of cumulative sums at order n, we extend the concept of dominance in the majorization sense Marshall and Olkin (1979) to any order.

Definition 4 (Dominance in the Majorization Sense). *We claim that x dominates y at order n in the majorization sense, and we write $x \succ^{[n]} y$, when*

$$\tilde{x}_t^{[n]} \geq \tilde{y}_t^{[n]} \quad \textit{for all} \quad t \leq T. \tag{3.6}$$

We can now state the core theoretical result of this section that extends Theorem 1 in Kuosmanen (2004) to any order. By observing that $\hat{F}^{[n]}$ and $\hat{G}^{[n]}$ are monotonically increasing piecewise linear functions with vertices located in $\tilde{x}_t^{[n]}$ and $\tilde{y}_t^{[n]}$, for $t = 1, ..., T$, we have:

Proposition 1. *Stochastic dominance of empirical distribution functions at order n is equivalent to dominance in the majorization sense at order n. In explicit terms:*

$$\hat{F}^{[n]}(r) \leq \hat{G}^{[n]}(r) \quad \forall r \in \mathbb{R} \quad \Leftrightarrow \quad \tilde{x}_t^{[n]} \geq \tilde{y}_t^{[n]} \quad \forall t \leq T, \tag{3.7}$$

A detailed proof of this result is provided in the appendix.

Empirically, we apply the comparison of the distributions in the majorization sense, in order to compare them in the stochastic dominance sense. We use the ranked returns of the sub-indices and the corresponding indices to compute the cumulative sums of lower order return series (up to order four) as in Equation (3.5). These cumulative sums of subindex returns are then compared with the cumulative sums of index returns in the spirit of the majorization theorem, reflecting the corresponding order of stochastic dominance.

3.2.2 Portfolio dominating sets

For a given portfolio of assets, the dominating set includes all the portfolios that can be constructed using the same assets and that dominate this portfolio at a given order n.

Consider a simple illustrative example: we construct the dominating set of a portfolio for which there are two return observations: $(1, 4)$. We look for all the pairs of returns (x_1, x_2) that satisfy $(x_1, x_2) \succ^{[n]} (1, 4)$, for $n = 1$ to 4. The cases $n = 1$ and $n = 2$ of first and second order stochastic dominance are studied in Kuosmanen (2004). For simplicity, we only consider the case where $x_1 < x_2$. For each order n, the case $x_1 > x_2$ is readily obtained by symmetry.

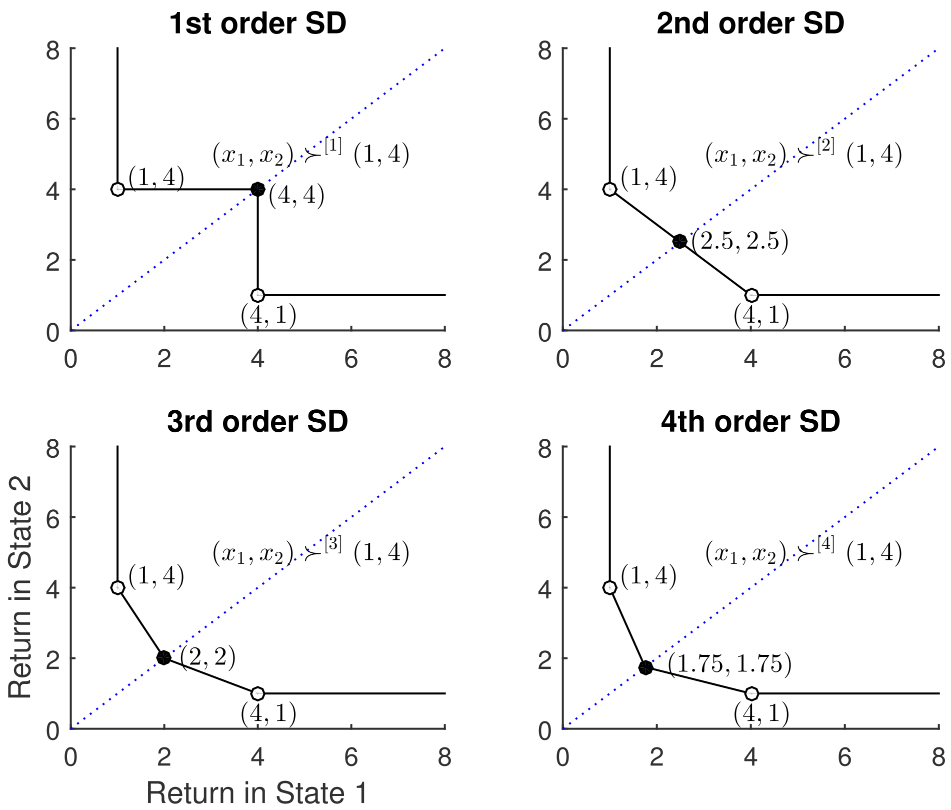
Using Equation 3.5, we see that a portfolio (x_1, x_2) dominates the portfolio $(1, 4)$ at order 1 in the majorization sense, and we write $(x_1, x_2) \succ^{[1]} (1, 4)$, when $x_1 > 1$ and $x_2 > 4$. Then, using Equation (3.5), we have that (x_1, x_2) dominates $(1, 4)$ at order 2 in the majorization sense, or $(x_1, x_2) \succ^{[2]} (1, 4)$, when $x_1 > 1$ and $x_1 + x_2 > 1 + 4$. The latter condition can be associated with the following limit segment: $x_2 = 5 - x_1$, which starts at $(1, 4)$ and stops on the straight line $x_2 = x_1$.

The third and fourth order dominating sets can be constructed in a similar way, using Equation (3.5). Specifically, a portfolio (x_1, x_2) dominates $(1, 4)$ at order 3 in the

[7]The cumulative sums for the first four orders are given, $\forall t \leq T$, by

$$\tilde{x}_t^{[1]} = \tilde{x}_t, \quad \tilde{x}_t^{[2]} = \sum_{j_1=1}^{t} \tilde{x}_{j_1}, \quad \tilde{x}_t^{[3]} = \sum_{j_2=1}^{t} \sum_{j_1=1}^{j_2} \tilde{x}_{j_1}, \text{ and } \quad \tilde{x}_t^{[4]} = \sum_{j_3=1}^{t} \sum_{j_2=1}^{j_3} \sum_{j_1=1}^{j_2} \tilde{x}_{j_1}. \tag{3.5}$$

FIGURE 3.1: Dominating sets at order one to four: an illustration



majorization sense, or $(x_1, x_2) \succ^{[3]} (1,4)$, when $x_1 > 1$ and $x_1 + x_1 + x_2 > 1 + 1 + 4$. The latter condition can be associated with the following limit segment: $x_2 = 6 - 2x_1$, which starts at $(1,4)$ and stops on the straight line $x_2 = x_1$. Finally, (x_1, x_2) dominates $(1,4)$ at order 4 in the majorization sense, or $(x_1, x_2) \succ^{[4]} (1,4)$, when $x_1 > 1$ and $x_1 + x_1 + x_1 + x_2 > 1 + 1 + 1 + 4$. The latter condition can be associated with the following limit segment: $x_2 = 7 - 3x_1$, which starts at $(1,4)$ and stops on the straight line $x_2 = x_1$.

Figure 3.1 summarizes these results. The sets that dominate $(1,4)$ are all convex except the dominating set at order 1. The figure confirms that dominating sets are increasing by inclusion: the dominating set at order m is included in the dominating set at order n, for $n \geq m$. This result has strong implications that are explored in the remainder of this text. If we cannot find empirical portfolios in a dominating set at order 3, for instance, then the dominating sets at order 1 and 2 are empty. Equivalently, empirical emptiness of dominating portfolios is a more powerful property when the order n increases.

3.2.3 SD efficiency of portfolios

The concept of pairwise stochastic dominance can be extended to stochastic dominance efficiency (SD efficiency). A portfolio of assets is SD efficient of order K relative to a given span of the underlying assets when it is not possible to fund any other linear combination of the assets that dominates this portfolio at order K or higher. On the contrary, if a portfolio is not efficient, it is dominated by at least one other portfolio. Consider, for example, second order stochastic dominance. If a portfolio is not second order efficient, one can construct another portfolio using the same assets, such that the latter portfolio second order stochastically dominates the former portfolio. All risk averse investors favor this latter portfolio.

From Hardy, Littlewood, and Pólya (1934), a portfolio dominates another portfolio at order two if the former portfolio can be expressed as the product of a doubly stochastic matrix by the latter portfolio.[8] By extension, we have:

$$\forall t \leq T \quad \tilde{x}_t^{[2]} \quad \geq \quad \tilde{y}_t^{[2]} \quad \Leftrightarrow \quad \exists W \in \Xi \mid x \geq Wy, \tag{3.8}$$

where Ξ is the set of all doubly stochastic matrices and W is one element of this set.

Replacing x by $x^{[n-1]}$ and y by $y^{[n-1]}$ at any order n, we have the generalized result:

Proposition 2 (High Order Stochastic Dominance and Doubly Stochastic Matrices)**.** *A portfolio x dominates a portfolio y at order n if and only if the cumulative sum at order $n-1$ of the returns of portfolio x is larger than the product of a doubly stochastic matrix by the cumulative sum at order $n-1$ of the returns of portfolio y:*

$$\forall t \leq T \quad \tilde{x}_t^{[n]} \quad \geq \quad \tilde{y}_t^{[n]} \quad \Leftrightarrow \quad \exists W \in \Xi \mid x^{[n-1]} \geq Wy^{[n-1]}. \tag{3.9}$$

Specifically, for third order stochastic dominance:

$$\forall t \leq T \quad \tilde{x}_t^{[3]} \quad \geq \quad \tilde{y}_t^{[3]} \quad \Leftrightarrow \quad \exists W \in \Xi \mid x^{[2]} \geq Wy^{[2]}, \tag{3.10}$$

and for fourth order stochastic dominance:

$$\forall t \leq T \quad \tilde{x}_t^{[4]} \quad \geq \quad \tilde{y}_t^{[4]} \quad \Leftrightarrow \quad \exists W \in \Xi \mid x^{[3]} \geq Wy^{[3]}. \tag{3.11}$$

While stochastic dominance comparisons are conducted with permutation matrices[9] at order 1 and with doubly stochastic matrices at order 2, Proposition 2 shows that stochastic dominance comparisons at any higher order can also be achieved using doubly stochastic matrices as well.

Denote by y a portfolio being tested for n^{th} order stochastic dominance efficiency. We want to compare this portfolio to a market represented by N assets for which we have T observations. This market is represented by the database $(y^1, ..., y^N)$, where each element y^j is a vector of T observations. We also construct a broader database Y comprised of the market database completed by the portfolio being tested: $Y = (y, y^1, ..., y^N)$.

Using the generalization (3.9) of (3.8), we extend Theorem 5 in Kuosmanen (2004) to an arbitrary order n:

Proposition 3 (n^{th} Order SD Efficiency, Necessary Condition)**.** *Denote*

$$\theta_n^{nec}(y) = \frac{1}{T} \max_{\lambda, W} \left(\sum_{t=1}^{T} \sum_{i=1}^{N+1} Y_{i,t} \lambda_i - \sum_{t=1}^{T} y_t \right),$$

such that

$$(Y\lambda)^{[n-1]} \geq Wy^{[n-1]},$$

[8] A doubly stochastic matrix is a square matrix with all entries being non-negative real numbers and with the sums of the elements along each row and column being equal to one.

[9] A permutation matrix is a square matrix with all entries being equal to zero or one and with the sums of the elements along each row and column being equal to one.

where W is a doubly stochastic matrix and λ a vector of portfolio weights in the portfolio being tested and in the reference market. Then, $\theta_n^{nec}(y) = 0$ is a necessary condition for the portfolio y to be n^{th} order SD efficient given the market information $(y^1, ..., y^N)$.

Similarly, using the generalization (3.9) of (3.8), we extend the sufficient condition of Theorem 6 in Kuosmanen (2004) to an arbitrary order n:

Proposition 4 (n^{th} order SD Efficiency, Sufficient Condition)**.** *Define*

$$\theta_n^{suf}(y) = \min_{W,\lambda,s^+,s^-} \sum_{j=1}^{T}\sum_{i=1}^{T}\left(s_{ij}^+ + s_{ij}^-\right),$$

such that

$$(Y\lambda)^{[n-1]} = Wy^{[n-1]},$$

where s_{ij}^+ and s_{ij}^- are non-negative numbers satisfying:

$$s_{ij}^+ - s_{ij}^- = W_{ij} - \frac{1}{2},$$

and where W is a doubly stochastic matrix and λ a vector of portfolio weights in the portfolio being tested and in the reference market.

Denote by d_t the number of occurrences where t values are identical in the portfolio being tested. Then, $\theta_n^{suf}(y) = \frac{T^2}{2} - \sum_{t=1}^{T} td_t$ is a sufficient condition for this portfolio to be n^{th} order SD efficient given the market information $(y^1, ..., y^N)$.

Propositions 3 and 4 give necessary and sufficient conditions for a candidate portfolio to be efficient with respect to a given market. As a by-product of Proposition 3, one also obtains the optimal portfolio weights, defining the SD efficient portfolio. These weights are given by the optimal values of λ, obtained as the solution of the optimization problem defined in Proposition 3.

3.3 The road map of the empirical analysis

We conduct our analysis in several steps. First, for each market index i in our sample and each year t for which the information on the index and its constituents is available we construct time series of sub-index returns. We sort all year-beginning components of a benchmark index into sector groups according to their ICB codes. The return time series for sub-indices are calculated with weighting scheme consistent with that of the benchmark index (so, for value weighted indices, the sub-indices are also value weighted).

Next, we test if each index i is SD efficient with respect to any other portfolio constructed as a combination of its sub-indices. We use the extended approach of Kuosmanen (2004) for higher order stochastic dominance, as detailed in Section 3.2.1. For those cases in which the index is not efficient, we compute portfolio weights for sub-indices to construct a dominating efficient portfolio with the highest mean improvement relative to the index under consideration. Then, we compare the performance of the indices with that of the dominating efficient portfolios to assess the maximum possible gain for investors that optimize their portfolios using the stochastic dominance approach. The technical details of the efficiency tests and portfolio construction are provided in Section 3.2.3.

When implementing the above approach, we obtain the optimal portfolios from the necessary stochastic dominance condition. We start by testing for the first order SD efficiency of each index against all possible combinations of its sub-indices and itself. If the index turns efficient at a given order, we move on to test for a higher order efficiency. Conversely, because inefficiency at a given stochastic dominance order also implies inefficiency at higher orders, there is no need to conduct tests at higher orders for a portfolio that is inefficient at a given order.

The necessary test we implement is a linear programming optimization problem that maximizes the mean return difference of a potentially dominating portfolio over the portfolio in test. This maximization is subject to the constraint that the returns of the potentially dominating portfolio are larger than the returns of a mean preserving anti-spread of the portfolio in test. This mean preserving anti-spread is formed as the portfolio in test is permuted by a non-negative doubly stochastic matrix. If the mean return difference is 0, it is straightforward that the portfolio in test is necessarily efficient and we can further conduct necessary stochastic dominance test at higher order.

Although it cab be possible to construct a dominating portfolio relative to an index using realized returns, frequent re-balancing may lead to high transaction costs. Further, relying on in-sample optimality may lead to overfitting and not necessarily stellar out-of-sample performance.[10] Thus, in a second step, we refrain from the in-sample optimization and perform a pairwise stochastic dominance comparison of each index with the underlying sub-indices every year.

Next we analyze the determinants of dominance of sub-indices over the corresponding market indices using logistic regression framework. Last but not least, we use a simple trading strategy that equally weights the sub-indices that have dominated the index in the past, up to the fourth-order of stochastic dominance, and evaluate the performance of this portfolio.

3.4 Data

In this paper we use seventeen equity indices from Datastream spanning most representative regions of the world (including American, Asian, European developed and developing markets) and their constituent industry-based sub-indices. We use the total return approach to calculating daily return series for each index in our sample.

In order to construct industry sub-indices, we first group the constituents of each of the equity indices according to Industry Classification Benchmark (ICB), which assigns each company to one of 10 macro industries. Then the return on an industry sub-index is calculated as a (weighted) average of the returns on all stocks classified in this industry. The methodology for computing sub-indices is consistent with that of the corresponding benchmark index. If the benchmark index is price weighted (market capitalization weighted), its sub-indices are also price weighted (market capitalization weighted). In order to assure that the constructed sub-indices are investible and are not subject to a look-ahead bias, we determine the constituents of each index at the beginning of a year and keep them unchanged until the end of a year. If any of the firms are delisted during that year (due to various corporate events such as a bankruptcy or a merger), we use their return series until the last

[10] See, for example,Hodder, Jackwerth, and Kolokolova (2014). Post, Karabati, and Arvanitis (2018) suggest an approach to improve the out-of-sample performance of the portfolios chosen using SD criteria up to the order 3, by jointly utilizing an empirical likelihood estimation method for the multivariate return distribution.

active trading day, and subsequently re-weight the sub-index after deletion of the concerned stocks without adding any new stock until the year end is reached.

Our sample traces each equity index back to its earliest complete year covered in Datastream. A few components do not have complete series of total return or market capitalization in certain years. We exclude such firms from our sub-indices composition. Table 3.1 lists the indices used in our study together with the starting date of their histories. Table 3.2 reports the corresponding return descriptive statistics.

TABLE 3.1: Equity indices

The table lists seventeen equity indices used in our study together with their country of origin. The source of the data is Datastream. "Start year" indicates the first full year when the index is available in our sample. The sample ends on December 31, 2015.

	Ticker	Short name	Country	Start year
S&P 100 Index	OEX	S&P 100	U.S.	1990
Dow Jones Industrial Average	DJI	DJIA	U.S.	2004
FTSE 100 Index	UKX	FTSE 100	U.K.	1996
CAC 40	PX1	CAC	France	2001
DAX Performance Index	DAX	DAX	Germany	2001
EURO STOXX 50	SX5E	Euro Stoxx 50	Eurozone	2001
Nikkei 225	NI225	Nikkei 225	Japan	2002
Andice Bovespa	IBOV	Indice Bovespa	Brazil	2007
RTS Index	RTSI	RTS Index	Russia	2008
S&P BSE SENSEX	SENSEX	BSE SENSEX	India	2005
Shanghai Stock Exchange 50 A Share Index	000016	SSE 50	China	2009
FTSE/JSE Top 40 Index	JTOPI	FTSE/JSE Top 40	South Africa	2003
S&P/ASX 50	AS31	S&P/ASX 50	Australia	2001
MERVAL Index	MERVAL	MERVAL Index	Argentina	2002
S&P/TSX 60	SPTSX60	S&P/TSX 60	Canada	2003
FTSE MIB	FTSEMIB	FTSE MIB	Italy	2010
KOSPI 50 Index	KOSPI50	KOSPI 50	South Korea	2003

3.5 Empirical results: market indices vs. sector sub-indices

3.5.1 SD index efficiency

We start the section by presenting an example of application of our methodology to a single index, the S&P 100 index. Then, we proceed with a discussion of the complete set of empirical results based on all seventeen domestic indices.

Worked example: the S&P 100 index

Table 3.3 reports the results of the efficiency test of the S&P 100 index. Efficiency of the index is tested against all possible portfolios which can be constructed using the index and its industry-based sub-indices using daily returns for each year from 1990 to 2015. The first row of the table reports the lowest order of index inefficiency. Note that dominance at order 2 implies dominance at order 3 and beyond. However, the converse is not true. So, a number 2 in the table means that the index is not efficient of order 2, that is, one can construct a portfolio that dominates the index at orders 2, 3, 4, and beyond. A number 3 in the table indicates that it is possible to

TABLE 3.2: Equity indices: descriptive statistics

The table reports the descriptive statistics of the daily returns of the seventeen equity indices used in our study. For each index, the sample starts when the complete data for index components becomes available and ends on December 31, 2015. The mean, standard deviation (Std), minimum (Min) return, maximum (Max) return, and Sharpe ratio are annualised, whereas skewness and kurtosis based on the original daily returns.

	Mean	Std	Median	Min	Max	Skewness	Kurtosis	Sharpe ratio
S&P 100	0.09	0.18	0.08	-23.97	27.81	-0.17	11.35	0.31
DJIA	0.07	0.18	0.09	-21.40	27.43	-0.07	14.64	0.31
FTSE 100	0.06	0.19	0.06	-24.18	24.49	-0.16	8.98	0.13
CAC	0.02	0.24	0.05	-24.72	27.65	0.03	8.09	-0.18
DAX	0.03	0.25	0.13	-23.16	28.18	-0.02	7.63	-0.10
Euro Stoxx 50	0.01	0.24	0.00	-21.37	27.24	0.01	7.57	-0.04
Nikkei 225	0.05	0.24	0.00	-31.61	34.54	-0.48	10.64	0.11
Indice Bovespa	0.00	0.29	0.00	-31.57	35.70	0.02	9.49	-0.40
RTS Index	-0.11	0.39	0.00	-55.33	52.73	-0.30	13.77	-0.44
BSE SENSEX	0.14	0.24	0.03	-30.29	41.73	0.08	11.88	0.36
SSE 50	0.10	0.27	0.00	-25.71	19.70	-0.34	7.16	0.31
FTSE/JSE Top 40	0.16	0.21	0.11	-20.46	20.22	-0.10	6.73	0.36
S&P/ASX 50	0.08	0.17	0.07	-22.52	15.84	-0.36	8.73	0.16
MERVAL Index	0.26	0.33	0.06	-33.80	32.88	-0.35	7.37	0.53
S&P/TSX 60	0.08	0.18	0.17	-26.90	25.65	-0.68	15.21	0.37
FTSE MIB	0.02	0.26	0.00	-18.39	27.89	-0.10	5.47	-0.04
KOSPI 50	0.09	0.23	0.00	-28.38	30.54	-0.30	8.88	0.28

construct a portfolio that dominates the index at orders 3, 4, and beyond, but not at order 2.

The middle part of the table reports the optimal portfolio weights for each sub-index for every year. Then, the descriptive statistics of the index and the optimal portfolio are reported. The last row of the table reports the improvement of the mean return which could have been achieved should an investor have invested in the optimal portfolio and not the index. The last column of Table 3.3 summarises the average performance of the optimal portfolios relative to the index across the last quarter of a century. The order of inefficiency reflect the inefficiency of the index in majorization sense over the complete sample period.

Almost always (with the exception of years 1994, 2000, and 2005) the S&P 100 index is not efficient of order 2. This implies that most of time risk averse investors would be better off by not tracking the index, but investing in a different portfolio in which some industries are overweighed relative to the index. In years 1994, 2000, and 2005 the index is inefficient at order 3, implying that risk averse and prudent investors should optimally deviate from holding the index. Taken together as one time series, the S&P 100 index is still inefficient of order 3 over the complete sample.

Optimal portfolios show substantially higher mean returns than the index and deliver a higher Sharpe ratio every year. Overall, over the 25 years the potential mean improvement is 15% annualized. At the same time, such attractive gains may be difficult to achieve in practice, as the optimal portfolio weights are rather volatile. For example, the health care sub-index has a weight of 54% in 1990, 77% in 1991, and then three consecutive years of zero weights until the weight increases again to 36% in 1995. Such a volatility of optimal weights makes the out-of-sample construction of SD efficient portfolios rather difficult. This result confirms findings in Hodder, Jackwerth, and Kolokolova (2014) based on Fama-French industry portfolios that the construction of out-of-sample SD efficient portfolios is challenging.

TABLE 3.3: (In-)Efficiency of the S&P 100 index

The table reports the results of the SD efficiency tests for the S&P 100 index. The first row reports the lowest order of index inefficiency for each year from 1990 to 2015. The middle part of the table report the optimal weights of sub-indices in each year, and the descriptive statistics of the index returns and that of the optimal portfolio. The last row or the table reports the difference in means between the optimal portfolio and the index. The mean return, return standard deviation (Std), and Sharpe ratio are annualised, whereas skewness and kurtosis are based on the original daily returns.

SP100 Domi Weights	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	Average
Inefficiency order	2	2	2	2	3	2	2	2	2	2	3	2	2	2	2	3	2	2	2	2	2	2	2	2	2	2	2
S&P 100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Oil and gas	0.15	0.00	0.07	0.00	0.00	0.18	0.27	0.00	0.00	0.14	0.00	0.00	0.00	0.00	0.08	1.00	0.24	0.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
Basic materials	0.00	0.00	0.00	0.00	0.13	0.00	0.20	0.00	0.00	0.23	0.00	0.39	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00	0.00	0.00	0.00	0.00	0.05
Industrials	0.00	0.00	0.00	0.71	0.00	0.16	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.50	0.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.00	0.00	0.09
Consumer goods	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.62	0.00	0.08	0.00	0.30	0.26	0.00	0.03	0.67	0.00	0.00	0.00	0.00	0.00	0.08
Health care	0.54	0.77	0.00	0.00	0.00	0.36	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.04	0.05	0.00	0.00	0.00	0.21	0.06	0.00	0.09
Consumer Services	0.00	0.13	0.00	0.00	0.00	0.00	0.00	0.21	0.00	0.26	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.96	0.00	0.00	0.00	0.74	0.28	0.00	0.83	0.13
Telecommunications	0.00	0.00	0.50	0.00	0.00	0.00	0.00	0.58	0.19	0.00	0.00	0.46	0.00	0.00	0.00	0.00	0.35	0.00	0.00	0.00	0.01	0.00	0.00	0.04	0.00	0.17	0.09
Utilities	0.00	0.10	0.08	0.29	0.00	0.06	0.00	0.00	0.24	0.00	1.00	0.00	0.00	0.24	0.25	0.00	0.11	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.62	0.00	0.15
Financials	0.00	0.00	0.34	0.00	0.00	0.20	0.26	0.21	0.00	0.00	0.00	0.00	0.38	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00	0.00	0.00	0.07
Technology	0.31	0.00	0.00	0.00	0.87	0.05	0.19	0.00	0.58	0.33	0.00	0.00	0.00	0.22	0.12	0.00	0.00	0.00	0.00	0.92	0.00	0.00	0.00	0.05	0.32	0.00	0.15
Index Mean	-0.04	0.25	0.06	0.11	0.03	0.34	0.23	0.26	0.29	0.28	-0.13	-0.15	-0.26	0.23	0.06	0.01	0.17	0.06	-0.43	0.20	0.12	0.03	0.15	0.27	0.12	0.03	0.09
Index Std	0.17	0.17	0.10	0.09	0.10	0.08	0.12	0.19	0.21	0.19	0.24	0.23	0.27	0.18	0.11	0.10	0.09	0.16	0.40	0.26	0.17	0.22	0.12	0.11	0.11	0.16	0.18
Index skewness	-0.16	0.52	0.04	0.00	-0.18	0.02	-0.59	-0.48	-0.69	-0.02	0.00	0.03	0.43	0.01	-0.08	0.03	0.02	-0.49	0.01	0.03	-0.23	-0.52	0.10	-0.24	-0.38	-0.16	-0.17
Index kurtosis	3.87	11.89	3.20	5.10	4.26	4.27	4.73	8.98	8.36	2.85	4.20	4.96	3.80	3.91	2.97	3.12	4.41	4.80	6.85	5.46	5.12	5.93	4.08	4.69	4.54	5.24	11.35
Sharpe ratio	-0.71	1.11	0.22	0.85	-0.17	3.43	1.42	1.10	1.14	1.24	-0.82	-0.79	-1.00	1.27	0.44	-0.20	1.29	0.10	-1.12	0.78	0.68	0.14	1.19	2.48	1.07	0.16	0.02
Opt. Port mean	0.08	0.41	0.19	0.19	0.15	0.40	0.28	0.34	0.41	0.35	0.47	-0.06	-0.12	0.28	0.15	0.17	0.22	0.16	-0.11	0.44	0.20	0.16	0.24	0.30	0.25	0.11	0.22
Opt. Port Std	0.17	0.17	0.10	0.09	0.19	0.08	0.13	0.18	0.21	0.19	0.29	0.23	0.22	0.17	0.11	0.23	0.10	0.15	0.35	0.26	0.18	0.15	0.13	0.11	0.11	0.15	0.18
Opt. Port skewness	-0.21	-0.12	0.12	-0.19	0.23	-0.04	-0.39	-0.72	-0.41	-0.06	-0.13	-0.03	0.30	0.03	-0.04	-0.36	0.00	-0.67	0.35	0.11	-0.08	-0.22	0.14	-0.21	0.08	-0.14	-0.03
Opt. Port kurtosis	3.48	4.90	2.83	3.39	5.43	3.99	3.76	7.39	5.68	2.77	3.25	4.29	4.35	3.58	3.11	3.38	3.68	4.16	8.21	5.31	4.79	6.05	3.52	4.43	3.19	4.99	8.82
Sharpe ratio	0.04	2.13	1.52	1.78	0.60	4.16	1.80	1.57	1.72	1.61	1.44	-0.42	-0.60	1.53	1.27	0.62	1.84	0.76	-0.37	1.69	1.11	1.05	1.92	2.85	2.16	0.73	0.06
Mean improvement	0.13	0.16	0.14	0.08	0.13	0.07	0.05	0.08	0.12	0.07	0.61	0.09	0.14	0.04	0.09	0.16	0.05	0.10	0.32	0.24	0.08	0.13	0.10	0.04	0.13	0.09	0.13
Std improvement	0.00	-0.01	0.00	0.00	0.09	0.00	0.00	-0.01	0.00	0.00	0.05	0.00	-0.05	0.00	0.00	0.13	0.00	0.00	-0.05	0.00	0.01	-0.07	0.00	0.00	0.00	-0.01	0.00
Skewness improvement	-0.05	-0.64	0.09	-0.19	0.40	-0.06	0.20	-0.24	0.28	-0.04	-0.13	-0.05	-0.13	0.02	0.04	-0.39	-0.02	-0.18	0.34	0.08	0.15	0.30	0.04	0.03	0.46	0.02	0.14
Kurtosis improvement	-0.38	-6.99	-0.37	-1.72	1.17	-0.28	-0.98	-1.59	-2.67	-0.08	-0.95	-0.67	0.55	-0.33	0.14	0.25	-0.73	-0.65	1.36	-0.15	-0.33	0.11	-0.56	-0.26	-1.35	-0.25	-2.53
Sharpe ratio improvement	0.75	1.02	1.30	0.93	0.76	0.74	0.39	0.47	0.59	0.37	2.26	0.37	0.39	0.26	0.84	0.82	0.55	0.66	0.75	0.91	0.44	0.92	0.73	0.37	1.09	0.57	0.04

Global indices results

Table 3.4 summarises the average results for the 17 global indices under study. For each index, we report the average values of the optimal portfolio weights[11], as well as the average descriptive statistics of the optimal portfolios and corresponding index across the years (similar to the structure of the last column of Table 3.3).[12] The SD inefficiency results for other indices are even more striking than those for the S&P100 index. Over the complete sample available for each index, all indices are not SD efficient at order two. Any risk-averse investor across the globe would be better off not investing in the well diversified index, but deviating from it. Table 3.5 further reports the orders of inefficiency of the market indices year by year. It indicates, that all indices have been consistently inefficient at least of order three over their histories.

[11]In Appendix 3.9.1 we further report the average correlations between optimal portfolio weights.
[12]The detailed results for every year are available from the authors upon request.

TABLE 3.4: SD optimal portfolios

The table reports the results of the SD efficiency tests for 17 indices under study. The first row reports the minimum order of SD inefficiency of the index. Then, the average optimal portfolio weights and the descriptive statistics of the optimal portfolio returns are reported. The mean return, return standard deviation (Std), and Sharpe ratio are annualised, whereas skewness and kurtosis are based on the original daily returns. The last rows report the average improvement of the optimal portfolio over the index.

	S&P 100	DJIA	FTSE 100	CAC	DAX	Euro Stoxx 50	Nikkei 225	Indice Bovespa	RTS	BSE SENSEX	SSE 50	FTSE JSE Top 40	S&P/ ASX 50	MERVAL	S&P/ TSX 60	FTSE MIB	KOSPI 50	
Inefficiency order	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Optimal portfolio average weights																		
Index	0.00	0.05	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.07	0.14	0.06	0.00	0.00	0.05	0.00	0.04	0.02
Oil and gas	0.10	0.11	0.01	0.06	-	0.08	0.14	0.04	0.22	0.05	0.00	-	0.07	0.07	0.11	0.17	0.15	0.09
Basic materials	0.05	0.02	0.11	0.10	0.18	0.09	0.04	0.03	0.13	0.01	0.00	0.04	0.15	0.24	0.04	0.00	0.08	0.08
Industrials	0.09	0.08	0.06	0.13	0.05	0.06	0.00	0.02	0.13	0.06	0.02	0.07	0.02	0.22	0.00	0.04	0.03	0.06
Consumer goods	0.08	0.02	0.23	0.19	0.10	0.08	0.02	0.22	0.10	0.16	0.27	0.28	0.10	0.19	0.14	0.39	0.23	0.16
Health care	0.09	0.25	0.14	0.15	0.35	0.20	0.11	0.00	0.03	0.23	0.00	0.19	0.19	0.00	0.07	0.21	-	0.14
Consumer Services	0.13	0.24	0.03	0.00	0.02	0.12	0.11	0.33	0.34	0.00	0.08	0.27	0.14	-	0.18	0.13	0.06	0.14
Telecommunications	0.09	0.08	0.09	0.13	0.06	0.15	0.20	0.21	0.03	0.04	0.29	0.10	0.12	0.17	0.17	0.12	0.18	0.13
Utilities	0.15	-	0.27	0.19	0.09	0.16	0.20	0.13	0.00	0.07	0.03	-	0.14	0.13	0.03	0.02	0.12	0.11
Financials	0.07	0.05	0.05	0.03	0.02	0.03	0.08	0.01	0.05	0.12	0.19	0.01	0.08	0.26	0.14	0.00	0.07	0.07
Technology	0.15	0.11	0.02	0.01	0.13	0.05	0.09	-	0.00	0.20	-	-	-	-	0.07	0.02	0.06	0.08
Optimal portfolio descriptive statistics																		
Opt. Port mean	0.22	0.17	0.20	0.21	0.22	0.16	0.21	0.24	0.34	0.31	0.27	0.33	0.25	0.51	0.25	0.26	0.26	0.26
Opt. Port Std	0.18	0.18	0.18	0.23	0.22	0.23	0.23	0.26	0.35	0.22	0.24	0.21	0.17	0.34	0.16	0.25	0.21	0.23
Opt. Port skewness	-0.03	0.12	-0.11	0.11	0.16	-0.02	-0.24	0.13	0.46	0.11	0.60	-0.08	0.88	0.57	0.02	-0.05	-0.14	0.15
Opt. Port kurtosis	8.82	10.75	7.10	6.60	8.96	6.95	8.65	7.77	23.20	9.32	9.44	6.22	24.51	13.36	6.49	4.45	5.33	9.88
Sharpe ratio	0.06	0.87	0.88	0.67	0.72	0.61	0.82	0.50	0.78	1.16	1.02	1.20	1.17	1.23	1.50	0.91	1.10	0.89
Index portfolio descriptive statistics																		
Index mean	0.09	0.07	0.06	0.02	0.03	0.01	0.05	0.00	-0.11	0.14	0.10	0.16	0.08	0.26	0.08	0.02	0.09	0.07
Index Std	0.18	0.18	0.19	0.24	0.25	0.24	0.24	0.29	0.39	0.24	0.27	0.21	0.17	0.33	0.18	0.26	0.23	0.24
Index skewness	-0.17	-0.07	-0.16	0.03	-0.02	0.01	-0.48	0.02	-0.30	0.08	-0.34	-0.10	-0.36	-0.35	-0.68	-0.10	-0.30	-0.19
Index kurtosis	11.35	14.64	8.98	8.09	7.63	7.57	10.64	9.49	13.77	11.88	7.16	6.73	8.73	7.37	15.21	5.47	8.88	9.62
Sharpe ratio	0.02	0.31	0.13	-0.20	-0.11	-0.04	0.12	-0.40	-0.44	0.36	0.31	0.36	0.17	0.53	0.37	-0.04	0.28	0.10
Optimal portfolio improvement																		
Mean	0.13	0.10	0.14	0.20	0.19	0.15	0.16	0.24	0.44	0.17	0.16	0.17	0.18	0.24	0.17	0.24	0.17	0.19
Std	0.00	0.00	-0.01	-0.01	-0.03	-0.02	-0.01	-0.03	-0.04	-0.02	-0.03	-0.01	0.01	0.01	-0.02	-0.01	-0.01	-0.01
Skewness	0.14	0.20	0.05	0.08	0.19	-0.03	0.24	0.12	0.76	0.03	0.94	0.02	1.24	0.92	0.70	0.05	0.16	0.34
Kurtosis	-2.53	-3.89	-1.87	-1.49	1.33	-0.62	-1.99	-1.71	9.43	-2.57	2.28	-0.51	15.78	5.99	-8.73	-1.02	-3.55	0.26
Sharpe ratio	0.04	0.57	0.75	0.86	0.83	0.65	0.70	0.90	1.22	0.80	0.71	0.84	1.00	0.70	1.13	0.96	0.81	0.79

TABLE 3.5: Time-series of orders of inefficiency of global indices

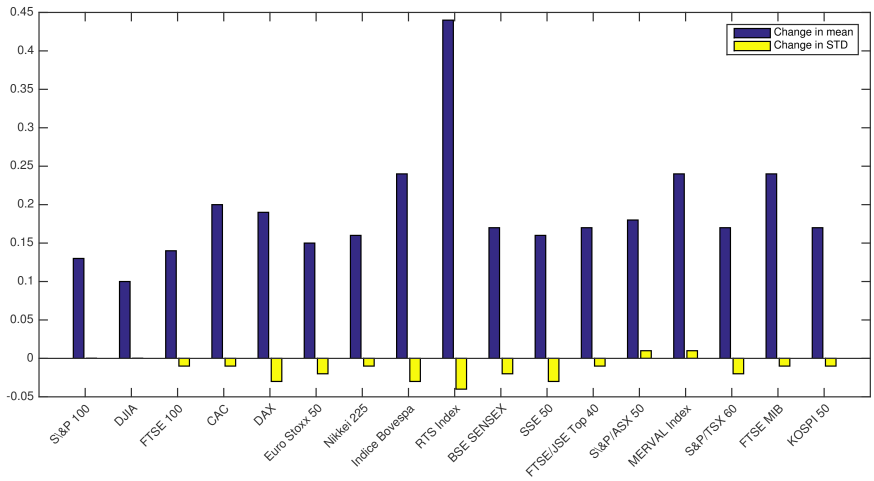
The tables reports the orders of inefficiency of each of the market indices during each of the years from 1990 to 2015. "-" indicates that the index is not available for this particular year in our sample.

	S&P 100	DJIA	FTSE 100	CAC	DAX	Euro Stoxx 50	Nikkei 225	Indice Bovespa	RTS	BSE SENSEX	SSE 50	FTSE JSE Top 40	S&P/ ASX 50	MERVAL	S&P/ TSX 60	FTSE MIB	KOSPI 50
1990	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1991	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1992	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1993	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1994	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1995	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1996	2	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1997	2	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1998	2	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1999	2	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2000	3	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2001	2	-	2	2	2	2	-	-	-	-	-	-	2	-	-	-	-
2002	2	-	2	2	2	2	2	-	-	-	-	-	2	2	-	-	-
2003	2	-	2	2	2	2	2	-	-	-	-	2	3	2	2	-	2
2004	2	2	2	2	2	2	2	-	-	-	-	2	2	2	2	-	2
2005	3	3	2	2	2	2	2	-	-	2	-	2	2	2	2	-	2
2006	2	2	2	2	2	2	2	-	-	2	-	2	2	2	2	-	2
2007	2	2	2	2	3	2	3	2	-	2	-	2	2	2	2	-	2
2008	2	2	2	2	2	2	2	2	2	2	-	2	2	2	2	-	2
2009	2	2	2	2	2	2	3	2	2	2	2	2	3	2	2	-	2
2010	2	2	2	3	2	2	2	2	2	2	2	2	3	2	2	2	2
2011	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2012	2	2	2	2	2	2	2	2	2	2	3	2	2	2	2	2	2
2013	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	2	2
2014	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2
2015	2	3	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2

The potential average gains from deviating from indices are large, but vary across countries. The minimum average gain of 10% annualized is associated with the DJIA index, and the maximum of 44% annualized is achievable for the RTS index. Together with decreasing return standard deviations, this translates into substantial gains in terms of the Sharpe ratios. Figure 3.2 and 3.3 depicts the average improvements in annualized mean returns, standard deviations, and daily Sharpe ratios for our 17 indices.

FIGURE 3.2: Average improvements in mean return, standard deviations

The figure plots the average changes in the annualized mean returns and return standard deviations for the 17 indices under study, when the market index portfolio is swapped for an optimal SD efficient portfolio.

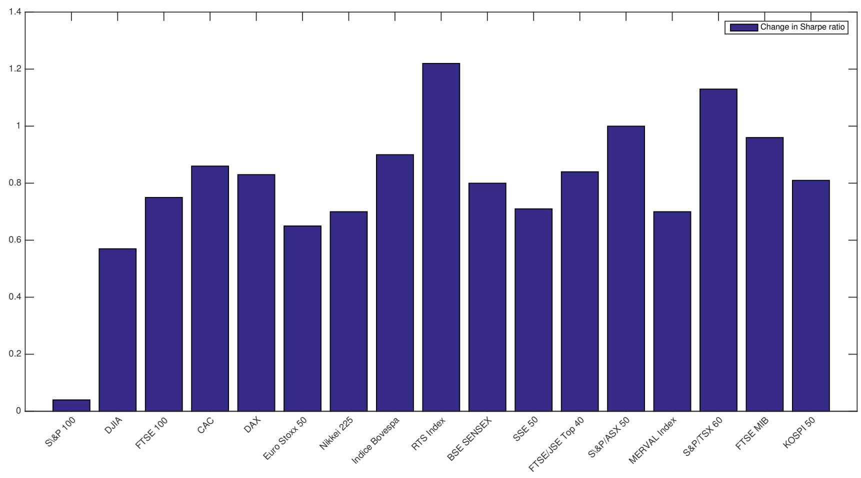


Following Kuosmanen (2004), the degree of portfolio inefficiency can be measures by the optimal parameter θ from Proposition (3). This parameter indicates the maximal possible improvement in the portfolio mean return that can be achieved by moving from an SD inefficient portfolio to an efficient one. Figure 3.4 plots the time series of the estimated θ-s for 17 market indices under study together with the average value of θ for each year. Overall, individual and average θ-s tend to spike during turbulent periods, such as year 2000 (the burst of Internet bubble) and 2008 (the pick of the financial crisis of 2007-2009), indicating serious SD inefficiency of the market indices during these periods. Individual θ-s also spike during market specific events. For example, in 2014 during Russian-Ukrainian geo-political crisis the θ of the RTS index reached a record level of 1.34, implying that during this year, investors who would optimally deviate from the Russian market index could generate 134% of return more per year, while also investing in an SD efficient portfolio. Importantly, however, one cannot identify any particularly trend that would suggest that market index SD efficiency improves over time. The degree of market SD inefficiency seems to be rather persistent, or even marginally increasing during the recent years.

We next consider the SD efficiency of the market indices from the point of view of a long-term investor. This investor does not re-balance their portfolio each year, but instead has been holding the market indices for decades. We, thus, test for the

FIGURE 3.3: Average improvements in Sharpe ratios

The figure plots the average changes in the Sharpe ratios for the 17 indices under study, when the market index portfolio is swapped for an optimal SD efficient portfolio.



SD efficiency of all the indices across the complete sample of monthly returns. We find that all but the S&P 100 and Nikkei 225 indices are not efficient at order 2 across their entire life, indicating, that most of market indicted across the globe have not been a good investment for any risk-averse individual.

3.5.2 Pairwise comparison with sub-indices

The precious sub-section discussed the *ex-post* analysis of SD efficiency of market indices. It provides insights into the maximum potential gains of deviating from an inefficient market portfolio to an efficient one. However, the relevant practical question remains of whether these gains can be realized by choosing portfolios *ex-ante*. This is especially challenging given that the optimal (in SD sense) portfolio weights are not persistent. Thus in this section we discuss a simple, but potentially more robust analysis – a pairwise comparison of the performance of each index and industry sub-indices. Again, we first present the case of the S&P 100 index in detail and then we summarize the results for the global indices.

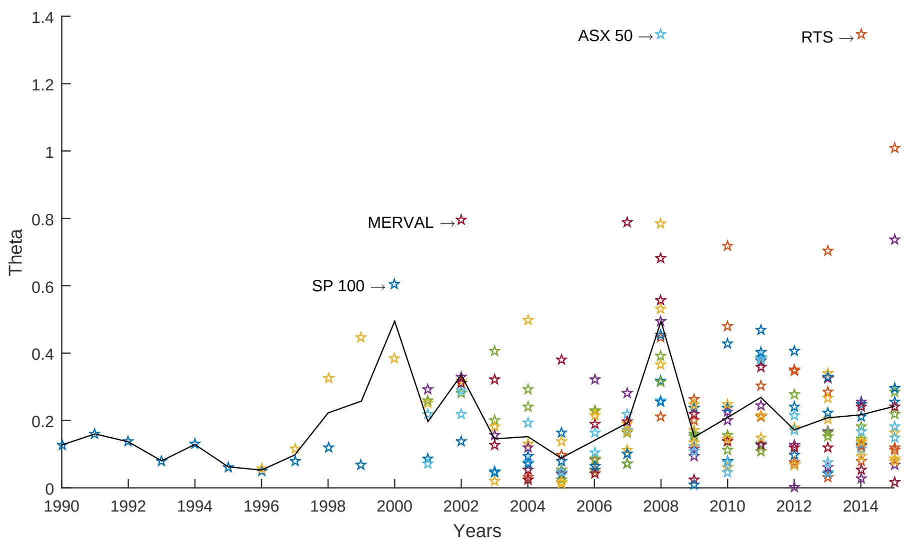
Worked example: the S&P 100 index

Table 3.6 reports the results of the pairwise comparison between the S&P 100 index and its sub-indices for each of the years from 1990 to 2015. The numbers in Panel A indicate the order of dominance of the sub-index (reported in columns) over the S&P 100 index in a given year (reported in rows). The last column of the table reports the minimum order of stochastic dominance of any sub-index over the index. The last row of the table reports the percentage of years in which a given sub-index dominates the index.

Notably, there are several industries which never dominate the index. These include Oil and gas, Basic material, Financial and Technology sectors. At the same time, other sectors, such as Consumer goods, Consumer services and Health care

FIGURE 3.4: Time series of maximal mean improvement of inefficient market indices

The figure plots the time series of the estimated θ-s from Proposition 3 for 17 indices under study. The pentacles indicate individual θ estimates. The solid line plots the average θ-s for each year. Each θ reflects the maximum annualized mean return improvement that can be achieved by moving from an SD inefficient portfolio to an efficient one.



(industries often described as countercyclical) often dominate the index. Consumer goods, for example, dominate the S&P 100 index in 42% of years. Generally, in 58% of years one can find a sub-index that dominates the index at order 3 or higher. This implies that all risk averse (and prudent) investors can still increase their utility functions by investing in sub-indices rather than in the index. Technically, this process is simpler and less costly than the frequent portfolio rebalancing of optimal portfolios implied by Table 3.3. Another emerging pattern is that the index efficiency decreases over the course of time, and the dominance by sub-indices is clustered after 2005. In particular, all risk averse investors would increase their utility function by investing in the Consumer goods sub-index after the year 2000, instead of tracking the diversified S&P 100 index.

We also conduct a reverse test to check if the index dominates its sub-indices. As reported in panel B of Table 3.6, the Consumer Goods sub-index is dominated by the S&P 100 index only in 31% of years. Thus, all risk averse (and prudent) investors would be better off investing only in Consumer goods, rather that in fully diversified portfolio in 42% of cases. They would be better off by sticking to the diversified index in 31% of cases, and in the remaining 17% of cases these portfolios lie in the same dominance class, implying that different types of investors can prefer one of them over another depending on the exact shape of their utility function.

TABLE 3.6: Pairwise comparison of the S&P 100 index and sub-indices

The table reports the results of the pairwise comparison of the S&P 100 index and its industry sub-indices for each year from 1990 to 2015. In Panel A, the numbers indicate the minimum order of dominance of the sub-index over S&P 100. The last row summarizes the percentage of years during which the index was dominated by a given sub-index. In Panel B, the numbers indicate the minimum order of dominance of the S&P 100 index over each sub-indices. The last row summarizes the percentage of years during which the index dominated a given sub-index.

	Oil &gas	Basic mat.	Ind.	Cons. goods	Health care	Cons.Services	Telecom.	Util.	Fin.	Tech.	Minimum SD order
						Panel A: Market index is dominated					
1990								2			2
1991								2			2
1992											
1993											
1994											
1995											
1996											
1997						3		3			3
1998								3			3
1999											
2000											
2001				2	3						2
2002											
2003				3				3			3
2004											
2005											
2006				3							3
2007			4	2	3						2
2008				2	2			2			2
2009				3	3	3		4			3
2010				2	3	2	2	3			2
2011				2	2	2	2	2			2
2012				3	2	2					2
2013											
2014				3							3
2015				2			3				2
Domi ratio	0	0	0.04	0.42	0.27	0.19	0.12	0.35	0	0	0.58
						Panel B: Market index is dominating					
1990		2	2	3	3	2	2		2	3	2
1991										2	2
1992	2	2	3	3	2	3	3	3	3	2	2
1993	2	2	4		2	2	3	3	3	2	2
1994	3	3		2	3	2	2	2	3	3	2
1995	2	2	3	2	3	2	2	2	3	3	2
1996	3		3	2	2	2	2	2	3	3	2
1997	3	2			2				3	2	2
1998				2	3	3				3	2
1999	2	3	2		2	3	2		2	3	2
2000		2		3		2	2		3	2	2
2001		3	3			3	3	2	3	2	2
2002	3	3	2				2	3	3	2	2
2003		3	3		2	2	2		3	3	2
2004	3	2	3		2	2	3	4	3	2	2
2005	3	2	3		2	2	2	3	3	2	2
2006	3	2	2		2		3	3	3	2	2
2007	3	2				3	3	3	2	3	2
2008	3	2							2		2
2009	2	3	2						2		2
2010	3	3	3						2	2	2
2011	3	2	2						2		2
2012	2	2	2				3		3	2	2
2013	2	3	3	2	3	3	2	2	3	2	2
2014	2	2	2		3	3	2	3	3	3	2
2015	2	2			3			2	2	3	2
Domi ratio	0.77	0.88	0.69	0.31	0.62	0.62	0.69	0.54	0.92	0.88	1

Global indices results

Across the globe, the results for the diversified equity indices are also not extremely favorable (Panel A of Table 3.7). For most of the indices, it is possible to find a dominating sub-index in over 50% of years. Surprisingly, two relatively efficient indices are the Indian BSE SENSEX and Korean KOSPI 50 indices, which are dominated in 27% and 31% of years respectively. On the other extreme are the Russian RTS and Italian FTSE MIB indices that are always dominated by at least one industry sub-index.[13]

In terms of the dominating industries, there is considerable cross-country variation. For example, Consumer goods sub-index is often dominating in different markets. However, it never dominates the German DAX, Chinese SSE 50, Canadian TSX 60, and Korean KOSPI 50 indices.

Oil and gas relatively rarely dominates diversified indices, with the exception of Russian RTS, for which risk-averse investors would be better off investing just in oil and gas in 75% of years in our sample, which manifests a strong dependence of the Russian economy on oil and gas exports.

Remarkably, despite the booming financial industry before the crisis of 2007-2008, the Financial sector sub-index very rarely dominates diversified indices, and never does so for developed markets. It is also dominated by the diversified indices in most of cases as reported in Panel B of Table 3.7. These results suggest that only investing in the Financial sector has been an inferior strategy for any risk-averse investor even before the financial crisis. The only exception is the Chinese SSE 50 index, which is dominated by the Financial sub-index in 71% of cases and dominates it only in 14% of cases.

As far as the time variation in index efficiency is concerned, Figure 3.5 plots the share of the diversified indices that are dominated by at least one sub-index during each year starting from 2003. We choose the year 2003 as the starting point, as this is the first year in our sample that covers more than 10 indices across the world. The figure clearly reveals that in the rump up to the financial crises of 2007-2008, more indices have become SD inefficient, making them an unsuitable investment for risk-averse investors.

3.5.3 Determinants of dominating industries

In this section we take a closer look at the drivers of stochastic dominance, and check if it is possible to forecast if a market index will be stochastically dominated by any of the sector sub-indices.

SD inefficiency of market indices. We estimate a logit model for the probability that a sub-index dominates its parent equity index in a given year, and relate it to the past index and sub-index performance, volatility, as well as several key macroeconomic indicators, which potentially can explain SD inefficiency of equity markets.

As macroeconomic factors, we choose a wide range of indicators, including the GDP annual growth rate, annual consumer prices index, total unemployment rate, gross domestic savings, current account balance, and real effective exchange rate

[13]The results discussed here cover SD at orders 4, 3, and 2. In Appendix 3.9.2 we tabulate the results for SD orders 3 and 2 separately. The interpretation of the results does not change, since in majority of cases the indices are dominated at least at the 3rd order, and 4th order SD can be detected only in a handful of cases.

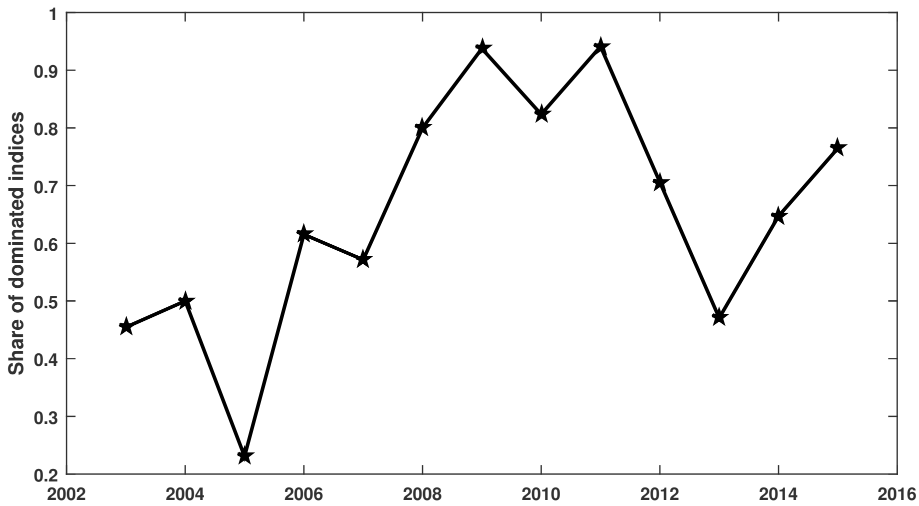
TABLE 3.7: Market indices dominated by/dominating sector sub-indices

Panel A of the table reports the fraction of years during which each of the 17 diversified equity indices is dominated by each of sector sub-indices at orders 4, 3 or 2. The last column reports the fraction of years during which the index is dominated by at least one sub-index. Panel B reports the fraction of years during which each of the 17 diversified equity indices dominated each of sector sub-indices at orders 4, 3 or 2. The last column reports the fraction of years during which the index dominates at least one sub-index.

	Oil&gas	Basic mat.	Ind.	Cons. goods	Health care	Cons. Services	Telecom.	Util.	Fin.	Tech.	Any
Panel A: Market indices are dominated											
S&P 100	0	0	0.04	0.42	0.27	0.19	0.12	0.35	0	0	0.58
DJIA	0	0	0	0.33	0.33	0.33	0.25	-	0	0	0.58
FTSE 100	0	0	0.1	0.55	0.2	0.3	0.05	0.45	0	0	0.75
CAC	0	0	0.07	0.4	0.13	0.33	0.27	0.07	0	0	0.67
DAX	-	0.07	0.07	0	0.33	0.07	0.07	0.2	0.07	0.07	0.53
Euro Stoxx 50	0.2	0	0	0.27	0.07	0.2	0.2	0.2	0	0.07	0.53
Nikkei 225	0.07	0.07	0	0.57	0.71	0.36	0.07	0.43	0	0	0.93
Indice Bovespa	0	0	0.22	0.89	0	0.33	0.33	0.56	0	-	0.89
RTS Index	0.75	0.38	0.13	0.25	0.13	0.5	0.75	0.13	0.13	0	1
BSE SENSEX	0	0	0	0.27	0	0	0	0	0	0	0.27
SSE 50	0.14	0	0.14	0	0	0	0.14	0.43	0.71	-	0.71
FTSE/JSE Top 40	-	0	0.31	0.38	0	0	0	-	0.31	-	0.69
S&P/ASX 50	0	0	0.13	0.2	0	0.4	0	0	0.13	-	0.6
MERVAL Index	0.21	0.36	0.07	0.14	-	-	0.07	0.07	0	-	0.50
S&P/TSX 60	0	0	0.08	0	0.08	0.77	0.23	0.08	0.38	0	0.92
FTSE MIB	0.5	0	0.83	0.83	0	0	0.17	0.83	0	0	1
KOSPI 50	0	0	0	0	-	0	0.23	0.15	0	0	0.31
Panel B: Market indices are dominating											
S&P 100	0.77	0.88	0.69	0.31	0.62	0.62	0.69	0.54	0.92	0.88	1.00
DJIA	1.00	0.92	0.83	0.33	0.50	0.33	0.67	-	0.92	0.75	1.00
FTSE 100	0.80	0.95	0.55	0.25	0.60	0.20	0.80	0.20	1.00	0.90	1.00
CAC	0.60	0.80	0.73	0.13	0.67	0.33	0.60	0.87	0.87	1.00	1.00
DAX	-	0.60	0.67	0.67	0.13	0.73	0.73	0.73	0.73	0.73	1.00
Euro Stoxx 50	0.53	0.73	0.87	0.27	0.73	0.33	0.40	0.47	1.00	0.73	1.00
Nikkei 225	0.71	0.57	0.79	0.00	0.07	0.14	0.57	0.36	0.79	0.57	1.00
Indice Bovespa	1.00	0.89	0.11	0.00	0.33	0.22	0.44	0.22	0.44	-	1.00
RTS Index	0.25	0.13	0.38	0.63	0.25	0.00	0.00	0.25	0.75	0.13	1.00
BSE SENSEX	1.00	0.91	0.73	0.27	0.73	0.09	0.91	0.73	0.91	0.73	1.00
SSE 50	0.14	0.86	0.71	0.14	0.00	0.71	0.43	0.14	0.14	-	0.86
FTSE/JSE Top 40	-	0.92	0.31	0.31	0.77	0.69	0.92	-	0.38	-	1.00
S&P/ASX 50	1.00	1.00	0.53	0.53	0.87	0.40	0.80	0.80	0.53	-	1.00
MERVAL Index	0.29	0.14	0.00	0.21	0.00	-	0.50	0.36	0.71	-	0.93
S&P/TSX 60	0.92	1.00	0.85	0.85	0.92	0.08	0.46	0.62	0.15	1.00	1.00
FTSE MIB	0.33	0.83	0.17	0.00	0.50	0.00	0.67	0.00	1.00	1.00	1.00
KOSPI 50	0.92	0.92	0.85	0.69	-	0.62	0.46	0.54	0.69	0.92	1.00

FIGURE 3.5: A share of of dominated indices

The figure plots a share of the market indices which are dominated at the order 3 or lower by at least one sector sub-index for each year from 2003 to 2015.



change. We obtain the data from the World Bank database. Apart from macroeconomic variable, we use several financial indicators, such as representative government bonds yield, and a central bank policy rate or main interest rate. These data are obtained from the IMF database and Datastream.

We also include several index and sub-index specific characteristics, such as index and sub-index annualized mean return and volatility over the previous year, and the volatility ratio defined as sector volatility over the index volatility, which measures a relative riskiness of the sub-index in normalized terms. We control for market liquidity and for each index we compute the Amihud (2002) illiquidity measure. Some indices, however, lack the trading volume data needed to calculate the measure. For example, Euro Stoxx 50 lacks volume data before 2005. Also, Argentina's government bonds yield data are unavailable before 2006. In our regression analysis we thus omit those index-years, for which we cannot construct all the required factors.

In addition to the aforementioned explanatory variables, we use index and sub-index fixed effects, and a dummy variable that takes a value of 1 if a given sub-index dominated the index during the previous year (or during any previous year) and zero otherwise.

There is substantial literature suggesting that there exist structural differences between advanced and developing economies, that impact firm productivity and, as a consequence, stock performance. These include, among others, differences in the infrastructure, regulations, human capital, financial constraints, adopted managerial practices and skillsBloom, Mahajan, McKenzie, and Roberts (2010) and Bruhn, Karlan, and Schoar (2010). Thus, we split all the market indices under study into two groups – advanced and developing markets – and estimate logit models for them separately. The first group includes the market indices from the U.S., UK, Eurozone, and Japan; all other countries are included in the second group. The results are reported in Table 3.8.

The model in general has a relatively food fit, with the Efron's pseudo R-squared

TABLE 3.8: Stochastic dominance determinants: advanced vs. developing economies

The table reports the estimation results for the Logit model for the probability of each sector sub-index to dominate its respective diversified equity index. The model is estimated for the advanced economies (the U.S., the U.K., Euro-zone, and Japan) and the rest of the regions separately.

Panel A: Advanced economies								
	Coef.	t-stat.	Coef.	t-stat.	Coef.	t-stat.	Coef.	t-stat.
Intercept			3.03	1.48	2.56	1.19	0.67	0.31
GDP growth	-0.14*	-1.86	-0.14**	-2.05	-0.12*	-1.71	-0.12*	-1.72
Inflation	-0.45***	-3.66	-0.42***	-3.61	-0.39***	-3.12	-0.39***	-3.12
Unemployment	0.00	0.07	-0.01	-0.14	-0.01	-0.13	0.00	-0.03
Gross savings	-0.04	-0.77	-0.02	-0.36	-0.02	-0.43	-0.01	-0.25
Current account balance	-0.11*	-1.67	-0.15**	-2.17	-0.16**	-2.26	-0.16**	-2.25
Real effective exchange	-0.02	-0.87	-0.01	-0.54	0.00	-0.19	0.00	-0.10
Government bonds yield	-0.04	-0.26	-0.07	-0.45	-0.10	-0.59	-0.05	-0.28
Central bank policy rate	0.10	0.79	0.13	1.03	0.12	0.94	0.12	0.91
Liquidity	-2.97	-0.32	-2.83	-0.32	-5.31	-0.59	-1.94	-0.21
Index return	-1.04	-1.07	-1.05	-1.11	-0.70	-0.71	-0.56	-0.56
Index volatility	18.32**	2.17	21.78**	2.48	24.13***	2.67	23.68***	2.68
Sector return	0.26	0.33	0.29	0.36	0.30	0.38	0.25	0.31
Sector volatility	-19.91**	-2.46	-24.81***	-2.93	-26.18***	-3.00	-24.84***	-2.93
Volatility ratio	-1.30	-0.84	-2.31	-1.42	-1.97	-1.16	-1.17	-0.72
D Dominance, last year					-0.07	-0.24		
D Dominance, any year							0.67**	2.40
Sector dummies	yes		no		no		no	
Efron's pseudo R-sq	0.33		0.29		0.28		0.29	
N obs	1025		1025		967		967	
Panel B: Developing economies								
Intercept			6.84***	3.43	6.30***	3.01	4.57**	2.17
GDP growth	0.13**	2.27	0.13**	2.29	0.11*	1.89	0.13**	2.10
Inflation	0.05	1.01	0.05	1.00	0.05	0.91	0.04	0.64
Unemployment	-0.05**	-2.03	-0.05**	-1.99	-0.05*	-1.90	-0.06**	-2.15
Gross savings	-0.11***	-3.85	-0.11***	-3.87	-0.10***	-3.52	-0.10***	-3.45
Current account balance	0.09*	1.89	0.09*	1.79	0.08*	1.71	0.09*	1.80
Real effective exchange	0.03	1.53	0.03	1.55	0.02	0.86	0.03	1.10
Government bonds yield	-0.06	-1.04	-0.06	-1.02	-0.05	-0.87	-0.05	-0.80
Central bank policy rate	-0.02	-0.25	-0.02	-0.26	0.00	-0.01	0.01	0.08
Liquidity	-3.63	-1.57	-3.48	-1.54	-3.42	-1.46	-4.69*	-1.96
Index return	0.92	1.52	0.79	1.33	0.63	0.96	0.78	1.17
Index volatility	11.91	1.50	12.07	1.55	10.37	1.30	11.39	1.44
Sector return	-0.73	-1.19	-0.51	-0.86	-0.57	-0.87	-0.68	-1.04
Sector volatility	-11.74	-1.52	-12.10	-1.58	-11.88	-1.47	-11.74	-1.47
Volatility ratio	-4.69***	-2.60	-4.87***	-2.73	-4.30**	-2.32	-3.35*	-1.83
D Dominance, last year					0.24	0.70		
D Dominance, any year							0.98***	3.03
Sector dummies	yes		no		no		no	
Efron's pseudo R-sq	0.36		0.34		0.33		0.34	
N obs	798		798		721		721	

being around 28–36%. The key significant variables are index and sub-index return volatilities, and the ration of the volatilities. More volatile indices are likely to be dominated by less volatile sub-indices in absolute and relative terms. The retaliations between index and sub-indices volatilities and the probability of future dominance over market indices are generally consistent for different types of economies, with individual index and sub-index measures having stronger statistical support for advanced economies, and the ratio of volatilities being statistically significant for the developing markets. Also, a market index is more likely to be dominated by a sub-index, which has been dominating in the past. A dummy for past dominance of a given sub-index over the market index is positive and highly significant for both groups of countries.

The results related to the macroeconomic factors, however, suggest that the actual information content of the aggregate economic indicator is quite different across developed and developing economies.

Three significant predictors of future dominance of the market index for the advanced economies are GDP growth, inflation, and current account balance. Higher values of these indicators reduce the probability that the market index will be dominated by any sector sub-indices. A higher GDP growth rate reflects overall growth of the economy; inflation usually increases on up marketers and goes hand in hand with higher growth; and higher current account balance reflects higher levels of export from the advanced economies. Overall, the results suggest that improving economic conditions in developed countries make diversified market indices more attractive options for risk averse and prudent investors.

For the developing countries, all these determinant change their signs. To begin with, the GDP growth rate and the current account balance are positively related to the probability of the market index to be dominated by sector sub-indices. A likely reason for such a sign flip is that the developed economies are more "homogeneous". A higher GDP growth rate reflects a balanced growth of all areas of a developed economy. In the developing markets, the growth is often driven by just a few key sectors, which also contribute to higher exports and higher resulting current account balance, and it does not translate into overall improved performance of other sectors as by Koren and Tenreyro (2007). This makes the aggregate market index SD dominated by the fast growing, GDP driving industries, and less suitable for risk-averse investors. Also, the link between inflation and economic growth does not seem to be pronounced in the developing markets. Consequently, inflation is not a statistically significant predictor for the probability of the market index to be dominated by sector sub-indices.[14]

3.6 An out-of-sample trading strategy

3.6.1 Market specific index-based trading strategy

So far we have established that diversified stock indices across the globe are often not SD efficient; past dominance of a sub-index over the corresponding market index is a strong and consistent predictor of future dominance. In this section we propose a genuinely simple trading rule that uses this past stochastic dominance information, and check if such a strategy allows us to outperform the indices. The trading rule

[14] As a robustness check we re-estimated the pooled logit regression jointly for all economies. Results reported in Table 3.16 in Appendix 3.9.3 are consistent with the ones discussed in this section for return volatilities and past dominance, but none of the marco-factors are significant, due to the discussed differential impact on developed and developing markets.

is index specific and only relies on those sub-indices that are available for the index under study.

We conduct the analysis for advanced economies only, due to several major reasons. First and foremost, the developed market indices have longer histories. Thus, we have a sufficient number of training years during which we evaluate the indices from the SD perspective and a sufficient number of remaining years to perform out-of-sample performance assessments. Second, indices from the developed economies show a much better industrial coverage, and usually all 10 sub-indices are available for all years. Only the Utilities sub-index is absent in the DJIA 30 index due to historical reasons, and the Oil and gas sub-index is absent from the DAX 30 index. The long history, industry coverage, and continuity of the coverage across years are often missing for emerging economies, making forecasts difficult and at times pointless. Last but not least, stock indices in advanced economies are free from most of investment barriers, they are relatively easily investible, and they are also much more systemically important, in the sense of size, financial integration and worldwide influence. Therefore, we choose the S&P 100 index and DJIA 30 index of the U.S., the FTSE 100 index of the U.K., the CAC 40 index of France, the DAX 30 index of Germany, the Euro Stoxx 50 index of the Euro area, and the Nikkei 225 index of Japan for this exercise. The chosen economies cover on average 75% of the global GDP over the period 2009 to 2015 and around 81% of total world stock market capitalization, according to the Word Bank database, thus attracting most of business attention and investments around the world.

For each index, its sample period is divided into two parts. In the training sample (which we vary from 3 to 7 years), we record every sub-index that has dominated the respective index at any order of SD up to fourth. Then, for the first year of the prediction sub-sample, we choose those sub-indices that dominated the index at least twice in the past and construct an equally weighted portfolio from these dominating indices. We hold this portfolio for one year. Next, we roll over the training sample by one year forward and repeat the analysis. If we cannot find any sub-index that dominated at least twice the index in the past, the portfolio is 100% invested in the index itself for the next year. In fact, in our analysis it happened only once with the DJIA 30 index for the year of 2010.

Table 3.9 reports the annualized means and standard deviations of daily returns for total indices and our past-SD based portfolio strategy, and the corresponding Sharpe ratios. Table 3.17 in the Appendix 3.9.4 reports more detailed results with first four moments of the return distribution for each available year and each index.

For all of the indices and estimation windows, the annualized volatility for the SD strategy is lower than the index volatility, implying a lower investment risk. The average returns are also often improved. For the American DJIA 30, British FTSE 100, and German DAX 30 indices, the SD strategies deliver consistently higher mean return than those of the corresponding indices for all estimation horizons. For the S&P 100 index the strategy performs best with a short estimation horizon of 3 years, and for the EURO STOXX 50 the 5-year estimation horizon is optimal.

The only notable exception is the Japanese Nikkei 225 index, for which our past-SD based strategy always fail to deliver higher or even comparable mean return. The detailed results of Table 3.17 based on a 5-year estimation horizon reveal that this pattern is also consistent across years. In most of the years with some rare exceptions, SD-based strategies deliver higher returns with lower volatilities for the U.S., U.K, and continental Europe, but fail to do so for Japan. The reason for such a poor performance for Japan, seems to be the fact that the past stochastic dominance pattern is not quite consistent over the sample. Also, the past SD dominance

TABLE 3.9: Average out-of-sample performance of past-SD based trading strategy

The table reports the average annualized means and return standard deviations (in % per year) and the corresponding annualized Sharpe ratios of seven equity indices of the developed economies (Index) as well as the descriptive statistics of the corresponding portfolios that include only those sector sub-indices that have dominated the index in the past at least twice (SD).

	Mean		Std	
	Index	SD	Index	SD
3-year estimation window				
S&P 100	8.71	9.84	17.05	15.70
DJIA 30	6.37	8.22	18.01	17.18
FTSE 100	3.72	5.84	18.19	16.77
CAC 40	5.76	4.47	21.04	20.33
DAX 30	8.33	12.08	20.56	19.99
EURO STOXX 50	5.30	4.89	21.12	20.72
NIKKEI 225	6.26	2.58	22.94	20.68
5-year estimation window				
S&P 100	8.91	6.75	17.77	16.66
DJIA 30	12.45	14.14	15.67	14.29
FTSE 100	3.54	6.83	18.15	15.22
CAC 40	3.47	3.06	22.73	21.90
DAX 30	6.88	13.29	21.86	21.16
EURO STOXX 50	3.14	4.22	22.83	22.26
NIKKEI 225	2.89	0.67	24.35	20.68
7-year estimation window				
S&P 100	6.88	3.97	18.55	16.66
DJIA 30	10.72	13.07	13.87	11.94
FTSE 100	7.18	11.11	17.15	13.98
CAC 40	1.45	1.57	24.44	23.33
DAX 30	3.60	10.84	23.45	21.33
EURO STOXX 50	0.52	-1.15	24.72	24.70
NIKKEI 225	12.75	6.04	22.16	19.31
All-past-years estimation window				
S&P 100	7.79	9.76	17.40	11.97
DJIA 30	11.12	13.09	8.45	4.90
FTSE 100	4.73	8.10	15.73	12.56
CAC 40	9.01	9.98	13.63	13.38
DAX 30	11.47	1.49	14.54	9.38
EURO STOXX 50	8.30	2.99	13.59	10.58
NIKKEI 225	12.75	6.04	19.98	21.13

of sub-indices over indices in Japan is mostly at order 3, and its effect on return improvement and risk reduction is not as remarkable as for lower order SD.

3.6.2 Global index-based trading strategy

We now make a step forward and consider if our simple past-SD allocation rule can improve the performance of a global equity portfolio. Specifically, we construct the index of indices using the developed economies' diversified equity indices, namely, S&P 100, DJIA 30, FTSE 100, CAC 40, DAX 30, Euro Stoxx 50 and Nikkei 225. We consider three weighting schemes while constructing the global index of indices: equally-weighted, GDP-weighted, and stock market capitalization weighted. Relevant data are from World Band database, Federal Reserve St. Louis, and knoema.com. We next apply our past-SD based rule and invest in those markets whose indices dominated the global index at least twice in the past. The allocation is rebalanced every year.

The results reported in Table 3.10 reveal that for our SD-based strategy the average return increases by about 2 percentage point annualized and return standard deviation declines by about 3 percentage point. These results are consistent across all three waiting schemes and hold for most of the years. Even during the pick of the financial crisis, the SD-based strategy helps to mitigate the losses of the global equity portfolio. In 2008, the equally weighted global index lost 38% of its value, and its GDP and market cap weighted counterparts lost 42% of their values. The SD-base alternatives limited the losses to about 25% while also decreasing return standard deviation by 8-9% annualized during this year.

TABLE 3.10: Index of indices out-of-sample performance

The table reports the annualized mean returns (in % per year) of global index-of-indices for each year starting from 2003 to 2015 and the corresponding mean returns of SD-based index. The index is based on seven indices of the developed economies: S&P 100, DJIA 30, FTSE 100, CAC 40, DAX 30, Euro Stoxx 50, and Nikkei 225. The index-of-indices is constructed using three different weighting schemes: equally weighted (Equal), weighted by the GDP of the economy it represents (GDP), and weighted by the total market capitalization the index represents (Cap). The last two rows report the average annualized mean and return standard deviation over the entire sample.

	Equal				GDP				Market Cap			
	Index		SD-based		Index		SD-based		Index		SD-based	
	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.
2003	0.23	0.18	0.23	0.17	0.23	0.18	0.23	0.17	0.23	0.18	0.23	0.17
2004	0.06	0.11	0.14	0.10	0.06	0.11	0.14	0.10	0.06	0.11	0.14	0.10
2005	0.01	0.10	0.08	0.11	0.01	0.10	0.08	0.11	0.01	0.10	0.08	0.11
2006	0.15	0.10	0.19	0.08	0.16	0.09	0.18	0.08	0.16	0.09	0.18	0.08
2007	0.07	0.15	0.13	0.13	0.06	0.15	0.16	0.14	0.06	0.15	0.16	0.14
2008	-0.38	0.34	-0.25	0.26	-0.42	0.37	-0.25	0.28	-0.42	0.37	-0.25	0.29
2009	0.22	0.22	0.11	0.15	0.22	0.22	0.08	0.16	0.21	0.22	0.08	0.15
2010	0.07	0.17	0.02	0.13	0.07	0.16	0.03	0.13	0.09	0.16	0.06	0.13
2011	-0.08	0.21	-0.04	0.17	-0.04	0.21	0.01	0.16	0.00	0.20	0.06	0.16
2012	0.17	0.14	0.09	0.10	0.16	0.13	0.10	0.10	0.14	0.12	0.10	0.09
2013	0.26	0.11	0.20	0.10	0.26	0.10	0.20	0.10	0.26	0.10	0.20	0.09
2014	0.06	0.11	0.11	0.10	0.08	0.11	0.11	0.10	0.09	0.10	0.12	0.09
2015	0.06	0.16	0.05	0.15	0.04	0.15	0.05	0.14	0.03	0.15	0.06	0.13
Average	0.07	0.16	0.08	0.13	0.07	0.16	0.08	0.14	0.07	0.16	0.09	0.13

3.6.3 Market specific individual stock-based trading strategy

Apart from past dominance of sub-index over the index, the most significant and consistent predictors of stochastic dominance are index and subindex volatilities and their ratio. We use this intuition and construct portfolios of individual stocks sorting them based on historical market beta. The market beta of each stock is proportional to the ratio the stock return volatility to the market return volatility. The results from Section 3.5.3 suggest that stocks having lower market beta are likely to stochastically dominate the index over the following year.

To implement this strategy, for each stock for each year end we download its historical beta from Datastream (code 897E). The beta is computed based on the previous five years of monthly data using a linear regression of the logarithmic adjusted returns onto the returns of the corresponding market index. For each sample year, we sort individual stocks within each index according to their previous year-end values of the market beta. The stocks are then sorted into three portfolios: a low beta portfolio, comprising 30% of stocks with the lowest betas, a medium beta portfolio, containing 40% of stocks with the medium beta, and high beta portfolio, that includes 30% of stocks with the highest historical beta. For each of the portoflios, the returns are calculated using the same methodology as that of the benchmark market index. That is, the portfolio returns market capitalization weighted average or price weighted average on the components, depending on the calculation methodology of the corresponding index. We then asses the SD relationship between these portfolios and the market index over the following year using the majorization theorem.

Table 3.11 reports the average descriptive statistics of the market betas for all indices. The average mean and median betas are all positive and close to 1 for all indices. The 30% and 70% thresholds used to construct portfolios are smaller and larger than 1, respectively. For example, for the S&P 100 index these thresholds are 0.81 and 1.23.

The results reported in Table 3.12 indicate that low-beta portfolios perform extremely well in terms of stochastic dominance over the corresponding market index. Almost always they dominate the corresponding market index in way more that 50% of years. For example, the low-beta portfolio dominates the S&P 100 index in 73% of years.[15] There are only few exceptions when low-beta portfolios dominate the index in less than 50% of years. These include Argentinean MERVAL (21%), Italian MIB (17%), and South Korean KOSPI 50 (38%).

Looking at the actual performance of the low-beta portfolios (Table 3.13), we see the low-beta portfolio reduces out-of-sample volatility on all markets, but it often comes at a cost of also reducing the mean. The resulting Sharpe ratio depends on the country of interest. The low-beta portfolio works quite good in Europe, and less so in the U.S.

[15] Over the past decade, the index was dominated by the low-beta portfolio in all years, except of 2013, at orders 3 or 2.

TABLE 3.11: Beta descriptive statistics

The table reports the time series averages of the descriptive statistics of individual stock market betas for the stocks that are constituents of the 17 market indices used in this paper. "Effective stocks" reports the average percentage of stocks in each index that are alive at the end of a calendar year and are used for portfolio construction over the following year.

	Effective Stocks	Mean	Median	Std	Min	Max	30% quantile	70% quantile
S&P 100	98%	1.06	1.03	0.50	0.05	2.75	0.81	1.23
DJIA	100%	1.01	0.94	0.50	0.18	2.21	0.71	1.19
FTSE 100	98%	1.06	0.98	0.61	-0.34	3.28	0.74	1.27
CAC	100%	1.12	1.06	0.54	0.22	2.58	0.83	1.34
DAX	100%	0.99	0.99	0.44	0.23	2.06	0.77	1.15
Euro Stoxx 50	97%	1.05	1.03	0.47	0.25	2.54	0.76	1.25
Nikkei 225	100%	1.10	1.08	0.52	-0.42	3.33	0.87	1.30
Indice Bovespa	99%	0.92	0.92	0.47	-0.02	2.32	0.64	1.12
RTS Index	99%	1.11	1.10	0.41	-0.19	2.07	0.94	1.30
BSE SENSEX	100%	0.96	0.91	0.37	0.36	1.72	0.75	1.11
SSE 50	98%	1.11	1.13	0.35	0.38	1.95	0.93	1.27
FTSE/JSE Top 40	100%	0.86	0.78	0.44	-0.13	1.86	0.63	1.02
S&P/ASX 50	99%	0.93	0.86	0.56	-0.09	2.67	0.63	1.10
MERVAL Index	94%	0.87	0.90	0.31	0.30	1.47	0.69	1.04
S&P/TSX 60	100%	0.89	0.78	0.66	-0.13	3.41	0.52	1.06
FTSE MIB	97%	0.96	0.95	0.43	-0.06	1.86	0.79	1.21
KOSPI 50	96%	1.05	1.07	0.46	0.04	2.13	0.83	1.29

TABLE 3.12: Market indices vs. beta-sorted portfolios

The table reports the fraction of years during which each of the 17 diversified equity indices is dominated by beta-sorted portfolios of individual stocks at orders 4, 3 or 2.

	Low-beta	Medium-beta	High-beta
S&P 100	0.73	0.08	0.00
DJIA	0.50	0.00	0.00
FTSE 100	0.70	0.05	0.00
CAC	0.67	0.20	0.00
DAX	0.60	0.00	0.00
Euro Stoxx 50	0.73	0.20	0.00
Nikkei 225	0.79	0.00	0.00
Indice Bovespa	0.89	0.22	0.00
RTS Index	0.75	0.63	0.13
BSE SENSEX	0.64	0.00	0.00
SSE 50	0.86	0.14	0.00
FTSE/JSE Top 40	0.77	0.38	0.00
S&P/ASX 50	0.67	0.13	0.00
MERVAL Index	0.21	0.21	0.07
S&P/TSX 60	0.77	0.38	0.00
FTSE MIB	0.17	0.67	0.17
KOSPI 50	0.38	0.00	0.00

TABLE 3.13: Out-of-sample performance of low-beta portfolios

The table report the average descriptive statistics of the low-beta portfolios for 17 market indices under study. The mean return, return standard deviation (Std), and Sharpe ratio are annualised, whereas skewness and kurtosis are based on monthly returns. The last rows report the average improvement of the low-beta portfolio over the corresponding market index.

	S&P 100	DJIA	FTSE 100	CAC	DAX	Euro Stoxx 50	Nikkei 225	Indice Bovespa	RTS	BSE SENSEX	SSE 50	FTSE JSE Top 40	S&P/ ASX 50	MERVAL	S&P/ TSX 60	FTSE MIB	KOSPI 50
Low-beta portfolio																	
Mean	0.06	0.05	0.06	0.02	0.05	0.02	0.00	0.07	-0.04	0.09	0.05	0.14	0.05	0.25	0.09	0.05	0.10
St.D	0.15	0.16	0.16	0.22	0.21	0.21	0.24	0.25	0.36	0.24	0.22	0.19	0.14	0.37	0.18	0.20	0.21
Dkewness	-0.20	0.06	-0.35	0.04	-0.11	0.10	-0.55	-0.21	-0.11	-0.38	-0.53	-0.12	-0.38	-0.56	-0.52	-0.07	-0.25
Kurtosis	13.42	16.63	16.49	9.82	13.07	10.91	15.77	14.51	33.80	13.91	11.00	9.05	9.11	12.93	20.13	5.22	8.07
Sharpe ratio	0.20	0.25	0.15	-0.23	-0.07	0.00	-0.10	-0.16	-0.27	0.18	0.12	0.29	-0.01	0.43	0.41	0.07	0.36
Difference between low-beta portfolio and the index																	
Mean	-0.03	-0.01	0.00	0.00	0.01	0.01	-0.05	0.07	0.07	-0.04	-0.06	-0.02	-0.03	-0.02	0.01	0.02	0.01
St.D	-0.03	-0.02	-0.02	-0.02	-0.04	-0.03	0.00	-0.04	-0.02	0.00	-0.05	-0.02	-0.03	0.04	0.00	-0.06	-0.02
Skewness	-0.03	0.14	-0.19	0.02	-0.09	0.09	-0.07	-0.22	0.19	-0.46	-0.19	-0.01	-0.02	-0.21	0.16	0.03	0.04
Kurtosis	2.07	1.99	7.51	1.73	5.44	3.34	5.13	5.03	20.03	2.03	3.84	2.32	0.38	5.56	4.91	-0.25	-0.81
Sharpe ratio	-0.12	-0.05	0.01	-0.03	0.03	0.04	-0.22	0.22	0.17	-0.18	-0.19	-0.07	-0.18	-0.10	0.04	0.11	0.07

We repeat the analysis sorting portfolios based on historical total return volatility (as opposed to market beta). The historical volatility is estimated as a standard deviation of the returns over the past five years (Datastream code 400E). The results tabulated in Appendix 3.9.5 are qualitatively similar to the ones based on beta sorts, but are at times slightly weaker.[16]

Our stochastic dominance results complement a body of literature on exceptionally good performance of low-beta and low-volatility stocksBlitz and Vliet (2007), Ang, Hodrick, Xing, and Zhang (2009), Baker, Bradley, and Wurgler (2011), Frazzini and Pedersen (2014), and Asness, Frazzini, and Pedersen (2014). We show that portfolios of these stocks not only perform well in the traditional mean-variance scenes, but often stochastically dominate the diversified equity market indices across the globe. Thus, these portfolios are more suitable for risk-averse and prudent investors as opposed to the market indices.

3.7 Conclusion

In this paper we extend the approach of Kuosmanen (2004) for testing for stochastic dominance efficiency of a given portfolio with respect to a set of underlying assets. The extended approach allows us to test for dominance efficiency of higher orders than two, and, similar to the original paper, to obtain the optimal weights for an efficient portfolio in case the test portfolio is proved to be inefficient.

We apply this approach to 17 stock market indices covering developed and developing markets across the globe, and find that in the majority of the years these indices are inefficient at least at order 3, and often at order 2. Thus, all prudent and most of risk averse investors should optimally deviate from equity indices, investing instead in portfolios that overweight individual industries. The average mean return improvement that could be achieved by investing in an SD efficient portfolio is 23% annualized. Such a high return improvement is hard to achieve in practice, as this result stems from the in-sample optimization and knowledge of the realized return distribution. At the same time, the magnitude of the potential improvement suggests that even moderate deviations from the well diversified indices towards the optimal portfolio can result is substantial gains for investors.

Then, we conduct pairwise comparisons of the market equity indices with their sector sub-indices. Since here we use the ex-ante industry classification, this strategy is practically implementable. We find that on average in 67% of years not only the indices are inefficient but they are dominated by at least one sub-index. The percentage of dominated indices is especially high during the years 2008 – 2012. On the aggregate level, counter-cyclical industries such as Consumer Foods and Services, Health Care and Telecommunication are more likely to dominate their diversified equity indices. At the same time, the types of industries that are likely to dominate vary across the countries. For example, the Oil and Gas industry often dominates the Russian RTS index but not the other indices and the Financial sector often dominates the Chinese SSE 50 index but almost never the other equity markets.

[16]There are potentially other fundamental factors, which could be related to stochastic dominance. Fama and French (1992), Fama and French (1993), McLean and Pontiff (2016), and Yan and Zheng (2017) provide a systematic overview on the predictability of various indicators, including value signals, of stock market returns. Following these research and given the data coverage we select additional fundamental indicators to perform portfolio sorts. They include 12 Month forward earnings per share (FEPS, Datastream I/B/E/S), 12 Month Forward Price/Earnings Ratio (Datastream I/B/E/S), and last available earnings per share (EPS). None of these indicators provides a strong signal for out-of-sample dominance. The dominance of portfolios sorted based on these fundamentals over the index happens rather rarely and randomly.

Further, we estimate a Logit model for the determinants of the probability of a sub-index to dominate its index. Remarkably, macro factors contain different information with respect to future dominance for developed and developing markets. For more homogeneous and balanced economies, aggregate indicators of growth (such as GDP growth rate, inflation, and the current account balance) predict lower likelihood of the market index to be dominated. However, for the developing economies, which often rely on just one or several key industries, such aggregate indicators predict a higher likelihood that the market index will be dominated. The most significant and consistent predictors of dominance are index and sub-index volatilities, with the former being positively related to the probability of a sub-index to dominate the index, and the latter being negatively related to the probability, as well as the ratio of volatilities. Also, past dominance of a sector sub-index over the market index predicts higher likelihood for the future dominance.

Given that past stochastic dominance is a strong predictor of the future dominance of a sub-index over the index, we further suggest a simple trading rule based on the information on past dominance, that invests only in those sub-indices that dominate the index at least twice during a given number of previous years. Applying this strategy to the developed markets, we find that the rule results in consistent mean return improvement and volatility reduction in the U.S., the U.K., and Europe, but does not perform that well in Japan. Applying this strategy to a global portfolio results in about 1–2% annualized return improvement and 2–3% decline in the annualized standard deviation. Such improvements in the return distribution are consistent across time. Our past SD based approach also substantially limits the losses during market downturns like the financial crisis of 2007–2008.

Last but not least, we sort individual stocks into tercile portfolios based on their market betas and volatility and show that low-beta and low-volatility portfolios stochastically dominate the market indices in majority of years at order 3 and often at order 2 across most of world economies considered. These results contribute to the discussion of a stellar performance of low-beta and low-risk stocks in the mean-variance sense, and suggest that these portfolios are likely to be preferred by risk-averse and prudent investors over diversified market indices.

Overall, our findings suggest that diversified equity indices across the globe are not SD efficient. Risk averse and prudent investors could benefit from switching between different industry sub-indices, by taking positions in those industries that were dominating in the past, or by investing in low-beta stocks. The sector-based strategies can rely on trading ETFs at low frequency, re-balancing portfolios once every year, thus, delivering improved return distributions with low transaction costs, which can be rather appealing for regulated long-term investors such as pension funds or insurance companies.

3.8 Appendix

The result of Proposition 1 is already known proved at orders 1 and 2. To illustrate how we derive it at higher orders, we start by briefly re-deriving it at order 2 with our notation. We readily have that $\hat{F}^{[2]}(r) \leq \hat{G}^{[2]}(r) \Leftrightarrow \bar{\hat{F}}^{[2]}(r) \geq \bar{\hat{G}}^{[2]}(r)$ for all r. Using an integration by parts, we have that $\bar{\hat{F}}^{[2]}(r) \geq \bar{\hat{G}}^{[2]}(r)$ for all r is equivalent to $E(X\mathbb{1}_{X\leq\tilde{x}_k}) \geq E(Y\mathbb{1}_{Y\leq y_k})$ for all k. Then, we compute $E(X\mathbb{1}_{X\leq\tilde{x}_k}) = \sum_{i=1}^{k} \frac{\tilde{x}_i}{n}$ and $E(Y\mathbb{1}_{Y\leq\tilde{y}_k}) = \sum_{i=1}^{k} \frac{\tilde{y}_i}{n}$. By multiplying both expressions by n, we obtain $\hat{F}^{[2]}(r) \leq \hat{G}^{[2]}(r)\ \forall r \Leftrightarrow \sum_{i=1}^{k} \tilde{x}_i \geq \sum_{i=1}^{k} \tilde{y}_i\ \forall k$. It remains to observe that $\sum_{i=1}^{k} \tilde{x}_i = x_k^{[2]}$ and $\sum_{i=1}^{k} \tilde{y}_i = y_k^{[2]}$ to conclude that

$$\hat{F}^{[2]}(r) \leq \hat{G}^{[2]}(r) \quad \forall r \qquad \Leftrightarrow \qquad x_k^{[2]} \geq y_k^{[2]} \quad \forall k.$$

Next, we derive the same result at order 3. We readily have that $\hat{F}^{[3]}(r) \leq \hat{G}^{[3]}(r) \Leftrightarrow \bar{\hat{F}}^{[3]}(r) \geq \bar{\hat{G}}^{[3]}(r)$ for all r. Using an integration by parts, we have that $\bar{\hat{F}}^{[3]}(r) \geq \bar{\hat{G}}^{[3]}(r)$ for all r is equivalent to $E\big(X_k^{[2]}\big) \geq E\big(Y_k^{[2]}\big)$ for all k, where $X_k^{[2]}$ and $Y_k^{[2]}$ are random variables associated with the first k weights of $\bar{\hat{F}}$ and $\bar{\hat{G}}$. Up to a proportionality constant that would need to be introduced to ensure that the total probability is equal to one but that is simplified here, we have: $E\big(X_k^{[2]}\big) \geq E\big(Y_k^{[2]}\big) \Leftrightarrow \sum_{i=1}^{k} \tilde{x}_i\big(1 - \frac{i-1}{k}\big) \geq \sum_{i=1}^{k} \tilde{y}_i\big(1 - \frac{i-1}{k}\big)\ \forall k$. Because $\sum_{i=1}^{k} \tilde{x}_i\big(1 - \frac{i-1}{k}\big) = \frac{1}{k}\sum_{h=1}^{k}\sum_{i=1}^{h} \tilde{x}_i = \frac{\tilde{x}_k^{[3]}}{k}$ and $\sum_{i=1}^{k} \tilde{y}_i\big(1 - \frac{i-1}{k}\big) = \frac{1}{k}\sum_{h=1}^{k}\sum_{i=1}^{h} \tilde{y}_i = \frac{\tilde{y}_k^{[3]}}{k}$, we can to conclude that

$$\hat{F}^{[3]}(r) \leq \hat{G}^{[3]}(r) \quad \forall r \qquad \Leftrightarrow \qquad x_k^{[3]} \geq y_k^{[3]} \quad \forall k.$$

3.9 Additional results

3.9.1 Optimal portfolio weights's correlations

The correlation coefficients of the optimal portfolio weights across the indices are reported in Table 3.14. The average correlation across all the indices is 21%. It ranges between 62% for the Argentinean MERVAL and Chinese SSE 50 indices and −19% for the DJIA and SSE 50 indices.

TABLE 3.14: Optimal weight correlation

The table reports the average correlation coefficients between optimal portfolio weights across 17 stock market indices used in this paper.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(1)	S&P 100	1.00	0.26	0.34	0.12	0.08	0.24	0.17	0.15	0.21	0.18	0.22	0.19	-0.02	0.14	0.10	0.12	-0.04
(2)	DJIA		1.00	0.00	-0.04	0.15	-0.05	0.38	-0.10	0.14	0.26	-0.19	0.18	0.11	-0.03	0.18	0.12	0.00
(3)	FTSE 100			1.00	0.39	0.34	0.35	0.12	0.18	-0.01	0.14	0.47	0.18	-0.01	0.24	-0.08	0.19	0.16
(4)	CAC				1.00	0.20	0.39	-0.04	-0.02	0.12	0.06	0.14	0.12	0.05	0.17	0.11	0.46	0.09
(5)	DAX					1.00	0.43	0.21	0.29	0.08	0.08	0.18	0.23	0.34	-0.11	0.02	0.22	0.01
(6)	Euro Stoxx 50						1.00	0.21	0.15	0.04	-0.01	0.18	0.02	0.14	-0.02	0.07	0.05	0.13
(7)	Nikkei 225							1.00	0.41	0.10	0.18	0.24	0.18	0.25	0.22	0.09	-0.19	-0.02
(8)	Indice Bovespa								1.00	0.29	0.14	0.31	0.20	0.17	0.00	0.11	0.18	0.08
(9)	RTS Index									1.00	-0.13	0.29	0.24	0.11	0.22	0.01	0.46	-0.16
(10)	BSE SENSEX										1.00	0.25	0.06	0.00	0.31	0.14	0.02	0.24
(11)	SSE 50											1.00	0.10	0.18	0.62	0.14	0.08	0.49
(12)	FTSE/JSE Top 40												1.00	0.03	0.11	-0.02	0.34	0.19
(13)	S&P/ASX 50													1.00	-0.11	0.21	0.49	-0.03
(14)	MERVAL Index														1.00	-0.11	-0.14	0.11
(15)	S&P/TSX 60															1.00	0.20	0.17
(16)	FTSE MIB																1.00	0.19
(17)	KOSPI 50																	1.00

3.9.2 Pairwise comparison of market indices and sector sub-indices, SD orders 3 and 2

In this appendix we report the results of the pairwise comparison of each index and corresponding sector sub-indices with respect to the 3rd order and 2nd order SD separately.

TABLE 3.15: Percentage of indices being dominated by sub-indices at orders 2 and 3

The table reports the percentage of years during which each of the 17 diversified equity indices is dominated by each of sector sub-indices at order 3 (Panel A) or 2 (Panel B). The last column reports the percentage of years during which the index is dominated by at least one sub-index.

	Oil&gas	Basic mat.	Ind.	Cons. goods	Health care	Cons. services	Telecom.	Util.	Fin.	Tech.	Any
Panel A: 3rd order stochastic dominance											
S&P 100	0.00	0.00	0.00	0.42	0.27	0.19	0.12	0.31	0.00	0.00	0.58
DJIA	0.00	0.00	0.00	0.33	0.33	0.33	0.25	-	0.00	0.00	0.58
FTSE 100	0.00	0.00	0.05	0.55	0.20	0.20	0.05	0.45	0.00	0.00	0.65
CAC	0.00	0.00	0.07	0.33	0.13	0.27	0.27	0.07	0.00	0.00	0.67
DAX	-	0.07	0.00	0.00	0.33	0.07	0.07	0.20	0.07	0.07	0.53
Euro Stoxx 50	0.20	0.00	0.00	0.27	0.07	0.20	0.20	0.20	0.00	0.07	0.53
Nikkei 225	0.00	0.07	0.00	0.50	0.64	0.36	0.07	0.43	0.00	0.00	0.93
Indice Bovespa	0.00	0.00	0.22	0.89	0.00	0.33	0.33	0.56	0.00	-	0.89
RTS Index	0.75	0.38	0.13	0.25	0.13	0.50	0.63	0.13	0.13	0.00	1.00
BSE SENSEX	0.00	0.00	0.00	0.27	0.00	0.00	0.00	0.00	0.00	0.00	0.27
SSE 50	0.14	0.00	0.14	0.00	0.00	0.00	0.14	0.43	0.71	-	0.71
FTSE/JSE Top 40	-	0.00	0.31	0.31	0.00	0.00	0.00	-	0.31	-	0.62
S&P/ASX 50	0.00	0.00	0.13	0.20	0.00	0.40	0.00	0.00	0.13	-	0.60
MERVAL Index	0.21	0.29	0.07	0.07	-	-	0.00	0.07	0.00	-	0.50
S&P/TSX 60	0.00	0.00	0.08	0.00	0.08	0.77	0.23	0.08	0.38	0.00	0.92
FTSE MIB	0.50	0.00	0.83	0.83	0.00	0.00	0.17	0.67	0.00	0.00	1.00
KOSPI 50	0.00	0.00	0.00	0.00	-	0.00	0.23	0.15	0.00	0.00	0.31
Panel B: 2rd order stochastic dominance											
S&P 100	0.00	0.00	0.00	0.23	0.12	0.12	0.08	0.15	0.00	0.00	0.35
DJIA	0.00	0.00	0.00	0.08	0.25	0.25	0.25	-	0.00	0.00	0.42
FTSE 100	0.00	0.00	0.00	0.50	0.10	0.10	0.05	0.35	0.00	0.00	0.55
CAC	0.00	0.00	0.07	0.27	0.07	0.07	0.07	0.07	0.00	0.00	0.53
DAX	-	0.00	0.00	0.00	0.13	0.00	0.00	0.13	0.07	0.07	0.27
Euro Stoxx 50	0.07	0.00	0.00	0.27	0.00	0.07	0.07	0.07	0.00	0.00	0.27
Nikkei 225	0.00	0.00	0.00	0.14	0.29	0.07	0.07	0.21	0.00	0.00	0.43
Indice Bovespa	0.00	0.00	0.22	0.78	0.00	0.33	0.22	0.44	0.00	-	0.89
RTS Index	0.50	0.25	0.13	0.25	0.13	0.38	0.38	0.13	0.13	0.00	0.75
BSE SENSEX	0.00	0.00	0.00	0.18	0.00	0.00	0.00	0.00	0.00	0.00	0.18
SSE 50	0.14	0.00	0.14	0.00	0.00	0.00	0.00	0.14	0.29	-	0.43
FTSE/JSE Top 40	-	0.00	0.23	0.23	0.00	0.00	0.00	-	0.15	-	0.54
S&P/ASX 50	0.00	0.00	0.07	0.13	0.00	0.20	0.00	0.00	0.07	-	0.33
MERVAL Index	0.14	0.00	0.07	0.07	-	-	0.00	0.00	0.00	-	0.21
S&P/TSX 60	0.00	0.00	0.00	0.00	0.00	0.38	0.15	0.00	0.15	0.00	0.54
FTSE MIB	0.50	0.00	0.67	0.83	0.00	0.00	0.17	0.33	0.00	0.00	1.00
KOSPI 50	0.00	0.00	0.00	0.00	-	0.00	0.08	0.08	0.00	0.00	0.15

3.9.3 Pooled logit regression for probability of stochastic dominance of a sub-index over the index

Table 3.16 reports the estimation results of a pooled logit regression for dominance of sub-indices over the index base on all markets under study. The key predictors of the dominance are market and index volatilities, as well as their ratio. It also emerges that several sectors, namely, Consumer goods, Health care, Consumer services, Telecommunications, and Utilities are more likely to dominate their respective diversified indices. Their significance is suppressed, however, when we include a broader factor indicating sub-index dominance in the past. We do not find a single macro- or financial variable that can consistently explain the propensity of a market index to be dominated by any of its sub-indices. The absence of any link between the performance of the real economy and financial markets is explained by substantially different information content of the aggregate market indicators for different types of the economies, as explained in the main body of the paper.

TABLE 3.16: Stochastic dominance determinants

The table reports the estimation results for the Logit model for the probability of each sector sub-index to dominate its respective diversified equity index.

	Coef.	t-stat.	Coef.	t-stat.	Coef.	t-stat.	Coef.	t-stat.	Coef.	t-stat.	Coef.	t-stat.
S&P 100	-0.67	-0.31			-1.70	-0.75			-2.00	-0.88		
DJIA	-0.34	-0.16			-1.01	-0.44			-1.13	-0.49		
FTSE 100	-0.38	-0.16			-1.42	-0.55			-1.83	-0.70		
CAC	-0.50	-0.26			-1.30	-0.64			-1.60	-0.78		
DAX	-0.65	-0.33			-1.29	-0.61			-1.61	-0.76		
Euro Stoxx 50	-0.45	-0.24			-1.13	-0.58			-1.44	-0.74		
Nikkei 225	-0.01	-0.01			-1.07	-0.49			-1.43	-0.66		
Indice Bovespa	0.38	0.16			-0.30	-0.12			-0.77	-0.31		
RTS Index	1.24	0.64			0.52	0.25			0.11	0.05		
BSE SENSEX	-1.40	-1.18			-1.77	-1.44			-1.84	-1.47		
SSE 50	0.00	–			0.00	–			0.00	–		
FTSE/JSE Top 40	-0.80	-0.36			-0.92	-0.40			-1.21	-0.52		
S&P/ASX 50	-0.64	-0.36			-1.31	-0.69			-1.48	-0.77		
MERVAL Index	2.48	1.01			3.75	1.41			3.44	1.28		
S&P/TSX 60	0.77	0.40			0.54	0.27			0.36	0.18		
FTSE MIB	0.31	0.14			-0.59	-0.26			-0.96	-0.42		
KOSPI 50	-0.92	-0.65			-1.73	-1.13			-1.87	-1.22		
Oil and gas	1.61	0.38	2.46*	1.85	3.26	0.73	2.44*	1.74	2.74	0.61	1.12	0.80
Basic materials	1.32	0.31	2.12	1.56	2.82	0.63	2.04	1.42	2.28	0.51	0.68	0.47
Industrials	1.69	0.40	2.36*	1.80	3.43	0.76	2.41*	1.74	2.86	0.64	1.04	0.75
Consumer goods	2.97	0.71	3.52***	2.73	4.59	1.02	3.44**	2.51	3.97	0.89	2.00	1.46
Health care	2.54	0.61	3.07**	2.36	4.20	0.94	3.07**	2.23	3.58	0.80	1.64	1.19
Consumer Services	2.61	0.62	3.25**	2.54	4.30	0.96	3.25**	2.40	3.64	0.81	1.78	1.31
Telecommunications	2.40	0.57	3.06**	2.32	3.97	0.89	2.95**	2.12	3.38	0.76	1.56	1.12
Utilities	2.28	0.54	2.95**	2.29	3.80	0.85	2.79**	2.04	3.13	0.70	1.27	0.92
Financials	1.99	0.48	2.73**	2.08	3.47	0.78	2.56*	1.84	2.92	0.65	1.20	0.87
Technology	0.96	0.23	1.47	0.99	2.62	0.58	1.47	0.95	2.25	0.50	0.32	0.21
GDP growth	0.02	0.43	0.00	0.06	0.02	0.30	-0.02	-0.38	0.03	0.54	-0.01	-0.22
Inflation	-0.06	-1.06	0.00	-0.10	-0.07	-1.22	0.01	0.15	-0.09	-1.50	-0.01	-0.14
Unemployment	0.01	0.07	-0.01	-0.57	-0.05	-0.54	-0.02	-0.71	-0.06	-0.61	-0.02	-0.85
Gross savings	-0.02	-0.23	-0.03*	-1.82	-0.05	-0.58	-0.03*	-1.66	-0.06	-0.74	-0.03	-1.46
Current account balance	-0.09	-1.17	0.00	-0.14	-0.10	-1.15	-0.01	-0.44	-0.08	-0.88	-0.02	-0.56
Real effective exchange	0.02	1.44	0.02	1.56	0.02	1.41	0.02	1.27	0.03	1.60	0.02	1.54
Government bonds yield	-0.03	-0.50	-0.05	-1.30	0.00	0.02	-0.04	-1.15	0.01	0.24	-0.04	-0.97
Central bank policy rate	0.02	0.44	0.05	1.21	-0.01	-0.13	0.05	1.14	0.01	0.14	0.06	1.28
Liquidity	-8.21	-1.48	0.50	0.26	-16.12**	-2.49	0.30	0.15	-17.71***	-2.73	-0.53	-0.26
Index return	0.93*	1.80	0.57	1.18	0.93	1.64	0.43	0.83	1.01*	1.78	0.54	1.02
Index volatility	17.1***	3.06	15.52***	2.79	18.76***	3.23	15.3***	2.68	18.66***	3.28	15.63***	2.78
Sector return	-0.82*	-1.69	-0.59	-1.24	-0.67	-1.30	-0.52	-1.02	-0.77	-1.51	-0.60	-1.18
Sector volatility	-16.46***	-3.02	-15.28***	-2.82	-18.05***	-3.18	-16.02***	-2.86	-17.14***	-3.09	-15.58***	-2.83
Volatility ratio	-2.79**	-2.44	-3.3***	-2.87	-2.47**	-2.08	-3.07***	-2.58	-1.85	-1.61	-2.34**	-2.01
D dominance, last year					-0.07	-0.29	-0.04	-0.16				
D dominance, any year									0.59***	2.67	0.63***	2.95
Efron's pseudo R-sq	0.32		0.30		0.31		0.29		0.32		0.29	
N obs	1823		1823		1688		1688		1688		1688	

3.9.4 Out-of-sample trading strategy: details

This appendix reports detailed results of the out-of-sample performance of our past-SD based trading strategy. The descriptive statistics of the resulting returns are reported for each index year by year.

TABLE 3.17: Out-of-sample index performance: detailed

The table reports the descriptive statistics mean (Mean), standard deviation (Std), skewness (Sk), and kurtosis (kr) of market indices (Index) and corresponding index that includes those industries that dominated the index in the past at least two times (SD). Mean returns and standard deviations are annualized.

	Mean (Index)	Mean (SD)	Std (Index)	Std (SD)	Sk (Index)	Sk (SD)	Kr (Index)	Kr (SD)
S&P 100								
2003	23.31%	23.06%	17.52%	16.67%	0.01	0.05	3.91	4.54
2004	6.21%	13.66%	11.10%	9.80%	-0.08	0.10	2.97	4.77
2005	1.17%	8.09%	9.85%	11.23%	0.03	-0.22	3.12	3.23
2006	17.01%	17.26%	9.47%	8.71%	0.02	-0.17	4.41	2.97
2007	5.94%	17.15%	15.74%	15.44%	-0.49	-0.53	4.80	4.65
2008	-43.39%	-25.68%	40.12%	30.75%	0.01	0.72	6.85	10.32
2009	20.12%	8.33%	25.54%	16.93%	0.03	-0.29	5.46	4.01
2010	11.78%	6.97%	17.24%	12.75%	-0.23	-0.33	5.12	5.64
2011	3.14%	12.41%	22.35%	15.87%	-0.52	-0.55	5.93	6.09
2012	14.89%	10.68%	12.47%	8.82%	0.10	0.14	4.08	3.99
2013	26.54%	18.91%	10.67%	10.43%	-0.24	-0.26	4.69	4.63
2014	11.99%	12.98%	11.18%	9.43%	-0.38	-0.19	4.54	4.34
2015	2.60%	3.06%	15.89%	13.63%	-0.16	-0.15	5.24	5.21
DJIA 30								
2010	13.16%	13.16%	16.16%	16.16%	-0.17	-0.17	5.29	5.29
2011	8.08%	14.95%	21.12%	16.79%	-0.53	-0.35	5.61	5.66
2012	9.75%	11.09%	11.72%	9.11%	0.05	0.09	4.02	3.83
2013	25.97%	21.26%	10.15%	10.42%	-0.19	-0.35	4.48	4.73
2014	9.57%	11.58%	10.91%	9.53%	-0.35	-0.35	4.27	4.86
2015	0.21%	6.50%	15.45%	13.85%	-0.14	-0.05	4.58	5.13
FTSE 100								
2006	13.53%	20.57%	12.61%	10.09%	-0.40	-0.34	4.59	3.89
2007	7.11%	9.61%	17.49%	14.32%	-0.36	-0.31	4.67	4.39
2008	-33.18%	-24.11%	37.50%	29.84%	0.12	0.10	6.59	7.04
2009	24.16%	14.67%	23.50%	15.55%	-0.21	-0.05	4.53	5.18
2010	11.89%	11.84%	17.42%	13.40%	0.05	-0.16	5.11	5.37
2011	-2.21%	7.75%	21.29%	16.22%	-0.24	-0.21	4.35	4.15
2012	9.51%	11.37%	13.96%	10.87%	-0.01	0.05	3.67	3.48
2013	17.11%	19.70%	12.12%	11.26%	-0.26	-0.42	4.78	6.21
2014	0.73%	5.27%	11.39%	11.24%	-0.35	-0.42	5.03	5.70
2015	-1.33%	4.32%	17.34%	15.78%	-0.28	-0.05	5.01	4.26
CAC 40								
2009	24.36%	29.41%	26.73%	26.18%	-0.01	0.06	3.89	3.71
2010	0.55%	9.08%	23.57%	19.34%	0.57	0.41	9.13	7.84
2011	-14.43%	-14.82%	28.84%	24.32%	-0.13	-0.08	4.57	4.24
2012	18.54%	5.19%	20.73%	19.26%	0.09	0.16	3.98	4.12
2013	20.06%	14.24%	16.19%	17.09%	-0.18	-0.35	4.40	4.15
2014	2.67%	16.28%	16.23%	15.92%	-0.21	-0.13	4.28	3.96
2015	11.28%	10.50%	22.56%	24.11%	-0.25	0.05	4.22	3.97
DAX 30								
2009	21.39%	9.67%	28.43%	20.59%	-0.07	-0.22	3.89	3.09
2010	14.89%	-3.38%	18.42%	13.90%	-0.02	0.00	4.73	4.10
2011	-15.95%	-8.69%	28.97%	25.33%	-0.15	-0.57	4.37	5.18
2012	25.51%	6.06%	18.91%	15.62%	-0.12	-0.09	4.29	4.04
2013	22.70%	6.80%	14.64%	14.10%	-0.30	-0.45	4.17	5.89
2014	2.62%	11.80%	16.76%	16.47%	-0.16	-0.46	3.86	4.43
2015	9.13%	-11.84%	23.59%	25.09%	-0.15	-0.11	3.57	3.23
EURO STOXX 50								
2009	23.96%	3.65%	28.14%	26.98%	-0.08	-0.14	3.92	4.53
2010	-1.92%	-11.77%	23.70%	20.65%	0.77	-0.04	10.66	5.66
2011	-14.16%	-11.05%	28.94%	25.37%	-0.16	-0.06	4.38	4.24
2012	17.87%	8.73%	20.76%	18.92%	0.19	0.26	4.30	4.40
2013	20.49%	16.48%	16.39%	15.81%	-0.15	-0.17	4.27	3.63
2014	4.82%	8.15%	17.08%	16.51%	-0.16	-0.13	4.20	4.33
2015	7.03%	6.76%	23.30%	23.28%	-0.24	-0.17	4.22	4.62
NIKKEI 225								
2009	19.17%	-0.74%	27.41%	18.93%	-0.04	0.05	3.75	3.33
2010	-1.33%	-8.73%	20.64%	14.29%	-0.24	-0.41	3.26	3.62
2011	-17.01%	-26.75%	23.54%	22.01%	-1.72	-4.05	16.57	40.68
2012	22.83%	9.92%	16.03%	13.82%	-0.09	0.12	3.00	3.48
2013	46.59%	39.85%	26.70%	25.43%	-0.76	-0.87	5.50	6.28
2014	8.57%	9.80%	20.08%	19.11%	-0.06	-0.19	4.49	4.61
2015	10.43%	18.93%	20.74%	21.61%	0.00	-0.14	7.98	8.11

3.9.5 Market specific individual-stock based trading strategy: historical volatility

This appendix reports the results for SD tests and the average performance of portfolios sorted on stock historical volatility. Overall, the portfolios including 30% of stocks with the lowest historical volatility perform well in SD sense out-of-sample, consistent with findings of Hodder, Jackwerth, and Kolokolova (2014) on good performance of a global minimum variance portfolio.

TABLE 3.18: Historical stock return volatility descriptive statistics

The table reports the time series averages of the descriptive statistics of individual stock return historical volatilities for stocks that are constituents of the 17 market indices used in this paper. "Effective stocks" reports the average percentage of stocks in each index that are alive at the end of a calendar year and are used for portfolio construction over the following year.

	Effective Stocks	Mean	Median	Std	Min	Max	30% quantile	70% quantile
S&P 100	98%	0.31	0.29	0.11	0.15	0.71	0.25	0.33
DJIA	100%	0.27	0.26	0.08	0.15	0.51	0.23	0.29
FTSE 100	99%	0.30	0.28	0.10	0.15	0.73	0.25	0.32
CAC	100%	0.34	0.32	0.12	0.18	0.71	0.27	0.38
DAX	100%	0.35	0.33	0.11	0.20	0.70	0.29	0.38
Euro Stoxx 50	100%	0.32	0.30	0.11	0.18	0.67	0.26	0.35
Nikkei 225	100%	0.37	0.36	0.10	0.16	0.79	0.32	0.41
Indice Bovespa	99%	0.41	0.37	0.14	0.22	0.91	0.33	0.44
RTS Index	99%	0.57	0.54	0.17	0.30	1.15	0.48	0.61
BSE SENSEX	100%	0.39	0.37	0.10	0.25	0.63	0.33	0.42
SSE 50	100%	0.48	0.46	0.13	0.20	0.76	0.40	0.54
FTSE/JSE Top 40	100%	0.31	0.29	0.09	0.17	0.59	0.26	0.34
S&P/ASX 50	99%	0.28	0.25	0.10	0.13	0.65	0.22	0.29
MERVAL Index	94%	0.54	0.53	0.14	0.31	0.84	0.46	0.60
S&P/TSX 60	100%	0.32	0.29	0.14	0.15	0.83	0.24	0.36
FTSE MIB	98%	0.36	0.36	0.11	0.13	0.66	0.30	0.42
KOSPI 50	96%	0.44	0.43	0.12	0.22	0.83	0.38	0.48

TABLE 3.19: Market indices vs. historical volatility-sorted portfolios

The table reports the fraction of years during which each of the 17 diversified equity indices is dominated by historical volatility-sorted portfolios of individual stocks at orders 4, 3 or 2.

	Low-vol	Medium-vol	High-vol
S&P 100	0.62	0.04	0.00
DJIA	0.50	0.00	0.00
FTSE 100	0.50	0.05	0.10
CAC	0.73	0.13	0.00
DAX	0.73	0.07	0.00
Euro Stoxx 50	0.87	0.20	0.00
Nikkei 225	0.86	0.00	0.00
Indice Bovespa	0.78	0.11	0.00
RTS Index	0.75	0.38	0.38
BSE SENSEX	0.64	0.00	0.00
SSE 50	0.71	0.43	0.00
FTSE/JSE Top 40	0.62	0.00	0.00
S&P/ASX 50	0.67	0.00	0.00
MERVAL Index	0.36	0.21	0.36
S&P/TSX 60	0.92	0.15	0.00
FTSE MIB	0.83	0.33	0.00
KOSPI 50	0.38	0.00	0.00

TABLE 3.20: Out-of-sample performance of low-historical volatility portfolios

The table report the average descriptive statistics of the low-historical volatility portfolios for 17 market indices under study. The mean return, return standard deviation (Std), and Sharpe ratio are annualised, whereas skewness and kurtosis are based on monthly returns. The last rows report the average improvement of the low-historical volatility portfolio over the corresponding market index.

	S&P 100	DJIA	FTSE 100	CAC	DAX	Euro Stoxx 50	Nikkei 225	Indice Bovespa	RTS	BSE SENSEX	SSE 50	FTSE JSE Top 40	S&P/ ASX 50	MERVAL	S&P/ TSX 60	FTSE MIB	KOSPI 50
Low-volatility portfolio																	
Mean	0.06	0.04	0.05	0.03	0.04	0.02	0.03	0.07	-0.02	0.11	0.06	0.17	0.08	0.09	0.07	0.04	0.07
St.D	0.16	0.16	0.17	0.21	0.21	0.21	0.20	0.25	0.37	0.23	0.23	0.19	0.16	0.35	0.17	0.22	0.21
Skewness	-0.19	0.03	-0.61	0.10	-0.18	0.06	-0.51	0.10	-0.14	-0.27	-0.47	-0.08	-0.19	0.12	-0.60	-0.05	-0.14
Kurtosis	12.07	18.18	15.16	9.48	12.78	10.44	13.47	13.28	28.21	13.85	10.68	8.28	9.32	8.40	21.91	5.27	7.58
Sharpe ratio	0.21	0.17	0.09	-0.18	-0.10	0.02	0.03	-0.14	-0.22	0.26	0.17	0.48	0.17	0.00	0.35	0.04	0.23
Difference between low-volatility portfolio and the index																	
Mean	-0.02	-0.03	-0.01	0.01	0.01	0.01	-0.02	0.07	0.09	-0.03	-0.05	0.02	0.00	-0.18	-0.01	0.02	-0.02
St.D	-0.02	-0.02	-0.02	-0.03	-0.04	-0.04	-0.04	-0.04	-0.02	-0.01	-0.04	-0.02	-0.01	0.01	-0.01	-0.05	-0.02
Skewness	-0.02	0.11	-0.45	0.07	-0.16	0.05	-0.03	0.08	0.16	-0.35	-0.13	0.02	0.17	0.47	0.08	0.05	0.15
Kurtosis	0.72	3.54	6.18	1.39	5.15	2.87	2.83	3.79	14.44	1.97	3.52	1.55	0.59	1.03	6.70	-0.20	-1.31
Sharpe ratio	-0.10	-0.14	-0.05	0.01	0.01	0.06	-0.09	0.23	0.22	-0.10	-0.14	0.12	0.01	-0.53	-0.02	0.08	-0.05

Chapter 4

DJIA is more efficient than thought

> *"It(the DJIA)'s actually a pretty bad metric," says John Shoven from the Stanford Institute for Economic Policy Research. "It's poorly constructed. Its only claim to fame is that it's been around for 120 years."*
>
> ——CBC News Report, Armstrong (2017)

4.1 Introduction

Published in 1896, the Dow Jones Industrial Average (DJIA) is the first stock index dedicated for overall industry market movement. From then on, DJIA has always been one of the most recognized market indices in the world, broadly followed and closely watched by full spectrum of business and investment participants. However, the last 120 years have saw tectonic shifts in both business landscape and financial theory, and more people regard DJIA as obsolete. The marching DJIA hit several milestones beyond 20,000 since 2017, but this history making performance has reignited a broad criticism on DJIA as a flawed indicator. The mass wave of skepticism mainly centers on its limited corporate coverage and price weighted methodology compared to its peer of S&P 500 index, as reported in, for example, Armstrong (2017) and Rosenbaum (2017).

The contradiction of DJIA's significant presence and confronted opprobrium highlights an important question that whether DJIA is an efficient index or not. If DJIA is shown systematically biased, then this inefficient indicator does not live up to its accumulated presence as premier stock indicator. In this case, related financial practices and products get to correspondingly change to reduce deadweight loss.

It is opportune for an in-depth study on the question of DJIA efficiency, with two key developments greatly facilitating this inspection. Theoretically, Lo (2016) proposes a modern framework for financial index, based on new investment reality where recent computing and trading technology advancements open up broad possibilities of financial innovations. To serve as performance benchmark, an index should be transparent, investable, and systematic. We find the DJIA satisfies all the three fundamental properties and falls into the category of "dynamic index" in Lo (2016). It is a perfect example of extant indices in the spirit of passive investing and active management. Practically, index ETF industry burgeons in last decades, making index investment more accessible and comparable. Besides, the total return has been widely accepted as a realistic return metric on actual investment experience than price index return, with authentic total return data for both stock indices and companies commonly provided by databases. With reliable total return data, the DJIA efficiency analysis is much more accurate. This is an important advantage for us over prior research, like Shoven and Sialm (2000), Clarke and Statman (2000), Haensly, Tripathy, and Peak (2001) where total return, as well as index portfolio rebalancing, can only be crudely approximated on monthly basis.

We primarily focus on the performance benchmark function of DJIA, since the information aggregation function can be conveniently repositioned according to business reality and composition scope. Specifically, the S&P 500 index with much broader industry coverage is a better representation for the overall stock market than the DJIA. However, it can still serve as an information aggregator for a selection of blue-chip industry leaders with "excellent reputation, sustained growth" and "of interest to a large number of investors" as indicated in its methodology.

We collect total return data for both the DJIA and its components in the period of 1988 to 2017 in daily basis, since the DJIA total return series begins from Sep 30, 1987 in Datastream. We construct equal weighted, market value weighted, price weighted indices as DJIA variants based on contemporaneous compositions to see if the price weighted characteristic systematically biases the index performance. Compared to prior literature, our paper innovatively recognizes two price weighted schemes for both adjusted price and unadjusted price. This distinction corresponds to the common criticism that stock split leads to meaningless weight drop even there is no change to the relative importance of aggregate corporate wealth. Since the adjusted price naturally takes into consideration of such capital actions, the adjusted price weighted scheme offers a complementary assessment for weighting structures. Contrary to common viewpoint of the superiority of market value weighted scheme, we find equal weighted and unadjusted price weighted indices have better return profile over it. Since these two entail rebalancing costs, we further compare Sharpe ratio after adjusting turnover costs. The result confirms that the default total return DJIA performs the best, a strong support for the DJIA efficiency.

To make sure such benefit is accessible rather than on paper, we then examine the resemblance between DJIA and its chief ETF product, the SPDR Dow Jones Industrial Average ETF, whose data coverage is from late January 1998. Over the life of this ETF, we find that it tracks the DJIA quite well with minimal return reduction as adjustment of operational costs. There is no significant difference between their return series, guaranteeing the investment accessibility to the efficient DJIA.

Additionally, we reaffirm the DJIA efficiency from the perspective of out-of-sample performance of in-sample optimal portfolios. For each sample year, we construct in-sample optimal portfolios by both mean variance approach and stochastic dominance approach. These optimal portfolios are minimum variance portfolio, maximum Sharpe ratio portfolio, minimum expected short-fall portfolio, and Kuosmanen (2004) second-order stochastic dominance (SSD) optimal portfolio. Although these optimal portfolios have appealing outperformance over the DJIA in sample, none of them shows consistent improvement in the out-of-sample evaluation. Actually, the in-sample optimality cannot carry on and the features of return enhancement and risk reduction expire next year. This out-of-sample evaluation adds evidence to the DJIA efficiency.

Our results that the DJIA is more efficient that thought have several important implications. First, this is an effort in legitimating the DJIA significance as prominent stock index. The corroborated efficiency warrants its role of performance benchmark for investment practice, which is critical to various kinds of market participants especially active managers. Second, its efficiency sets a safe ground for DJIA index ETFs, derivatives and other potential financial innovations centered in DJIA without incurring deadweight loss, and stimulates more new DJIA financial products and services. Third, our paper adds new subtle insight to the debate of indexation paradigm anchored in market value or non market value, as in Treynor (2005), Hsu (2006), Perold (2007), Reinganum (2014), Fuller, Giovinazzo, and Tung (2014), and Arnott, Beck, and Kalesnik (2015). Our price weighted results niche the engulfing nuance between the two, since stock price is closely related to its market value but price weighted index is more similar to non market value weighted indices as dynamic strategies. Our results facilitates further understanding and practice of modern index in both academia and industry. Fourth, we highlight an interesting alteration of financial obsolescence and modernization, evincing how this dichotomy dances with technology advancements. As Lo (2016) annotates, DJIA used to be the first equal weighted index, and later price weighting was chosen in its 1928 revision over value weighting despite economists' suggestion. Its basic calculation renders DJIA enduringly caricatured as a relic from old days with primitive technology. Enlightened by Lo (2016), we show that DJIA is not only a first dynamic index reinvigorated by the technology-leveraged investing but also an efficient performance benchmark.

4.2 Data and methodology

Total return from Datastream is the default return type throughout our study. To make index variants of different weighting schemes, we assemble daily return data for DJIA components from 1988 to 2017 considering component changes, and then combine return series with equal weights (Eq), market value weights (MV), adjusted price weights (P) and unadjusted price weights (UP).

The general index variant method is

$$R_t^S = \sum w_{it} r_{it},$$

where $S \in Eq, MV, P, UP$ is the weighting scheme set, R_t is the index variant return at time t, $w_i t$ is the weight of component i at time t and $r_i t$ its return.

Specifically, for the equal weighted DJIA,

$$R_t^{Eq} = \sum \frac{r_{it}}{30}.$$

For the market value weighted DJIA,

$$R_t^{MV} = \sum \frac{MV_{it-1}}{\sum MV_{it-1}} r_{it}.$$

For the adjusted price weighted DJIA,

$$R_t^{P} = \sum \frac{P_{it-1}}{\sum P_{it-1}} r_{it}.$$

For the unadjusted price weighted DJIA,

$$R_t^{UP} = \sum \frac{UP_{it-1}}{\sum UP_{it-1}} r_{it}.$$

When the component change is effective from the beginning of trading day t, we instantly make corresponding composition change at the same day to minimize tracking error.

Note that we use the term "variants" for these indices made from underlying components, because the actual DJIA weighting is slightly different from the price weighted scheme by an additional adjustment factor called "divisor". Specifically, suppose the price index for DJIA at time t is PI_t and the divisor is d_t, then the index goes as

$$PI_t = \frac{\sum UP_{it}}{d_t}.$$

The divisor changes when the index rebalances, so that the index level right after rebalancing PI_t^+ is equal to the level right before rebalancing PI_t^-. For continuity, the return on price index DJIA goes as

$$R_t^{PI} = \frac{PI_t}{PI_{t-1}} = \frac{d_{t-1}}{d_t} \sum \frac{UP_{it-1}}{\sum UP_{it-1}} \frac{UP_{it}}{UP_{it-1}}.$$

Comparing R_t^{PI} and R_t^{UP}, we notice two important differences. The first is return type, where R_t^{PI} only takes capital appreciation and R_t^{UP} uses total return. The second is for the comprehensive adjustment $\frac{d_{t-1}}{d_t}$, while R_t^{UP} implies a constant divisor. This assumption is partially true, since only upon index rebalancing varies the divisor. The total return DJIA is a refined version of price index DJIA, replacing capital appreciation with total return including dividend reinvestment. Note R_t^{RI} involves reinvestment of dividend from components into the index, while r_{it} involves dividend reinvestment into the component itself. They are the same in trading days without dividends, and return discrepancy between component and index is usually insignificant. Therefore, we can still regard R_t^{UP} as a good approximation of R_t^{RI} by construction methodology.

For these variants, we also consider turnover costs associated with daily rebalancing similar to DeMiguel, Garlappi, and Uppal (2007) as following:

$$Turnover = \frac{1}{T-1} \sum_{t=2}^{T} \sum_{i=1}^{30} \mid w_{it} - w_{it-1} \mid$$

under the cost assumption of 50 basis points per transaction unit. Note that the turnover measure underestimates true turnover volume around component changes, which entails clearing the deleted component stocks w^- and buying the added one w^+. The quantity difference $\mid w^+ - w^- \mid$ cannot reflect this complete weight replacement $\mid w^+ \mid + \mid w^- \mid$.

Besides the four DJIA variants, we also collect the price return and total return for DJIA index, total return for the S&P 500 index, and total return for the 3-month T-Bill index from Datastream over sample period. As SPDR Dow Jones Industrial Average ETF incepted in mid-January 1998 is the main and earliest ETF product on DJIA, we then gather its total return since Jan 20, 1998 for further comparison.

After time series evaluation for index efficiency, we conduct out-of-sample analysis of in-sample optimal portfolios within DJIA components. To minimize the effect of trading costs, we set the investment horizon as one year. This setting of holding period are congruent to the finding of Almadi, Rapach, and Suri (2014) that annual rebalancing delivers the best gain at transaction costs of 50 basis points per turnover unit or higher. At each year-end from 1988 to 2017, we calculate the optimal portfolios with the return series of contemporaneous composition. The in-sample optimal portfolio weights are used in next year return scenario on the same composition as out-of-sample performance. We construct optimal portfolios by both the mean variance approach and the stochastic dominance approach. For mean variance optimization, we compute minimum variance portfolio, maximum revised Sharpe ratio portfolio, and minimum expected short-fall portfolio at 5% level. For stochastic dominance optimality, we adopt the necessary test for SSD efficiency in Kuosmanen (2004), which favorably returns the optimal weights giving maximum return improvement over the benchmark. Specifically, the Kuosmanen optimal portfolio is from the following program,

$$\begin{aligned} \max_{w,W} \quad & (\sum_{t=1}^{T}\sum_{i=1}^{30} r_{it}w_i - \sum_{t=1}^{T} R_t)/T \\ \text{s.t.} \quad & \sum_{i=1}^{30} r_{it}w_i \geq \sum_{j=1}^{T} W_{tj}R_t \\ & w \in \Lambda \\ & W \in \Xi \end{aligned}$$

where w is the optimal portfolio weight vector and Λ the set of long only portfolios, W is a doubly stochastic matrix of nonnegative elements with unit sum for each row and each column and Ξ its set. The intuition is to identify as optimal the portfolio with highest return given its SSD over the DJIA index return series R.

4.3 DJIA variants and index comparison

The price weighted DJIA is peculiar to the market value dominating index zoo. CAPM and EMH underpins the popularity of market value weighted indices, where component weights are proportional to their relative importance by market share and automatically rebalanced as market capitalization changes. Many argue that price weighted scheme systematically skews the performance of DJIA and market value weighted scheme should be the default setting.

Comparing the equal weighted, market value weighted, adjusted price weighted and unadjusted price weighted DJIA variants, we find that unadjusted price weighted scheme does not understate the index performance. Rather, it achieves substantial performance advantage over the ostensibly superior market value weighted scheme. As Table 4.1 shows, equal weighted and unadjusted price weighted schemes have the best return-risk profile. Specifically, equal weighted variant has the highest mean return of 14.11%, with a mean-variance improvement over the adjusted price weighted variant. Unadjusted price weighted variant has the lowest volatility of 16.65%, with a mean-variance improvement over the market value weighted variant. This point is also confirmed by Sharpe ratio. Equal weighted and unadjusted price weighted variants have the best Sharpe ratio at about 3.5%, much higher than the market value weighted and adjusted price weighted versions. Note that adjusted price circumvents the volatile weight movement associated with stock splits, which usually inculpated as an infamous weakness for DJIA even after the divisor adjustment. Nonetheless, we find the adjusted price weighted variant considerably underperforms its primitive counterpart, the unadjusted price weighted version, along almost all indicators implying that the price smoothness also leads to performance distortion.

When we further include the market indices for comparison, the DJIA efficiency gets clearer. The price index DJIA is based on capital appreciation, and unsurprisingly it underperforms the total return DJIA by about 3% per year. This size agrees to the finding of 2.9% dividend return from 1979 to 2010 by Chaves and Arnott (2012). Moreover, the unadjusted price weighted variant and total return DJIA have highly similar investment profile along all indicators. They both beat the total return S&P 500 index in the evaluation. Taking the S&P 500 index as the market index, we see market value weighted variant have the highest correlation of 96.61%, as they share the same weighting scheme. Equal weighted and adjusted price weighted variants have the lowest correlation, as their weighting schemes are less sensitive to concurrent market information. Jensen's alpha by Sharpe-Lintner CAPM shows the same story. Adjusted price weighted variant is inferior with significantly negative alpha, and market value weighted variant is mediocre with insignificant alpha. Unadjusted price weighted variant and total return DJIA have higher statistical significance than equal weighted variant, although their point estimates are slightly lower.

We also notice that weight rebalancing in the implementation incurs trading costs, which may impact the performance of DJIA variants. To capture this implication, we use turnover indicator defined as average weight change across components over the whole sample period. As Table 4.1 shows, market value weighted variant has 0 turnover due to its feature of automatic adaptation. Adjusted price weighted variant has lower turnover than unadjusted price weighted variant, because price cascade by stock split is attenuated in the adjusted price series but not in the unadjusted price series. After the consideration of turnover costs, the unadjusted price weighted variant still dominates the market value weighted variant, a repulse to the claim that market value weighted DJIA would have been much more efficient.

Table 4.2 gives confirming evidence from pairwise comparisons. The price index DJIA and total return DJIA have the highest correlation of 99.98%, since they are based on the synthesized DJIA index with divisor. The correlation of total return DJIA and unadjusted price weighted variance comes next, as unadjusted price weighted is the most close to the actual DJIA weighting scheme methodologically. The adjusted price weighted variant is statistically inferior to unadjusted price variant by 1.39 basis points, total return DJIA by 1.35 basis points, and total return S&P 500 index by 1.07 basis points. Market value weighted variant is also dominated by equal weighted variant with 0.72 basis points and unadjusted price weighted variant with 0.44 basis points. Despite higher point estimate, equal weighted variant is not statistically different from unadjusted price weighted variant, total return DJIA index, and total return S&P 500 index, an unambiguous indication of DJIA efficiency.

TABLE 4.1: Descriptive return statistics for DJIA variants and market indices

	DJIA Variants				Market Indices			
	Eq	MV	P	UP	PI	RI	S&P 500	3M T-Bill
Mean	14.11%	11.99%	9.27%	13.29%	10.35%	13.18%	12.35%	3.14%
Std	17.87%	17.22%	18.73%	16.65%	16.71%	16.71%	17.41%	0.23%
Skewness	0.02	-0.01	-0.24	-0.07	-0.07	-0.07	-0.12	2.01
Kurtosis	14.16	11.83	12.90	12.22	12.16	12.15	12.55	7.87
Min	-9.12%	-8.03%	-11.76%	-7.87%	-7.87%	-7.87%	-9.03%	0.00%
Max	12.14%	11.05%	11.35%	11.08%	11.08%	11.08%	11.58%	0.10%
Sharpe ratio	3.51%	2.97%	1.91%	3.50%	2.51%	3.45%	3.05%	
Correlation	95.54%	96.61%	93.27%	96.24%	96.24%	96.26%	100.00%	0.03%
Beta	0.98	0.96	1.00	0.92	0.92	0.92		
Alpha	1.74%*	0.06%	-2.77%**	1.53%*	-1.13%	1.39%*		
Turnover	0.98%	0.00%	0.95%	0.96%				
Turnover adjusted Sharpe ratio	3.07%	2.97%	1.50%	3.03%				

Data applied to the DJIA variants and of market indices are from Datastream over the sample period since 1988 to 2017. Mean return, standard deviation, and alpha are annualized. Alpha and beta are calculated in the Sharpe-Lintner CAPM. * denotes significance at 10% level and ** at 5%.

TABLE 4.2: Pairwise comparison between indices

	Eq	MV	P	UP	PI	RI	SP	TB
Eq		0.72**	1.67***	0.28	1.29***	0.32	0.60	3.89***
MV	96.50		0.95**	-0.44*	0.57**	-0.40	-0.12	3.17***
P	96.30	94.10		-1.39***	-0.38	-1.35***	-1.07**	2.22*
UP	98.37	97.50	95.94		1.01***	0.04	0.32	3.61***
PI	98.22	97.42	95.80	99.87		-0.97***	-0.69**	2.6**
RI	98.23	97.44	95.81	99.89	99.98		0.28	3.57***
SP	95.54	96.61	93.27	96.24	96.24	96.26		3.29***
TB	2.03	2.78	1.93	2.57	2.42	2.53	1.36	

Data applied to the DJIA variants and of market indices are from Datastream over the sample period since 1988 to 2017. The lower triangular part of the exhibit is for pairwise correlation, expressed in percentage form. The upper triangular part for the pairwise daily return difference of row identify over column identify, expressed in basis point. * denotes statistical significance at 10% level, ** at 5%, and *** at 1%.

As exhibited in, for example, Kuosmanen (2004) and Shalit (2010), stochastic dominance another is also frequently used for decision making and performance evaluation. We further compare these indices in terms of SSD, shown in Table 4.3. Obviously, the adjusted price weighted variant is SSD dominated by all the rest stock indices, consistent with the results revealed in previous exhibits. Market value weighted variant is SSD dominated by unadjusted price weighted variant and total return DJIA. This evidence seals to be misleading the common belief that market value weighted scheme is optimal and stock split stirs meaningless weight change in DJIA. Equal weighted and unadjusted weighted variants, as well as total return DJIA are the three stock indices that are SSD efficient. This conforms to their statistical indifference manifested in Table 4.2. Unadjusted price weighted variant and total return DJIA are with the most dominated indices. From the perspective of stochastic dominance, the DJIA is also efficient.

TABLE 4.3: Second order stochastic dominance pairwise comparison

	Eq	MV	P	UP	PI	RI	SP	TB
Eq			$\succ$					
MV			$\succ$					
P								
UP		$\succ$	$\succ$		$\succ$		$\succ$	
PI			$\succ$					
RI		$\succ$	$\succ$				$\succ$	
SP			$\succ$					
TB								

Data applied to the DJIA variants and of market indices are from Datastream over the sample period since 1988 to 2017. The results of pairwise comparison is expressed as the second order stochastic dominance of row identity over column identity.

To much extent, equal weighted and unadjusted price weighted variants have the best performance of the four variants along traditional evaluation indicators. Comparatively, equal weighted variant has slightly higher return but also higher risk than unadjusted price weighted variant, judging from its greater even moments, wider range, and bigger beta. Its minimal return advantage cannot eclipse the efficiency of unadjusted price weighted variant, not to mention the total return DJIA which has much higher Sharpe ratio. This point is reaffirmed in the SSD pairwise comparison, where unadjusted price weighted variant is with most dominated indices and its risk reduction edge is very straightforward.

As current DJIA greatly resembles the unadjusted price weighted variant in its methodological essence, their dominance out of the traditional performance indicators and SSD results endorses that DJIA is more efficient than thought, despite the popular delusion that DJIA is lamed by its price weighted nature.

4.4 DJIA ETF garnering the benefit

As previously shown, unadjusted price weighted scheme does not produce DJIA inefficiency as performance benchmark. On the contrary, it is more efficient than the most approving market value weighted scheme by both mean variance comparison and stochastic dominance comparisons. As the total return DJIA has the highest Sharpe ratio after adjusting the turnover costs, a direct investment in it can be very attractive for saving institutional investors and individual investors from operational matters. Now the critical question is if such index investment is possible.

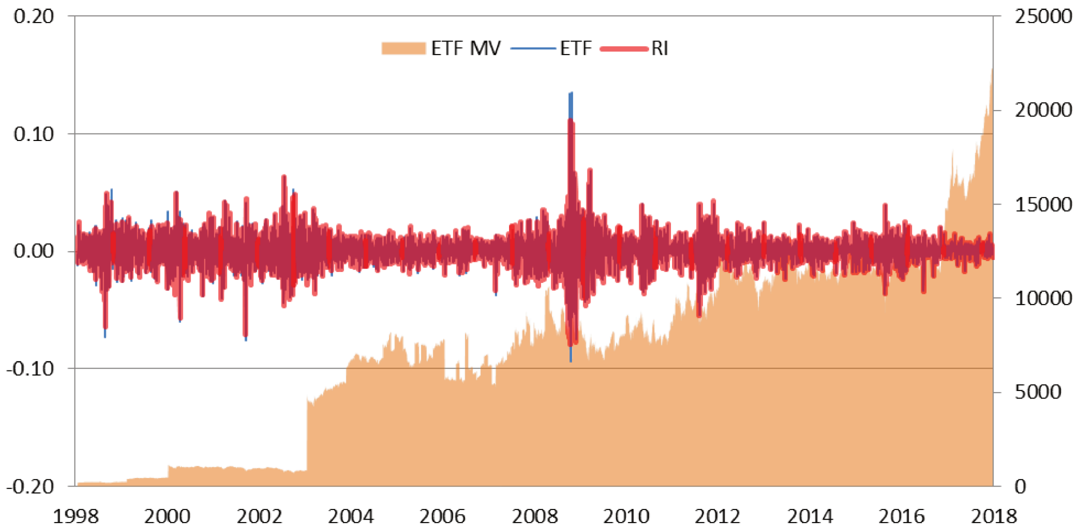
Since ETFs are the most practical way to follow stock indices, we then focus on the possibility of an ETF to garner the aforementioned DJIA benefit. There are some DJIA related ETFs, among which SPDR Dow Jones Industrial Average ETF is the most important one. Commenced in Jan 14, 1998, it grows rapidly in size along the last two decades as shown in Figure 4.1. Its market value snowballs from $198 million at the inception to $22.4 billion last year, at a startling annual increase of 28%. Aimed to track the DJIA yield performance, the ETF is highly correlated with the total return DJIA at a coefficient of 98.3%. This resemblance can be seen from the overlapping total return series between them in 4.1. They are statistically indifferent from each other, and only in few trading days during the 2008 financial crisis can we distinguish them as the ETF has more extreme return observations.

The annualized return for the ETF is 10.04%, slightly lower than the contemporaneous annualized DJIA return of 10.14%. Three reasons mainly account for this insignificant return difference. The first is due to the transaction costs for ETF rebalancing, but the situation is much alleviated than the case of aforementioned turnover for DJIA variants. There we take 50 basis points per turnover unit and the cost adjusted return for UP variant can only realize 90% of the total return DJIA. With this ETF, it's possible to realize 99% of the total return DJIA, implying that the turnover cost for the ETF is much lower. To great extent, this cost reduction thanks to the economies of scale, confirming that the benefit of DJIA is accessible despite implemental expenses. The second is by imperfect composition replication. When an index change is scheduled, the ETF is not in lockstep with it instantly. The ETF adjustment is made within 3 trading days around the effective date rather than immediately, therefore leading to tracking error. The third is about dividend payments and distributions. The total return DJIA assumes dividend reinvestment immediate to the actual payment, while the ETF distributes dividend net of fees before reinvestment monthly. This operational friction slightly reduces return realized.

Besides the unadjusted price weighted scheme, equal weighted variant also has better performance than the market value weighted and adjusted price weighted variants. However, equal weighted DJIA index is not officially launched by the S&P

FIGURE 4.1: Return series of DJIA and SPDR DJIA EFT, and the ETF market value series

Data of total returns for SPDR DJIA ETF and total return DJIA as well as market value for the ETF are from Datastream over the period since 1998 to 2017. The primary vertical axis is for daily total return of DJIA and this ETF, and the secondary vertical axis is for the ETF's market value in the unit of $ Million.



Dow Jones Indices until February 2017, and it is rebalanced on a crude quarterly basis. As a consequence, the related ETF products are not abundant and the principle one, First Trust Dow 30 Equal Weight ETF, comes half-year later in Aug 8, 2017. Therefore, the historical data for the evaluation of equal weighted DJIA is limited and the index benefit collection by this ETF is far from clear as in the total return DJIA.

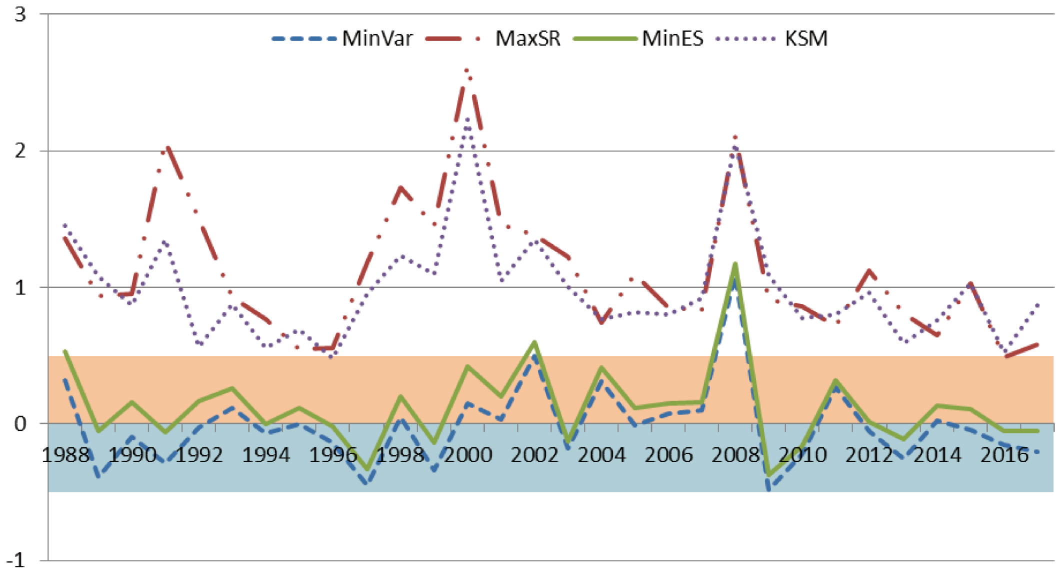
4.5 Out-of-sample performance for optimal DJIA portfolios

Apart from the aforementioned total sample return analysis for DJIA efficiency, we conduct an optimal portfolio study to complement the evaluation. The intuition is that if the in-sample optimal portfolio of constituents has consistent superior out-of-sample performance over DJIA, then we have confidence that DJIA as performance benchmark is an inefficient index relative to its components.

To echo the two perspectives of total sample return analysis, we construct in-sample optimal portfolios with corresponding mean variance approach and stochastic dominance approach. Specifically, in the category of mean variance efficient portfolios, we set up minimum variance portfolio, maximum Sharpe ratio portfolio, and minimum expected short-fall portfolio at 5% level. In addition, we use Kuosmanen (2004) SSD necessary test to get stochastic dominance optimal portfolio. Among all performance indicators, we concentrate on mean return, volatility, Sharpe ratio, and minimum return for their strong implications on performance evaluation. The former two are fundamental elements in performance measurement, and Sharpe ratio characterizes the tradeoff between risk and return, and minimum return is paramount in the revelation of extreme risk.

FIGURE 4.2: Return of in-sample optimal portfolios over total return DJIA

Optimal portfolios are constructed over the period since 1988 to 2017as in the text, and their mean daily return differences over the contemporaneous total return DJIA are expressed in the unit of permil. The light orange band is over [0, 0.5], while the light blue over [-0.5, 0].



As expected, the four in-sample optimal portfolios display their comparative advantages over the total return DJIA. Figure 4.2 gives the in sample return performance for the optimal portfolios against the index. Maximum Sharpe ratio portfolio resembles Kuosmanen SSD portfolio in terms of superior return, with a surplus of about 0.1% over the index on average. The minimum variance portfolio and minimum expected shortfall portfolio have barely such return edge, vacillating around the total return DJIA level. Their strength is on risk reduction, as shown by Figure 4.3. The two portfolios share great similarity across the whole period, gaining a risk reduction of 20% than the index.

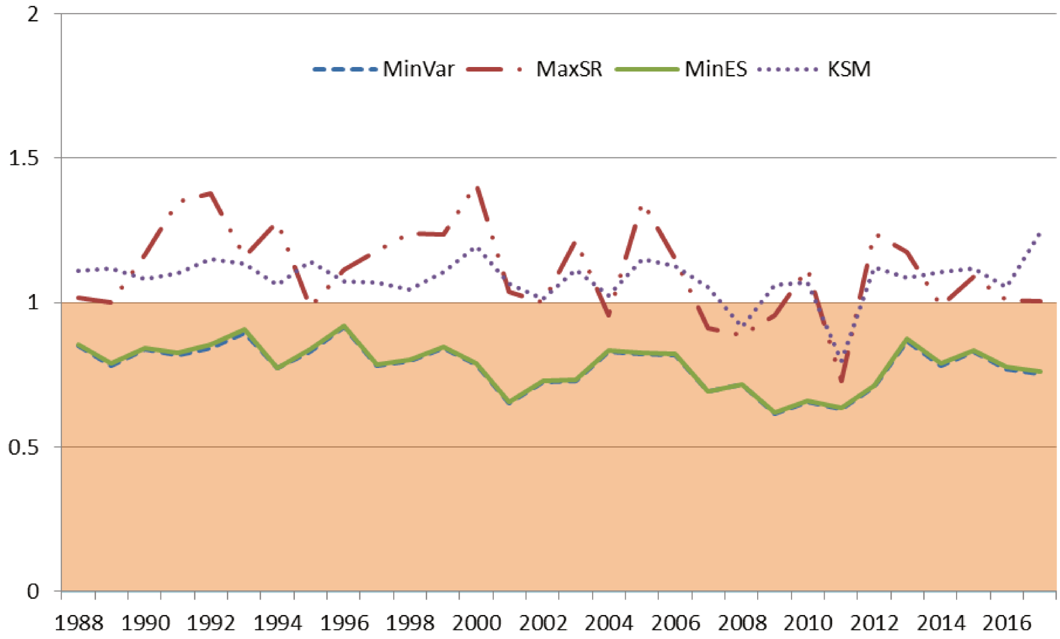
As a comprehensive demonstration of the improved tradeoff between risk and return by the optimal portfolios, Figure 4.4 plots their performance in terms of Sharpe ratio. Despite higher volatility, maximum Sharpe ratio portfolio and Kuosmanen SSD portfolio acquire significant Sharpe ratio increase, and the former one performs the best, by definition. The rest two risk reduction portfolios also have somewhat higher Sharpe ratio than the index.

We also interest in the worst case of investment, as impressed by the global financial crisis that extreme losses are devastating for investment maintenance. Figure 4.5 reveals the performance of optimal portfolios in this scenario. Notably, Kuosmanen SSD portfolio has consistent improvement than the index, which is due to the nature of stochastic dominance in extreme risk restriction. This is because a necessary condition for the SSD is that the worst return for the dominating portfolio is still better than that of the dominated portfolio. On average, minimum variance portfolio and minimum expected shortfall portfolio for their essence in risk reduction have better minimum return, and the maximum Sharpe ratio portfolio is also on a par with the index.

At first glance, it is reasonable to infer that the total return DJIA is not efficient

FIGURE 4.3: Volatility of in-sample optimal portfolios over total return DJIA

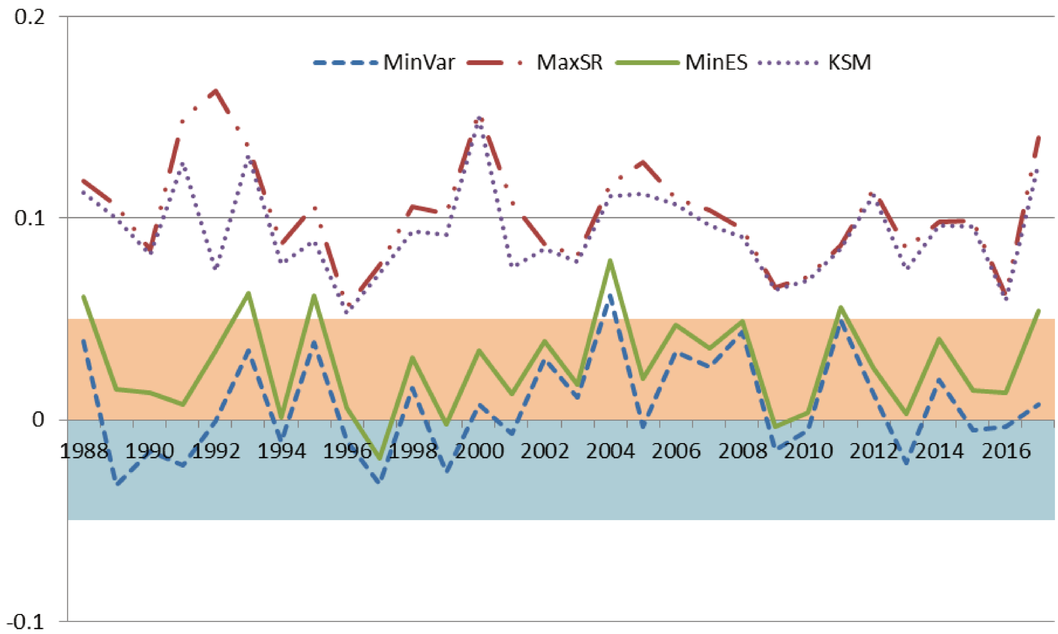
Optimal portfolios are constructed over the period since 1988 to 2017 as in the text, and their volatility is scaled against the contemporaneous total return DJIA. The light orange band is over [0, 1], implying a lower volatility level than the total return DJIA.



since the optimal portfolios have consistent better performance over it. Note that we test the efficiency with hindsight and an out-of-sample analysis is necessary for the examination of optimality continuation. We first see if the superior return performance by maximum Sharpe ratio portfolio and Kuosmanen SSD portfolio can carry on next year. Figure 4.6 manifests that this return superiority is not sustainable, with all of them oscillating around the index level. In fact, only maximum Sharpe ratio portfolio has slightly higher return than the total return DJIA, but the surplus of 0.1 basis point is neither economically nor statistically significant. And even for this optimal portfolio, in quite some years its loss goes beyond the low band of 5 basis points. Besides the impotent return enhancement, the risk reduction advantage of minimum variance portfolio and minimum expected shortfall portfolio is also diminished by half, and the volatility for maximum Sharpe ratio portfolio and Kuosmanen SSD portfolio is about 20% higher than the index revealed in Figure 4.7. As advantages of return enhancement and risk reduction expire considerably out of sample, we expect the Sharpe ratio performance is also much impaired, validated by Figure 4.8. The used-to-be winners of maximum Sharpe ratio portfolio and Kuosmanen SSD portfolio in Sharpe ratio are losers, abdicating all the in sample benefit. Moreover, they commit much bigger loss in worst case, and in 2003 the maximum loss is even 3 times than the index as shown in Figure 4.9. The property of stochastic dominance in limiting maximum loss does not hold any more out of sample, and the minimum variance portfolio and minimum expected shortfall portfolio cannot achieve similar goal of maximum loss protection as in sample. Actually, they are not statistically different from the index when it comes to maximum loss.

FIGURE 4.4: Sharpe ratio of in-sample optimal portfolios over total return DJIA

Optimal portfolios are constructed over the period since 1988 to 2017 as in the text, and the differences are expressed as Sharpe ratio for optimal portfolios over the contemporaneous total return DJIA. The light orange band is over [0, 0.05], while the light blue over [-0.05, 0].



Overall, the mediocre out-of-sample performance of optimal portfolios contradicts their excellent in-sample performance, vindicating that the optimality is not systematically predictive and the in-sample outperformance is transient. In other words, the results from optimal portfolio approach do not approve the asserted inefficiency of DJIA.

4.6 Conclusion

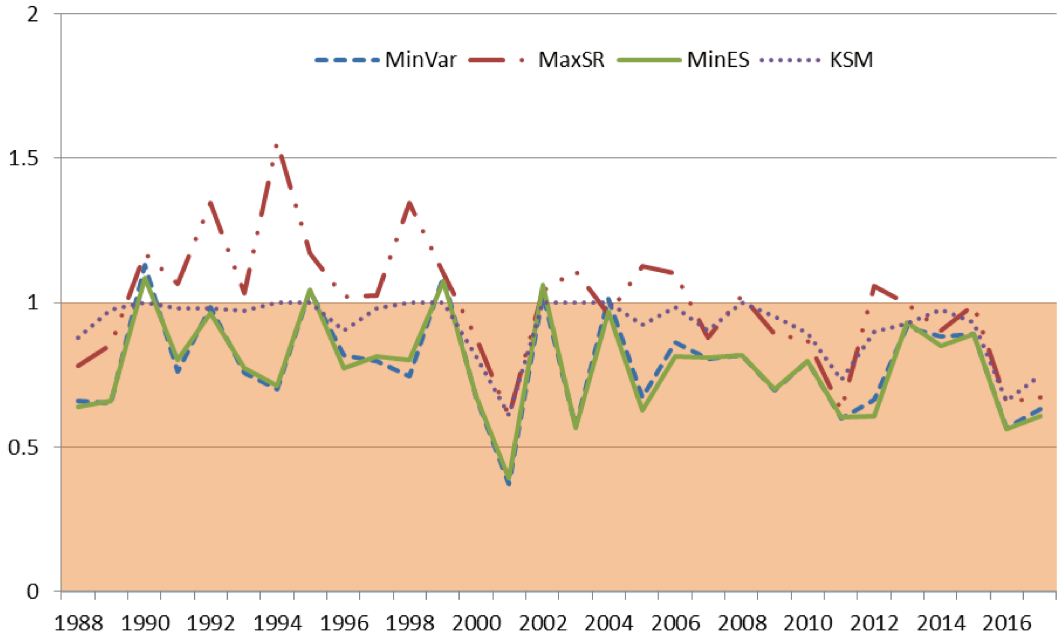
The DJIA's epic gain of about 5,000 points in 2017 highlights one of the most success developments in its history. However, the millstone has rekindled a wide disapproval and doubts that the DJIA is "flawed" and "meaningless" by its price weighted nature and limited coverage.

To answer the question of whether DJIA is an efficient index or not, we examine its role as performance benchmark, because its role as information aggregator can be easily reoriented. We find the index with "obsolete" price weighted methodology serendipitously turns out to be "dynamic," according to the modern definition of index by Lo (2016). It is definitely investable, as all of its constituents are blue-chip companies with great liquidity; it is transparent, as its composition changes and component weights are public information; and it is systematic, since its construction is by rules and the selection criteria are consistent.[1] Born in the old days

[1]Some may think that the DJIA is much dependent on the discretion of the DJIA committee and not governed exclusively by the quantitative rules, therefore not systematic. In fact, its index universe and its selection criteria are all explicit. There is some discretion to maintain "adequate sector representation" but this is not unique to the DJIA. The S&P 500 index also explicates in methodology that sector

FIGURE 4.5: Minimum return of in-sample optimal portfolios over total return DJIA

Optimal portfolios are constructed over the period since 1988 to 2017 as in the text, and their minimum returns are scaled against the contemporaneous total return DJIA. The light orange band is over [0, 1], implying a better minimum return situation than the total return DJIA.



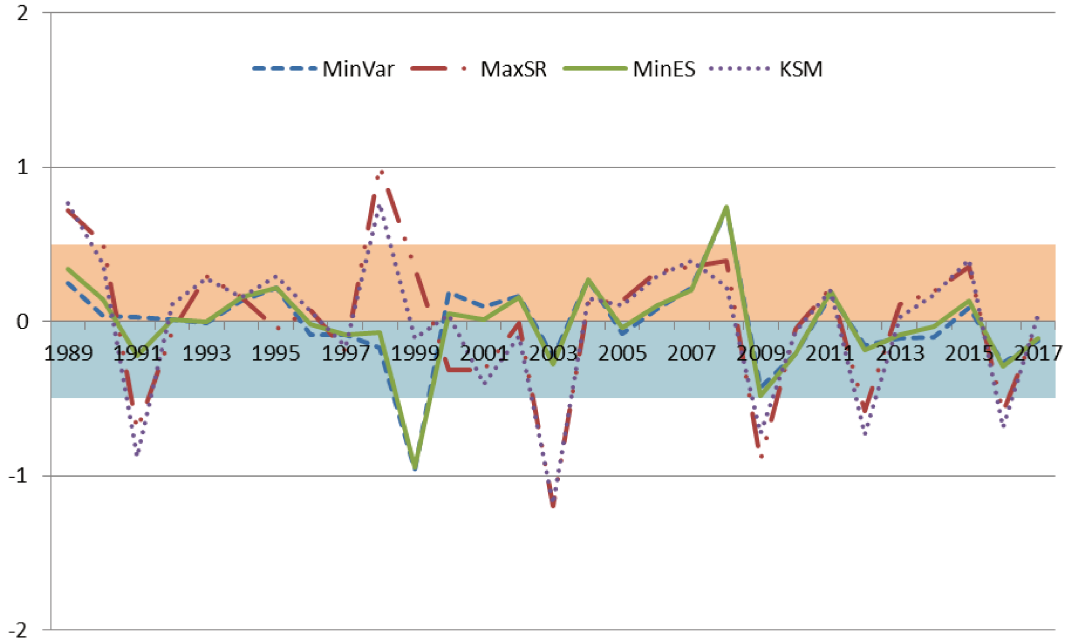
without computer, DJIA receives its renaissance in the technology-leveraged new financial reality. To examine its efficiency, we construct equal weighted, market value weighted, unadjusted price weighted, and adjusted price weighted variants with the underlying total return of components since 1988 to 2017. We find that equal weighted and unadjusted price weighted variants have a considerably better performance than the market value weighted and adjusted price weight variants. After adjusting trading costs for all of them, we do not find any evidence against the efficiency of unadjusted price weighted scheme. This result disapproves the common understanding that DJIA is badly constructed as a price weighted index. We then include the market indices of price index DJIA, total return DJIA and total return S&P 500 index for comparison, and find the total return DJIA has the highest cost-adjusted Sharpe ratio, a tenable indication of the efficiency of DJIA. The SSD pairwise comparison result confirms previous findings on the DJIA efficiency.

Moreover, we take an optimal portfolio approach to complement the efficiency analysis. If the in-sample optimal portfolio of DJIA components consistently outperforms the index out of sample, then the index is inefficient. However, our results show that all the optimal portfolios by mean variance optimization and stochastic dominance optimization lose their investment advantages out of sample, and the in-sample optimality cannot carry on next year. This reaffirms the DJIA efficiency as a performance benchmark.

balance is a consideration. As the S&P 500 index is the most representative static index satisfying the condition of being systematic as reaffirmed by Lo (2016), we safely reach that DJIA also meets this systematic condition.

FIGURE 4.6: Out-of-sample return of optimal portfolios over total return DJIA

Optimal portfolios are constructed over the period since 1989 to 2017 as in the text, and their mean daily return differences over the contemporaneous total return DJIA are expressed in the unit of permil. The light orange band is over [0, 0.5], while the light blue over [-0.5, 0].



Our results manifest that the repudiation of DJIA efficiency is too harsh. Probably the DJIA is not a comprehensive and proper index for the overall stock market, but as a perfect example of dynamic indices, it is still efficient as performance benchmark for the investment in elite blue-chip stocks.

FIGURE 4.7: Out-of-sample volatility of optimal portfolios over total return DJIA

Optimal portfolios are constructed over the period since 1989 to 2017 as in the text, and their volatility is scaled against the contemporaneous total return DJIA. The light orange band is over [0, 1], implying a lower volatility level than the total return DJIA.

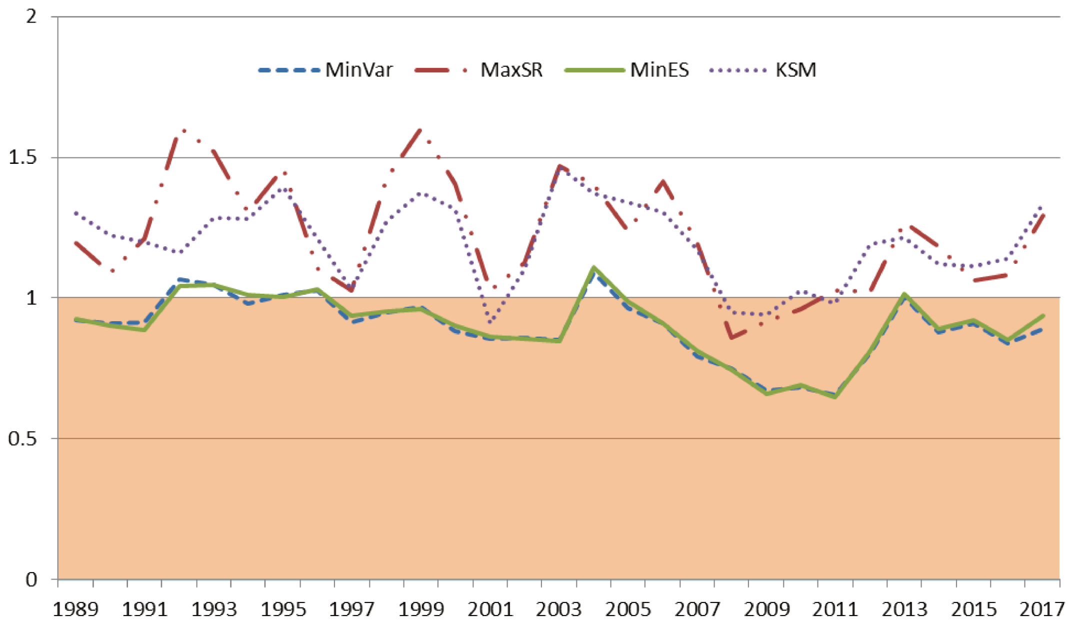


FIGURE 4.8: Out-of-sample Sharpe ratio of optimal portfolios over total return DJIA

Optimal portfolios are constructed over the period since 1989 to 2017 as in the text, and the differences are expressed as Sharpe ratio for optimal portfolios over the contemporaneous total return DJIA. The light orange band is over [0, 0.05], while the light blue over [-0.05, 0].

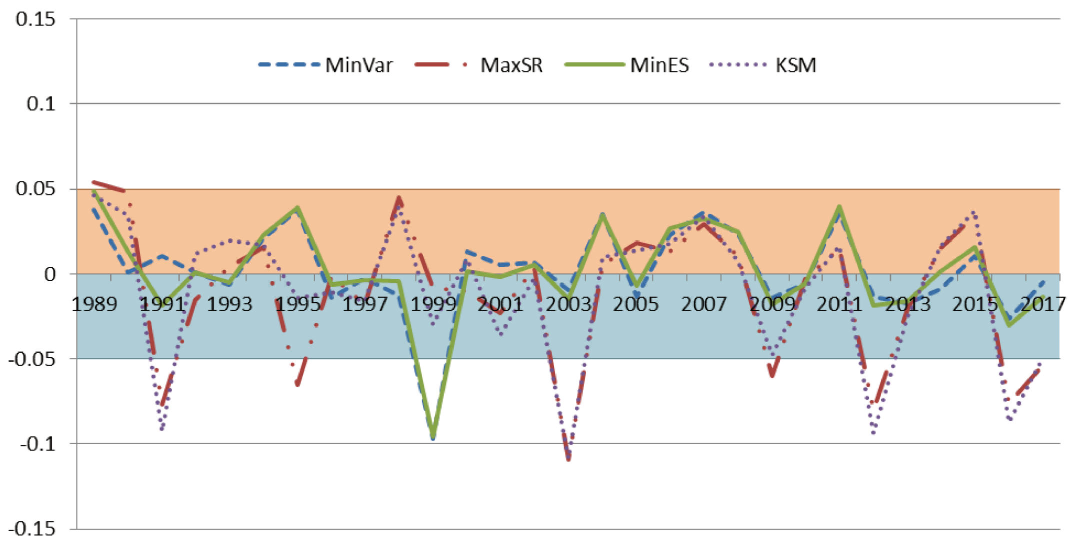
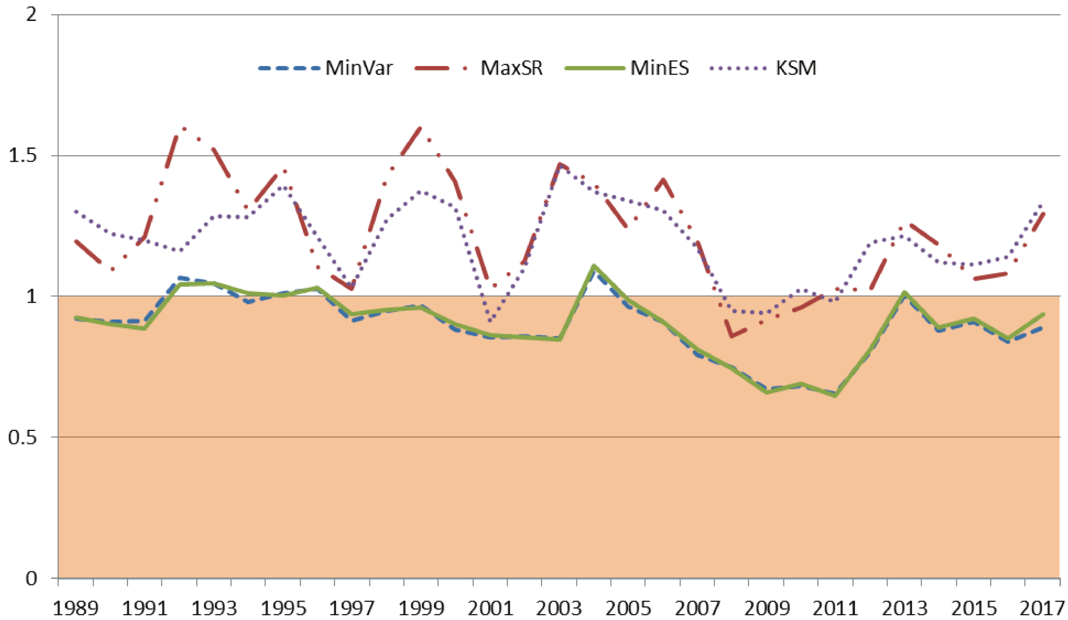


FIGURE 4.9: Minimum return ratio of out-of-sample optimal portfolios over total return DJIA

Optimal portfolios are constructed over the period since 1989 to 2017as in the text, and their minimum returns are scaled against the contemporaneous total return DJIA. The light orange band is over [0, 1], implying a better minimum return situation than the total return DJIA.



4.7 Appendics for in-sample and out-of-sample results

4.7.1 In-sample investment performance from 1988 to 2017

TABLE 4.4: Total return DJIA

	Mean	Std	Skewness	Kurtosis	Min	Max	Sharpe ratio
1988	0.000636	0.011243	-0.81927	9.800222	-0.06851	0.039549	0.033613
1989	0.001098	0.008802	-1.73959	18.28003	-0.06905	0.03489	0.088673
1990	2.88E-05	0.010132	-0.2361	4.017111	-0.03208	0.031145	-0.02604
1991	0.000876	0.00911	0.465951	6.518427	-0.03927	0.045677	0.073206
1992	0.00029	0.006409	0.092652	3.463532	-0.01891	0.02133	0.02456
1993	0.000615	0.005401	-0.34234	5.279893	-0.02445	0.019045	0.09232
1994	0.000211	0.006748	-0.28447	4.309607	-0.02428	0.022835	0.006836
1995	0.001224	0.005392	-0.18851	4.296766	-0.01961	0.019745	0.187556
1996	0.000991	0.007433	-0.55104	4.767676	-0.03035	0.020209	0.107196
1997	0.00092	0.011605	-0.71561	9.076299	-0.07184	0.047083	0.062368
1998	0.000714	0.012281	-0.46226	7.214977	-0.06367	0.049807	0.043043
1999	0.000972	0.010006	0.083532	2.981752	-0.02595	0.028379	0.079183
2000	-0.00011	0.012857	-0.20967	4.779717	-0.05655	0.049786	-0.02607
2001	-0.00013	0.013112	-0.45485	6.878257	-0.07073	0.044688	-0.01994
2002	-0.0005	0.015821	0.563548	4.44512	-0.0464	0.06348	-0.03547
2003	0.001007	0.010268	0.168192	4.26136	-0.03606	0.035981	0.094284
2004	0.00022	0.006716	0.035238	2.975194	-0.01614	0.017901	0.024868
2005	8.61E-05	0.0064	0.024736	3.129465	-0.01861	0.020907	-0.00563
2006	0.000689	0.00611	-0.06263	4.292088	-0.01961	0.019884	0.082685
2007	0.000367	0.008996	-0.55469	4.691641	-0.03293	0.025901	0.021862
2008	-0.00119	0.023442	0.434004	7.373139	-0.07873	0.110803	-0.05322
2009	0.000895	0.014999	0.17916	5.484782	-0.04619	0.068689	0.059312
2010	0.000554	0.010001	-0.1013	5.278182	-0.03604	0.038987	0.054879
2011	0.000395	0.013035	-0.44756	5.485163	-0.05499	0.042415	0.030116
2012	0.0004	0.00726	0.085217	4.011717	-0.02245	0.023945	0.0546
2013	0.001015	0.006288	-0.15616	4.477108	-0.02332	0.023835	0.161097
2014	0.000389	0.006746	-0.31506	4.254957	-0.02077	0.024357	0.057535
2015	5.36E-05	0.009556	-0.08669	4.584927	-0.03575	0.039515	0.005409
2016	0.000615	0.007787	-0.41147	5.247079	-0.03389	0.024683	0.077475
2017	0.000961	0.004088	-0.08577	5.234946	-0.01677	0.014574	0.226445

TABLE 4.5: Minimum variance portfolio

	Mean	Std	Skewness	Kurtosis	Min	Max	Sharpe ratio
1988	0.000952	0.009545	-0.4487	6.383145	-0.04512	0.031696	0.072649
1989	0.000703	0.006881	-1.28169	9.988097	-0.04507	0.015611	0.0561
1990	-6.31E-05	0.008485	-0.52904	4.461026	-0.03626	0.021455	-0.04192
1991	0.000585	0.007456	0.060737	4.188439	-0.02988	0.025147	0.050449
1992	0.000263	0.005409	-0.08335	3.704059	-0.01865	0.016586	0.02399
1993	0.000728	0.00484	-0.10359	4.017528	-0.01853	0.016857	0.126615
1994	0.000144	0.005209	-0.52367	3.819222	-0.01706	0.014556	-0.00392
1995	0.001224	0.004482	-0.37275	5.296213	-0.02039	0.016498	0.225641
1996	0.000853	0.006795	-0.32772	3.546427	-0.02478	0.018533	0.096879
1997	0.000471	0.009066	-0.97537	8.890821	-0.0574	0.029021	0.030268
1998	0.00076	0.009813	-0.41887	5.513203	-0.04745	0.036137	0.058534
1999	0.000629	0.008427	-0.09119	3.281678	-0.02832	0.024737	0.053247
2000	4.11E-05	0.010095	-0.15302	4.032688	-0.03797	0.03474	-0.01834
2001	-9.60E-05	0.008569	-0.14643	3.747418	-0.02646	0.025263	-0.02678
2002	5.37E-07	0.011494	-0.07803	4.852516	-0.04779	0.041861	-0.00537
2003	0.000831	0.0075	0.122671	3.150949	-0.02052	0.023514	0.105593
2004	0.000534	0.005577	-0.16307	3.242847	-0.01639	0.014633	0.086282
2005	7.58E-05	0.005261	0.195527	3.058124	-0.01255	0.017241	-0.00879
2006	0.000767	0.004996	0.035613	3.864906	-0.01693	0.016825	0.116637
2007	0.000468	0.006216	-0.35979	5.276525	-0.02649	0.024851	0.047949
2008	-0.00011	0.016821	0.599051	9.003865	-0.06457	0.082949	-0.00965
2009	0.000415	0.009258	-0.16642	4.390841	-0.03224	0.035835	0.044209
2010	0.000332	0.006571	-0.50494	5.58729	-0.02874	0.020805	0.049701
2011	0.00066	0.008253	-0.25466	5.42125	-0.03296	0.031655	0.079717
2012	0.000351	0.005139	0.048017	3.551172	-0.01488	0.015041	0.06767
2013	0.000763	0.005449	-0.14836	4.446615	-0.02133	0.019489	0.139677
2014	0.000409	0.005284	-0.15392	4.310616	-0.0183	0.019631	0.077224
2015	5.50E-06	0.007934	-0.06798	4.514955	-0.03191	0.030027	0.000447
2016	0.000458	0.006006	-0.16446	4.120067	-0.0192	0.018987	0.074303
2017	0.000756	0.003076	-0.2114	4.369709	-0.01057	0.010652	0.234192

TABLE 4.6: Minimum expected short-fall portfolio at 5%

	Mean	Std	Skewness	Kurtosis	Min	Max	Sharpe ratio
1988	0.001166	0.009606	-0.38041	5.916582	-0.04387	0.030376	0.094489
1989	0.001043	0.006968	-1.21809	10.06502	-0.04561	0.016137	0.103966
1990	0.000185	0.008554	-0.47893	4.236057	-0.03486	0.022474	-0.0126
1991	0.000814	0.007513	0.032811	4.48397	-0.03157	0.026075	0.08059
1992	0.000456	0.005468	-0.05805	3.656034	-0.01822	0.016465	0.0591
1993	0.000874	0.004897	-0.07428	3.912613	-0.01891	0.01666	0.15483
1994	0.000206	0.005234	-0.52686	3.871414	-0.01732	0.014627	0.007886
1995	0.00134	0.004521	-0.35333	5.46859	-0.02054	0.017238	0.249342
1996	0.000969	0.006836	-0.38383	3.589961	-0.02346	0.016975	0.113277
1997	0.000592	0.009102	-1.01243	9.3633	-0.05845	0.029034	0.043478
1998	0.000912	0.009864	-0.49614	6.338673	-0.05097	0.03664	0.07364
1999	0.000834	0.008484	-0.08288	3.194819	-0.0279	0.024927	0.077124
2000	0.000309	0.010178	-0.0807	4.024656	-0.03834	0.03508	0.008125
2001	7.18E-05	0.00862	-0.17322	3.936729	-0.02787	0.024468	-0.00714
2002	0.000103	0.011542	-0.08082	4.946152	-0.04933	0.042126	0.003551
2003	0.000879	0.007515	0.143174	3.135691	-0.02051	0.023498	0.11183
2004	0.000634	0.005608	-0.11094	3.168345	-0.01565	0.015342	0.10369
2005	0.000201	0.005292	0.21558	3.043617	-0.01167	0.017862	0.014941
2006	0.000837	0.00502	0.062627	3.786445	-0.01596	0.017187	0.129939
2007	0.000526	0.00623	-0.34527	5.233656	-0.02675	0.024765	0.057143
2008	-2.48E-05	0.016841	0.642353	9.212951	-0.0645	0.084362	-0.00464
2009	0.000526	0.009299	-0.12733	4.450574	-0.0323	0.037068	0.055905
2010	0.000392	0.006588	-0.46837	5.403885	-0.02874	0.021367	0.058707
2011	0.000714	0.00827	-0.25725	5.509264	-0.03323	0.031894	0.086137
2012	0.000418	0.005162	0.092485	3.48861	-0.01367	0.015155	0.080298
2013	0.000904	0.005491	-0.16619	4.550198	-0.02178	0.018779	0.164182
2014	0.00052	0.005321	-0.11942	4.03728	-0.01769	0.019052	0.097551
2015	0.000162	0.007976	-0.03153	4.565574	-0.03193	0.030229	0.020017
2016	0.000563	0.006051	-0.07726	4.298381	-0.01911	0.021681	0.090984
2017	0.000908	0.003111	-0.10868	4.395559	-0.01018	0.011069	0.28044

TABLE 4.7: Maximum Sharpe ratio portfolio

	Mean	Std	Skewness	Kurtosis	Min	Max	Sharpe ratio
1988	0.001995	0.011426	-0.21066	5.371078	-0.05349	0.036552	0.151896
1989	0.002035	0.008805	-1.06797	11.37044	-0.05899	0.031682	0.194876
1990	0.000979	0.011813	0.024932	4.223926	-0.03754	0.042305	0.05822
1991	0.002946	0.012294	0.177159	4.166749	-0.04188	0.047629	0.222702
1992	0.001794	0.008843	0.089611	2.932516	-0.02544	0.024848	0.187738
1993	0.001541	0.006261	-0.18478	4.030441	-0.02528	0.018327	0.227861
1994	0.000976	0.00863	-0.27606	5.228191	-0.03777	0.027603	0.094167
1995	0.001773	0.005312	-0.16388	5.462105	-0.02298	0.02116	0.29382
1996	0.001546	0.008278	-0.29626	3.919298	-0.03095	0.023816	0.163254
1997	0.002101	0.013691	-0.14625	8.040685	-0.07358	0.068394	0.139264
1998	0.002449	0.01521	-0.61639	8.177245	-0.08564	0.066328	0.148752
1999	0.002426	0.012369	0.164694	2.767289	-0.02863	0.035358	0.181592
2000	0.002516	0.01812	0.603318	4.672599	-0.04964	0.078924	0.126513
2001	0.00133	0.013618	0.402382	4.660512	-0.04312	0.05684	0.08784
2002	0.000883	0.015821	0.418741	4.139781	-0.04827	0.057375	0.05184
2003	0.002228	0.012471	0.338628	4.138302	-0.04004	0.047653	0.175559
2004	0.000961	0.00642	-0.06506	3.00623	-0.01548	0.018659	0.141471
2005	0.00118	0.008645	0.463256	4.424788	-0.02099	0.036756	0.122356
2006	0.001538	0.007032	0.186275	3.622595	-0.02158	0.023058	0.192419
2007	0.001203	0.008211	-0.35636	3.660343	-0.029	0.02108	0.125896
2008	0.000909	0.020763	0.578972	8.000306	-0.08064	0.110731	0.041227
2009	0.001804	0.014362	0.430657	6.457711	-0.04124	0.068249	0.125202
2010	0.001411	0.011193	0.156315	4.375279	-0.03134	0.046104	0.12555
2011	0.001114	0.009514	-0.12362	5.38748	-0.03458	0.039411	0.116863
2012	0.001522	0.008993	0.255495	3.717269	-0.02375	0.03218	0.168819
2013	0.001822	0.007392	-0.14258	3.638296	-0.02322	0.027178	0.246158
2014	0.00104	0.00667	-0.03309	3.739072	-0.01876	0.021863	0.155732
2015	0.001083	0.010406	0.398751	5.022677	-0.03549	0.043357	0.103933
2016	0.001108	0.007867	0.04224	3.842537	-0.02223	0.025174	0.139269
2017	0.001543	0.004112	0.369239	4.42962	-0.01127	0.016022	0.366564

TABLE 4.8: Kuosmanen SSD optimal portfolio

	Mean	Std	Skewness	Kurtosis	Min	Max	Sharpe ratio
1988	0.002088	0.012503	-0.07587	5.768005	-0.06025	0.044098	0.146292
1989	0.002177	0.009849	-0.83951	12.64841	-0.06744	0.043886	0.188743
1990	0.0009	0.010946	0.062835	3.986344	-0.03208	0.039316	0.055625
1991	0.002226	0.010041	0.337806	4.671588	-0.03845	0.04362	0.20087
1992	0.000857	0.007368	0.228112	3.515574	-0.01855	0.02607	0.098265
1993	0.001488	0.006136	-0.07562	3.834789	-0.02375	0.017902	0.223689
1994	0.000768	0.007171	-0.23108	3.496399	-0.02428	0.018247	0.084365
1995	0.001917	0.006171	0.255618	4.007219	-0.01961	0.024896	0.276196
1996	0.001474	0.00798	-0.13129	4.09859	-0.02738	0.025685	0.160305
1997	0.001872	0.012418	-0.20612	9.05322	-0.07055	0.064326	0.13506
1998	0.001944	0.012852	-0.47589	6.025878	-0.06367	0.047417	0.136775
1999	0.002071	0.011051	0.236329	2.848504	-0.02595	0.0326	0.171062
2000	0.002123	0.015351	0.548529	4.460738	-0.04609	0.059885	0.123741
2001	0.000914	0.013978	0.369103	4.913137	-0.04328	0.062945	0.055844
2002	0.000855	0.016031	0.491032	4.120484	-0.0464	0.056884	0.049417
2003	0.002011	0.011422	0.481033	4.265937	-0.03606	0.045235	0.172631
2004	0.000988	0.006898	0.093788	3.208641	-0.01614	0.024147	0.135635
2005	0.000905	0.007352	0.487191	3.915606	-0.01721	0.030141	0.106492
2006	0.001492	0.006893	0.274612	3.683057	-0.01932	0.023689	0.189585
2007	0.00129	0.009489	-0.14039	4.070739	-0.02974	0.034698	0.118066
2008	0.000861	0.021493	0.427722	7.043965	-0.07873	0.110377	0.037558
2009	0.001982	0.015954	0.650515	7.286506	-0.04389	0.083077	0.123894
2010	0.001327	0.010679	0.143908	4.697648	-0.03221	0.044201	0.12374
2011	0.001199	0.010417	0.040156	5.844553	-0.0404	0.046889	0.114909
2012	0.001361	0.008161	0.324486	3.731552	-0.02021	0.028483	0.166369
2013	0.001607	0.006818	0.047331	4.057529	-0.02163	0.027753	0.235306
2014	0.001148	0.007455	0.066506	3.897593	-0.02025	0.025175	0.153792
2015	0.001087	0.010697	0.479211	4.859129	-0.03335	0.041991	0.10139
2016	0.001135	0.008192	0.262406	4.27117	-0.02239	0.031451	0.13702
2017	0.001834	0.005084	0.906076	6.22644	-0.01254	0.026582	0.353765

4.7.2 Out-of-sample investment performance from 1989 to 2017

TABLE 4.9: Minimum variance portfolio

	Mean	Std	Skewness	Kurtosis	Min	Max	Sharpe ratio
1989	0.001343	0.008102	-1.37942	11.99012	-0.05593	0.021392	0.126423
1990	6.30E-05	0.00923	-0.3014	4.044101	-0.03217	0.031579	-0.02488
1991	0.000906	0.008328	0.252515	4.611323	-0.03035	0.036291	0.083625
1992	0.000307	0.006839	0.255433	3.646525	-0.01952	0.022923	0.02539
1993	0.0006	0.005642	-0.418	5.396545	-0.02883	0.018987	0.085888
1994	0.000348	0.006606	-0.35318	4.423942	-0.02454	0.020041	0.027722
1995	0.001445	0.005462	-0.03022	4.567897	-0.02238	0.016766	0.225391
1996	0.000905	0.007632	-0.39303	4.001506	-0.02894	0.020945	0.093029
1997	0.000834	0.010609	-1.31089	14.67709	-0.07763	0.044308	0.060159
1998	0.000541	0.011631	-0.03979	6.738415	-0.04961	0.059741	0.030607
1999	7.95E-06	0.009705	-0.07726	3.534477	-0.03169	0.029229	-0.01774
2000	7.88E-05	0.011326	0.022287	4.752486	-0.03901	0.043858	-0.01302
2001	-3.22E-05	0.011197	-0.85335	6.839333	-0.05873	0.034056	-0.01478
2002	-0.00033	0.013608	-0.18099	7.210494	-0.07284	0.057371	-0.02889
2003	0.000777	0.008717	-0.05065	3.29291	-0.0297	0.024015	0.084635
2004	0.000493	0.007321	0.033518	3.471589	-0.02345	0.028744	0.060083
2005	6.98E-06	0.006164	0.012946	3.146384	-0.01926	0.018213	-0.01869
2006	0.000772	0.005555	-0.12391	4.116261	-0.02262	0.015799	0.105872
2007	0.000584	0.007109	-0.48884	4.890561	-0.03113	0.024336	0.058257
2008	-0.00047	0.017564	1.245472	13.34267	-0.07195	0.114515	-0.02958
2009	0.000463	0.010085	-0.26895	4.998668	-0.03436	0.042403	0.045337
2010	0.000343	0.006821	-0.40597	5.685458	-0.02837	0.021945	0.049491
2011	0.000565	0.008544	-0.04984	5.415266	-0.03229	0.036863	0.065851
2012	0.000243	0.00584	0.088378	4.388527	-0.01962	0.020816	0.041024
2013	0.000907	0.006304	-0.36081	4.482524	-0.02414	0.019187	0.143541
2014	0.000289	0.005938	-0.15636	5.034857	-0.02078	0.023521	0.048468
2015	0.000139	0.008686	-0.04458	4.992062	-0.03768	0.034782	0.015803
2016	0.000346	0.006529	-0.51614	4.234909	-0.02528	0.017498	0.051068
2017	0.000841	0.003636	0.097299	3.460317	-0.00922	0.011211	0.221364

TABLE 4.10: Minimum expected short-fall portfolio at 5%

	Mean	Std	Skewness	Kurtosis	Min	Max	Sharpe ratio
1989	0.001438	0.008138	-1.38078	11.65204	-0.05547	0.021809	0.137503
1990	0.000173	0.009155	-0.4381	4.134118	-0.03348	0.028033	-0.01304
1991	0.000652	0.008071	0.235396	4.835655	-0.02984	0.035004	0.054886
1992	0.000303	0.006678	0.244296	3.648149	-0.01968	0.021586	0.025421
1993	0.00061	0.005654	-0.53984	6.152237	-0.03055	0.017813	0.087337
1994	0.000365	0.00683	-0.2684	4.351124	-0.02472	0.020223	0.029436
1995	0.00144	0.005409	-0.08833	4.691785	-0.02265	0.016742	0.226786
1996	0.000971	0.007674	-0.39774	4.017545	-0.02827	0.02157	0.101128
1997	0.000833	0.010869	-1.22376	14.12042	-0.07837	0.046868	0.058602
1998	0.000641	0.011684	-0.09019	7.081531	-0.05256	0.059725	0.038995
1999	2.38E-05	0.009608	-0.10723	3.265019	-0.03	0.026921	-0.01627
2000	-5.71E-05	0.01159	-0.16155	5.20583	-0.04634	0.044461	-0.02446
2001	-0.00011	0.011329	-0.9747	7.534453	-0.06267	0.034276	-0.02167
2002	-0.00034	0.013511	-0.22741	7.471314	-0.0747	0.055399	-0.02995
2003	0.000732	0.0087	-0.11186	3.449642	-0.02971	0.024463	0.079617
2004	0.000493	0.007449	0.053947	3.50366	-0.02371	0.029976	0.059072
2005	4.31E-05	0.006324	0.087597	3.046669	-0.01706	0.019461	-0.0125
2006	0.00079	0.005547	-0.13783	4.116264	-0.02278	0.015533	0.109111
2007	0.000567	0.00731	-0.53577	4.942026	-0.03269	0.024207	0.054332
2008	-0.00045	0.017508	1.317175	13.66325	-0.07155	0.115776	-0.02865
2009	0.000415	0.009868	-0.25238	4.960833	-0.03369	0.041136	0.041473
2010	0.000348	0.00691	-0.40678	5.724226	-0.02923	0.02223	0.049551
2011	0.000593	0.008454	-0.09959	5.412917	-0.03203	0.035255	0.069887
2012	0.000217	0.005897	-0.00706	4.52921	-0.0207	0.020854	0.036168
2013	0.000926	0.006371	-0.35858	4.433835	-0.02395	0.019489	0.145019
2014	0.000356	0.005994	-0.18838	4.806931	-0.02142	0.023271	0.059195
2015	0.000189	0.008808	-0.10383	4.938293	-0.03859	0.034491	0.021258
2016	0.000325	0.006619	-0.52154	4.224853	-0.02599	0.017188	0.047283
2017	0.000851	0.003825	0.198646	3.71637	-0.00959	0.013694	0.213088

TABLE 4.11: Maximum Sharpe ratio portfolio

	Mean	Std	Skewness	Kurtosis	Min	Max	Sharpe ratio
1989	0.001818	0.010501	-0.71135	8.05658	-0.06214	0.037309	0.142756
1990	0.000538	0.011019	-0.35987	5.051253	-0.04758	0.039141	0.022266
1991	0.000171	0.011039	-0.29633	7.442318	-0.06156	0.049693	-0.00347
1992	0.000227	0.010278	0.075512	3.445912	-0.03336	0.032859	0.009177
1993	0.000911	0.008216	-0.10293	4.834895	-0.03448	0.031349	0.09667
1994	0.000365	0.008811	-0.15369	4.58759	-0.03629	0.029075	0.022781
1995	0.001178	0.007877	0.122538	3.181928	-0.0201	0.027505	0.122368
1996	0.001066	0.008202	-0.45004	4.391064	-0.03186	0.029586	0.106131
1997	0.00073	0.011929	-0.74264	10.68421	-0.07685	0.057406	0.044725
1998	0.001721	0.017491	-0.44643	6.989968	-0.09669	0.065988	0.087772
1999	0.001319	0.016032	0.136196	3.649571	-0.06108	0.055783	0.071069
2000	-0.00042	0.018082	0.229636	4.083627	-0.06559	0.058136	-0.03599
2001	-0.00045	0.013481	-0.33528	3.873389	-0.04367	0.038693	-0.04293
2002	-0.00051	0.017811	0.331332	4.300238	-0.06847	0.058847	-0.0322
2003	-0.00019	0.015069	-1.62563	15.92783	-0.10883	0.052897	-0.01505
2004	0.00034	0.009407	0.093406	2.892027	-0.02337	0.027105	0.030481
2005	0.000222	0.007923	0.058512	3.270018	-0.02248	0.024127	0.012614
2006	0.001016	0.008645	0.328708	3.698687	-0.0222	0.031981	0.096143
2007	0.00072	0.010739	-0.52589	4.198147	-0.04162	0.027791	0.051187
2008	-0.0008	0.020105	1.033402	9.216614	-0.0672	0.10895	-0.04246
2009	-5.06E-06	0.013788	-0.2683	6.672197	-0.0749	0.046101	-0.00078
2010	0.000503	0.009617	-0.03128	4.744663	-0.03602	0.032001	0.051728
2011	0.00061	0.01339	-0.21233	5.582583	-0.05879	0.050228	0.045412
2012	-0.00018	0.007435	-0.89563	6.098045	-0.03742	0.018759	-0.02514
2013	0.001125	0.007984	0.009986	4.089605	-0.02313	0.029383	0.140609
2014	0.000581	0.007985	-0.3208	4.124295	-0.02616	0.024259	0.072651
2015	0.000409	0.010158	-0.11168	4.159457	-0.0358	0.039065	0.040056
2016	2.60E-05	0.008417	-0.59413	5.375658	-0.04075	0.022632	0.001642
2017	0.000949	0.00528	0.716012	4.731462	-0.01159	0.02495	0.172864

TABLE 4.12: Kuosmanen SSD optimal portfolio

	Mean	Std	Skewness	Kurtosis	Min	Max	Sharpe ratio
1989	0.001862	0.011452	-0.64647	8.756942	-0.06961	0.043492	0.13481
1990	0.000397	0.012377	-0.11988	5.12699	-0.04734	0.045588	0.008441
1991	-2.55E-06	0.010909	-0.45109	7.783328	-0.06336	0.047126	-0.0194
1992	0.000403	0.007425	-0.00301	3.348334	-0.02121	0.020336	0.036337
1993	0.000893	0.006942	-0.43385	4.933982	-0.02891	0.021299	0.111903
1994	0.00037	0.008643	-0.17274	4.665633	-0.03577	0.029247	0.023751
1995	0.00152	0.007523	0.002307	2.806767	-0.02013	0.021229	0.173583
1996	0.001061	0.008998	-0.39522	4.639912	-0.03642	0.032247	0.09612
1997	0.000755	0.011977	-0.81804	12.01083	-0.08024	0.059024	0.046676
1998	0.00148	0.015618	-0.52839	8.197014	-0.09088	0.05942	0.082891
1999	0.00086	0.01376	-0.05674	3.87295	-0.05772	0.045796	0.049411
2000	-6.62E-05	0.016921	-0.21949	5.641835	-0.07652	0.058222	-0.01729
2001	-0.00053	0.011917	-0.45325	4.319348	-0.04583	0.035649	-0.05575
2002	-0.00059	0.017406	0.464427	3.856689	-0.04514	0.055466	-0.03769
2003	-0.00017	0.015032	-1.54921	15.26121	-0.1071	0.051675	-0.01416
2004	0.000373	0.009212	0.06394	3.314159	-0.02923	0.030151	0.03473
2005	0.000194	0.008562	0.057305	3.459732	-0.02576	0.029142	0.008354
2006	0.000986	0.007967	0.15784	3.837622	-0.02486	0.029532	0.100561
2007	0.000756	0.010535	-0.5536	4.329241	-0.04149	0.028293	0.055661
2008	-0.00097	0.022212	0.44498	6.782125	-0.08352	0.105175	-0.04589
2009	0.000168	0.014106	-0.02249	6.383343	-0.07169	0.053344	0.011484
2010	0.000487	0.010272	0.000561	4.733601	-0.03666	0.034724	0.0469
2011	0.000596	0.012788	-0.26352	5.370326	-0.05388	0.045709	0.046425
2012	-0.00033	0.008647	-0.98309	6.505551	-0.04458	0.026807	-0.03913
2013	0.001041	0.007646	0.017927	3.953942	-0.02121	0.028391	0.135792
2014	0.000558	0.007554	-0.27406	4.046854	-0.0253	0.023132	0.07372
2015	0.000456	0.010633	-0.02739	4.009917	-0.03351	0.041116	0.042671
2016	-7.05E-05	0.008868	-0.42443	4.862059	-0.03858	0.023094	-0.00933
2017	0.001005	0.005467	0.768567	5.541872	-0.01654	0.02729	0.177252

Chapter 5

Corporate name changes of M&As among S&P 500 index

What is a name? It is exactly the label that summarizes the physical attributes, past behavior, and other characteristics of the carrier of that name. In our language-based society this is our way of representing a large amount of information in a word or two. We label anything we can perceive or recognize with a unique name in order to distinguish it from everything else in our world. This is also true for firms: once a firm is established, it is recognized by its name, which is uniquely associated with its characteristics and past performance.

——Tadelis (1999)

5.1 Introduction

Mergers and acquisitions (M&A) change the landscape of business world. For the year of 2015, the Institute for Mergers, Acquisitions and Alliances (IMAA) reports 12,894 M&A transactions in the U.S. with a total value of $2.4 trillion, equivalent to about 10% of the national total market capitalization of $25 trillion. The M&A market is so important that Jensen (1988) lauds its benefits "in helping the American economy adjust to major changes in competition and regulation." Especially, M&As among large companies impact industry competition and market dynamics, entail fundamental adjustments for market portfolio, which is usually not the case for M&As among small companies or between small and medium companies. A vivid example is the historic formation of United States Steel by the merger of Carnegie Steel and Federal Steel, which recast not only the steel industry but also the national economy by its market dominance. Consequently, the market portfolio adjustments upon the M&As among large companies is more substantial and pivotal than any other M&As across sizes by involving higher composition weights and greater composition adjustments. These systematic M&As are best confined within the S&P 500 index, the most widely recognized indicator of U.S. large-cap equities. It is also an ideal proxy for the market portfolio since its 500 members of leading companies have broad industry coverage and capture about 80% of the total market capitalization.

Among these systematic M&As by the S&P 500 index members, we focus on the most noteworthy aspect of corporate reorientation: name change. Corporate name most directly impresses investors and stakeholders and is extremely important especially after substantial business consolidation. Normally, the combined company is still under the acquirer's name, an indication of its predominance in ownership and control. However, the combined company sometimes adopts the target's name, or a name combination, or a new name. This is more of interest for the integration of two S&P 500 companies, since both of them are blue chips and their corporate names crystalize the most important asset, corporate reputation.[1] A famous name change M&A deal is the case of SBC and AT&T: after acquiring AT&T in 2005, SBC adopted AT&T's name, brand, and trading ticker. In fact, SBC had already been a leading telecommunications brand and joined the Dow Jones Industrial Average as a prominent player before the merger, but its name change plainly demonstrated its ambition to expand business nationwide and its endeavor to revive AT&T's historical magnificence in championing inspiration, innovation, and domination. Although "rose by any other name is the same sweet, " a brand of AT&T rather than SBC after integration still differently excites the investors' and stockholders' belief on its reputation, business strategy, enterprising management and prosperous outlook. Similarly, any name change among systematic M&As at the cost of the acquirer's current name is a valuable signal of future return distribution, expecting a significant change of the return, risk, and correlation profile. Therefore, we study the name change effect on M&As among the S&P 500 index members.

It is always a primary challenge to isolate the interested event from other confounding information to avoid mutual contamination in empirical studies. In the context of M&As involving S&P 500 companies, there are several value drivers upon the information revelation, with M&A announcement effect and S&P 500 index change effect the most relevant ones. Along the strand of M&A literature, Jensen and Ruback (1983) and Betton, Eckbo, and Thorburn (2008) offer representative reviews on previous M&A studies. They summarize that corporate takeovers benefit target firm shareholders and create value for the target and bidder shareholders combined. Along the strand of S&P 500 index change literature, Lynch and Mendenhall (1997) and Patel and Welch (2017), for example, typically report significant positive abnormal return at index change announcement for addition and negative abnormal return for deletion. Besides, price pressure effect is a rising concern around M&As, as shown by Mitchell, Pulvino, and Stafford (2004) and Edmans, Goldstein, and Jiang (2012), for instance. Especially, Mitchell, Pulvino, and Stafford (2004) relate price pressure to mergers and stock index rebalancing, suggesting that the acquirer's return on M&A announcement is biased downward by merger arbitrage and indexing stimulates acquirer's positive merger closing return. The strong price pressure effect from S&P index trading is challenging evidence to previous M&A studies which hardly distinguish systematic deals involving S&P 500 companies from others. This indiscriminate treatment is analogous to nullifying the influence of S&P 500 index membership on stock return patterns, against numerous S&P 500 index change studies such as Beneish and Whaley (2002), Chen, Noronha, and Singal (2004) , Chen, Noronha, and Singal (2006) and Chan, Kot, and Tang (2013).

To attenuate the aforementioned concerns of confounding information, we choose the S&P 500 index change announcements associated with systematic M&As as event group to study name change effect. As name change information is a part of M&A information, our sample choice is innovative and different from any previous M&A studies with the following key advantages. First, it is free from the S&P 500 index change effect, as both the acquirer and the target are S&P 500 components and the combined company remains an index member, providing no new information about the membership change. Second, when Standard and Poor's announces index changes, M&A deals have already been proposed and approved. Thus, index

[1]Besides academic papers and books, an interesting article on the prominent of corporate name and brand is"Your Brand Reputational Value Is Irreplaceable. Protect It!" by *Forbes* in February 2010. It reminds that reputation generally represents 75% of the corporate value.

announcements provide scarcely new information on the M&A deals since related information has already been public. Third, it is robust to price pressure influences. As S&P 500 index change announcement generally follows or coincides with the M&A consummation, it is irrelevant to the impact of price pressure on takeover attempt or takeover probability highlighted by Edmans, Goldstein, and Jiang (2012). It is also irrelevant to price pressure effect from index trading since this systematic sample is non index rebalancing mergers, which is confirmed by Mitchell, Pulvino, and Stafford (2004). The price pressure effect from merger arbitrage is relevant but not a serious concern. As shown by Mitchell, Pulvino, and Stafford (2004), this effect is significant on fixed-exchange-ratio stock mergers around deal closing, but not on any other merger types such as cash, floating-exchange-ratio and collar stock mergers. This effect is even economically and statistically insignificant over the entire event window. Since the short position is automatically closed without additional trading in fixed-exchange-ratio stock mergers, and in our analysis we take group average then compare the return difference between name change group and non name change group, we have confidence that the impact of merger arbitrage on our sample is marginal and unsystematic. We are somehow reassured by the fact that the assets indexed to the S&P 500 index is 130 times larger than the merger arbitrage funds in terms of asset under management, because at least our sample is not affected by the major source of price pressure effects.[2]

We argue that name change contains valuable information, a signal from acquirer management for favorable belief update. However, name change is additional M&A information unique to name change systematic M&A deals but not common to all. The common M&A information categories, such as involved companies, associated industries, deal amount, and payment structure, are more myopic and immediate. If the investor cannot process all M&A information at once, we expect investors underreact to name change information as more myopic and immediate M&A information overwhelms an investor's attention, due to investor inattention problem. As a consequence of deal materialization, S&P 500 index change announcement inspirits information rumination about name change, triggered by investor's direct interest in the combined company's trading ticker, the paramount stock identity in trading activities.

From the perspective of information efficiency, if investors have fully assimilated name change information at M&A announcement, by the efficient market hypothesis we should not expect a significant abnormal return difference between name change group and non name change group at the index change announcement. If this difference is significant, the most likely reason is the name change effect. As name change is information surplus for name change group of systematic M&As over non name change group, it is reasonable to assume it as main driver of significant difference between the two groups. To a great extent, the existence of name change effect is an examination of investor behavior and a test of market efficiency.

Existing literature support the concatenation of name change and investor inattention underlying this logic of name change effect on our systematic M&As sample. Empirical name change studies endorse the positive effect of name change on stock return performance. For example, Cooper, Dimitrov, and Rau (2001) and Cooper, Khorana, Osobov, Patel, and Rau (2005) document that name change consonant with

[2]Values for assets are by the end of 2016. S&P Dow Jones Indices surveyed that $8.7 trillion in assets is benchmarked to the S&P 500 index while BarclayHedge documented the size of $66.1 billion on merger arbitrage. Even in consideration of leverage, the size of merger arbitrage funds is still small relative to that of S&P 500 index funds.

Internet industry cycle, i.e., adding Internet elements into corporate name during Internet boom and removing them during Internet bust, leads to remarkable positive abnormal return. Moreover, Cooper, Gulen, and Rau (2005) add evidence that mutual funds changing name to capture hot investment style also experience a notable cumulative abnormal inflow in the following year, albeit no performance improvement is observed. Green and Jame (2013) find corporate name changes generally improve name fluency and increase firm value. These positive findings echo the insight of Tadelis (1999) on the link between name change and belief update. Name change is an attempt from management to favorably update investors' and stakeholders' belief about the adaption of business strategy, the willingness of management improvement and the outlook for company prosperity, conveying valuable information of future return distribution. In perfect efficient market, the information of name change will be soon assimilated into stock price. However, the information efficiency is challenged by a number of studies inspired by the intuition of Kahneman (1973) that attentional resources are limited. Hirshleifer (2001) surveys investor's focus on subsets of available information forced by limited attention. Huberman and Regev (2001) find past good news amplified by mass media can catch public attention and still induce a significant price rise. Corwin and Coughenour (2008) show that the limited attention of individual NYSE specialists significantly impacts liquidity provision when they trade securities. Hirshleifer and Teoh (2003), Cohen and Frazzini (2008), DellaVigna and Pollet (2009), Hirshleifer, Lim, and Teoh (2009), and Lou (2014) add evidence that investors cannot process all information instantly and only react to part of the information. Particularly, Peng and Xiong (2006) show that investors are inattentive to firm-specific information and explicitly link the name change effect with this limited attention.

The studies of investor inattention also give a hint on information rumination. The classic cancer example of Huberman and Regev (2001) further indicates that investors ruminate on value information which is not fully discounted into stock price, once the value information recoups investors' attention. Hou and Moskowitz (2005), DellaVigna and Pollet (2007), DellaVigna and Pollet (2009), Hirshleifer, Lim, and Teoh (2009), Tetlock (2011), Hirshleifer, Lim, and Teoh (2011) also register a delay of price adjustment to information or an effect of information rumination. Unlike Huberman and Regev (2001), Engelberg and Parsons (2011) or other media induced information rumination, name change effect is independent of information amplification. Index change announcement kindles investors' direct interest in the combined company's trading ticker, an epitome of name change information. Trading ticker is uniquely assigned to a stock to facilitate trading and is the stock identity universally acknowledged by traders and investors. It is so consequential for investment choice that Rashes (2001) even documents a comovement of two different firms with similar ticker symbols. Any change of trading ticker calls for special attention. For example, the ticker symbol of SBC changed from "SB" to the classic "T" used by AT&T after its acquisition of AT&T to form the new AT&T in 2005, and the ticker symbol of Dow Chemical changed from "DOW" to "DWDP" for the combined DowDuPont after its acquisition of du Pont in 2017. As we can see, name change systematic M&A deals often lead to a trading identity modification upon consummation, nudging investors to fully recognize the stale information of name change, which results in the rumination effect on name change.

We find significant evidence for the existence of name change effect in our sample of systematic M&As among S&P 500 companies since 1976 to 2017. The sample size is 329, among which 266 events entail no name change and 63 entail name change. For each event, we primarily collect the deal summary information from Eikon M&A

database and relevant stock return series from Datastream in its best availability. We adopt event study methodology for the examination of name change effect upon index change announcement, and test abnormal return both in short term with an event window of 1 week (5 trading days) and in long term of 1 year (250 trading days). In the short term, the abnormal return for name change acquirer group is statistically and economically significant at an annualized rate of about 60%, much higher than that of non name change acquirer group. In the long term, the buy and hold abnormal return for the name change group is 10% over the non name change group, significant even after adjustment for skewness bias.

Our paper contributes in several aspects. First, it properly distinguishes S&P 500 mergers from the others for their significant effect on market portfolio adjustments, pioneering a discriminatory treatment in sample construction. This differentiation fully acknowledges the influence of S&P 500 membership on stock return patterns and eliminate price pressure concern by index trading emphasized in Mitchell, Pulvino, and Stafford (2004). Second, it provides new perspective for both M&A studies and name change studies, since the existing literature hardly research on the conjunction. By choosing index change announcement as event date, we filter much of M&A information and focus on the information surplus of name change. This context is also different from corporate environments without control shifts and substantial restructuring where prior name change studies are situated. Third, it adds new evidence to investor inattention and market efficiency literature. The significant name change effect echoes with the insight of Kahneman (1973) and associated findings in finance. It reveals how investors actually react on the name change information, implying that the market is not mechanically information efficient at name change and leaves a niche.

The remainder of the paper proceeds as follows. Section 5.2 describes the data and methodology. Section 5.3 examines general characteristics and descriptive statistics of systematic M&As. Section 5.4 explores name change effect in the short term. Section 5.5 studies name change effect in the long term. Section 5.6 interprets these results and relates them to prior studies of M&A announcement effect and S&P 500 index change effects. Section 5.7 concludes.

5.2 Data and methodology

We specify all index component changes for the S&P 500 index from 1979 to 2017. Before 2000, the component changes are obtained from Barberis, Shleifer, and Wurgler (2005) with confirming checks from available Standard and Poor's press releases. Since 2000 on, we primarily rely on Standard and Poor's announcements of changes to U.S. indices to ascertain component additions and deletions.

To double-check these index changes, we perform external and internal corroboration for above sources, and make a few corrections. The external corroboration refers to other media reports or information sources, like archives of *The Wall Street Journal*, *Washington Post*, *New York Times*, *Los Angeles Times*, *Chicago Tribune* and SEC EDGAR documents, as well as the Eikon M&A database and Datastream, on relevant M&A activities. For instance, the three deletions of ASA, Ideal Basic and Warner Communications from the S&P 500 index in August 1989 is characterized as due to "merger or takeover" in LexisNexis. However, this characterization is probably mistaken in the ASA case. No pertinent M&A information can be found from any of the above archive; NYSE documents common stock removals for the latter two as a consequence of M&A other than for ASA; there are no M&A announcement records for ASA from Jan 1, 1986 to Sep 1, 1989 in Eikon; the ASA price series in Datastream has been continuously registered with the code of "ASA"; and the company changes name from ASA to ASA Gold and Precious Metals on a much later date of Mar 28, 2011. Therefore we reason that the deletion of ASA cannot be ascribed to M&A activities. The internal corroboration is a check of component change flows, based on the fact that a component's deletion should be preconditioned by its membership and double membership should be precluded upon an addition. For example, the S&P's explains the deletion of General Instrument from the S&P 500 index in January 2000 as due to its acquisition by S&P (MidCap) 400 component Motorola. But Motorola has never been deleted from the S&P 500 index since the index inception. The absence of a previous index member replacement for Motorola staples that S&P misclassifies Motorola here.

We also standardize the series of index announcement date and change effective date. The announcement date and effective date are the same until October 1989 when S&P changed to preannounce index changes. Henceforth, the two dates are different with a lag of several trading days in between. The announcement date is easily verifiable as the date of associated press release but not more accurate in hours for the entire sample period. We primarily presume the announcing hour to be prior to the trading close, but it is still possible to be after close. We complete date information in the part of Barberis, Shleifer, and Wurgler (2005) in case of missing, correct dates if relevant information is found to be public earlier, and concatenate it with S&P's press releases information in the same format. The effective date in the change summary for S&P's press releases can be misleading since S&P's can use one of the phrases like "(replacement) after the close of trading on", "effective at the open of trading on" or "effective prior to the open on" for a date. Therefore we make it consistent to be after the close of trading for index change effectiveness across the period.

We further impose restrictions on the M&A activities in our sample construction:

1. Strictly among S&P 500 index components except Case 3 detailed below. There are several M&A events involving a company outside of the index. For instance, Placer Development, a non S&P 500 company, merges with two S&P 500 components Dome Mines and Campbell Red Lake Mines in 1987. The combined company, Placer Dome, takes up one position in the index after adjustment. Although in this case the index membership adjustment takes on the form of "two for one" and Dome Mines and Campbell Red Lake Mines shareholders own about 55% of Placer Dome, Placer Development is the largest single stakeholder and is more often classified as the acquirer and an index joiner. We omit such cases to keep our sample deals totally free from S&P 500 index addition effect.
2. The acquirer should be a unique component, rather than a team of components, to facilitate the analysis of a direct M&A effect and to keep consistent with most other takeover activities in our sample. For example, CSX and Norfolk Southern, two S&P 500 index components, team up to acquire Conrail, another component, in 1997. This kind of acquisition by parts mingles market perception of M&A effects, so we exclude such kind of events.
3. We allow the acquirer to be a company that is not included in the S&P 500 index but is a subsidiary fully controlled by its parent S&P 500 company. It's a common practice in a reverse merger that the ultimate acquirer creates a

wholly-owned subsidiary, the so called "merger sub", to merge it with the target immediately. Essentially, we regard this involvement as an M&A arrangement between the ultimate acquirer and the target. As a M&A example among S&P 500 companies, Intel acquired McAfee via its wholly-owned subsidiary Jefferson Acquisition Corporation (JAC) in February 2011. Though JAC is the immediate acquirer, we readily consider the acquisition as from JAC's parent, Intel. However, we exclude cases where the acquirer is not fully controlled by its S&P 500 parent. For instance, Laidlaw Environmental Services, majority-owned by S&P 500 component Laidlaw, acquires another index component Satety-Kleen Services in 1998. We don't consider this takeover directly to be from Laidlaw since Laidlaw Environmental Services is not a merger sub and has its own stock listed.

4. In contrast to Case 3, the target must be an S&P 500 index component itself or its core business, not its peripheral subsidiary fully controlled or not. The sale of core business is a vital corporate change and amounts to the sale of itself to a great extent. For example, in June 2017, a subsidiary of Verizon acquired the paramount operational business of Yahoo, which transforms the reminder into Altaba as a closed end fund. Since the brand of Yahoo continues as a subsidiary of Verizon, here we regard this acquisition as a M&A between two S&P 500 companies. Note that there is an asymmetrical structure associated with M&A deals by subsidiary. Acquired by a fully controlled subsidiary immediately, the target component is set to be a part of the subsidiary's parent component. Nonetheless, acquiring a subsidiary of the target component does not indicate in any sense the takeover of its parent company. For instance, Comcast acquires the newly spun-off AT&T Broadband from AT&T in 2002, but this takeover doesn't affect the membership of AT&T in the index at all. Therefore we omit any case like this one.
5. The company combining two components must also be a component after to be free from the deletion effect. Such events are quite rare. In November 2008, one S&P 500 constituent, Hercules, was acquired by another component, Ashland. This led to a low market capitalization and the deletion of Ashland from the S&P 500 index. We ignore this situation in our sample assembly.

Within all S&P 500 component changes, we collect 329 events due to systematic M&As during the sample period. Among them, 266 events entail no name change where the combined company still follows the acquirer's name and 63 entail a name change in the form of either a name combination (36), or a target name (18), or a new name (9). What we mean by name change is a direct name change, immediately pertinent to the systematic M&A activities rather than the corporate decision post this integration. For example, Westinghouse acquired CBS in 1995 but changed its name to CBS two years later. We do not regard this kind of name change for its irrelevant timing to systematic M&As.

We identify most of the related components and systematic M&A events in both Datastream and Eikon databases. Because our sample spreads over about four decades and may involve companies that have been delisted for quite a long time, some events are missing in the databases. Indeed, out of the total 329 systematic M&A deals, 3 events are unavailable for their summary tearsheets in the Eikon M&A

database. We make summary tearsheets for them manually, collecting key information such as deal amount, payment structure, and so on, from external information sources.[3] For the industry classification information, we refer to the M&A records of the same company adjacent to the original event time in Eikon to keep consistency. We also check this industry classification by searching the company's business and operating income description at that time from aforementioned information archives. External corroboration also performs on M&A information beyond industry classification, rendering a few corrections for the tearsheets. For example, Eikon registers the effective date for Southern Railway's acquisition of Norfolk and Western Railway as Mar 25, 1982. However, we detect a report from *New York Times* archive about rail consolidation in April 28, 1982, stating that the consolidation would be effective on June 1. Therefore we choose June 1, 1982, as the effective date instead of the Eikon record. In terms of return series, 24 events are unavailable in Datastream because the data for the pairs of acquirer and target are not complete, including 3 name change events. We drop these events in the time series analysis. We use total return data for both stocks and the S&P 500 index, since closing price or price index data cannot capture the impact of dividend reinvestment.

We follow event study methodology to examine the name change effect upon index change announcements of systematic M&As. For a better assessment, we conduct abnormal return tests in both the short term with an event window of 1 week (5 trading days) and the long term with an event window of 1 year (250 trading days). We choose 1 year as the length of long term window to keep most acquirers in our sample available in the long term analysis. If we extend this window to, say, 3 years, it is highly probable that an acquirer gets delisted from the S&P 500 index over this time period due to various reasons, like acquisition by another company. In this case, the associated short-run negative reaction and long-run reversion may bias the estimation of name change effect. A 3-year window also means the sample period for long term abnormal return analysis has to be truncated at 2014, reducing the sample size in use.

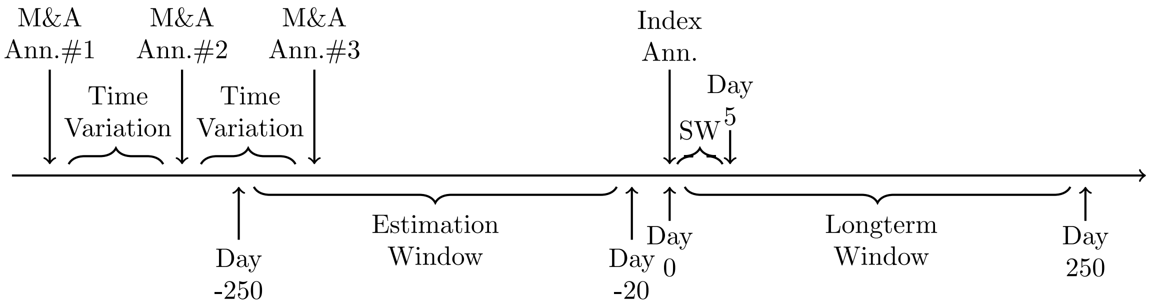
Figure 5.1 illustrates the timeline of M&A announcement, index change announcement, and the construction of estimation window and event window. We set index change announcement date as Day 0. The time lag between M&A announcement and associated index change announcement is case specific. It can be longer than a year, or much shorter. For short term analysis, the year before index announcement is set to be the estimation window for market model parameters. In case of information leakage or contamination, we truncate the month immediately prior to the index announcement. Therefore, the estimation window includes all trading days from Day -20 back to Day -250. Correspondingly, the short term event window is from Day 0 to Day 5. Note that we cannot ascertain if the index change is announced before the close of trading or not especially for early events, so the announcement information can be impounded into return in Day 0 or Day 1. To fully capture the effect, our short term event window is consequently a little bit longer than the common 3-day event window in previous studies. The long term event window is a year after the index announcement, including trading days from Day 0 to Day 250.

The abnormal return of stock i at time t, $AR_{i,t}$, is the difference between the actual return $R_{i,t}$ and its reference, the expected return $E[R_{i,t}]$. See, for example, Kothari

[3]Indeed, there were 39 events unavailable in Eikon M&A database at first. We made manual tearsheets for all of them. We also contacted Thomson Reuters data team for enquiries about the 39 events and shared the information we collected. Eikon M&A database later on added 36 events.

FIGURE 5.1: Timeline of announcements and event windows

This figure presents the timeline of M&A announcement, index change announcement, estimation window and event window used in the analysis. Index change announcement date is set to be Day 0. The period of previous 250 trading days except the most recent 20 trading days is estimation window for market model used in short term analysis. "SW" is short for the short term event window, which is the period of post 5 trading days is. The long term event window is from Day 0 to Day 250. M&A announcement date can be before or after Day -250.



and Warner (2008) and Patel and Welch (2017). That is

$$AR_{i,t} = R_{i,t} - E[R_{i,t}]. \tag{5.1}$$

The cumulative abnormal return $CAR_{i,[t_1,t_2]}$ is the sum of abnormal returns over the time period of $[t_1, t_2]$ for stock i:

$$CAR_{i,[t_1,t_2]} = \sum_{t=t_1}^{t_2} AR_{i,t}. \tag{5.2}$$

The reported mean cumulative abnormal return for group G is

$$CAR^G_{[t_1,t_2]} = \frac{\sum_{i \in G} CAR_{i,[t_1,t_2]}}{n}, \tag{5.3}$$

where n is the number of stocks falling into the group G by classification. The associated statistical significance is by the standard t test **p**[for a recent summary of event study methods, see][]patel2017extended.

In the short term analysis, we use market model and market return model to specify the expected return $E[R_t]$ in the detection of abnormal return. In the long term analysis, we use buy and hold abnormal return method to specify $E[R_t]$ as the return on the matching portfolio to test the significance of name change abnormal return.

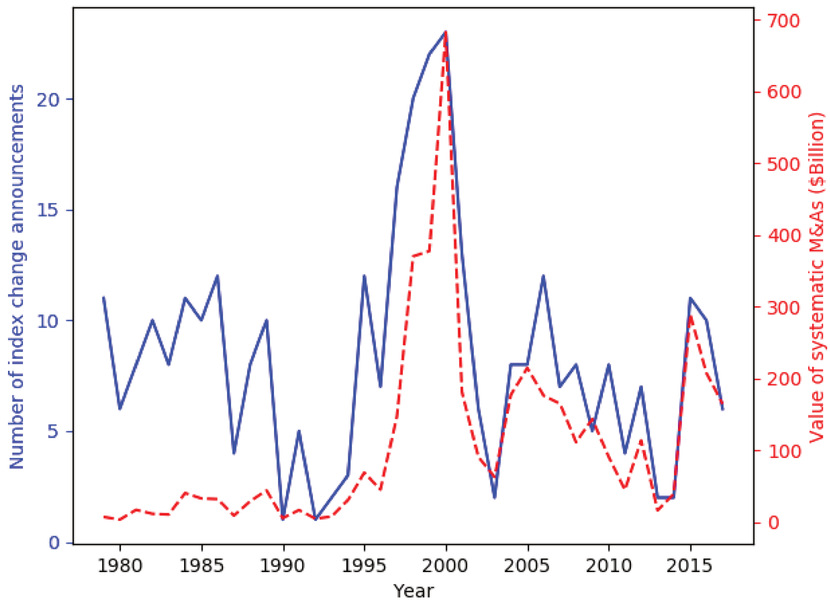
5.3 Characteristics of systematic M&As

Generally, large companies can better react to economic, technological, and regulatory shocks by M&A to adapt to the new business environment. As industry leaders, the S&P 500 companies usually pioneer aggregate M&A activities. In the peak period of the aggregate M&A activities, large companies energetically promote systematic M&As, facilitating associated index changes. In the trough period, large companies are less involved and the index composition tends to keep. In short, index changes by systematic M&As epitomize merger waves.

Figure 5.2 shows the number of S&P 500 index change announcements due to systematic M&As. The time series is volatile, with the acme of 23 events in 2000 and the nadirs of 1 event in 1990 and 1992. It is similar to prior studies with much bigger M&A samples, suggesting the general pattern for the aggregate M&A activities. For example, the trend of our index change number series is consistent with the fraction of delisting series in Betton, Eckbo, and Thorburn (2008) Figure 1 and in Eckbo (2014) Figure 1, which they define as the annual fraction of all public CRSP companies in January of each year that delists due to merger during the year. Interestingly, our index change series implicitly serve as fraction since the changes are over the fixed index size of 500 companies. Moreover, this pattern is accordant with that of total merger number series in Harford (2005) Figure 1 and Ahern and Harford (2014) Figure 2, no matter if the merger number is contingent on merger size. In fact, the pattern of systematic M&As value series in our Figure 5.2 also agrees with the merger value series in Ahern and Harford (2014) Figure 2, both reaching the maximum around 2000. Therefore, we are assured that the index change of systematic M&As is a good indicator for the aggregate merger activities.

FIGURE 5.2: Number and value of the S&P 500 index changes by systematic M&As, 1979–2017

This figure presents the number of index change announcements by systematic M&As across the period of 1979 to 2017 on the left axis. The total transaction value of systematic M&As are on the right axis with unit of nominal $billion.

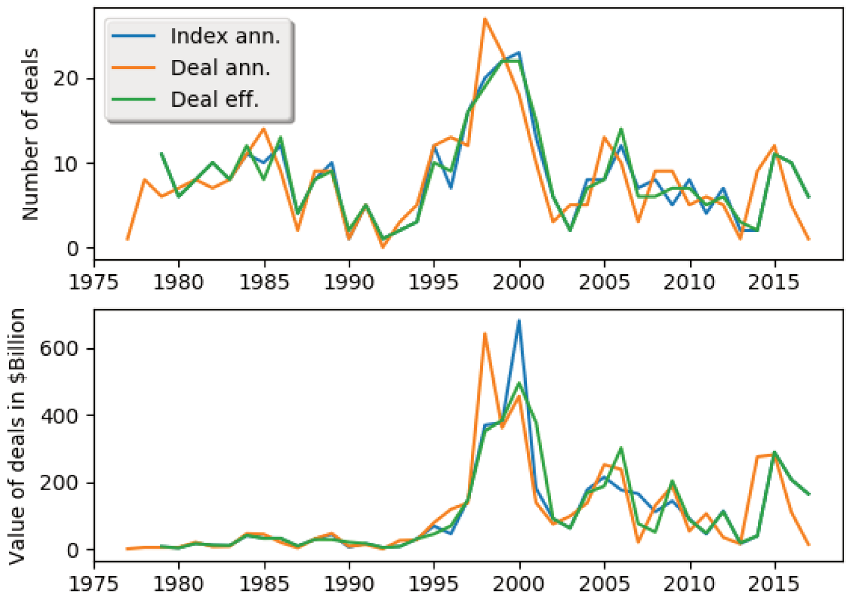


We acknowledge that the index change announcement date maybe different from the systematic M&A announcement date or effective date. Deal effective date is close to index change announcement date, since the target stock is about to be delisted upon the deal consummation. The approaching is more complicated for deal announcement date, because many issues can delay the process to deal finalization, such as regulatory check, financial deterioration, and shareholder rejection. Therefore, the time lag between systematic M&A announcement date to effective date, and also to index change announcement date, varies in months, quarters, or even years.

For a better inspection on this point, we plot time series of number and value for the systematic M&As in Figure 5.3, where they are grouped respectively by associated index change announcement year, deal announcement year, and deal effective year. We see that the number and value series are highly similar for the classification of index announcement and deal effectiveness across the whole period, and the series for deal announcement behave as leftward shifting the series for index announcement to some extent. This obvious time lag suggests that for systematic M&As, investors have considerable time to react on the deal announcement information before index change release. We examine this implication in the following sections.

FIGURE 5.3: Number and value of systematic M&As by various grouping methods, 1979–2017

This figure presents the number of systematic M&As as well as the total transaction value with unit of nominal $billion across the period of 1979 to 2017. The systematic M&As are grouped in three ways, by associated index change announcement year, by deal announcement year, and by deal effective year. Time period for the classification of deal announcement year goes further to 1977 since 9 systematic M&As are announced before our sample beginning year of 1979. Index announcement years and deal effective years are the same and from 1979.



Our systematic M&As sample covers a significant proportion of total M&A market in U.S. with a straightforward and tractable size. Figure 5.4 exhibits this advantage by four panels, where the first three are about plain comparisons in deal number and value, and the last is in terms of proportion comparisons in deal number and value. The total M&A transactions grow both in deal number and aggregate value. Specifically, there are 2,308 transactions in total of $306 billion in 1985, with a remarkable increase to 13,909 transactions in total of $1,636 billion in 2017. The number of systematic M&As are relatively stable around the average of 8 events across the whole period. However, as the dynamic criteria of systematic M&As are embedded in the S&P 500 index, the value for systematic M&As in fact increases from $33 billion in 1985 to $164 billion in 2017, a similar growth to that of total

M&A transactions. Therefore, the average value per deal for systematic M&As are increasing, which is confirmed by the third panel. The average size of systematic M&As is $3,263 million in 1985 and $27,393 million in 2017, considerably greater than the contemporaneous average size of total transactions, which is $133 million in 1985 and $118 million in 2017. Actually, it is more than 141 times bigger across the whole sample, a clear evidence of the predominance of systematic M&As over average M&As for their great economic influence. The last panel is a persuasive demonstration of the advantage of systematic M&As: this sample, only 0.1% out of the total M&A transactions, is accountable for over 10% of the total value of M&A transactions.[4] Certainly, our systematic M&As sample is not complete and exhaustive, but it obviates a cumbersome and tortuous sample coverage and still takes up a material proportion out of the total value. Figure 5.4 also suggests the imperative to distinguish the systematic sample from other deals for being "too big to ignore" or "too big for indifference."

Besides transaction sizes, industry participation is another most important aspect in assessing M&A influence. We use SIC to classify target and acquirer participants in systematic M&As. 9 industry divisions are effective in our analysis, and for the industry titles we make instructive abbreviations, which are more informative and straightforward that the official divisions by alphabet letters. For example, Division H covers the SIC code from 6000 to 6799 with the industry title of "Finance, Insurance and Real Estate." We refer to it with the abbreviation of "Fin" for the sake of convenience. More information about SIC structure and industry abbreviations are in Table 5.1.

TABLE 5.1: SIC divisions and abbreviations

Code	Division	Abbr.	Industry
0100-0999	A	Agri	Agriculture, Forestry and Fishing
1000-1499	B	Mine	Mining
1500-1799	C	Cons	Construction
2000-3999	D	Mfg	Manufacturing
4000-4999	E	Infra	Transportation, Communications, Electric, Gas and Sanitary service
5000-5199	F	Whol	Wholesale Trade
5200-5999	G	Retl	Retail Trade
6000-6799	H	Fin	Finance, Insurance and Real Estate
7000-8999	I	Svc	Services

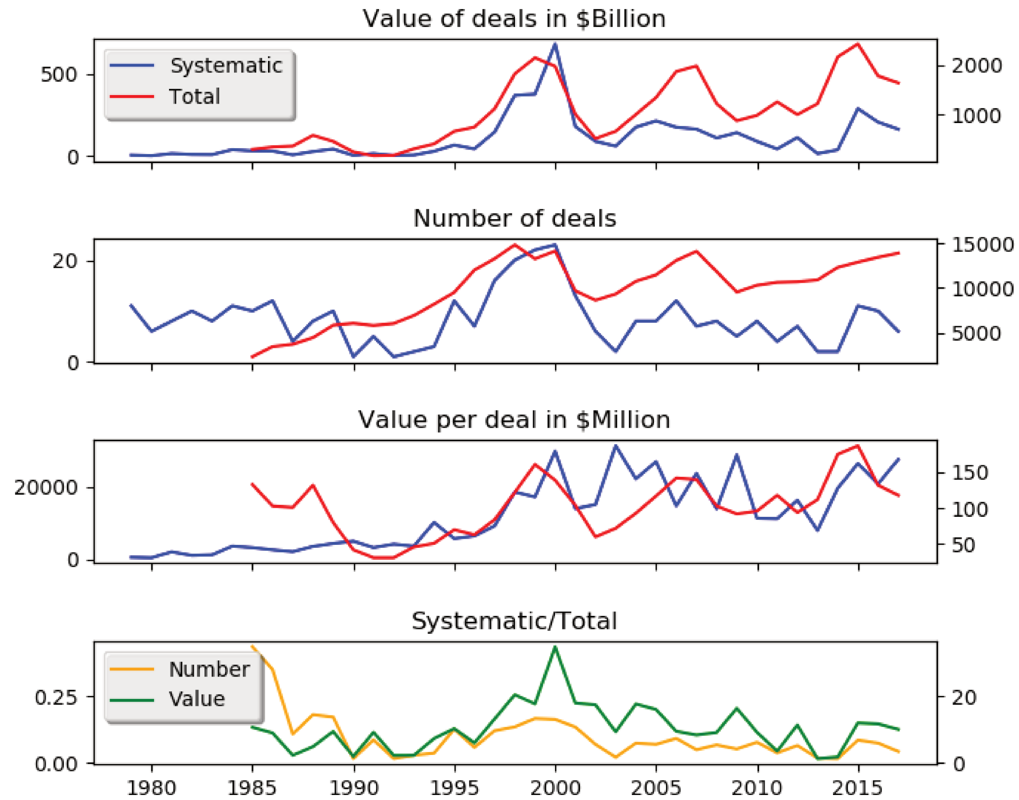
This table gives detailed information about SIC structure used in our paper. Out of the total 12 SIC divisions, we only use 9 relevant ones. The range of SIC codes of 1800-1999 is not used, the range of 9100-9729 specific to Public Administration, and the range of 9900-9999 nonclassifiable. Therefore we safely ignore these three ranges in our study. The column of "Abbr." lists the abbreviations for each industry title. More information refer to the SIC manual at the SEC EDGAR website.

For a panorama of systematic M&A industry distribution, we examine the target industry and acquirer industry separately first, and then examine the target and acquirer industries in pair. Table 5.2 shows the number of deals classified by SIC industry division for target and acquirer individually. For example, the number of 26 in the second industry row of Mining means that out of the total 329 targets, 26

[4] Here the importance of systematic sample over total transactions in value is biased downward since the IMAA data are for announced deals which do not necessarily arrive at success as systematic M&As do.

FIGURE 5.4: Systematic M&As and total M&A transactions in U.S., 1979–2017.

This figure presents the number and value of systematic M&A deals and total M&A transactions in the period of 1979 to 2017. Systematic M&As are organized by associated index change announcement year. Data for total M&A transactions are from IMAA, and the availability is from 1985 to 2017. For the first three panels, data for systematic M&As are on the left axis and data for the total transactions are on the right axis. The last panel is about the deal number proportion and transaction value proportion of systematic M&As in total transactions, expressed in percentage.



targets are from Mining industry. Similar explanation goes to the industry classification for acquirers. We see that almost 50% of the targets and acquirers are from Manufacturing, about 20% from Finance, Insurance and Real Estate, and about 15% from Transportation, Communications, Electric, Gas and Sanitary. More than that, the general pattern of industry distribution for targets is highly similar to that of acquirers, and only slightly different in the bottom three ranking industries.

This industry distribution reveals some unique characteristics of systematic M&As over the aggregate M&As. Betton, Eckbo, and Thorburn (2008) Figure 4 summarizes the number of contests by industry sectors, where Services is the industry of most targets, then Manufacturing, and Finance, Insurance and Real Estate. Their finding is complementary to Netter, Stegemoller, and Wintoki (2011) Table 7 about industry distribution of aggregate acquirers, where Services is also the leading industry of acquirers. Both are in sharp contrast to our finding of the paramount presence by Manufacturing for being targets and acquirers. Harford (2005) Table 3 lists deregulatory events stimulating merger wave but none is about Manufacturing, which contributes half of all the systematic targets and acquirers. These industry distribution inconsistencies of systematic M&As with aggregate M&As once again highlight the necessity of separate treatment for the systematic M&A sample.

TABLE 5.2: M&A participant by individual, 1979–2017

Division	Target			Acquirer		
	Number	%	Rank	Number	%	Rank
Agri	2	0.6	8	2	0.6	7
Mine	26	7.9	4	27	8.2	4
Cons	1	0.3	9	1	0.3	8
Mfg	146	44.4	1	148	45.0	1
Infra	48	14.6	3	51	15.5	3
Whol	3	0.9	7	1	0.3	9
Retl	11	3.3	6	11	3.3	6
Fin	67	20.4	2	66	20.1	2
Svc	25	7.6	5	22	6.7	5
Total	329	100		329	100	

This table classifies targets and acquirers of systematic M&As from 1979 to 2017 individually by use of SIC divisions. The abbreviations for industry titles are detailed in Table 5.1. The rank columns are determined by the number of targets or acquirers falling into a certain industry from large to small.

The symmetric industry distribution pattern on target and acquirer suggests a highly probable domination of horizontal mergers, i.e., both the target and acquirer are from the same industry, as confirmed by Table 5.3. This table exhibits the number of deals classified by SIC industry division for target and acquirer in pair simultaneously. For instance, the number of 1 in the third industry row and second industry column means that there is only 1 deal with target from Construction and acquirer from Mining. More than 80% of the systematic M&As are horizontal mergers, in accumulation along the diagonal. Only 62 deals are across industry. Again, Manufacturing has the highest contribution, with 125 horizontal deals. Finance, Insurance and Real Estate has 62 horizontal deals, and Transportation, Communications, Electric, Gas and Sanitary has 43 horizontal deals. The pattern of industry distribution in pair is comparable to those independent patterns of target and acquirer. Actually, the marginal industry distributions in pair is exactly the distributions for target and acquirer, noting the Total row and the Total column of Table 5.3 in regard to Table 5.2.

The findings on industry distribution in pair reaffirm the sample uniqueness. Betton, Eckbo, and Thorburn (2008) Figure 4 also reports the percentage of horizontal deals by industry sectors, where Manufacturing is the most inactive industry and the synthesized group of Agriculture, Forestry and Fishing, Mining and Construction the most active. This is remarkably different from the fact about systematic M&As that Manufacturing is far ahead of any other industry in horizontal merger, while the above synthetized group only accounts for 7% of total horizontal mergers. This contrast does not change no matter we define horizontal deals by identical 2-digit SIC code or 4-digit SIC code. Ahern and Harford (2014) Table 4 lists most central industries as well as sub-industries for merger connection estimated from aggregate merger data, and the top connections are all across industries with the only one connection about Manufacturing ranked the 6th place. It conflicts with the superiority of Manufacturing in terms of horizontal merger and the inferiority of vertical merger in Table 5.3, echoing the stark comparison between systematic M&As and aggregate M&As inferred from Table 5.2.

TABLE 5.3: M&A participants in pair, 1979–2017

Target	Acquirer									
	Agri	Mine	Cons	Mfg	Infra	Whol	Retl	Fin	Svc	Total
Agri	2									2
Mine		17		6	2				1	26
Cons		1								1
Mfg		9		125	3		2	1	6	146
Infra		1		2	43				2	48
Whol				3						3
Retl					1		8	1	1	11
Fin				3				62	2	67
Svc				9	2	1	1	2	10	25
Total	2	28	0	148	51	1	11	66	22	329

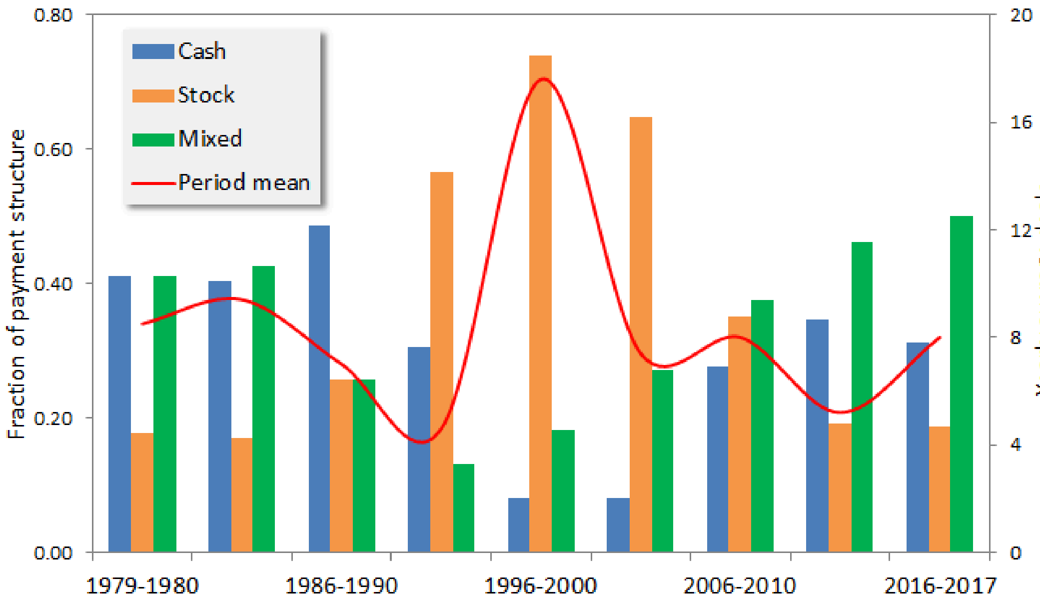
This table classifies targets and acquirers of systematic M&As from 1979 to 2017 in pair by use of SIC divisions. The abbreviations for industry titles are detailed in Table 5.1. The industry for target is in a row wise, while the industry for acquirer is in a column wise. The last row shows the total number of systematic M&As by various acquirer industries, regardless of target industry information. Similarly, the last column shows the total number of systematic M&As by various target industries, regardless of acquirer industry information.

The choice of M&A payment method reveals important information, as relevant theories and hypotheses reviewed by Betton, Eckbo, and Thorburn (2008). Figure 5.5 demonstrates the fraction of cash merger, stock merger, and mixed payment merger for our systematic sample in 9 sub-periods. Across the whole period, each method accounts for about one third of all systematic M&As, specifically cash for 30%, stock for 37%, and mixed payment for 33%. However, they have quite different shares along sub-periods. Cash payment is the choice by half of the mergers in late 1980s, and about 10% in the late 1990s, then recovering its average level at about 30% gradually. Stock payment fuels the merger waves from 1991 to 2005. Especially in late 1990s when the merger wave reaches its prime suggested by the yearly average deal number, three quarters of the deals are stock financed. Mixed payment is the most stable choice and gains dominance in the last decade. This result resembles the finding in Betton, Eckbo, and Thorburn (2008) Figure 7 based on about 16 thousand deals, and extends to most recent years not covered by them.

In short, these key aspects prove that our systematic M&As sample is of great importance and interest. The gigantic transaction size warrants a special attention to this sample, since it constitutes 0.1% of the aggregate M&As by deal number but contributes more than 10% of the aggregate M&A value. Also, its industry distribution patterns, both by individual and in pair, are materially different from patterns based on the aggregate M&A deals. Besides, the value and number growth of systematic M&As pioneers the trend of aggregate M&A dynamics. The payment methods by systematic M&As share a lot common with the aggregate M&A behaviors. Therefore, it is productive and worthwhile to specify this unique sample from the aggregate M&A activities. This sample is in a tractable size covering significant portion, and is highly receptive and responsive to market portfolio rebalancing.

FIGURE 5.5: Systematic M&As payment methods, 1979–2017

This figure presents the payment method distribution by systematic M&A deals over the period of 1979 to 2017. The columns are fraction of payment methods in sub-periods, against the left axis. The line is the yearly average number of systematic M&As over the sub-periods, against the right axis.



We classify this sample into two groups, name change group and non name change group. Name change is intuitively identified as any material change of the combined company's name from the acquirer's name, immediate upon the M&A. It is possible that the combined company takes up the target's name, or a name combination, or a new name at the consolidation. Table 5.4 gives examples on various cases on name change. For example, after the acquisition of Mobil, Exxon changes its name to ExxonMobil, a name combination with the acquirer's name preceding. After Chase Manhattan's acquisition of J.P. Morgan, it changes name to JP Morgan Chase, with the target's name coming first. Both forms are substantial changes from the single acquirer's name, hence falling into the name change group. Our group classification is robust to name change concerns such as alphabetic climb and merger of equals, detailed in Section 5.6.3.

5.4 Short-term abnormal returns

For the specification of short term abnormal returns, we first use the market model as follows,

$$E[R_{i,t}] = \alpha_i + \beta_i R_{M,t}, \tag{5.4}$$

where α and β are the regression intercept and slope respectively, and R_M is the return of the market portfolio, here the S&P 500 index. Note that the choice of S&P 500 index or CRSP value weighted stock market index as market portfolio is not consequential, as indicated by Patel and Welch (2017). The estimation window for the two parameters is exhibited in Figure 5.1 as the time period of $[-250, -20)$. We leave out recent trading days to the announcement day to make sure our estimation window is clean from the event impact.

TABLE 5.4: Examples of corporate name changes in systematic M&As

Acquirer	Target	Combined				
A	B	A	B	AB	BA	New
Pfizer	Wyeth	Pfizer				
SBC	AT&T		AT&T			
Exxon	Mobil			ExxonMobil		
Chase Manhattan	JP Morgan				JP Morgan Chase	
Standard Oil of California	Gulf Oil					Chevron

This table gives 5 examples for the corporate name change scenarios in the systematic M&A deals. The item of "Combined" refers to the combined company upon systematic M&A. The alphabetic letters of A and B, as well as the letter combinations refer to company name forms for the combined company. For example, AB means the name combination with the acquirer's name preceding.

For each event i of our sample I, the acquirer is denoted as ACQ^i and the target as TGT^i. Correspondingly, the acquirer group ACQ is the collection of all acquirers as $\{ACQ^i : \forall i \in I\}$ and the target group TGT as $\{TGT^i : \forall i \in I\}$. Further, we assign a dummy variable X for each event to structure sub-collections based on name change classification. That is to say, we have

$$X^i = \begin{cases} 1, & ACQ^i \text{ changes name} \\ 0, & \text{otherwise} \end{cases} \tag{5.5}$$

Now, the group of name change acquirers $ACQ1$ is the sub-collection of acquirers with dummy value of 1, as $\{ACQ^i : \forall X^i = 1, i \in I\}$. Similarly, we have the group of non name change acquirers $ACQ0$ as $\{ACQ^i : \forall X^i = 0, i \in I\}$. Apparently, $ACQ0$ and $ACQ1$ are the name change partition for ACQ. The same is done for the group of name change targets $TGT1$, and the group of non name change targets $TGT0$.

The return for each group and subgroup is the cross-sectional equal-weighted average return of the stocks falling into the category. For example, the abnormal return of $ACQ1$ is

$$AR_t^{ACQ1} = \overline{AR}_{i,t} \text{ , } fori \in ACQ1 \tag{5.6}$$

and the corresponding cumulative abnormal return for the group of name change acquirers is

$$CAR_{[t_1,t_2]}^{ACQ1} = \sum_{t=t_1}^{t_2} AR_t^{ACQ1} \text{ .} \tag{5.7}$$

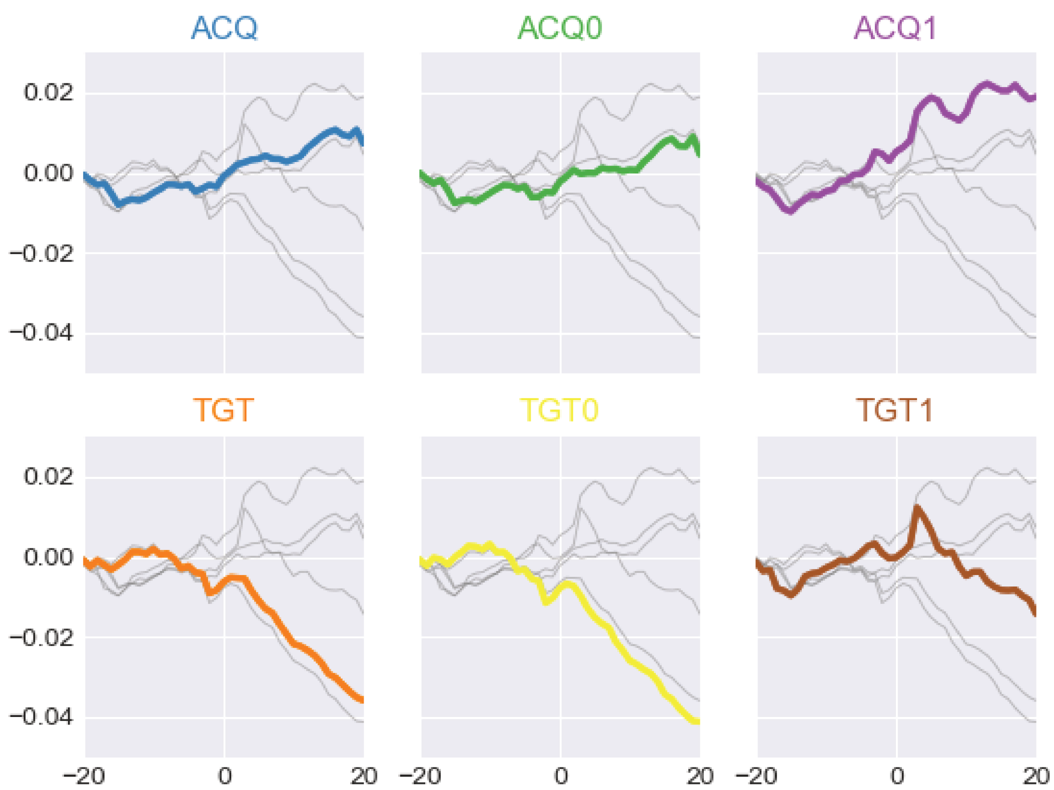
The abnormal return for other classification groups is similarly calculated.

Figure 5.6 provides a primary view of the cumulative abnormal returns in the time period of [-20, 20]. For the same event classification, i.e. vertical comparisons, acquirer groups have higher abnormal returns than their counterparts, especially after Day 0. For example, the cumulative abnormal return for the acquirer group till Day 0, $CAR_{[-20,0]}^{ACQ}$, is 0.55% higher than the cumulative abnormal return for the target group $CAR_{[-20,0]}^{TGT}$. The surplus grows over time, and $CAR_{[-20,20]}^{ACQ}$ is 4.32% higher than $CAR_{[-20,20]}^{TGT}$. The pattern is similar for the comparisons between non name change acquirers and targets, as well as between name change acquirers and targets. The average acquirer CAR surplus for the three group comparisons is 4.08% at the last day.

For the same group classification, i.e. horizontal comparisons, name change groups have higher post event abnormal returns than non name change groups and the aggregate groups. For instance, $CAR^{ACQ}_{[-20,0]}$, is 0.75% higher than $CAR^{ACQ0}_{[-20,0]}$ till the event day. This advantage gets bigger and bigger, as $CAR^{ACQ1}_{[-20,20]}$ is 1.44% higher than $CAR^{ACQ0}_{[-20,20]}$ till Day 20. Parallel pattern is found in the comparisons between name change and non name change targets, as well as between name change and aggregate targets. The average excess CAR for name change events is 1.87% at the last day.

FIGURE 5.6: Cumulative abnormal returns, [-20, 20]

This figure presents the cumulative abnormal return series for the 6 group specifications by market model. Abnormal returns are accumulated from Day -20 to Day 20, with Day 0 as the index change announcement date.



With the intuitive manifestation by Figure 5.6 in the period of [-20, 20], we further zoom in for the event widow of [0, 5]. The Panel A in Table 5.5 offers CARs for the 6 groups during the event window with the specification of market model for expected return. For the same event classification, acquirer groups still have higher abnormal returns at almost all horizons, in terms of mean cumulative abnormal return and median cumulative abnormal return. The surplus is 0.95% for $CAR^{ACQ}_{[0,5]}$ over $CAR^{TGT}_{[0,5]}$, 0.98% for $CAR^{ACQ0}_{[0,5]}$ over $CAR^{TGT0}_{[0,5]}$, and 0.90% for $CAR^{ACQ1}_{[0,5]}$ over $CAR^{TGT1}_{[0,5]}$, generally with statistical significance as suggested in Table 5.6 Panel A. Moreover, the CAR^{ACQ} and CAR^{ACQ1} are significantly positive at the end of event window for both mean and median, while CARs for the target groups are hardly

TABLE 5.5: Cumulative abnormal returns and t-statistics

Panel A: Reference return specification with market model						
Day	Acquirer Groups			Target Groups		
	ACQ	ACQ0	ACQ1	TGT	TGT0	TGT1
0	0.26%***	0.27%***	0.23%	0.20%	0.25%	0.02%
	[0.26%]***	[0.34%]***	[0.09%]	[-0.09%]	[-0.08%]	[-0.15%]
1	0.4%***	0.41%***	0.34%	0.32%*	0.36%*	0.16%
	[0.38%]***	[0.42%]***	[0.12%]	[0.07%]	[0.04%]	[0.27%]
2	0.56%***	0.57%***	0.50%	0.31%	0.30%	0.34%
	[0.47%]***	[0.53%]***	[0.21%]	[0.07%]	[-0.04%]	[0.26%]
3	0.61%***	0.46%**	1.23%**	0.29%	0.05%	1.27%**
	[0.64%]***	[0.40%]**	[1.41%]***	[-0.12%]	[-0.17%]	[0.83%]**
4	0.68%***	0.5%**	1.45%***	-0.01%	-0.26%	1.02%*
	[0.68%]***	[0.50%]**	[1.42%]***	[-0.47%]*	[-0.63%]***	[0.14%]
5	0.7%***	0.49%*	1.59%***	-0.25%	-0.49%*	0.69%
	[0.71%]***	[0.50%]**	[1.50%]***	[-0.57%]***	[-0.79%]***	[0.08%]
Panel B: Reference return specification with market return model						
Day	Acquirer Groups			Target Groups		
	ACQ	ACQ0	ACQ1	TGT	TGT0	TGT1
0	0.27%***	0.28%**	0.26%	0.27%**	0.31%**	0.12%
	[0.20%]***	[0.21%]***	[0.06%]	[0.01%]	[0.04%]	[-0.10%]
1	0.39%***	0.41%***	0.32%	0.46%***	0.51%**	0.30%
	[0.43%]***	[0.44%]***	[0.20%]	[0.20%]***	[0.23%]**	[0.20%]
2	0.56%***	0.55%***	0.58%	0.58%***	0.56%**	0.66%*
	[0.45%]***	[0.45%]***	[0.50%]	[0.42%]**	[0.42%]*	[0.28%]*
3	0.60%***	0.44%**	1.26%***	0.67%***	0.45%**	1.57%***
	[0.58%]***	[0.42%]**	[1.27%]***	[0.32%]***	[0.21%]	[1.02%]***
4	0.65%***	0.46%*	1.41%***	0.44%**	0.20%	1.4%***
	[0.51%]***	[0.21%]**	[0.96%]***	[0.13%]	[-0.15%]	[0.62%]***
5	0.69%***	0.47%*	1.58%***	0.32%	0.15%	1.01%**
	[0.78%]***	[0.53%]**	[1.48%]***	[-0.22%]	[-0.39%]	[0.91%]**

This table provides the cumulative abnormal returns for the 6 group specifications in the event window of [0, 5]. The CARs are cross-sectional equal-weighted average return, calculated using market model within the estimation window of $[-250, -20)$ in Panel A, and using market return model in Panel B. The median CARs are shown below mean CAR estimates with brackets. The corresponding t statistics are based on the cross-sectional standard deviation, and Wilcoxon signed rank test is used to test for median against 0. * denotes significance at 10% level, ** at 5%, and * at 1%.

so. For the same group classification, name change groups gain the highest abnormal returns. $CAR_{[0,5]}^{ACQ1}$ is 1.10% higher than $CAR_{[0,5]}^{ACQ0}$, and the excess return is at a similar size of 1.18% for $CAR_{[0,5]}^{TGT1}$ over $CAR_{[0,5]}^{TGT0}$, both are of statistical significance as in Table 5.6 Panel A. Compared with the aggregate groups, name change groups also have an average of 0.92% higher abnormal return. Despite smaller point estimates, the CAR^{ACQ} in general has higher statistical significance because this group involves more companies and risk is better diversified.

TABLE 5.6: Return difference between groups

Panel A: Return difference with market model					
Minuend	Subtrahend				
	ACQ	ACQ0	ACQ1	TGT	TGT0
ACQ0	-0.20% [-0.21%]				
ACQ1	0.89% [0.79%]	1.10%* [1.00%]			
TGT	-0.95%*** [-1.28%]	-0.75%** [-1.07%]	-1.84%*** [-2.07%]		
TGT0	-1.18%*** [-1.50%]	-0.98%** [-1.29%]	-2.07%*** [-2.29%]	-0.23% [-0.22%]	
TGT1	-0.00% [-0.63%]	0.20% [-0.42%]	-0.90% [-1.42%]	0.95% [0.65%]	1.18%** [0.87%]
Panel B: Return difference with market return model					
Minuend	Subtrahend				
	ACQ	ACQ0	ACQ1	TGT	TGT0
ACQ0	-0.22% [-0.25%]				
ACQ1	0.89% [0.70%]	1.11%* [0.95%]			
TGT	-0.37% [-1.00%]	-0.15% [-0.75%]	-1.26%** [-1.70%]		
TGT0	-0.54% [-1.17%]	-0.32% [-0.92%]	-1.43%** [-1.87%]	-0.17% [-0.17%]	
TGT1	0.32% [0.13%]	0.54% [0.38%]	-0.57% [-0.57%]	0.69% [1.13%]	0.86% [1.30%]

This table provides the cumulative abnormal return differences among the 6 group specifications in the event window of [0, 5]. The $CAR_{[0,5]}$s are cross-sectional equal-weighted average return, calculated using market model within the estimation window of $[-250, -20)$ in Panel A, and using market return model in Panel B. The return difference is calculated as the minuend group return minus the subtrahend group return. The differences between median $CAR_{0,5}$s are shown below the mean $CAR_{0,5}$ differences with brackets. The corresponding t statistics for the $CAR_{0,5}$ differences are based on two sample test with equal means. We do not calculate the significance for the median return differences. * denotes significance at 10% level, ** at 5%, and * at 1%.

The return advantage of name change groups is statistically and economically significant for acquirers, and economically significant for targets. $CAR^{ACQ1}_{[0,5]}$ is annualized at 66.17% with 250 trading days in a year, and $CAR^{TGT1}_{[0,5]}$ at 28.79%. We see that the annualized name change effect for acquirers is 45.83%, and the effect for targets is a similar 49.17%. The outperformance by the group of name change acquirers and the group of name change targets is an evidence for the presence of a rumination effect on name change information.

Some may argue that the estimation window may contain the M&A announcement date, so the abnormal return estimated by the reference model possibly gets biased. We do not regard it as a serious concern. One reason is that the length between the M&A announcement date and the index change announcement date varies a lot among events, ranging from years to weeks or even days as illustrated in Figure 5.1. Because of such case-specific variation, we cannot take a unified truncation across events to exclude most of the M&A announcement dates. Also, there is no asymmetric impact of M&A announcement effect on our groups of acquirers and targets, since returns are equal-weighted across hundreds of stocks. In short, the diversification by portfolio construction and random timing ameliorates the concern that the potential inclusion of the M&A announcement date may impact our abnormal return estimation by the reference model. On the other hand, we cannot make such adjustment since M&A announcement is an important aspect influencing recent risk dynamics and return profile for the stock. The deletion of M&A announcement will corrupt the data integrate in the estimation window.

To reaffirm previous findings, we further use a modified market model for reference return. Setting α to 0 and β to 1, the market model becomes the market return model. The abnormal return is then expressed as

$$AR_{i,t} = R_{i,t} - R_{M,t}. \tag{5.8}$$

As highlighted in Fuller, Netter, and Stegemoller (2002), the advantage of market return model is that the market parameters do not need to be estimated so there is no worry on the accuracy of β estimation or α estimation. So, the abnormal return by the market return model is immune to aforementioned worry.

Panel B in Table 5.5 gives CARs for the 6 groups during event window with the specification of market return model. The results are generally the same with the findings by market model. Acquirer groups and name change groups still have benefits over their peers. $CAR^{ACQ1}_{[0,5]}$ again is highly statistically significant, with an annualized rate at 65.77%. Interestingly, $CAR^{TGT1}_{[0,5]}$ is more than economically significant, gaining statistical significance with an annualized rate at 42.19%. This is because the estimated target β is lower than 1, and the ratio $\frac{\alpha}{1-\beta}$ is higher than the daily market return. Actually, the mean β is 0.84, and the median is 0.80, not to mention the case of negative market return. Therefore, the expected return by market model is typically higher than market return, suggesting that the abnormal return by market model is lower than that by market return model. The name change effect for acquirers is again significant with an annualized rate of 46.25%, as suggested in Table 5.6 Panel B. This effect for targets is also economically significant at the annualized difference of 35.83%.

We notice that the overlap of events maybe a concern for our analysis, since S&P's sometimes announces several membership adjustments in the same index change press release. To better assess the potential overlapping issue in our sample, we calculate the number of affected events by various time length as displays in Table 5.7 Panel A. 25 events share index change announcement date with others, and

TABLE 5.7: Overlapping events and correlation adjusted CARs

Panel A: Overlapping events by announcement date difference						
	0	1	2	6	7	8
Number	25	31	34	55	64	71
%	8	10	11	18	21	23

Panel B: Correlation adjusted cumulative abnormal return						
Day	Acquirer Groups			Target Groups		
	ACQ	ACQ0	ACQ1	TGT	TGT0	TGT1
0	0.27%***	0.28%**	0.26%	0.27%*	0.31%*	0.12%
	(2.84)	(2.49)	(1.36)	(1.91)	(1.80)	(0.67)
1	0.39%***	0.41%***	0.32%	0.46%**	0.51%**	0.30%
	(2.75)	(2.64)	(0.85)	(2.50)	(2.35)	(0.90)
2	0.56%***	0.55%***	0.58%	0.58%***	0.56%**	0.66%*
	(3.27)	(3.10)	(1.15)	(2.79)	(2.35)	(1.67)
3	0.6%***	0.44%**	1.26%**	0.67%***	0.45%*	1.57%***
	(3.10)	(2.11)	(2.45)	(2.97)	(1.80)	(3.30)
4	0.65%***	0.46%*	1.41%**	0.44%*	0.20%	1.4%***
	(2.92)	(1.92)	(2.44)	(1.88)	(0.78)	(2.77)
5	0.69%***	0.47%*	1.58%***	0.32%	0.15%	1.01%*
	(2.74)	(1.68)	(2.74)	(1.25)	(0.52)	(1.93)

This table provides the number and percentage of overlapping events in our sample by announcement date difference in Panel A. The date difference is calculated as the time lag between two adjacent events. For example, 1 means that the event comes the day next to previous event. The cross-sectional correlation bias is corrected in Panel B with the method by Kolari and Pynnönen (2010) under market return model. * denotes significance at 10% level, ** at 5%, and * at 1%. The corresponding t statistics are based on the cross-sectional standard deviation with correlation adjustment, and are shown below CAR estimates with parentheses.

31 events come next to previous events in the following day. As our event window is [0, 5], any date difference within 5 days are involved in the overlapping concern. In total, we have 50 events, or about 16% of our sample, affected. We also list date difference over 5 days to incorporate the weekend effect. After this extension, we have 64 events affected in our sample, not very substantial. We adjust this concern with cross-sectional correlation correction proposed by Kolari and Pynnönen (2010). Panel B in Table 5.7 reports the t statistics adjusted for correlation with market return model. Although corresponding t statistics go slightly downward, the results do not change. The outperformance of acquirer groups and name change groups are still notable with economic and statistical significance.

In total, our results of significant name change effect in short term are robust to model specification and event clustering. The annualized $CAR^{ACQ1}_{[0,5]}$ is more than 65% with obvious statistical and economic significance, and the annualized $CAR^{TGT1}_{[0,5]}$ is 42% with statistical and economic significance under market return model and is 28% with economic significance under market model.

5.5 Long-term abnormal returns

After the reaffirmation of name change effect in the short term, we further focus on the one year abnormal return that follows index change announcement over the long term event window of $[0, 250]$. This means we have to truncate the latest year of 2017 to guarantee the window coherence across events.

We primarily use the buy-and-hold abnormal return (BHAR) approach to specify abnormal return, as the difference of the event return over its matching portfolio return **p**[see, e.g.][]barber1997detecting, kothari2008econometrics. The matching portfolio can consist of one company or several companies to parallel certain event company characteristics. In our context, the expression for BHAR of company i is

$$BHAR_{i,[t_1,t_2]} = R_{i,[t_1,t_2]} - R_{Matching^i,[t_1,t_2]}, \tag{5.9}$$

where $i \in ACQ1$, $R_{i,[t_1,t_2]} = \Pi_{t=t_1}^{t_2}(1 + R_{i,t}) - 1$, and $R_{Matching^i,[t_1,t_2]} = \Pi_{t=t_1}^{t_2}(1 + R_{Matching^i,t}) - 1$. The first term in the right hand side is the buy and hold return on the name change acquirer i over the period of $[t_1, t_2]$, and the second term is the buy and hold return on the matching portfolio of i from the pool of non name change acquirers over the same period. That is to say, $Matching^i \subset ACQ0$. Our matching portfolio construction is an adjusted mixture of Barber and Lyon (1997) single control firm matching and Savor and Lu (2009) portfolio matching. We set the size of matching portfolio to be dependent on the quality of the matching pool rather than strictly predetermined.

Specifically, the single control firm matching method and the portfolio matching method exclusively concentrate on identifying all available firms with some key company characteristics such as company size, book-to-market ratio and so on, to match the event firm, which is the acquirer. Due to this exclusivity, both methods are inadequate for the analysis of our sample where the characteristics of the target are of great interest in evaluating the integrity of systematic M&As. Still take the example of SBC's acquisition of AT&T. The single control firm matching method and the portfolio matching method principally consider the characteristics of SBC, and the characteristics of AT&T are not explicitly regarded in the matching process. However, the target makes a difference because it is easy to recognize the disparity between SBC's acquisition of AT&T and SBC's acquisition of Gillette, even though the matching firm or portfolio for SBC is the same by the two methods. This disparity is crucial since the benefits of systematic M&As are more closely associated with industry matching for both the target and the acquirer, especially when it comes to name change specification where the role of target is better highlighted in the business consolidation.

Therefore, we prioritize the complete industry matching, a process for both the acquirer and the target. For each event, we collect the following industry information for the acquirer and the target: the four-digit primary SIC code, the Thomson Reuters Business Classification (TRBC) economic and business sector classification, as well as the Thomson Financial (TF) macro and mid industry classification. SIC, TRBC, and TF are subtly different industry classification benchmarks, and we synthesize them if there is a main classification disagreement among them. For instance, Berkshire Hathaway has a SIC code of 6331, which means Fire, Marine & Casualty Insurance within the broad industry of Finance, Insurance, Real Estate. Its TRBC economic and business sector is classified as Financials and Insurance, while its TF macro and mid industry is classified as Industrials and Industrial Conglomerates. The difference is that SIC and TRBC emphasize Berkshire Hathaway's main business of insurance but TF considers the company as a whole business across industries. Therefore, when we consider matching firms for Berkshire Hathaway, acquirers from Financials/Insurance and Industrials/Industrial Conglomerates are all candidates. The same process is used for the target industry matching.

Besides the complete industry matching, we have contemporary matching imposed on the pool of $ACQ0$ so that the index change announcements of $Matching^i$ should be within 1 year around the index change announcements of event i. If there is no availability, we extend the searching radius. This selection criterion of contemporaneousness greatly alleviates the worry that the return difference may just stem from market comovements. That is to say, a good matching firm for an acquirer with its index change announcement in 2009 should also be considered in this time period of financial crisis rather than in 2006 when the stock market showed exuberance nor in 2013 when the stock market recovered. Otherwise, the abnormal return is plausibly more a reflection of business cycle or economic trend. As the pool of non name change candidates is not very large, we concentrate on the industry and contemporaneousness selection criteria. If we get more than a few candidates left, we drop those with disparate M&A deal size. Assume the name change acquisition of Mellon by Bank of New York with a deal amount of \$15.7 billion in 2007. A candidate matching time and industry criteria is the acquisition of Bear Stearns by JPMorgan Chase in 2008 but the deal amount is a dwarfed amount of \$1.2 billion. We exclude this deal from our final matching portfolio due to the substantial economic asymmetry between the matching pair.

In short, industry matching for target and acquirer plus contemporaneousness matching within a reasonable searching radius guarantees that our matching portfolio construction is robust to merger clustering, industry wave and economic trend. For an abundant matching pool, we dismiss candidates of acute economic asymmetry as shown by the pair of Mellon and Bear Stearns. The initial size of candidates depends on their availability, and the final size of matching portfolio ranges from 1 to 3 after selection procedures.

The statistical test for BHAR given by Barber and Lyon (1997) is

$$t_{BHAR_{[t_1,t_2]}} = \sqrt{n}\frac{\overline{BHAR}_{[t_1,t_2]}}{\sigma_{[t_1,t_2]}(BHAR)}, \tag{5.10}$$

where $\overline{BHAR}_{[t_1,t_2]}$ is the average buy-and-hold abnormal return of a group of n firms.

To tackle the positive skewness concern, Lyon, Barber, and Tsai (1999) introduce a skewness adjusted t statistic

$$t^{SkAdj}_{BHAR_{[t_1,t_2]}} = \sqrt{n}\left(S + \frac{1}{3}\gamma S^2 + \frac{1}{6n}\gamma\right), \tag{5.11}$$

where $S = \frac{\overline{BHAR}_{[t_1,t_2]}}{\sigma_{[t_1,t_2]}(BHAR)}$ and $\gamma = \frac{\sum_{i=1}^{n}[BHAR_{i,[t_1,t_2]} - \overline{BHAR}_{[t_1,t_2]}]^3}{n\sigma^3_{[t_1,t_2]}(BHAR)}$.

The results of the long term name change abnormal return are shown in Table 5.8. We calculate the correlation matrix of name change abnormal return series in Panel A, and note that the mean correlation coefficient among abnormal return series is about 0, as well as the median. Even the maximum correlation coefficient is still below 60%, and the frequency of positive correlation coefficients and negative coefficients is about the same. These statistics reassure that our matching portfolio construction method is free from comovement bias, otherwise a large industry or economy comovement would have delivered highly positive correlations between abnormal return series.

TABLE 5.8: BHAR for name change acquirers

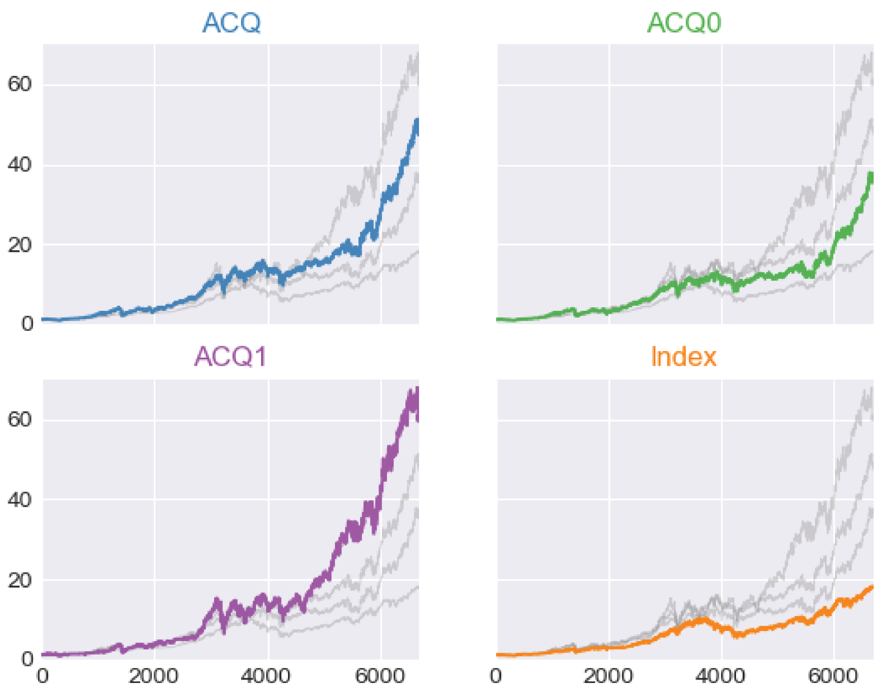
Panel A: BHAR Series Correlations			
Mean	0.00	Volatility	0.08
Skewness	1.56	Kurtosis	10.69
Min	-0.21	Max	0.58
Median	0.00	Size	1711
Positive	49.68	Negative %	50.32
Panel B: 1 Year BHAR for ACQ1			
Mean	0.10	Volatility	0.34
Skewness	0.37	Kurtosis	2.95
Min	-0.61	Max	0.97
Median	0.08	Size	59
Positive	55.93	Negative	44.07
t value	2.37	p value	0.0210
Sk-Adj t value	2.47	p value	0.0165

This table gives descriptive statistics of correlations about 1-year abnormal return series and for $BHAR^{ACQ1}_{[0,250]}$ in the long term event window. Positive % means the percentage of non-negative values out of the total observations, while negative % means the percentage of negative values. The definition for BHAR t statistic and $SkAdj\ t$ statistic are in the text, followed with corresponding p value.

Panel B gives more concrete performance statistics for the buy-and-hold abnormal return on the name change acquirer group. We see that the mean abnormal return $\overline{BHAR}_{[0,250]}$ is about 10%, with highly economic and statistical significance. Recall that the annualized short term abnormal return of name change is about 45%, greater than the long term abnormal return. This indicates a reversion of the abnormal return in economic size but the effect of name change is permanent. Now with more return observations, the confidence level for the existence of name change abnormal return is strongly improved. Since the skewness is very close to 0 and the frequency of positive and negative values is roughly equal, we are confident that the skewness bias indicated in **t***barber1997detecting is not a challenge to our findings. We also report the skewness adjusted *t* statistic which is slightly higher than the normal *t* statistic and is enough to support the existence of long term name change abnormal return.

FIGURE 5.7: Buy and hold performance for pseudo calendar time portfolios

This figure presents the performance of pseudo calendar time portfolios in terms of buy and hold wealth accumulation. The initial wealth of $1 is invested in the four strategies of systematic acquirers, non name change systematic acquirers, name change systematic acquirers, as well as the S&P 500 index. The horizontal axis is for the common trading days, and the vertical axis is for the wealth accumulation by unit of $1.



We consolidate the previous findings of significant name change effect in long term by examining the buy and hold performance for pseudo calendar time event portfolios. We construct portfolios of index investment, name change acquirers, non name change acquirers, and aggregate acquirers following the calendar time portfolio approach. The portfolios roll forward on a daily basis, and the length of any stock addition to investment is 1 year, consistent with the length in BHAR approach. Due to event availability, we trim return series and only keep data of coexistence to facilitate contemporary comparison. The advantage of pseudo calendar time portfolios, compared to the BHAR approach in our context, is that the portfolio formation has no discretion in the assembly of matching events. This formation only evolves respect to event arrival by calendar time, providing a complementary perspective to aforementioned matching portfolio creation. Figure 5.7 shows the evolution of an initial investment of $1 on various portfolio strategies. After 6,680 common trading days, the S&P 500 index investment earns about $18 in the end, while an investment in the group of systematic acquirers, ACQ, gains $47. An investment in the group of non name change systematic acquirers, $ACQ0$, rewards $35 eventually, about twice as large as index investment. The most attractive one is the investment in the group of name change systematic acquirers, $ACQ1$, which harvests a much higher return of about $60. We can easily detect the value generation edge of $ACQ1$ investment over any other investments especially in the second half period. The results of buy and hold performance of pseudo calendar time portfolios corroborate the results diagnosed in the BHAR approach that investment in the name change acquirers garners superior profit.

We use buy and hold investment performance for pseudo calendar time portfolios as an alternative to calendar time portfolio approach due to its inapplicability in our context. The calendar time portfolio approach is frequently used to detect long-term abnormal returns, although Loughran and Ritter (2000) argue that BHAR approach is better at capturing true abnormal returns. The key reason for its inapplicability is that we cannot build continuous return series for an event portfolio. Therefore, it is impossible to inject the excess return into an asset pricing model like the Fama-French three-factor model for abnormal return identification. The discontinuity comes from the fact that any time interval between two adjacent events exceeding one year inevitably leads to null return for the calendar time portfolio. For example, June 25, 2007, is the index announcement date for the name change acquisition of Mellon by Bank of New York, but it takes about two years for the subsequent index announcement by name change acquisition of Embarq by CenturyTel in May 21, 2009. So, no return is observed between June 2008 and May 2009. Subsequently, we cannot include this return series into a regression to test Jensen's alpha, the abnormal return in the calendar time portfolio.

It is possible to merge an investment in $ACQ1$ or $ACQ0$ with an investment in the S&P 500 index to make the integrated return series continuous. In practice, we can employ two alternative integration methods. The first method combines the index investment with $ACQ1$ unconditionally, where we always invest in the index and make additional investment in name change events once they are available. The other method includes an investment in $ACQ1$ plus a contingent investment in the index, where we primarily invest in the name change events but invest in the index instead upon their unavailability. Within the example of Mellon and Embarq, the two integration methods are common in the period of June 2008 to May 2009, when the index investment is the only choice. For the remaining period, the two methods are different in whether to take a full investment in $ACQ1$ or a combination of $ACQ1$ and the index. An important issue with these integration methods is that the index investment inevitably dilutes the real event performance and the time series discontinuity distorts the performance evaluation across the whole profile, leading to a serious bias against name change effect.

We illustrate this bias by a simple investment example in Table 5.9. Suppose we have a 7-period investment strategy choice over the index, $ACQ1$, $ACQ0$, and ACQ. For simplicity, assume identical daily return for the first three strategies. However, the investment of $ACQ1$ is discontinuous over the whole period, with observed return in Period 2, 3, and 6. The return is missing for the $ACQ0$ investment in Period 6. ACQ investment equally combines the investment of $ACQ1$ and $ACQ0$ and is continuous over the whole period. The average daily return of index investment is

0.1%. Among the three acquirer groups, the $ACQ1$ investment has the highest return of 1%, $ACQ0$ the lowest of 0.7% and ACQ the middle of 0.79%. The pattern is like our short term abnormal return results in Table 5.5. By combination method, the missing returns are filled with the index return and the observed returns for $ACQ1$ and $ACQ0$ are diluted. The return for $ACQ1$ in Period 1 is now 0.1% and the return in Period 2 shrinks to 0.55%. Consequently, the mean return for $ACQ1$ is deflated to 0.29%. The mean return for $ACQ0$ and ACQ also decreases by half. Now, the previous lowest return by $ACQ0$ is the highest, and the previous highest return by $ACQ1$ is the lowest. This overturn is a bias against $ACQ1$, embedded in the construction of counterfeit continuity for return series. The contingent method also suffers from such a bias, although the infection degree is slightly less. Table 5.10 is an exhibition of this bias from the perspective of wealth accumulation, where the $ACQ1$ is obviously the disadvantaged choice by its null observations.

The discontinuity of return series is the root of the incompetence of calendar time portfolio approach in capturing accurate abnormal returns in our study for name change effect. In combination method, we give material weight to the index as if it was a "shadow constant event", while in contingent method, we weight the index as mutual exclusivity to real event. We see the difference is quite remarkable for the mean return and final wealth, yet we have no indication of the true or appropriate weight in an index-integrated investment. The ambiguity of index weight for the counterfeit continuity is compounded in case of many null return observations, and the rendered bias distances the calendar time portfolio approach from a sound excess return input in the asset pricing model for abnormal return identification.

5.6 Announcement effects revisited

5.6.1 The S&P 500 index change announcement effect

The typical results of S&P 500 index change announcement effect in the previous studies are positive short term price response to index additions and negative response to index deletions. For index deletions, Lynch and Mendenhall (1997) register a return reaction of -1.46% with significance upon the index announcement in their sample covering March 1990 to April 1995. Based on their Table 4, the 5-day cumulative abnormal return is -9.15%. Beneish and Whaley (2002) study index changes since 1996 to 2001, and document a significant mean abnormal return of -6.21% upon announcement in their Exhibit 6. Chen, Noronha, and Singal (2004) also find negative abnormal return upon deletion announcement for the period of July, 1962 to 2000. Especially in the sub-period since October, 1989 to 2000, the return is a highly significant -8.46%, shown in their Table 1. Patel and Welch (2017) examine index changes from 1979 to 2015, and confirm the negative reaction to deletions. In their Table 7, the discretionary removals lead to a return of -5.39% upon announcement and the forced removals with a return of -1.04%.

Note that Lynch and Mendenhall (1997), Beneish and Whaley (2002) and Patel and Welch (2017) use market return model, so results in Table 5.5 Panel B is a more appropriate reference also to the common 3-day event window. It is interesting to contrast these results with our findings of target returns in Table 5.5, where the price response to the target group CAR_0^{TGT} is significantly positive. Specifically, our sample period coverage shares a lot in common with Patel and Welch (2017). We further compare our results with their findings on forced removals since systematic M&As induce forced deletions. Corresponding to their significant -1.04% in the window of (0,1), our $CAR_{[0,1]}^{TGT}$ is a highly significant 0.46%. Our effective sample size is 305, and

TABLE 5.9: Discontinuity and integration bias: an example of return

Strategy	Time							Mean
	1	2	3	4	5	6	7	
Panel A: Raw return data with discontinuity								
Index	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%
ACQ1		1.00%	1.00%			1.00%		1.00%
ACQ0	0.70%	0.70%	0.70%	0.70%	0.70%		0.70%	0.70%
ACQ	0.70%	0.85%	0.85%	0.70%	0.70%	1.00%	0.70%	0.79%
Panel B: Combination method for continuity								
Index	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%
ACQ1	0.10%	0.55%	0.55%	0.10%	0.10%	0.55%	0.10%	0.29%
ACQ0	0.40%	0.40%	0.40%	0.40%	0.40%	0.10%	0.40%	0.36%
ACQ	0.25%	0.48%	0.48%	0.25%	0.25%	0.33%	0.25%	0.33%
Panel C: Contingent method for continuity								
Index	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%
ACQ1	0.10%	1.00%	1.00%	0.10%	0.10%	1.00%	0.10%	0.49%
ACQ0	0.70%	0.70%	0.70%	0.70%	0.70%	0.10%	0.70%	0.61%
ACQ	0.40%	0.85%	0.85%	0.40%	0.40%	0.55%	0.40%	0.55%

This table illustrates the integration bias originating from return discontinuity in producing essential input to the calendar time portfolio approach. Panel A gives an example of 7-period investment strategies with return discontinuity and associated average daily returns. Panel B shows the combination method in dealing with null returns by combinating unconditional index investment. Panel C displays the contingent method in dealing with null returns by integrating conditional index investment on event unavailability.

TABLE 5.10: Discontinuity and integration bias: an example of wealth

Panel A: Raw wealth data with discontinuity								
Strategy	Time							Final
	0	1	2	3	4	5	6	
Index	1	1.001	1.002	1.003	1.004	1.005	1.006	1.007
ACQ1	1	1.000	1.010	1.020	1.020	1.020	1.030	1.030
ACQ0	1	1.007	1.014	1.021	1.028	1.035	1.035	1.043
ACQ	1	1.007	1.016	1.024	1.031	1.039	1.049	1.056
Panel B: Combination method for continuity								
Strategy	Time							Final
	0	1	2	3	4	5	6	
Index	1	1.001	1.002	1.003	1.004	1.005	1.006	1.007
ACQ1	1	1.001	1.007	1.012	1.013	1.014	1.020	1.021
ACQ0	1	1.004	1.008	1.012	1.016	1.020	1.021	1.025
ACQ	1	1.003	1.007	1.012	1.015	1.017	1.020	1.023
Panel C: Contingent method for continuity								
Strategy	Time							Final
	0	1	2	3	4	5	6	
Index	1	1.001	1.002	1.003	1.004	1.005	1.006	1.007
ACQ1	1	1.001	1.011	1.021	1.022	1.023	1.033	1.034
ACQ0	1	1.007	1.014	1.021	1.028	1.035	1.037	1.044
ACQ	1	1.004	1.013	1.021	1.025	1.029	1.035	1.039

This table illustrates the effect of return discontinuity and integration bias on wealth accumulation. Panel A gives the wealth accumulation corresponding to the raw return data in Table 5.9 Panel A. Null return is considered as 0 return to mimic the investment experience across the full period. Panel B shows the wealth accumulation by combination method for return continuity corresponding to return series in Table 5.9 Panel B. Panel C displays the wealth accumulation by contingent method for return continuity corresponding to return series in Table 5.9 Panel C.

they indicate 506 involuntary removals in their sample, while the return difference is a material 1.5%. This suggests that the index change behavior of systematic M&As is different from others, and again shows the necessity of special treatment in the sample construction.

As to index additions, Shleifer (1986) finds the return upon announcement is a significant 2.79%, and the return over (0,5) period is 2.22% with significance in his full sample from September, 1976 to 1983 in Table 2. Lynch and Mendenhall (1997) report a significant return of 2.86% upon announcement. Inferred from their Table 2, the 5-day cumulative abnormal return is 6.09%. Beneish and Whaley (2002) also report return significance in their Exhibit 5, where the announcement return is 8.15% for those with immediately subsequent effectiveness and 5.76% for those with latent effectiveness. Chen, Noronha, and Singal (2004) document a significant return of 5.45% on announcement for their sample period since October 1989 to 2000 in their Table 1. Patel and Welch (2017) register a significant return of 3.5% in (0,1) window by various methods in their Table 4. Since these studies use market model and market return model for abnormal return specification, we can readily compare these results with our Table 5.5.

What is striking is not that our CAR^{ACQ} is in smaller economic size compared to the addition abnormal returns, despite the acquirers are not actual index joiners. It is the fact that CAR^{ACQ} is kind of abnormal return after "hidden deletion" and still significantly positive. Mitchell, Pulvino, and Stafford (2004) refer the situation where a S&P 500 component acquirers a non-component by stock as "hidden addition," since its weight in the index is bound to increase. We argue that for our systematic sample, acquirers experience "hidden deletion" where its weight in index is somehow bound to decrease.

We demonstrate this point by a hypothetical index change in Table 5.11. Suppose we have a Small Portfolio 5 index, SP 5 index for short, consisting in company A, B, C, D, and E. Each one has a market capitalization of $20 million, so the total market capitalization covered by the SP 5 index is $100 million. The index weights are easy to compute, simply 20% for each one. And the price of each component stock is simplified as $20. So the total investment in one SP 5 index portfolio is $100. Suppose we have a hypothetical homemade index fund with $100 stake equivalent to one index portfolio, so this fund has one share of stock for each component. Now, assume a stock merger between D and E and the associated SP 5 index addition of company F. The combined company D+E doubles its market capitalization to $40 million, and the newly added F has a capitalization of $18 million. The total market capitalization covered by this index increases to $118 million, and corresponding weights are calculated. Note that the weight for D+E is not 40%, a simple weight sum of D and E, while the actual weight is 34%. Since the total capitalization and shares outstanding all double, the price is still $20. Consequently, the cost for one index portfolio is $98 not the initial stake of $100. The homemade index fund has three options on its portfolio rebalancing. The first one is still to keep one index portfolio and a fund outflow of $2. The rest $98 are reassigned to each component according to their weights. Note that the original components experience a shareholding reduction and the new component a shareholding increase. Specifically, the number of D+E shares decreases from 2 to 1.66 upon the index adjustment. This adjustment method is Fund share 1.

The index fund can also insist on its initial investment stake of $100 for additional share of index portfolio. This is equal to less holding reduction compared to Fund share 1. Or, the fund can avoid the holding reduction, sticking to the initial fund share stake by holding 1 share of F in addition. This necessitates a fund inflow of

TABLE 5.11: Systematic M&A and hidden deletion: an example of SP 5 index adjustment

Panel A: SP 5 index before systematic merger						
	Index components					Total
	A	B	C	D	E	
Market cap, $M	20	20	20	20	20	100
Weight	20%	20%	20%	20%	20%	100%
Price, $	20	20	20	20	20	100
Fund share	1	1	1	1	1	1

Panel B: SP 5 index after systematic merger						
	Index components					Total
	A	B	C	D+E	F	
Market cap, $M	20	20	20	40	18	118
Weight	17%	17%	17%	34%	15%	100%
Price, $	20	20	20	20	18	98
Fund share 1	0.83	0.83	0.83	1.66	0.83	1.00
Fund share 2	0.85	0.85	0.85	1.69	0.85	1.02
Fund share 3	1	1	1	2	1	1.20

This table illustrates the hidden deletion effect by systematic merger among the hypothetical Small Portfolio (SP) 5 index components. Market capitalization for each component is by $Million. Fund share is calculated for a hypothetical index fund with $100 investment stake. The index fund possesses 1 index portfolio, with identical unit share in each component. Panel B displays a case of systematic merger between D and E, with F the newly added. Fund share 1 is calculated for a unit index portfolio stake, so the investment is $98. Fund share 2 is calculated for the initial investment stake of $100, so the index portfolio share is 1.02. Fund share 3 is calculated for the initial fund share stake, so the investment is $118 and the index portfolio share is 1.20.

$18, the price of stock F. Therefore, the total investment is $118, analogous to 1.20 index portfolio. In arrangement of Fund share 1 and Fund share 2, the shareholding of D+E has to decline. This is not necessary in arrangement of Fund share 3, but additional fund inflow must arrive to settle the extra holding of F. Usually the inflow is substantial, and in our example it is 18% of the initial investment stake.

Taking all of the three arrangements into consideration, we show that the weight decline by systematic merger acts as hidden deletion, especially when Fund share 3 is not 100% certain in place. Here we specify the merger as stock merger. In cash merger, as explained in Mitchell, Pulvino, and Stafford (2004), the market capitalization does not change for D+E since it is just a trade of one asset (cash) for another asset (the target). As discussed in our Section 5.3, cash mergers only account for 30% of our sample. Therefore, we believe that the hidden deletion is an actual concern.

Given this concern, we may expect the abnormal return for ACQ behaves toward the negative response of deletion effect. The expectation is not consistent with our findings in Table 5.5, which implies that CAR^{ACQ} should be higher than the estimates in economic size, ceteris paribus. So, even ACQ is already the group of components, the price response for their index announcement is still significantly positive.

Here we summarize our contribution to the literature of S&P 500 index change effect. Analysis of our special sample of systematic M&As reveals positive abnormal return on index deletion, different from previous studies on full sample. Our paper is the first, to our best knowledge, to pay attention to the hidden deletion led by systematic mergers. This is a bias against abnormal return for ACQ, despite its current economic and statistical significance.

5.6.2 The M&A announcement effect

Studies of M&A announcement effect have a consensus on the target abnormal return but not on the acquirer abnormal return. For example, Jensen and Ruback (1983) review previous literature and suggest that corporate takeovers benefit target firm shareholders and do not deteriorate bidding firm shareholders. Martynova and Renneboog (2008) and Bhagat, Dong, Hirshleifer, and Noah (2005) also confirm that M&As bring value creation for the target and bidder shareholders combined. However, Roll (1986) pioneers the hubris hypothesis, based on the appealing intuition that bidding firm managers tend to be overconfident and overpay of the target since the neutral evaluation for the target should just be its market capitalization. Malmendier and Tate (2008), Huang and Kisgen (2013), and Masulis, Wang, and Xie (2007) support this hypothesis, and Moeller, Schlingemann, and Stulz (2004) find that bidding firm shareholders make significant negative gains.

Our results in Table 5.5 indicate positive price response to both the acquirer and the target with significance. Note that this positive abnormal return is not upon M&A announcement but rather upon M&A closing. Mitchell, Pulvino, and Stafford (2004) have some discussion of closing period abnormal return, offering proper reference for the interpretation of our results and our contribution to the M&A literature.

Synthesizing the results in Mitchell, Pulvino, and Stafford (2004) Table 2 across merger classifications, we estimate that the $CAR_{[-1,1]}$ for acquirers in their sample is 0.69%. Correspondingly, $CAR^{ACQ}_{[-1,1]}$ in our sample is 0.35%. The return difference most probably results from the sample distinction. In fact the payment composition for both samples is similar, with about 30% of cash mergers. However, our sample is robust to price pressure from index trading, which is not the case for their sample.

Therefore, we get an estimate of 0.34% as the abnormal return for the price pressure effect by index trading for nonsystematic M&A deals.

Our systematic sample is the most prominent and representative division out of the aggregate M&A deals that entail no index rebalancing. The acquirer cumulative abnormal return evolution in the period of [-20,20] in our Figure 5.6 is very similar to their analysis of acquirer CAR pattern for the non index rebalancing group over the same window of [-20,20] their in Figure 4. CAR is negative in most of the time before Day 0, and then gradually increases toward 1% as approaching Day 20. However, the non index rebalancing classification by Mitchell, Pulvino, and Stafford (2004) can eclipse the true importance of systematic M&As among S&P 500 components. Their Table 4 reports that the acquirer CAR of stock mergers within the same S&P index is 0.26% over the period of [-3,1], so we get an estimate of 0.18% for total mergers within the same S&P index in their sample. This estimate is only about one third of our $CAR^{ACQ}_{[-3,1]}$ of 0.52%. Our estimate is more reliable for two reasons. First, the sample size for their non index rebalancing mergers among the same S&P index is about 120, while our sample has 305 systematic mergers. Second, S&P 500 index is predominant over S&P MidCap 400 index and S&P SmallCap 600 index in market capitalization. As the end of 2017, the total market capitalization is \$24 trillion for S&P 500 index, \$1.9 trillion for S&P 400 index and \$0.8 trillion for S&P 600 index.[5] Simply grouping them is equivalent to assigning equal weights to mergers within S&P 500 index and mergers within the rest two indices in the CAR calculation, ignoring any effect of company size on merger. This is violently against the finding of size effect in the M&A performance evaluation by Moeller, Schlingemann, and Stulz (2004), which echoes our point in Section 5.3 that our systematic sample is special.

In short, our results of short term abnormal return are more valid and indicative to interprete the price responce of non index rebalancing mergers. We give 0.34% as an estimate of price pressure effect by index trading.

Now we reconcile the non index rebalancing to hidden deletion effect. We describe the hidden deletion by systematic M&As in previous section, and our sample should be classified as non index rebalancing by Mitchell, Pulvino, and Stafford (2004) for subduing index fund trading. This seemingly paradox comes since systematic M&As necessarily induce hidden deletion, but the hidden deletion effect diminishes as the index component size gets large, where non index rebalancing is a convenient approximation of hidden deletion. In the case of S&P 500 index, assume equal weight scheme for simplicity. The combination of D and E has a 2/500 weight just before rebalancing, which becomes 2/501 later. The weight difference is quite trivial, a deviance from 0.4% to 0.3992%, which is usually not enough to trigger a substantial index fund rebalancing. But in two cases of significant composition adjustment, this approximation may lead to mistakes. The first case is when the newly added component is much greater than the average component size in the S&P 500 index. A perfect example is the inclusion of Berkshire Hathaway in February 2010 after improving its stock liquidity. This addition entails a serious revision on the market capitalization covered by the index, and the weight for any previous component need a trim. The second case is when S&P's announces a handful of index changes at a same time. For instance, 5 component replacements are made in the single day of Feb 7, 1979, which brings a big change to the index and a review of weigh scheme. In these cases, the hidden deletion needs a careful investigation.

Our results also provide subtle evidence for the test of hubris hypothesis. As

[5] Numbers are from S&P Dow Jones Indices Factsheets, released as of Dec 29, 2017.

Moeller, Schlingemann, and Stulz (2004) confirm the firm size effect on merger abnormal return and dollar return, no other sample is more ideal than our systematic M&As in the exploration of hubris hypothesis and overpaying in terms of firm size. They find small acquirer makes positive return and large acquirer makes negative gain. Their definition of large companies is above the 25th percentile of NYSE listed companies that year. Since the number of NYSE listed companies in 1979 is 1,565 and then keeps growing,[6] it is guaranteed that the S&P 500 companies are within the 75th upper percentile. Because the company size in our sample is typically greater than their definition of large companies, we probably expect that the acquirer merger gain is negative. However, as indicated in our Table 5.5, the cumulative abnormal return is significantly positive. In other words, the mega acquirer behaves different from large acquirer as they make positive gain. Although Moeller, Schlingemann, and Stulz (2004) study announcement abnormal return and our study is more about effectiveness abnormal return, the evidence of positive gain is still not perfectly consistent with the hubris hypothesis from the megamerger perspective.

5.6.3 Few further concerns

Some may argue that name change is just an attempt by the company to climb the alphabet so that its name can appear sooner in the alphabetic sorting, or to be more visible and recognizable for consumers and stakeholders. Hence, name changes do not necessarily associate with valuable information revelation. However, this allegation is not really applicable to our sample of systematic M&As. Since S&P 500 companies have already accumulated reputation for their brands with broad name recognition, it is not rewarding for them to change name simply out of alphabetic climb or better name visibility. Verizon is a case in point against this argument. After the acquisition of GTE, Bell Atlantic changes name to Verizon. The new name appears much later in the alphabetic sorting than previous names, which are of great brand acknowledgement with glorious history. The choice of separation from tradition is an determination for "a global image and a high-tech image" as a young technology dynamo.[7] This marks an important business strategy reorientation, manifesting that name change conveys valuable information.

Moreover, the direct cost for name change is usually very high for large companies, not to mention the hidden cost of separating from traditional image. In previous example, marketing the new brand of Verizon costs $300 million. Any rational company weighs the cost and benefit of name change, and alphabetic climb or better visibility is not tenable enough for a large company to change name if it can not create tradeoff value. On the other hand, if these motivations really create value more than the total cost for the company, this argument is actually a support for our intuition that name change conveys valuable information since the motivations themselves bring value. In total, this argument is not a main name change motive for our sample, thus is not a challenge to our analysis.

Another concern is about merger of equals, which is a special case of merging two companies in similar size. The two companies are consolidated into a new entity, trying to play down the role of acquirer or target in the combination. Therefore, name change effect may result from this specific type of takeover, rather than from the information conveyed by name change.

[6]Data are from NYSE historical archive of Facts and Figures in the official website of NYSE.

[7]This quote is by a Bell Atlantic executive in the *New York Times* report titled "Bell Atlantic and GTE Pick Post-Merger Name" in April 2000.

We maintain that this concern of merger of equals is not a serious problem for our study, based on the following reasons.

1. Merger of equals is a euphemism for friendly acquisition. After all, it is impossible to share a consolidated company's control and ownership in perfectly equal measure between shareholders from two companies. For example, DaimlerChrysler is often considered as an exemplary international merger of equals since its announcement. However, Jürgen Schrempp, the CEO of Daimler-Benz and the later DaimlerChrysler, admits that "merger of equals" term is just for "psychological reasons" and the merger is in fact a Daimler takeover.[8] His remarks reveal the very essence of merger of equals. Another example is AOL Time Warner, the biggest M&A deal in U.S. history which is officially announced as "a strategic merger of equals." Nonetheless, AOL is commonly regarded as the acquirer, like by Savor and Lu (2009). Moreover, Eikon M&A database classifies the acquirer and target for these mergers of equals, offering helpful information to determine the actual roles under the euphemism. The very essence of merger of equals is the main reason that we can safely treat merger of equals as more common M&A deal.
2. Merger of equals does not necessarily lead to name combination or new name for the combined company. For example, Eikon classifies the combination of Bell Atlantic and NYNEX as merger of equals, the former as acquirer. Still, the combined company bears the name of Bell Atlantic. Therefore, merger of equals is not a sufficient condition for name change, suggesting that it has bilateral influence on name change group and non name change group. In our long term analysis of buy and hold abnormal return, the return difference of the two groups attenuates any potential impact from merger of equals.
3. Merger of equals as deal type is important information at the deal announcement. It is more immediate and myopic than name change information, especially when it is not in lockstep with name change information. Therefore, we expect investors have already discounted this information before index change announcement, which usually comes with or after the merger effectiveness. Hence, its influence is not substantial.

In short, our analysis is robust to the concern that name change does not convey value information or that merger of equals stipulates the origin of name change value information.

5.7 Conclusion

Our paper examines name change effect upon M&As among S&P 500 index. The sample is systematically important because both the acquirer and the target are prominent components in the market portfolio and their merger entails substantial market portfolio adjustments. Our sample includes all the systematic M&As among index components from 1979 to 2017, representing the development of aggregate M&A activities in terms of deal number and transaction value. It has distinguishing features such as robustness to price pressure, gigantic transaction size and symmetric industry distribution, calling for special treatment from the aggregate M&A activities other than equal treatment as in previous studies.

[8] Related articles are "DaimlerChrysler dawns" by *CNN* in May 1998, "Scenes From A Marriage" by *New York Times* in August 2001, and "Why Corporate Mergers of Equals Almost Never Work" by *Forbes* in June 2014.

We argue that name change conveys valuable information about corporate strategy and is an attempt from the management to favorably update stakeholders' belief. Due to investor inattention, this information is not fully reflected in stock price upon M&A announcement and the index change announcement triggers information rumination on it. We find support for this hypothesis both in the short term and in the long term. The cumulative abnormal return for name change acquirer is statistically and economically significant, and the name change target is also significantly better than its peer upon the index change announcement. In the long term, the name change acquirer typically has a significant abnormal return of 10% over its matching non name change acquirers. The name change pseudo calendar time portfolio also has a superior buy and hold performance than non name change portfolio and the index portfolio.

Our paper contributes to the existing literature in multiple aspects. First, it shows that the deletion effect is not negative upon index announcement for this systematic sample. Second, it explicates the hidden deletion effect by systematic merger, which is a return bias against the acquirer. Third, our results refine the non rebalancing merger abnormal return in Mitchell, Pulvino, and Stafford (2004), giving an estimate of 0.34% for the influence of price pressure by index trading. Fourth, our paper is a new application of investor inattention to analyze return pattern in the cross context of M&A events and S&P 500 index change events. Fifth, our result of positive name change abnormal return is an empirical support for the intuition that name change carries valuable information.

Chapter 6

General conclusion

Extreme events have a material impact on return distributions and investment decisions. However, the role of event risks is understated in popular financial decision making approaches. This thesis includes event risks (understood in a broad sense) into investment decisions to improve global investment optimality. We examined event risks in two different but coherent financial settings: portfolio selection and corporate finance. In the portfolio selection setting, we primarily focused on the incorporation of higher order information to capture the impact of event risks on portfolio construction. Higher order extensions were implemented on two main portfolio optimization methods: the classic framework of mean variance optimization and CAPM in Chapter 2, and the stochastic dominance approach in Chapter 3. We found that the inclusion of higher order information improves global portfolio optimality given the presence of event risks. In the corporate finance setting, we principally identified corporate name changes of M&As among the S&P 500 index, and examined how the name change events impact the return patterns for the acquirers and the targets. This corporate event study was conducted in Chapter 5. We found that name change events substantially affect return dynamics, and that the abnormal return difference between name change events and non name change events is economically and statistically significant. Generally, our studies showed that the inclusion of event risks in decision processes brings important benefits to the asset allocation optimization.

Chapter 2 conducted the higher order moment extension of the traditional mean variance optimization and CAPM framework. Kurtosis captures the impact of extreme returns, thus we used it as a measure of extreme risks. It complements variance for a comprehensive depiction of investment risk, as investment with relatively low variance may well have relatively large kurtosis. We then considered the portfolio construction problem for those investors with extreme risk aversion. The preference of extreme risk aversion is instructive and useful given the presence of extreme financial events, especially during the global financial crisis and the European sovereign debt crisis recently. Therefore, we considered the portfolio development in the mean variance kurtosis space, to highlight the impact of extreme risks on portfolio selection. We proposed the Pareto improvement method for the mean variance kurtosis efficient portfolio specification. Compared to current methods for higher order moment efficient frontier construction, i.e., the constrained variance minimization program by De Athayde and Flôres (2004) and the shortage function method by Briec, Kerstens, and Jokung (2007), the Pareto improvement method has a key advantage that it is able to detect marginal improvements for portfolio profiles. This advantage is important because the failure of the two current methods in specifying marginal improvements leads to a serious potential mistake: the misclassification of inefficient portfolios as efficient. Moreover, the Pareto improvement method is simple and efficient. By pairwise comparison among the portfolio profiles, this method classifies portfolios with inferior profiles as inefficient and only those portfolios without any profile improvement are labeled as efficient. To implement the Pareto improvement method, a proper approximation of the feasible portfolio set is necessary. We approximated this set by using the Dirichlet distribution. The Dirichlet distribution produces vectors of nonnegative elements in sum of 1. This feature makes the Dirichlet distribution a proper program for portfolio weight simulation. We illustrated the use of the Pareto improvement method and Dirichlet simulation with an empirical implementation of the S&P 500 index sectors with total return data from 1995 to 2015. The market implied risk aversion parameters and equilibrium sector returns were obtained from the Black Litterman model. The equilibrium sector returns were combined with Dirichlet weight simulations for Pareto improvement comparisons. The Dirichlet parameterization utilized two properties of the mean variance efficient set and the mean variance kurtosis efficient set: mean variance efficient portfolios are mean variance kurtosis efficient portfolios, and mean variance inefficient portfolios can be mean variance kurtosis efficient portfolios. With regard to these two properties, we had 5.1 million weight vectors by Dirichlet simulation. We then produced the mean variance kurtosis efficient frontier, which behaves like two line segments at the two ends, and a band spread in the middle. Further work will be on the mean variance kurtosis utility specification, the generalized Sharpe ratio for return and risk tradeoff, and the tangency of utility surface and efficient frontier in the mean variance kurtosis space.

Chapter 3 extended the approach of Kuosmanen (2004) for testing for stochastic dominance efficiency of a given portfolio with respect to a set of underlying assets. The extended approach allows us to test for dominance efficiency of higher orders than two, and, similar to the original paper, to obtain the optimal weights for an efficient portfolio in case the test portfolio is proved to be inefficient. We applied this approach to 17 stock market indices covering developed and developing markets across the globe, and found that in the majority of the years these indices are inefficient at least at order 3, and often at order 2. Thus, all prudent and most of risk averse investors should optimally deviate from equity indices, investing instead in portfolios that overweight individual industries. The average mean return improvement that could be achieved by investing in an SD efficient portfolio is 23% annualized. Such a high return improvement is hard to achieve in practice, as this result stems from the in-sample optimization and knowledge of the realized return distribution. At the same time, the magnitude of the potential improvement suggests that even moderate deviations from the well diversified indices towards the optimal portfolio can result is substantial gains for investors. Then, we conducted pairwise comparisons of the market equity indices with their sector sub-indices. Since here we used the ex-ante industry classification, this strategy is practically implementable. We found that on average in 67% of years not only the indices are inefficient but they are dominated by at least one sub-index. The percentage of dominated indices is especially high during the years 2008 – 2012. On the aggregate level, counter-cyclical industries such as Consumer Foods and Services, Health Care and Telecommunication are more likely to dominate their diversified equity indices. At the same time, the types of industries that are likely to dominate vary across the countries. For example, the Oil and Gas industry often dominates the Russian RTS index but not the other indices and the Financial sector often dominates the Chinese SSE 50 index but almost never the other equity markets. Further, we estimated a Logit model for the determinants of the probability of a sub-index to dominate its index. Remarkably, macro factors contain different information with respect to future dominance for developed and developing markets. For more homogeneous and balanced economies, aggregate indicators of growth (such as GDP growth rate, inflation, and the current account balance) predict lower likelihood of the market index to be dominated. However, for the developing economies, which often rely on just one or several key industries, such aggregate indicators predict a higher likelihood that the market index will be dominated. The most significant and consistent predictors of dominance are index and sub-index volatilities, with the former being positively related to the probability of a sub-index to dominate the index, and the latter being negatively related to the probability, as well as the ratio of volatilities. Also, past dominance of a sector sub-index over the market index predicts higher likelihood for the future dominance. Given that past stochastic dominance is a strong predictor of the future dominance of a sub-index over the index, we further suggested a simple trading rule based on the information on past dominance, that invests only in those sub-indices that dominate the index at least twice during a given number of previous years. Applying this strategy to the developed markets, we found that the rule results in consistent mean return improvement and volatility reduction in the U.S., the U.K., and Europe, but does not perform that well in Japan. Applying this strategy to a global portfolio results in about 1–2% annualized return improvement and 2–3% decline in the annualized standard deviation. Such improvements in the return distribution are consistent across time. Our past SD based approach also substantially limits the losses during market downturns like the financial crisis of 2007–2008. Last but not least, we sorted individual stocks into tercile portfolios based on their market betas and volatility and showed that low-beta and low-volatility portfolios stochastically dominate the market indices in majority of years at order 3 and often at order 2 across most of world economies considered. These results contribute to the discussion of a stellar performance of low-beta and low-risk stocks in the mean-variance sense, and suggest that these portfolios are likely to be preferred by risk-averse and prudent investors over diversified market indices. Overall, our findings suggest that diversified equity indices across the globe are not SD efficient. Risk averse and prudent investors could benefit from switching between different industry sub-indices, by taking positions in those industries that were dominating in the past, or by investing in low-beta stocks. The sector-based strategies can rely on trading ETFs at low frequency, re-balancing portfolios once every year, thus, delivering improved return distributions with low transaction costs, which can be rather appealing for regulated long-term investors such as pension funds or insurance companies.

The two extensions have shown that the higher order information helps to improve investment optimality at the presence of event risks. However, they do not necessarily dismiss traditional applications of the mean variance optimization and stochastic dominance method when the research focus is out of event risks. In Chapter 4, we combined the use of mean variance optimization and stochastic dominance analysis to examine the index efficiency of DJIA. The DJIA's epic gain of about 5,000 points in 2017 remarks one of the most success developments in its history. However, this millstone has rekindled a wide disapproval and doubts that the DJIA is "flawed" and "meaningless" by its price weighted nature and limited coverage. To answer the question of whether DJIA is an efficient index or not, we examined its role as performance benchmark, because its role as information aggregator can be easily reoriented. We found the index with "obsolete" price weighted methodology serendipitously turns out to be "dynamic," according to the modern definition of index by Lo (2016). It is definitely investable, as all of its constituents are blue-chip companies with great liquidity; it is transparent, as its composition changes and component weights are public information; and it is systematic, since its construction is by rules and the selection criteria are consistent. Born in the old days without computer, DJIA receives its renaissance in the technology-leveraged new financial reality. To examine its efficiency, we constructed equal weighted, market value weighted, unadjusted price weighted, and adjusted price weighted variants with the underlying total return of components since 1988 to 2017. We found that equal weighted and unadjusted price weighted variants have a considerably better performance than the market value weighted and adjusted price weight variants. After adjusting trading costs for all of them, we did not find any evidence against the efficiency of unadjusted price weighted scheme. This result disapproves the common understanding that DJIA is badly constructed as a price weighted index. We then included the market indices of price index DJIA, total return DJIA and total return S&P 500 index for comparison, and found the total return DJIA has the highest cost-adjusted Sharpe ratio, a tenable indication of the efficiency of DJIA. The SSD pairwise comparison result confirmed previous findings on the DJIA efficiency. Moreover, we took an optimal portfolio approach to complement the efficiency analysis. If the in-sample optimal portfolio of DJIA components consistently outperforms the index out of sample, then the index is inefficient. However, our results showed that all the optimal portfolios by mean variance optimization and stochastic dominance optimization lose their investment advantages out of sample, and the in-sample optimality cannot carry on next year. This reaffirmed the DJIA efficiency as a performance benchmark. Our results manifested that the repudiation of DJIA efficiency is too harsh. Probably the DJIA is not a comprehensive and proper index for the overall stock market, but as a perfect example of dynamic indices, it is still efficient as performance benchmark for the investment in elite blue-chip stocks.

Chapter 5 examined name change effect upon M&As among S&P 500 index. The sample is systematically important because both the acquirer and the target are prominent components in the market portfolio and their merger entails substantial market portfolio adjustments. Our sample includes all the systematic M&As among index components from 1979 to 2017, representing the development of aggregate M&A activities in terms of deal number and transaction value. It has distinguishing features such as robustness to price pressure, gigantic transaction size and symmetric industry distribution, calling for special treatment from the aggregate M&A activities other than equal treatment as in previous studies. We argued that name change conveys valuable information about corporate strategy and is an attempt from the management to favorably update stakeholders' belief. Due to investor inattention, this information is not fully reflected in stock price upon M&A announcement and the index change announcement triggers information rumination on it. We found support for this hypothesis both in the short term and in the long term. The cumulative abnormal return for name change acquirer is statistically and economically significant, and the name change target is also significantly better than its peer upon the index change announcement. In the long term, the name change acquirer typically has a significant abnormal return of 10% over its matching non name change acquirers. The name change pseudo calendar time portfolio also has a superior buy and hold performance than non name change portfolio and the index portfolio. Our paper contributes to the existing literature in multiple aspects. First, it showed that the deletion effect is not negative upon index announcement for this systematic sample. Second, it explicated the hidden deletion effect by systematic merger, which is a return bias against the acquirer. Third, our results refined the non rebalancing merger abnormal return in Mitchell, Pulvino, and Stafford (2004), giving an estimate of 0.34% for the influence of price pressure by index trading. Fourth, our paper is a new application of investor inattention to analyze return pattern in the cross context of M&A events and S&P 500 index change events. Fifth, our result of positive name change abnormal return is an empirical support for the intuition that name change carries valuable information.

Bibliography

Acharya, Viral V and Lasse Heje Pedersen (2005). "Asset pricing with liquidity risk". *Journal of Financial Economics* 77.2, pp. 375–410.

Affleck-Graves, John and Bill McDonald (1990). "Multivariate tests of asset pricing: The comparative power of alternative statistics". *Journal of Financial and Quantitative Analysis* 25.2, pp. 163–185.

Ahern, Kenneth R and Jarrad Harford (2014). "The importance of industry links in merger waves". *The Journal of Finance* 69.2, pp. 527–576.

Aït-Sahalia, Yacine, Julio Cacho-Diaz, and Tom R Hurd (2009). "Portfolio choice with jumps: A closed-form solution". *The Annals of Applied Probability*, pp. 556–584.

Aït-Sahalia, Yacine and Felix Matthys (2017). "Robust consumption and portfolio policies when asset prices can jump". Working Paper.

Almadi, Himanshu, David E Rapach, and Anil Suri (2014). "Return Predictability and Dynamic Asset Allocation: How Often Should Investors Rebalance?" *Journal of Portfolio Management* 40.4, pp. 16–27.

Amihud, Yakov (2002). "Illiquidity and stock returns: cross-section and time-series effects". *Journal of Financial Markets* 5.1, pp. 31–56.

Ang, Andrew et al. (2009). "High idiosyncratic volatility and low returns: International and further US evidence". *Journal of Financial Economics* 91.1, pp. 1–23.

Annaert, Jan, Sofieke Van Osselaer, and Bert Verstraete (2009). "Performance evaluation of portfolio insurance strategies using stochastic dominance criteria". *Journal of Banking & Finance* 33.2, pp. 272–280.

Armstrong, Peter (2017). "Why the Dow Jones hitting 20,000 is meaningless". *CBC News*. [Online; posted January 17, 2017; accessed March 30, 2017]. URL: http://www.cbc.ca/news/business/dow-jones-20-000-peter-armstrong-1.3929025.

Arnott, Robert, Noah Beck, and Vitali Kalesnik (2015). "Rip Van Winkle indexing". *Journal of Portfolio Management* 41.4, pp. 50–67.

Arrow, Kenneth Joseph (1965). *Aspects of the theory of risk-bearing*. Yrjö Jahnssonin Säätiö.

Asness, Clifford S, Andrea Frazzini, and Lasse H Pedersen (2014). "Low-risk investing without industry bets". *Financial Analysts Journal* 70.4, pp. 24–41.

Back, Kerry (2010). *Asset Pricing and Portfolio Choice Theory*. Oxford University Press.

Bae, Kee-Hong, G Andrew Karolyi, and René M Stulz (2003). "A new approach to measuring financial contagion". *The Review of Financial Studies* 16.3, pp. 717–763.

Baker, Malcolm, Brendan Bradley, and Jeffrey Wurgler (2011). "Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly". *Financial Analysts Journal* 67.1, pp. 40–54.

Bakshi, Gurdip and Dilip Madan (2006). "A theory of volatility spreads". *Management Science* 52.12, pp. 1945–1956.

Barber, Brad M and John D Lyon (1997). "Detecting long-run abnormal stock returns: The empirical power and specification of test statistics". *Journal of Financial Economics* 43.3, pp. 341–372.

Barber, Brad M et al. (2008). "Just how much do individual investors lose by trading?" *The Review of Financial Studies* 22.2, pp. 609–632.

Barberis, Nicholas, Andrei Shleifer, and Jeffrey Wurgler (2005). "Comovement". *Journal of Financial Economics* 75.2, pp. 283–317.
Barro, Robert J (2006). "Rare disasters and asset markets in the twentieth century". *The Quarterly Journal of Economics* 121.3, pp. 823–866.
Bates, David S (2008). "The market for crash risk". *Journal of Economic Dynamics and Control* 32.7, pp. 2291–2321.
— (2012). "US stock market crash risk, 1926–2010". *Journal of Financial Economics* 105.2, pp. 229–259.
Bekaert, Geert et al. (2011). "What segments equity markets?" *The Review of Financial Studies* 24.12, pp. 3841–3890.
Beneish, Messod D and Robert E Whaley (2002). "S&P 500 index replacements". *The Journal of Portfolio Management* 29.1, pp. 51–60.
Benzoni, Luca, Pierre Collin-Dufresne, and Robert S Goldstein (2011). "Explaining asset pricing puzzles associated with the 1987 market crash". *Journal of Financial Economics* 101.3, pp. 552–573.
Betton, Sandra, B Espen Eckbo, and Karin S Thorburn (2008). "Corporate takeovers". *Handbook of Empirical Corporate Finance*. Ed. by B Espen Eckbo. Vol. 2. Elsevier. Chap. 15, pp. 289–427.
Bhagat, Sanjai et al. (2005). "Do tender offers create value? New methods and evidence". *Journal of Financial Economics* 76.1, pp. 3–60.
Black, Fischer (1986). "Noise". *The Journal of Finance* 41.3, pp. 528–543.
Black, Fischer and Robert Litterman (1992). "Global portfolio optimization". *Financial Analysts Journal* 48.5, pp. 28–43.
Blitz, David C and Pim van Vliet (2007). "The volatility effect". *The Journal of Portfolio Management* 34.1, pp. 102–113.
Bloom, Nicholas et al. (2010). "Why do firms in developing countries have low productivity?" *American Economic Review* 100.2, pp. 619–23.
Board, Simon and Moritz Meyer-ter Vehn (2013). "Reputation for quality". *Econometrica* 81.6, pp. 2381–2462.
Bollerslev, Tim and Viktor Todorov (2011). "Tails, fears, and risk premia". *The Journal of Finance* 66.6, pp. 2165–2211.
Briec, Walter and Kristiaan Kerstens (2010). "Portfolio selection in multidimensional general and partial moment space". *Journal of Economic Dynamics and Control* 34.4, pp. 636–656.
Briec, Walter, Kristiaan Kerstens, and Octave Jokung (2007). "Mean-variance-skewness portfolio performance gauging: A general shortage function and dual approach". *Management Science* 53.1, pp. 135–149.
Brockett, Patrick L and Yehuda Kahane (1992). "Risk, return, skewness and preference". *Management Science* 38.6, pp. 851–866.
Bruhn, Miriam, Dean Karlan, and Antoinette Schoar (2010). "What capital is missing in developing countries?" *American Economic Review* 100.2, pp. 629–33.
Canina, Linda et al. (1998). "Caveat compounder: A warning about using the daily CRSP equal-weighted index to compute long-run excess returns". *The Journal of Finance* 53.1, pp. 403–416.
Chan, Kalok, Hung Wan Kot, and Gordon YN Tang (2013). "A comprehensive long-term analysis of S&P 500 index additions and deletions". *Journal of Banking & Finance* 37.12, pp. 4920–4930.
Chaves, Denis B and Robert D Arnott (2012). "Rebalancing and the value effect". *Journal of Portfolio Management* 38.4, pp. 59–74.

Chen, Honghui, Gregory Noronha, and Vijay Singal (2004). “The price response to S&P 500 index additions and deletions: Evidence of asymmetry and a new explanation”. *The Journal of Finance* 59.4, pp. 1901–1930.

— (2006). “Index changes and losses to index fund investors”. *Financial Analysts Journal* 62.4, pp. 31–47.

Chotikapanich, Duangkamon and William E Griffiths (2002). “Estimating Lorenz curves using a Dirichlet distribution”. *Journal of Business & Economic Statistics* 20.2, pp. 290–295.

Clarke, Roger G and Meir Statman (2000). “The DJIA crossed 652,230”. *The Journal of Portfolio Management* 26.2, pp. 89–92.

Cohen, Lauren and Andrea Frazzini (2008). “Economic links and predictable returns”. *The Journal of Finance* 63.4, pp. 1977–2011.

Conine, Thomas E and Maurry J Tamarkin (1981). “On diversification given asymmetry in returns”. *The Journal of Finance* 36.5, pp. 1143–1155.

Constantinides, George M et al. (2011). “Are options on index futures profitable for risk-averse investors? Empirical evidence”. *The Journal of Finance* 66.4, pp. 1407–1437.

Cont, R (2001). “Empirical properties of asset returns: stylized facts and statistical issues”. *Quantitative Finance* 1.2, pp. 223–236.

Cooper, Michael J, Orlin Dimitrov, and P Raghavendra Rau (2001). “A rose. com by any other name”. *The Journal of Finance* 56.6, pp. 2371–2388.

Cooper, Michael J, Huseyin Gulen, and P Raghavendra Rau (2005). “Changing names with style: Mutual fund name changes and their effects on fund flows”. *The Journal of Finance* 60.6, pp. 2825–2858.

Cooper, Michael J et al. (2005). “Managerial actions in response to a market downturn: Valuation effects of name changes in the dot. com decline”. *Journal of Corporate Finance* 11.1, pp. 319–335.

Corwin, Shane A and Jay F Coughenour (2008). “Limited attention and the allocation of effort in securities trading”. *The Journal of Finance* 63.6, pp. 3031–3067.

Coval, Joshua and Erik Stafford (2007). “Asset fire sales (and purchases) in equity markets”. *Journal of Financial Economics* 86.2, pp. 479–512.

Crainich, David, Louis Eeckhoudt, and Olivier Le Courtois (2014). “Decreasing downside risk aversion and background risk”. *Journal of Mathematical Economics* 53, pp. 59–63.

Cremers, Martijn, Michael Halling, and David Weinbaum (2015). “Aggregate Jump and Volatility Risk in the Cross-Section of Stock Returns”. *The Journal of Finance* 70.2, pp. 577–614.

Cvitanić, Jakša, Levon Goukasian, and Fernando Zapatero (2003). “Monte Carlo computation of optimal portfolios in complete markets”. *Journal of Economic Dynamics and Control* 27.6, pp. 971–986.

Cvitanić, Jakša, Vassilis Polimenis, and Fernando Zapatero (2008). “Optimal portfolio allocation with higher moments”. *Annals of Finance* 4.1, pp. 1–28.

Damodaran, Aswath (2003). *Investment Philosophies: Successful Strategies and the Investors Who Made Them Work*. Vol. 185. John Wiley & Sons.

Davidson, Russell (2009). “Testing for restricted stochastic dominance: Some further results”. *Review of Economic Analysis* 1.1, pp. 34–59.

De Athayde, Gustavo M and Renato G Flôres (2004). “Finding a maximum skewness portfolio – A general solution to three-moments portfolio choice”. *Journal of Economic Dynamics and Control* 28.7, pp. 1335–1352.

De Giorgi, Enrico and Thierry Post (2008). "Second-order stochastic dominance, reward-risk portfolio selection, and the CAPM". *Journal of Financial and Quantitative Analysis* 43.2, pp. 525–546.

DellaVigna, Stefano and Joshua M Pollet (2007). "Demographics and industry returns". *The American Economic Review*, pp. 1667–1702.

— (2009). "Investor inattention and Friday earnings announcements". *The Journal of Finance* 64.2, pp. 709–749.

DeMiguel, Victor, Lorenzo Garlappi, and Raman Uppal (2007). "Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy?" *The Review of Financial studies* 22.5, pp. 1915–1953.

Detemple, Jérôme B, René Garcia, and Marcel Rindisbacher (2003). "A Monte Carlo method for optimal portfolios". *The Journal of Finance* 58.1, pp. 401–446.

Dittmar, Robert F (2002). "Nonlinear pricing kernels, kurtosis preference, and evidence from the cross section of equity returns". *The Journal of Finance* 57.1, pp. 369–403.

Drechsler, Itamar (2013). "Uncertainty, time-varying fear, and asset prices". *The Journal of Finance* 68.5, pp. 1843–1889.

Eckbo, B Espen (2014). "Corporate takeovers and economic efficiency". *Annual Review of Financial Economics* 6.1, pp. 51–74.

Edmans, Alex, Itay Goldstein, and Wei Jiang (2012). "The real effects of financial markets: The impact of prices on takeovers". *The Journal of Finance* 67.3, pp. 933–971.

Eeckhoudt, Louis and Harris Schlesinger (2008). "Changes in risk and the demand for saving". *Journal of Monetary Economics* 55.7, pp. 1329–1336.

Engelberg, Joseph E and Christopher A Parsons (2011). "The causal impact of media in financial markets". *The Journal of Finance* 66.1, pp. 67–97.

Eraker, Bjørn, Michael Johannes, and Nicholas Polson (2003). "The impact of jumps in volatility and returns". *The Journal of Finance* 58.3, pp. 1269–1300.

Eun, Cheol S, Wei Huang, and Sandy Lai (2008). "International diversification with large-and small-cap stocks". *Journal of Financial and Quantitative Analysis* 43.2, pp. 489–524.

Fama, Eugene F (1991). "Efficient capital markets: II". *The Journal of Finance* 46.5, pp. 1575–1617.

Fama, Eugene F and Kenneth R French (1992). "The cross-section of expected stock returns". *The Journal of Finance* 47.2, pp. 427–465.

— (1993). "Common risk factors in the returns on stocks and bonds". *Journal of Financial Economics* 33.1, pp. 3–56.

Fama, Eugene F et al. (1969). "The adjustment of stock prices to new information". *International Economic Review* 10.1, pp. 1–21.

Fang, Hsing and Tsong-Yue Lai (1997). "Co-kurtosis and capital asset pricing". *Financial Review* 32.2, pp. 293–307.

Fang, Yi and Thierry Post (2017). "Higher-degree stochastic dominance optimality and efficiency". *European Journal of Operational Research* 261.3, pp. 984–993.

Frazzini, Andrea and Lasse Heje Pedersen (2014). "Betting against beta". *Journal of Financial Economics* 111.1, pp. 1–25.

Frijns, Bart et al. (2013). "Uncertainty avoidance, risk tolerance and corporate takeover decisions". *Journal of Banking & Finance* 37.7, pp. 2457–2471.

Fuller, Kathleen, Jeffry Netter, and Mike Stegemoller (2002). "What do returns to acquiring firms tell us? Evidence from firms that make many acquisitions". *The Journal of Finance* 57.4, pp. 1763–1793.

Fuller, Russell J, Raife Giovinazzo, and Yining Tung (2014). "The stable ROE portfolio: An alternative equity index strategy based on common sense security analysis". *Journal of Portfolio Management* 40.5, pp. 135–145.

Gabaix, Xavier (2012). "Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance". *The Quarterly Journal of Economics* 127.2, pp. 645–700.

Garlappi, Lorenzo and Georgios Skoulakis (2011). "Taylor series approximations to expected utility and optimal portfolio choice". *Mathematics and Financial Economics* 5.2, p. 121.

Green, T Clifton and Russell Jame (2013). "Company name fluency, investor recognition, and firm value". *Journal of Financial Economics* 109.3, pp. 813–834.

Griffin, John M (2002). "Are the Fama and French factors global or country specific?" *The Review of Financial Studies* 15.3, pp. 783–803.

Grinblatt, Mark and Matti Keloharju (2001). "How distance, language, and culture influence stockholdings and trades". *The Journal of Finance* 56.3, pp. 1053–1073.

Grullon, Gustavo, Sébastien Michenaud, and James P Weston (2015). "The real effects of short-selling constraints". *The Review of Financial Studies* 28.6, pp. 1737–1767.

Guiso, Luigi, Paola Sapienza, and Luigi Zingales (2009). "Cultural biases in economic exchange?" *The Quarterly Journal of Economics* 124.3, pp. 1095–1131.

Hadar, Josef and William R Russell (1969). "Rules for ordering uncertain prospects". *The American Economic Review* 59.1, pp. 25–34.

Hadar, Josef and Tae Kun Seo (1990). "The effects of shifts in a return distribution on optimal portfolios". *International Economic Review* 31.3, pp. 721–36.

Haensly, Paul J, Niranjan Tripathy, and Daniel Peak (2001). "Tracking error in the Dow Jones industrial average versus alternative market indices: New evidence". *Quarterly Journal of Business and Economics*, pp. 101–116.

Hanoch, Giora and Haim Levy (1969). "Efficiency analysis of choices involving risk". *Review of Economic Studies* 36.3, pp. 335–46.

Hardy, Godfrey Harold, John Edensor Littlewood, and George Pólya (1934). *Inequalities*. Cambridge University Press.

Harford, Jarrad (2005). "What drives merger waves?" *Journal of Financial Economics* 77.3, pp. 529–560.

Harvey, Campbell R et al. (2010). "Portfolio selection with higher moments". *Quantitative Finance* 10.5, pp. 469–485.

Hirshleifer, David (2001). "Investor psychology and asset pricing". *The Journal of Finance* 56.4, pp. 1533–1597.

Hirshleifer, David, Sonya S Lim, and Siew Hong Teoh (2009). "Driven to distraction: Extraneous events and underreaction to earnings news". *The Journal of Finance* 64.5, pp. 2289–2325.

— (2011). "Limited investor attention and stock market misreactions to accounting information". *The Review of Asset Pricing Studies* 1.1, pp. 35–73.

Hirshleifer, David and Siew Hong Teoh (2003). "Limited attention, information disclosure, and financial reporting". *Journal of Accounting and Economics* 36.1, pp. 337–386.

Hodder, James E, Jens Carsten Jackwerth, and Olga Kolokolova (2014). "Improved portfolio choice using second-order stochastic dominance". *Review of Finance* 19.4, pp. 1623–1647.

Hou, Kewei and Tobias J Moskowitz (2005). "Market frictions, price delay, and the cross-section of expected returns". *The Review of Financial Studies* 18.3, pp. 981–1020.

Hsu, Jason C (2006). "Cap-weighted portfolios are sub-optimal portfolios". *Journal of Investment Management* 4.3, pp. 1–10.

Huang, Chi-Fu and Robert H Litzenberger (1988). *Foundations for Financial Economics*. Prentice Hall.

Huang, Jiekun and Darren J Kisgen (2013). "Gender and corporate finance: Are male executives overconfident relative to female executives?" *Journal of Financial Economics* 108.3, pp. 822–839.

Huberman, Gur and Tomer Regev (2001). "Contagious speculation and a cure for cancer: A nonevent that made stock prices soar". *The Journal of Finance* 56.1, pp. 387–396.

Idzorek, Thomas M (2011). "A step-by-step guide to the Black-Litterman model". *Forecasting Expected Returns in the Financial Markets*. Ed. by Stephen Satchell. Academic Press, New York. Chap. 2, pp. 17–38.

Jensen, Michael C (1988). "Takeovers: Their causes and consequences". *The Journal of Economic Perspectives* 2.1, pp. 21–48.

Jensen, Michael C and Richard S Ruback (1983). "The market for corporate control: The scientific evidence". *Journal of Financial Economics* 11.1-4, pp. 5–50.

Jobson, John D and Bob Korkie (1982). "Potential performance and tests of portfolio efficiency". *Journal of Financial Economics* 10.4, pp. 433–466.

Jondeau, Eric and Michael Rockinger (2006). "Optimal portfolio allocation under higher moments". *European Financial Management* 12.1, pp. 29–55.

Jorion, Philippe and William N Goetzmann (1999). "Global stock markets in the twentieth century". *The Journal of Finance* 54.3, pp. 953–980.

Jurczenko, Emmanuel, Bertrand Maillet, and Paul Merlin (2006). "Hedge fund portfolio selection with higher-order moments: A nonparametric mean–variance–skewness–kurtosis efficient frontier". *Multi-moment Asset Allocation and Pricing Models*. Ed. by Emmanuel Jurczenko and Bertrand Maillet. John Wiley & Sons. Chap. 3, pp. 51–66.

Kahneman, Daniel (1973). *Attention and Effort*. Prentice-Hall Englewood Cliffs, NJ.

Karpoff, Jonathan M and Graeme Rankine (1994). "In search of a signaling effect: The wealth effects of corporate name changes". *Journal of Banking & Finance* 18.6, pp. 1027–1045.

Kerstens, Kristiaan, Amine Mounir, and Ignace Van de Woestyne (2011). "Geometric representation of the mean–variance–skewness portfolio frontier based upon the shortage function". *European Journal of Operational Research* 210.1, pp. 81–94.

Kimball, Miles S (1990). "Precautionary Saving in the Small and in the Large". *Econometrica*, pp. 53–73.

— (1991). *Precautionary motives for holding assets*. Tech. rep. National Bureau of Economic Research.

— (1992). "Precautionary motives for holding assets". *New Palgrave Dictionary of Money and Finance*. Ed. by Peter Newman, Murray Milgate, and John Eatwell. Vol. 3. Stockton Press, New York, pp. 158–161.

Klibanoff, Peter, Owen Lamont, and Thierry A Wizman (1998). "Investor reaction to salient news in closed-end country funds". *The Journal of Finance* 53.2, pp. 673–699.

Kolari, James W and Seppo Pynnönen (2010). "Event study testing with cross-sectional correlation of abnormal returns". *The Review of Financial Studies* 23.11, pp. 3996–4025.

Kolokolova, Olga, Olivier Le Courtois, and Xia Xu (2017). "Is it efficient to buy the index? A worldwide tour with stochastic dominance". Working Paper.

Kopa, Miloš and Thierry Post (2009). "A portfolio optimality test based on the first-order stochastic dominance criterion". *Journal of Financial and Quantitative Analysis* 44.5, pp. 1103–1124.
— (2015). "A general test for SSD portfolio efficiency". *OR spectrum* 37.3, pp. 703–734.
Koren, Miklós and Silvana Tenreyro (2007). "Volatility and development". *The Quarterly Journal of Economics* 122.1, pp. 243–287.
Kothari, SP and Jerold B Warner (2008). "Econometrics of event studies". *Handbook of Empirical Corporate Finance*. Ed. by B Espen Eckbo. Vol. 2. Elsevier. Chap. 1, pp. 3–36.
Kraus, Alan and Robert H Litzenberger (1976). "Skewness preference and the valuation of risk assets". *The Journal of Finance* 31.4, pp. 1085–1100.
Kumar, Alok, Alexandra Niessen-Ruenzi, and Oliver G Spalt (2015). "What's in a name? Mutual fund flows when managers have foreign-sounding names". *The Review of Financial Studies* 28.8, pp. 2281–2321.
Kuosmanen, Timo (2004). "Efficient diversification according to stochastic dominance criteria". *Management Science* 50.10, pp. 1390–1406.
La Porta, Rafael et al. (1998). "Law and finance". *Journal of Political Economy* 106.6, pp. 1113–1155.
Lai, Tsong-Yue (1991). "Portfolio selection with skewness: a multiple-objective approach". *Review of Quantitative Finance and Accounting* 1.3, pp. 293–305.
Lamont, Owen A and Richard H Thaler (2003). "Anomalies: The law of one price in financial markets". *Journal of Economic Perspectives* 17.4, pp. 191–202.
Le Courtois, Olivier and Xia Xu (2017a). "Corporate name changes of M&As among S&P 500 index". Working Paper.
— (2017b). "Portfolio optimization when bond market can crash". Working Paper.
— (2018). "DJIA is more efficient than thought". Working Paper.
Lehmann, EL et al. (1955). "Ordered Families of Distributions". *The Annals of Mathematical Statistics* 26.3, pp. 399–419.
Levy, Haim (1992). "Stochastic dominance and expected utility: survey and analysis". *Management Science* 38.4, pp. 555–593.
— (2016). *Stochastic dominance: Investment decision making under uncertainty*. 3rd ed. Springer.
Levy, Haim and Harry M Markowtiz (1979). "Approximating expected utility by a function of mean and variance". *American Economic Review* 69.3, pp. 308–17.
Lintner, John (1965). "Security prices, risk, and maximal gains from diversification". *The Journal of Finance* 20.4, pp. 587–615.
Liu, Jun, Francis A Longstaff, and Jun Pan (2003). "Dynamic asset allocation with event risk". *The Journal of Finance* 58.1, pp. 231–259.
Lo, Andrew W (2007). *Efficient Markets Hypothesis The New Palgrave: A Dictionary of Economics*.
— (2016). "What is an index?" *Journal of Portfolio Management* 42.2, pp. 21–36.
Longarela, Iñaki R (2015). "A characterization of the SSD-efficient frontier of portfolio weights by means of a set of mixed-integer linear constraints". *Management Science* 62.12, pp. 3549–3554.
Longin, Francois and Bruno Solnik (2001). "Extreme correlation of international equity markets". *The Journal of Finance* 56.2, pp. 649–676.
Lou, Dong (2014). "Attracting investor attention through advertising". *The Review of Financial Studies* 27.6, pp. 1797–1829.
Loughran, Tim and Jay R Ritter (2000). "Uniformly least powerful tests of market efficiency". *Journal of Financial Economics* 55.3, pp. 361–389.

Lynch, Anthony W and Richard R Mendenhall (1997). "New evidence on stock price effects associated with changes in the S&P 500 Index". *The Journal of Business* 70.3, pp. 351–383.

Lyon, John D, Brad M Barber, and Chih-Ling Tsai (1999). "Improved methods for tests of long-run abnormal stock returns". *The Journal of Finance* 54.1, pp. 165–201.

Mailath, George J and Larry Samuelson (2001). "Who wants a good reputation?" *The Review of Economic Studies* 68.2, pp. 415–441.

Malmendier, Ulrike and Stefan Nagel (2011). "Depression babies: do macroeconomic experiences affect risk taking?" *The Quarterly Journal of Economics* 126.1, pp. 373–416.

Malmendier, Ulrike and Geoffrey Tate (2008). "Who makes acquisitions? CEO overconfidence and the market's reaction". *Journal of Financial Economics* 89.1, pp. 20–43.

Mann, Henry B and Donald R Whitney (1947). "On a test of whether one of two random variables is stochastically larger than the other". *The Annals of Mathematical Statistics* 18.1, pp. 50–60.

Markowitz, Harry (1952). "Portfolio selection". *The Journal of Finance* 7.1, pp. 77–91.

— (1959). *Portfolio Selection: Efficient Diversification of Investments*. John Wiley & Sons, New York.

Marshall, Albert W and Ingram Olkin (1979). *Inequalities: Theory of Majorization and Its Applications*. Academic Press, New York.

Martynova, Marina and Luc Renneboog (2008). "A century of corporate takeovers: What have we learned and where do we stand?" *Journal of Banking & Finance* 32.10, pp. 2148–2177.

Masulis, Ronald W, Cong Wang, and Fei Xie (2007). "Corporate governance and acquirer returns". *The Journal of Finance* 62.4, pp. 1851–1889.

McLean, R David and Jeffrey Pontiff (2016). "Does academic research destroy stock return predictability?" *The Journal of Finance* 71.1, pp. 5–32.

Mencía, Javier and Enrique Sentana (2009). "Multivariate location–scale mixtures of normals and mean–variance–skewness portfolio allocation". *Journal of Econometrics* 153.2, pp. 105–121.

Merton, Robert C (1992). *Continuous-Time Finance*. reprint. Wiley-Blackwell.

Michaud, Richard O and Robert O Michaud (2008). *Efficient Asset Management: A Practical Guide to Stock Portfolio Optimization and Asset Allocation*. 2nd ed. Oxford University Press.

Mitchell, Mark, Todd Pulvino, and Erik Stafford (2004). "Price pressure around mergers". *The Journal of Finance* 59.1, pp. 31–63.

Mitton, Todd and Keith Vorkink (2007). "Equilibrium underdiversification and the preference for skewness". *The Review of Financial Studies* 20.4, pp. 1255–1288.

Moeller, Sara B, Frederik P Schlingemann, and René M Stulz (2004). "Firm size and the gains from acquisitions". *Journal of Financial Economics* 73.2, pp. 201–228.

Mossin, Jan (1966). "Equilibrium in a capital asset market". *Econometrica* 34.4, pp. 768–783.

Netter, Jeffry, Mike Stegemoller, and M Babajide Wintoki (2011). "Implications of data screens on merger and acquisition analysis: A large sample study of mergers and acquisitions from 1992 to 2009". *The Review of Financial Studies* 24.7, pp. 2316–2357.

Palomino, Frederic, Luc Renneboog, and Chendi Zhang (2009). "Information salience, investor sentiment, and stock returns: The case of British soccer betting". *Journal of Corporate Finance* 15.3, pp. 368–387.

Patel, Nimesh and Ivo Welch (2017). "Extended Stock Returns in Response to S&P 500 Index Changes". *The Review of Asset Pricing Studies* 7.2, pp. 172–208.

Peng, Lin and Wei Xiong (2006). "Investor attention, overconfidence and category learning". *Journal of Financial Economics* 80.3, pp. 563–602.

Perold, André F (2007). "Fundamentally flawed indexing". *Financial Analysts Journal* 63.6, pp. 31–37.

Poon, Ser-Huang, Michael Rockinger, and Jonathan Tawn (2003). "Extreme value dependence in financial markets: Diagnostics, models, and financial implications". *The Review of Financial Studies* 17.2, pp. 581–610.

Porter, R Burr and Roger P Bey (1974). "An evaluation of the empirical significance of optimal seeking algorithms in portfolio selection". *The Journal of Finance* 29.5, pp. 1479–1490.

Porter, R Burr and Jack E Gaumnitz (1972). "Stochastic dominance vs. mean-variance portfolio analysis: an empirical evaluation". *The American Economic Review* 62.3, pp. 438–446.

Post, Thierry (2003). "Empirical tests for stochastic dominance efficiency". *The Journal of Finance* 58.5, pp. 1905–1931.

— (2016). "Empirical tests for stochastic dominance optimality". *Review of Finance* 21.2, pp. 793–810.

Post, Thierry, Selcuk Karabati, and Stelios Arvanitis (2018). "Portfolio optimization based on stochastic dominance and empirical likelihood". *Journal of Econometrics* forthcoming.

Post, Thierry and Miloš Kopa (2013). "General linear formulations of stochastic dominance criteria". *European Journal of Operational Research* 230.2, pp. 321–332.

— (2017). "Portfolio choice based on third-degree stochastic dominance". *Management Science* 63.10, pp. 3381–3392.

Post, Thierry and Haim Levy (2005). "Does risk seeking drive stock prices? A stochastic dominance analysis of aggregate investor preferences and beliefs". *The Review of Financial Studies* 18.3, pp. 925–953.

Post, Thierry and Philippe Versijp (2007). "Multivariate tests for stochastic dominance efficiency of a given portfolio". *Journal of Financial and Quantitative Analysis* 42.2, pp. 489–515.

Pratt, John W (1964). "Risk aversion in the small and in the large". *Econometrica* 32.1/2, pp. 122–136.

Quirk, James P and Rubin Saposnik (1962). "Admissibility and measurable utility functions". *The Review of Economic Studies* 29.2, pp. 140–146.

Rashes, Michael S (2001). "Massively confused investors making conspicuously ignorant choices (MCI–MCIC)". *The Journal of Finance* 56.5, pp. 1911–1927.

Reinganum, Marc R (2014). "Anchored in reality or blinded by a paradigm: The role of cap-weighted indices in the future". *Journal of Portfolio Management* 40.5, pp. 119–125.

Rob, Rafael and Arthur Fishman (2005). "Is bigger better? Customer base expansion through word-of-mouth reputation". *Journal of Political Economy* 113.5, pp. 1146–1162.

Roll, Richard (1977). "A critique of the asset pricing theory's tests Part I: On past and potential testability of the theory". *Journal of Financial Economics* 4.2, pp. 129–176.

— (1986). "The hubris hypothesis of corporate takeovers". *Journal of Business* 59.2, pp. 197–216.

Rosenbaum, Eric (2017). "Dow at 22,000: All eyes on an index few investors actually put money in". *CNBC*. [Online; posted August 2, 2017; accessed March 30, 2017].

URL: https://www.cnbc.com/2017/08/02/dow-at-22k-everyone-watches-market-index-but-no-one-invests-in-it.html.

Rothschild, Michael and Joseph E Stiglitz (1970). "Increasing risk: I. A definition". *Journal of Economic Theory* 2.3, pp. 225–243.

Santa-Clara, Pedro and Shu Yan (2010). "Crashes, volatility, and the equity premium: Lessons from S&P 500 options". *The Review of Economics and Statistics* 92.2, pp. 435–451.

Savor, Pavel G and Qi Lu (2009). "Do stock mergers create value for acquirers?" *The Journal of Finance* 64.3, pp. 1061–1097.

Shalit, Haim (2010). "Finding better securities while holding portfolios: Is stochastic dominance the answer?" *Journal of Portfolio Management* 37.1, pp. 31–42.

Sharpe, William F (1964). "Capital asset prices: A theory of market equilibrium under conditions of risk". *The Journal of Finance* 19.3, pp. 425–442.

Shefrin, Hersh and Meir Statman (2000). "Behavioral portfolio theory". *Journal of Financial and Quantitative Analysis* 35.2, pp. 127–151.

Shleifer, Andrei (1986). "Do demand curves for stocks slope down?" *The Journal of Finance* 41.3, pp. 579–590.

Shoven, John B and Clemens Sialm (2000). "The dow jones industrial average: the impact of fixing its flaws". *Journal of Wealth Management* 3.3, pp. 9–18.

So, Eric C (2013). "A new approach to predicting analyst forecast errors: Do investors overweight analyst forecasts?" *Journal of Financial Economics* 108.3, pp. 615–640.

Stein, Jeremy C (2009). "Presidential address: Sophisticated investors and market efficiency". *The Journal of Finance* 64.4, pp. 1517–1548.

Sunder, Shyam (1980). "Stationarity of market risk: Random coefficients tests for individual stocks". *The Journal of Finance* 35.4, pp. 883–896.

Tadelis, Steven (1999). "What's in a name? Reputation as a tradeable asset". *American Economic Review* 89.3, pp. 548–563.

Tehranian, Hassan (1980). "Empirical studies in portfolio performance using higher degrees of stochastic dominance". *The Journal of Finance* 35.1, pp. 159–171.

Tetlock, Paul C (2011). "All the news that's fit to reprint: Do investors react to stale information?" *The Review of Financial Studies* 24.5, pp. 1481–1512.

Thaler, Richard (2009). "Markets can be wrong and the price is not always right". *Financial Times*. [Online; posted August 4, 2009; accessed March 30, 2017]. URL: https://www.ft.com/content/efc0e92e-8121-11de-92e7-00144feabdc0.

The Economist (2014). "What are brands for?" *The Economist*. [Online; posted August 30, 2014; accessed March 30, 2017]. URL: https://www.economist.com/news/business/21614150-brands-are-most-valuable-assets-many-companies-possess-no-one-agrees-how-much-they.

Todorov, Viktor (2009). "Variance risk-premium dynamics: The role of jumps". *The Review of Financial Studies* 23.1, pp. 345–383.

Treynor, Jack (2005). "Why market-valuation-indifferent indexing works". *Financial Analysts Journal* 61.5, pp. 65–69.

Tversky, Amos and Daniel Kahneman (1986). "Rational choice and the framing of decisions". *Journal of Business*, S251–S278.

Uppal, Raman and Tan Wang (2003). "Model misspecification and underdiversification". *The Journal of Finance* 58.6, pp. 2465–2486.

Wachter, Jessica A (2013). "Can time-varying risk of rare disasters explain aggregate stock market volatility?" *The Journal of Finance* 68.3, pp. 987–1035.

Weber, Elke U and Christopher Hsee (1998). "Cross-cultural differences in risk perception, but cross-cultural similarities in attitudes towards perceived risk". *Management Science* 44.9, pp. 1205–1217.

Yan, Xuemin and Lingling Zheng (2017). “Fundamental analysis and the cross-section of stock returns: A data-mining approach”. *The Review of Financial Studies* 30.4, pp. 1382–1423.