Notices

OF THE AMERICAN MATHEMATICAL SOCIETY

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Notices

OF THE

AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

CONTENTS

MEETINGS

Calendar of Meetings

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts*
684	April 7-10, 1971	New York, New York	Feb. 22, 1971
685	April 24, 1971	Monterey, California	Feb. 22, 1971
686	June 19, 1971	Corvallis, Oregon	May 5, 1971
687	August 30-September 3, 1971 (76th Summer Meeting)	University Park, Pennsylvania	July 7, 1971
688	October 30, 1971	Cambridge, Massachusetts	
	November 19-20, 1971	Auburn, Alabama	
	November 27, 1971	Milwaukee, Wisconsin	
	January 17-21, 1972 (78th Annual Meeting)	Las Vegas, Nevada	

*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadlines for by-title abstracts will be April 28, 1971, and June 30, 1971.

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Six Hundred Eighty-Third Meeting
University of Illinois at Chicago Circle
Chicago, Illinois
March 26-27, 1971

The six hundred eighty-third meeting of the American Mathematical Society will be held at the University of Illinois at Chicago Circle, Chicago, Illinois, on Friday and Saturday, March 26-27, 1971. All sessions of the meeting will be held in the Lecture Center of the university. The university is located approximately one mile west and one-half mile south of the intersection of State and Madison Streets, the origin of coordinates in the Chicago street numbering system.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be four one-hour addresses. Professor Marvin I. Knopp of the University of Wisconsin and the University of Illinois at Chicago Circle will speak on Friday, March 26, at 11:00 a.m.; his topic will be "Eichler cohomology and the Fourier coefficients of automorphic forms." Professor John H. Walter of the University of Illinois at Urbana-Champaign will address the Society on Friday, March 26, at 1:45 p.m.; his subject will be "The structure of the centralizers of involutions in finite simple groups." Professor Robert Ellis of the University of Minnesota will speak on Saturday, March 27, at 11:00 a.m.; his talk will be entitled "Recent results in the algebraic theory of minimal sets." Professor Allen L. Shields of the University of Michigan will address the Society on Saturday, March 27, at 1:45 p.m.; his topic will be "Spaces of analytic functions: some recent results." All four hour talks will be presented in Room A1 of the Lecture Center.

By invitation of the same committee there will be five special sessions of selected twenty-minute papers. Professor Eben Matlis of the Northwestern University has arranged one such session for Friday on the subject of Commutative Algebra; the speakers will be Shreeram S. Abhyankar, John A. Eagon, Robert M. Fossum, William J. Heinzer, Eben Matlis, Tsuong-Tsieng Moh, David L. Shannon, Michael R. Stein, and Roger P. Ware. Another special session has been arranged by Professor Jim Douglas, Jr., of the University of Chicago on the subject of Numerical Solution of Partial Differential Equations, to be held on Friday afternoon and Saturday morning; the speakers will be Ivo Babuška, James H. Bramble, Jim Douglas, Jr., Todd Dupont, George J. Fix, Henry H. Rachford, Jr., Marco Antonio Raupp, Martin H. Schultz, W. Gilbert Strang, and Mary Fanett Wheeler. Professor Edward R. Fadell of the University of Wisconsin has organized a session for Friday afternoon and Saturday morning on Fixed Point and Coincidence Theory; the speakers will be Robin B. S. Brooks, Felix E. Browder, Andrzej Granas, John Philip Huneke, Ronald J. Knill, Kalyan K. Mukherjea, Roger D. Nussbaum, and Richard B. Thompson. Another session, to be held on Saturday, has been arranged by Professor Steven B. Bank of the University of Illinois at Urbana-Champaign on the subject of Growth, Oscillation, and Asymptotic Properties of Solutions of Ordinary Differential Equations; the speakers will be Steven B. Bank, Harold E. Benzinger, William A. Harris, Jr., Po-Fang Hsieh, Donald A. Lutz, Zeev Nehari, William T. Reid, Thomas L. Sherman, Gilbert Stengle, Walter C. Strodt, Robert K. Wright, and Chung-Chun Yang. Finally, Professor Leon W. Green of the University of Minnesota has arranged a special session for Saturday on the subject of Flows; the speakers will be Kenneth R. Berg, Hsin Chu, Charles C. Conley, Patrick B. Eberlein, Charles C. Pugh, Leonard D. Shapiro, William A. Veech, and Peter Walters. There will also be eleven sessions for the presentation of contributed ten-minute papers.

REGISTRATION

The registration desk will be located in the Illinois Room Lobby on the third level of the Chicago Circle Center, the student center of the University of Illinois at Chicago Circle. The Center is located on the west side of Halsted Street, opposite the point at which Polk Street comes to a dead end. (The Lecture Center adjoins the Chicago Circle Center on the west.) The registration desk will be open from 8:30 a.m. to 4:30 p.m. on Friday, March 26, and from 8:30 a.m. to 3:30 p.m. on Saturday, March 27.

ACCOMMODATIONS

The hotel headquarters for the meeting will be the Pick-Congress Hotel. However, those coming by car may prefer to stay at either the Holiday Inn (Kennedy Expressway) or the Ramada Inn (Downtown). Detailed information about these three hotels is given below. The rates quoted are subject to an 8% tax. A form which may be used for making reservations at the Pick-Congress Hotel can be found on page 304 of the January *Notices*. Those making reservations at either the Holiday Inn or the Ramada Inn should simply specify that they will be attending the meeting of the American Mathematical Society as guests of the University of Illinois at Chicago Circle.

1. Pick-Congress Hotel, 520 South Michigan Avenue, Chicago, Illinois 60605. One and one quarter miles east of the university. Room rates are $18.00 to $27.00 for single occupancy and $26.00 to $35.00 for double occupancy. There is a delivery service garage.

2. Holiday Inn (Kennedy Expressway), 1 South Halsted Street, Chicago, Illinois 60606. One-half mile north of the university. Room rates are $15.00 for single occupancy and $19.00 for double occupancy. There is a free garage.

3. Ramada Inn (Downtown), 506 West Harrison Street, Chicago, Illinois 60607. One-half mile east of the university. Room rates are $15.00 for single occupancy and $17.00 for double occupancy. Free delivery-service parking is available.

FOOD SERVICE

A cafeteria in the Chicago Circle Center will be open for lunch on both days of the meeting. There are several Greek restaurants on Halsted Street just north of the campus. All three hotels listed have dining facilities. In addition, there are several restaurants on Wabash Avenue near the Pick-Congress Hotel. For those wishing to venture farther afield the list of dining possibilities is endless.

TRAVEL AND LOCAL INFORMATION

The University of Illinois at Chicago Circle is named after, served by, and located at the Chicago Circle cloverleaf formed by the three major expressways into the downtown area of Chicago from the north, south, and west. Those coming from the west on the Dwight D. Eisenhower Expressway should use the Racine Avenue Exit. Those coming from the north on the John F. Kennedy Expressway should turn west on the Eisenhower Expressway and take the Morgan Street Exit. Those coming from the south on the Dan Ryan Expressway should exit at Taylor Street. There will be free parking for those attending the meeting in the two parking lots at the corner of Polk and Halsted Streets (across Halsted Street from the Chicago Circle Center). Those wishing to avail themselves of this privilege should present a copy of the *Notices*.

There is direct limousine service between O'Hare Airport and the Pick-Congress Hotel. All three hotels and the Chicago Circle Campus are close to the major railroad stations.

The Holiday Inn and the Ramada Inn are within walking distance of the campus. There are two easy methods of getting to the campus by public transportation from the Pick-Congress Hotel: (a) Walk two blocks west and then take the Number 7 bus from its starting point on the west side of State Street between Harrison and Congress Streets, (b) Walk three blocks west and two blocks north, enter the subway on Dearborn Street north of Van Buren Street, and take either the Congress or the Douglas train to the

Halsted Street stop. Those preferring to go by taxi should ask to be taken to the intersection of Polk and Halsted Streets.

ENTERTAINMENT

The Chicago Symphony Orchestra will give performances of Verdi's Requiem in Orchestra Hall on Thursday, March 25, at 8:15 p.m. and on Saturday, March 27, at 8:30 p.m. Tickets are priced at $5.50, $7.00, $8.00, $9.00, $9.50, and $10.00 and may be ordered from the Box Office, Chicago Symphony Orchestra, Orchestra Hall, 220 South Michigan Avenue, Chicago, Illinois 60604. Orchestra Hall is three blocks north of the Pick-Congress Hotel.

Pianist Byron Janis will give a recital in Orchestra Hall on Sunday, March 28, at 3:00 p.m. Tickets are priced at $3.50, $4.50, $5.50, $6.50, and $7.50 and may be ordered from Allied Arts Corporation, 20 North Wacker Drive, Chicago, Illinois 60606.

The American Ballet Theatre will perform on Thursday, Friday, and Saturday evenings at 8:30 p.m. and on Sunday afternoon at 3:00 p.m. in the Auditorium Theatre, which is next door to the Pick-Congress Hotel. Tickets are priced at $2.50, $3.50, $5.00, $7.00, $8.00, $9.00, and $10.00 and may be ordered from the Auditorium Theatre, 70 East Congress Parkway, Chicago, Illinois 60605.

The Art Institute of Chicago is located three blocks north of the Pick-Congress Hotel. It is open from 10:00 a.m. to 8:30 p.m. on Thursday, from 8:30 a.m. to 5:00 p.m. on Friday and Saturday, and from 1:00 p.m. to 6:00 p.m. on Sunday. At the time of the meeting it will have a special exhibit of nineteenth century German paintings.

The Chicago Circle Campus contains the original site of the Jane Addams' Hull House and Residents' Dining Room. Both have been restored by the University of Illinois and are designated as National Historic Landmarks by the U. S. Department of the Interior. They are open to visitors from 10:00 a.m. to 4:00 p.m. every day but Sunday.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper in the general sessions is 10 minutes; they are scheduled at 15 minute intervals. To maintain the schedule for both the general and special sessions, time limits will be strictly enforced.

FRIDAY, 9:10 A.M.

Special Session on Commutative Algebra, Room C1

9:10-9:30

(1) On analytic independence
Mr. Tsuong-Tsieng Moh, Purdue University (683-A24)
(Introduced by Professor Paul T. Bateman)

9:35-9:55

(2) Uniqueness of the coefficient ring in a polynomial ring
Professor Shreeram S.Abhyankar and Professor William J. Heinzer*, Purdue University, and Mr. Paul Eakin, University of Kentucky (683-A16)

10:00-10:20

(3) Epimorphisms of polynomial rings
Professor Shreeram S. Abhyankar, Purdue University (683-A15)

10:25-10:45

(4) The category of maps; coherence and Gorenstein properties
Professor Robert M. Fossum* and Professor Phillip A.Griffith, University of Illinois at Champaign-Urbana (683-A12)

FRIDAY, 9:15 A.M.

Session on General Topology, Room C3

9:15-9:25

(5) Triods, commuting functions, and FPP-less plane continua. Preliminary report
Dr. William M. Boyce, Bell Telephone Laboratories, Murray Hill, New Jersey (683-G15)

9:30-9:40

(6) Subnormal and normal spaces. Preliminary report
Professor J. M. Boyte* and Professor Ernest P. Lane, Appalachian State University (683-G2)

9:45-9:55

(7) No nondegenerate space satisfying Axiom Ω is connected
Mr. Jerrel K. Yates, University of Mississippi (683-G16)

10:00-10:10

(8) Pushing apart a locally finite collection of disjoint closed subsets in a normed linear space
Professor Douglas W. Curtis, Louisiana State University (683-G23)

10:15-10:25

(9) Some fixed point theorems in a reflexive Banach space
Mr. R. Kannan, Purdue University (683-G20)

10:30-10:40

(10) Fixed point theorems for set valued transformations on compact sets
Mr. Nadim A. Assad, University of Iowa (683-G1)
(Introduced by Professor William A. Kirk)

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*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

FRIDAY, 9:15 A.M.

Session on Algebraic Topology, Room C4

9:15-9:25

(11) A generalized Mayer-Vietoris sequence. Preliminary report
Mr. Ming-Jung Lee, University of Rochester (683-G3)

9:30-9:40

(12) Free smooth actions of S^1 and S^3 on homotopy spheres
Mr. Kai Wang, University of Chicago (683-G5)

9:45-9:55

(13) Metastable annihilation in the homotopy groups of spheres. Preliminary report
Professor Henry H. Glover and Mr. Guido Mislin*, Ohio State University (683-G13)

10:00-10:10

(14) Common fixed points of commuting periodic maps. Preliminary report
Professor Henry H. Glover, Ohio State University (683-G24)

10:15-10:25

(15) The fundamental group of the space of conjugacy classes of a central analytic group
Mr. Dennis R. Daluge, University of Minnesota (683-G21)

FRIDAY, 9:15 A.M.

Session on Partial Differential Equations, Room C6

9:15-9:25

(16) On nonlinear equations of Hammerstein type in Banach spaces
Dr. Peter Hess, University of Chicago (683-B45)

9:30-9:40

(17) Initial value problems in L_p for systems with variable coefficients. Preliminary report
Professor Philip Brenner, Case Western Reserve University (683-B34)
(Introduced by Professor Gerald W. Hedstrom)

9:45-9:55

(18) Analogues of the Schwarz lemma for elliptic partial differential equations
Professor Herbert W. Hethcote, University of Iowa (683-B4)

10:00-10:10

(19) An a priori estimate for Poisson's equation
Professor Alan R. Elcrat, Wichita State University (683-B35)

10:15-10:25

(20) The singular Cauchy problem for a quasilinear hyperbolic equation
Dr. Seymour Singer, University of Notre Dame (683-B11)

10:30-10:40

(21) On the Cauchy problem for the nonlinear biharmonic equation
Dr. Philip W. Schaefer, University of Tennessee (683-B3)

FRIDAY, 11:00 A.M.

Invited Address, Room A1

Eichler cohomology and the Fourier coefficients of automorphic forms
Professor Marvin I. Knopp, University of Wisconsin and University of Illinois at Chicago Circle

FRIDAY, 1:45 P.M.

Invited Address, Room A1

The structure of the centralizers of involutions in finite simple groups
Professor John H. Walter, University of Illinois at Urbana-Champaign

FRIDAY, 3:00 P.M.

Special Session on Commutative Algebra, Room C1

3:00-3:20

(22) On monoidal transforms of regular local rings

Mr. David L. Shannon, Purdue University (683-A14)

3:25-3:45

(23) Cohen-Macaulay rings and resolutions of perfect ideals

Dr. John A. Eagon, University of Minnesota (683-A23)

(Introduced by Professor Paul T. Bateman)

3:50-4:10

(24) Relativizing functors on rings and algebraic K-theory

Professor Michael R. Stein, Northwestern University (683-A17)

4:15-4:35

(25) On the structure of Witt rings of forms

Professor Roger P. Ware, Northwestern University (683-A13)

4:40-5:00

(26) Rings of type I

Professor Eben Matlis, Northwestern University (683-A25)

(Introduced by Professor Paul T. Bateman)

FRIDAY, 3:00 P.M.

Special Session on Numerical Solution of Partial Differential Equations, Room D1

3:00-3:20

(27) Some new projection methods for the approximation of solutions of elliptic and parabolic problems

Professor James H. Bramble, Cornell University (683-C16)

3:25-3:45

(28) The effect of interpolating coefficients in nonlinear parabolic Galerkin methods

Professor Todd Dupont, University of Chicago (683-C12)

3:50-4:10

(29) Finite element approximations to parabolic problems

Professor George J. Fix, Harvard University (683-C13)

(Introduced by Professor Jim Douglas, Jr.)

4:15-4:35

(30) A priori L_2-error estimates for Galerkin approximations to parabolic partial differential equations

Mrs. Mary Fanett Wheeler, Rice University (683-C3)

(Introduced by Professor Jim Douglas, Jr.)

4:40-5:00

(31) Galerkin methods for the unsteady 2-dimensional flow of an inviscid incompressible fluid

Mr. Marco Raupp, University of Chicago (683-C14)

(Introduced by Professor Jim Douglas, Jr.)

FRIDAY, 3:00 P.M.

Special Session on Fixed Point and Coincidence Theory, Room F1

3:00-3:20

(32) On some generalizations of the Leray-Schauder theory

Professor Andrzej Granas, University of Montreal (683-G6)

(Introduced by Professor Edward R. Fadell)

3:25-3:45

(33) On the homology of a fixed point set

Professor Ronald J. Knill, Tulane University (683-G8)

3:50-4:10

(34) New methods in coincidence theory

Professor Kalyan K. Mukherjea, University of California, Los Angeles (683-G9)

4:15-4:35

(35) A lower bound for the number of solutions of f(x) = a

Professor Robin Brooks, Bowdoin College (683-G12)

FRIDAY, 3:00 P.M.

Session on Real Analysis, Room C6

3:00-3:10

(36) Weighted spaces of vector-valued continuous functions

Professor Joao B. Prolla, University of Rochester (683-B14)

3:15-3:25

(37) The approximation problem

Dr. Robert S. Borden, Knox College (683-B15)

3:30-3:40

(38) On the inverse of a measurable function

Professor Maurice Machover, St. John's University (683-B9)

3:45-3:55

(39) Necessary and sufficient conditions that an operator be a limit

Professor William C. Bennewitz, Southern Illinois University at Edwardsville (683-B33)

4:00-4:10

(40) On an inequality for Stieltjes integrals

Dr. James D. Baker, Honeywell Corporate Research Center, Hopkins, Minnesota (683-B40)

4:15-4:25

(41) A generalization of the Silverman-Toeplitz theorem

Professor David F. Dawson, North Texas State University (683-B5)

4:30-4:40

(42) Weighted convergence in length. Preliminary report

Professor William R. Derrick, University of Utah (683-B10)

4:45-4:55

(43) Fixed point theorems for lipschitzian pseudo-contractive mappings

Mr. Juan A. Gatica* and Professor William A. Kirk, University of Iowa (683-B19)

FRIDAY, 3:00 P.M.

Session on Ordinary Differential Equations, Room C4

3:00-3:10

(44) Local linearization without differentiability

Professor Aaron Strauss*, University of Maryland, and Professor A. Lasota, Jagiellonian University, Krakow, Poland (683-B7)

3:15-3:25

(45) Global weak asymptotic stability for dynamical polysystems

Professor L. David Sabbagh, Bowling Green State University (683-B38)

3:30-3:40

(46) Pointwise recurrent solutions of nonautonomous differential equations

Dr. Alan J. Heckenbach, Iowa State University (683-B36)

3:45-3:55

(47) Periodic solutions for perturbed nonlinear differential equations. II

Professor Thomas G. Proctor, Clemson University (683-B37)

4:00-4:10

(48) The existence of oscillatory solutions for a nonlinear differential equation
Professor John W. Heidel* and Professor Don Hinton, University of Tennessee (683-B13)

4:15-4:25

(49) Quickly oscillating solutions of nonlinear ordinary differential equations. Preliminary report
Professor Stephen R. Bernfeld*, University of Missouri-Columbia, and Professor A. Lasota, Jagiellonian University, Krakow, Poland (683-B6)

4:30-4:40

(50) On asymptotic behavior of perturbed nonlinear systems
Professor Robert E. Fennell* and Professor Thomas G. Proctor, Clemson University (683-B1)

4:45-4:55

(51) Invariant sets in the Monkey saddle
Professor David L. Rod, University of Calgary (683-B20)

FRIDAY, 3:00 P.M.

Session on Topology and Dynamical Systems, Room C3

3:00-3:10

(52) Riemann surfaces are unions of two open disks
Mr. Ralph Jones, University of Wisconsin (683-G4)

3:15-3:25

(53) Manifolds with monotone union and monotone intersection properties. Preliminary report
Mr. Orville L. Bierman, University of Utah (683-G14)

3:30-3:40

(54) Periodic and almost periodic orbits in a central force problem
Dr. Carl P. Simon, University of California, Berkeley (683-C7)

3:45-3:55

(55) Separatrices and cross-sections in dynamical systems
Professor Lawrence M. Franklin* and Professor N. P. Bhatia, University of Maryland, Baltimore (683-G22)

4:00-4:10

(56) Proximally equicontinuous regular minimal sets over the circle
Professor Richard Freiman, University of Maryland, Baltimore (683-G18)

4:15-4:25

(57) F-minimal sets
Professor Nelson G. Markley, University of Maryland (683-G19)

4:30-4:40

(58) Substitution minimal flows
Mr. John C. Martin, Rice University (683-G17)

4:45-4:55

(59) Weak attraction, minimality, recurrence and almost periodicity in semiflows
Professor Nam P. Bhatia*, University of Maryland, and Mr. Shui-Nee Chow, Michigan State University (683-B41)

SATURDAY, 8:20 A.M.

Special Session on Growth, Oscillation, and Asymptotic Properties of Solutions of Ordinary Differential Equations, Room C1

8:20-8:40

(60) The L^2 behavior of eigenfunction expansions. Preliminary report
Professor Harold E. Benzinger, University of Illinois (683-B21)

8:45-9:05

(61) On the reduction of rank of linear differential systems
Professor Donald A. Lutz, University of Wisconsin-Milwaukee (683-B22)

9:10-9:30

(62) General solution of a system of nonlinear equations at an irregular type singularity
Professor Po-Fang Hsieh, Naval Research Laboratory, Washington, D. C. and Western Michigan University (683-B23)

9:35-9:55

(63) Asymptotic distribution of eigenvalues for a boundary value problem
Professor William A. Harris, Jr.*, University of Southern California, and Professor Yasutaka Sibuya, University of Minnesota (683-B24)

10:00-10:20

(64) Conjugate points for higher order equations
Professor Thomas L. Sherman* and Dr. Jerry R. Ridenhour, Arizona State University (683-B25)

10:25-10:45

(65) Oscillation criteria for systems of linear differential equations
Professor Zeev Nehari, Carnegie-Mellon University (683-B26)

SATURDAY, 8:45 A.M.

Special Session on Numerical Solution of Partial Differential Equations, Room D1

8:45-9:05

(66) Approximation by hill functions. Preliminary report
Professor Ivo Babuška, University of Maryland (683-C10)
(Introduced by Professor Jim Douglas, Jr.)

9:10-9:30

(67) Application of variational methods to approximate the transient response of gas transmission systems
Professor Henry H. Rachford, Jr., Rice University (683-C2)

9:35-9:55

(68) The finite element method
Professor Gilbert Strang, Massachusetts Institute of Technology (683-C1)

10:00-10:20

(69) Quadrature-Galerkin approximations to solutions of elliptic differential equations
Professor Martin H. Schultz, Yale University (683-C15)

10:25-10:45

(70) Galerkin methods for nonlinear parabolic equations with nonlinear Neumann boundary conditions
Professor Jim Douglas, Jr., University of Chicago (683-C11)

SATURDAY, 9:00 A.M.

Session on Number Theory, Group Theory, and Geometry, Room C4

9:00-9:10

(71) Diophantine approximations for quaternions and Cayley numbers. Preliminary report
Professor Francis A. Roach, University of Houston (683-A8)

9:15-9:25

(72) Sums of polynomials as permutation polynomials
Professor Harald G. Niederreiter, Southern Illinois University (683-A18)

9:30-9:40

(73) Primary elements in the Schur subgroup
Professor Mark Benard, Tulane University (683-A9)

9:45-9:55

(74) A generalized Frattini subgroup of a finite group

Dr. Hasso C. Bhatia, Michigan State University (683-A5)

10:00-10:10

(75) On Schein semigroups

Professor Naoki Kimura, University of Arkansas (683-A20)

10:15-10:25

(76) An "extra" characterization of Moufang loops. Preliminary report

Professor Daniel A. Robinson, Georgia Institute of Technology (683-A10)

10:30-10:40

(77) A characterization of the Lüneburg planes

Mr. Robert Allen Liebler, Dartmouth College (683-D1)

SATURDAY, 9:00 A.M.

Session on Ring Theory, Room C6

9:00-9:10

(78) Lattice isomorphisms between Morita-related modules. Preliminary report

Professor Timothy V. Fossum, University of Utah (683-A22)

9:15-9:25

(79) Meta-Cayley hypercomplex algebras must include at least one hypercomplex square root of positive unity

Mr. K. Demys, Santa Barbara, California (683-A3)

9:30-9:40

(80) Ideals, inverses and conductors

Mr. Stephen J. McAdam, University of Texas (683-A4)

9:45-9:55

(81) The finiteness of I when $D[X]/I$ is flat. Preliminary report

Professor Jack E. Ohm and Mr. David Eugene Rush*, Louisiana State University (683-A7)

10:00-10:10

(82) Essential valuation overrings of $D[[X]]$. Preliminary report

Professor Jimmy T. Arnold, Virginia Polytechnic Institute and State University, and Professor James W. Brewer*, University of Kansas (683-A1)

10:15-10:25

(83) A theory of multiplicity for multiplicative filtrations

Mr. Wayne Bishop, Western Michigan University (683-A19)

SATURDAY, 9:10 A.M.

Special Session on Fixed Point and Coincidence Theory, Room F1

9:10-9:30

(84) Normal solvability and the solutions of nonlinear equations in Banach spaces

Professor Felix E. Browder, University of Chicago (683-G25)

9:35-9:55

(85) Fixed point theory via semicomplexes. Preliminary report

Professor Richard B. Thompson, University of Arizona (683-G11)

10:00-10:20

(86) Commuting functions and their fixed points

Professor Phil Huneke, Ohio State University (683-G7)

10:25-10:45

(87) An asymptotic fixed point theorem

Professor Roger D. Nussbaum, Rutgers University (683-G10)

SATURDAY, 9:10 A.M.

Special Session on Flows, Room C3

9:10-9:30

(88) Quasi-disjointness, products, and inverse limits

Professor Kenneth R. Berg, University of Maryland (683-H6)

9:35-9:55

(89) A continuous flow acting on S^n

Professor Hsin Chu, University of Maryland (683-H1)

10:00-10:20

(90) The index of an isolated invariant set of a flow

Professor Charles C. Conley, University of Wisconsin (683-H8)

10:25-10:45

(91) Geodesic flows in manifolds of negative curvature

Professor Patrick Eberlein, University of California, Berkeley (683-H3)

SATURDAY, 11:00 A.M.

Invited Address, Room A1

Recent results in the algebraic theory of minimal sets

Professor Robert Ellis, University of Minnesota

SATURDAY, 1:45 P.M.

Invited Address, Room A1

Spaces of analytic functions: some recent results

Professor Allen L. Shields, University of Michigan

SATURDAY, 3:00 P.M.

Special Session on Growth, Oscillation, and Asymptotic Properties of Solutions of Ordinary Differential Equations, Room C1

3:00-3:20

(92) Asymptotic expansions and structure theorems. Preliminary report

Professor Walter C. Strodt, St. Lawrence University (683-B27)

3:25-3:45

(93) Asymptotic compatibility of second-order solutions with coefficient fields of logarithmic type

Professor Robert K. Wright, University of Vermont (683-B28)

3:50-4:10

(94) On meromorphic solutions of generalized algebraic differential equations. Preliminary report

Dr. Chung-Chun Yang, Naval Research Laboratory, Washington, D. C. (683-B29)

4:15-4:35

(95) Asymptotics of some random second order differential equations

Professor Gilbert Stengle, Lehigh University (683-B30)

(Introduced by Professor Steven B. Bank)

4:40-5:00

(96) Variational aspects of oscillation phenomena for higher order differential equations

Professor William T. Reid, University of Oklahoma (683-B32)

5:05-5:25

(97) A representation theorem for large and small solutions of algebraic differential equations in sectors

Professor Steven B. Bank, University of Illinois (683-B31)

SATURDAY, 3:00 P.M.

Special Session on Flows, Room C3

3:00-3:20

(98) On normally hyperbolic flows

Professor Charles C. Pugh, University of California, Berkeley (683-H2)
(Introduced by Professor Leon W. Green)

3:25-3:45

(99) Solving ordinary differential equations on homogeneous spaces

Professor Leonard Shapiro, University of Minnesota (683-H5)

3:50-4:10

(100) Substitutions of nonconstant length

Professor William A. Veech, Rice University (683-H4)

4:15-4:35

(101) Some invariant σ-algebras for measure-preserving transformations

Professor Peter Walters, University of Maryland and University of Warwick, Coventry, England (683-H7)

SATURDAY, 3:00 P.M.

Session on Algebra and Logic, Room C4

3:00-3:10

(102) A variation of the friendship problem

Professor Helen Skala, University of Massachusetts, Boston (683-A21)

3:15-3:25

(103) On the genera of graphs of group presentations. III

Professor Henry W. Levinson, Rutgers University (683-A2)

3:30-3:40

(104) A note on distributive filter lattices. Preliminary report

Professor Robert S. Smith, Miami University (683-A11)

3:45-3:55

(105) A natural transformation of $\mathrm{Ext}_{\mathcal{O}}$

Mr. Joel A. Winthrop, University of California, Davis (683-A6)

4:00-4:10

(106) The Friedberg-Muchnik theorem re-examined

Professor Robert I. Soare, University of Illinois at Chicago Circle (683-E1)

SATURDAY, 3:00 P.M.

Session on Complex Analysis, Room C6

3:00-3:10

(107) Study of the transport operator for the infinite slab. Preliminary report

Dr. Ahmed N. Currim, Western Carolina University (683-B18)
(Introduced by Professor S. Manickam)

3:15-3:25

(108) Linear transformations on the power-series convergent on the unit disc which have matrix representations. Preliminary report

Professor Philip C. Tonne, Emory University (683-B39)

3:30-3:40

(109) Holomorphic idempotents and common fixed points on the 2-disk

Professor Dan J. Eustice, Ohio State University (683-B16)

3:45-3:55

(110) On generalized convexity in conformal mappings. Preliminary report

Professor Petru Mocanu, Babès-Bolyai University, Cluj, Romania, and Professor Maxwell O. Reade*, University of Michigan (683-B12)

4:00-4:10

(111) Quasibounded and singular functions

Professor Maynard G. Arsove and Dr. Heinz Leutwiler*, University of Washington (683-B17)

4:15-4:25

(112) One-sided boundary behavior for analytic and bounded functions. Preliminary report

Dr. James R. Choike, Oklahoma State University (683-B44)

(113) WITHDRAWN

SATURDAY, 3:00 P.M.

Session on Numerical Analysis, Room D1

3:00-3:10

(114) Association of nonlinear partial differential equation with a linear system of Schroedinger equations

Professor M. Z. v. Krzywoblocki, Michigan State University (683-C8)

3:15-3:25

(115) Multiple asymptotic expansions and singular problems

Dr. Kenneth D. Shere, U. S. Naval Ordnance Laboratory, Silver Spring, Maryland (683-B2)

(Introduced by Dr. A. H. Van Tuyl)

3:30-3:40

(116) Numerical solutions of a hyperbolic problem. Preliminary report

Mr. Fred H. Brink and Professor Arthur O. Garder*, Southern Illinois University (683-C9)

3:45-3:55

(117) Some projection schemes for second order nonlinear boundary value problems

Dr. Thomas R. Lucas, University of North Carolina, and Mr. George W. Reddien*, Georgia Institute of Technology (683-C5)

4:00-4:10

(118) Two higher-order projection schemes for second order nonlinear boundary value problems

Dr. Thomas R. Lucas*, University of North Carolina, and Mr. George W. Reddien, Georgia Institute of Technology (683-C6)

4:15-4:25

(119) Triangular spline interpolation in the plane

Professor Paul O. Frederickson, Lakehead University (683-B42)

4:30-4:40

(120) A new algorithm for the nonlinear wave equation

Professor Gerald W. Hedstrom, Case Western Reserve University (683-C4)

Paul T. Bateman
Associate Secretary

Princeton, New Jersey

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

Six Hundred Eighty-Fourth Meeting
The Waldorf-Astoria
New York, New York
April 7-10, 1971

The six hundred eighty-fourth meeting of the American Mathematical Society will be held at The Waldorf-Astoria in New York on April 7-10, 1971.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be four hour addresses. Professor Daniel Gorenstein of Rutgers University will speak on "Simple groups of low 2-rank" on Friday, April 9, at 11:00 a.m.; Professor Simon Kochen of Princeton University will speak on Friday, April 9, at 2:00 p.m. on "Local Diophantine equations"; Professor John Mather of Harvard University will speak on "Singularities of mappings" on Saturday, April 10, at 11:00 a.m.; and Professor Ted E. Petrie of Rutgers University will present a lecture entitled "Smooth actions of compact groups on manifolds" at 2:00 p.m. on Saturday, April 10.

There will be sessions for ten-minute contributed papers both mornings and afternoons of Friday and Saturday. There will be provision for late papers.

The Council of the Society will meet on Friday, April 9, at 5:00 p.m. There will be an intermission for dinner.

SYMPOSIUM ON MATHEMATICAL ASPECTS OF STATISTICAL MECHANICS

With the expected support of the National Science Foundation, there will be a symposium on Mathematical Aspects of Statistical Mechanics on April 7-8. The SIAM-AMS Committee on Applied Mathematics, which at the time consisted of Hirsh G. Cohen, Jim Douglas, Jr., Joaquin B. Diaz, William H. Reid, Richard S. Varga, and Calvin H. Wilcox (chairman), chose the topic of the symposium and appointed the Organizing Committee which includes Mark Kac, O. E. Lanford III, James C. T. Pool (chairman), Robert T. Powers, and Seymour Sherman. The hour speakers at the symposium will be Professor Freeman Dyson of the Institute for Advanced Study; Professor Jean Ginibre of the Laboratoire de Physique Theorique, France, and the Institute for Advanced Study; Professor Robert B. Griffiths of Carnegie-Mellon University; Professor David Ruelle of the Institute for Advanced Study; Professor Masamichi Takesaki of the University of California, Los Angeles; and possibly four others. The names of these additional speakers will be announced later.

MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

An open Register will be maintained in the Jade Room on April 8-9 from 9:00 a.m. to 5:00 p.m. on both days. A full announcement will be found on page 345 of these *Notices*.

REGISTRATION

The registration desk will be open from 8:30 a.m. to 4:30 p.m. on Wednesday through Friday, April 7-9, and 8:30 a.m. to 2:30 p.m. on Saturday, April 10.

ACCOMMODATIONS

Persons intending to stay at The Waldorf-Astoria should make their own reservations with the hotel. A reservation blank and a listing of room rates will be found on the last page of these *Notices*.

MAIL ADDRESS

Registrants at the meeting may receive mail addressed in care of the American Mathematical Society, The Waldorf-Astoria, New York, New York 10022.

Walter Gottschalk
Associate Secretary
Middletown, Connecticut

Six Hundred Eighty-Fifth Meeting
Naval Postgraduate School
Monterey, California
April 24, 1971

The six hundred eighty-fifth meeting of the American Mathematical Society will be held at the Naval Postgraduate School in Monterey, California, on Saturday, April 24, 1971.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited hour addresses. Professor James Dugundji of the University of Southern California will lecture at 11:00 a.m. His lecture is entitled "Topics in the theory of retracts." Professor Ray A. Kunze of the University of California, Irvine, will speak at 2:00 p.m. on "Some results and problems concerning uniformly bounded representations of semisimple Lie groups." Both of these addresses will be given in the auditorium in Ingersoll Hall. There will be sessions for contributed papers on Saturday morning and afternoon. The deadline for abstracts of contributed papers to be published in the program of the meeting is February 22, 1971. Late papers will be accepted for presentation until a few days before the meeting. All sessions for contributed papers will be held in Spanagel Hall.

The registration desk for the meeting will be located in Room 101a of Spanagel Hall. Registration will begin at 8:30 a.m. on Saturday. The Visitors and Convention Bureau of Monterey has blocked off rooms for this meeting in the following motels at the specified rates:

EL CASTELL MOTEL

Single	$12.00
Double	14.00
Twin	16.00

MARK THOMAS INN

Single	$17.00
Double	23.00
Twin	23.00

MONTEREY MOTOR LODGE

Single	$12.00
Double	14.00
Twin	16.00

STAGE COACH LODGE

Single	$13.00
Double	14.00
Twin	16.00

All of these motels except the El Castell are within one-half mile of the Naval Postgraduate School campus. Requests for reservations should be addressed to Visitors and Convention Bureau, P. O. Box 1770, Monterey, California 93940 (phone: 408-375-2252). Requests for reservations should indicate first, second, and third preferences and type of accommodation desired; should indicate arrival and departure times; should refer to the meeting of the American Mathematical Society; and must be accompanied by a deposit for the first night. Requests for reservations should be received no later than April 6, 1971.

The Monterey Airport is served by United Airlines, Cal-State Airline, and Air West Airlines. Greyhound Bus Lines serves the Monterey area. There is also passenger train service connecting Monterey with San Francisco.

K. A. Ross
Eugene, Oregon — Associate Secretary

SPECIAL MEETINGS INFORMATION CENTER

SPECIAL MEETINGS INFORMATION CENTER

The Symposia Information Center was established in 1969 on the recommendation of the Committee to Monitor Problems in Communication. Since October 1969, the Symposia Information Center has maintained a file on prospective symposia, many of these in the very early planning stages, and information has been made available to any organization or individual planning a conference. If conflicts in subject matter, dates, or geographical area have become apparent, the Center has notified the organizers. The announcements of definitely scheduled symposia have, of course, continued to be carried in the *Notices* as News Items. The name of the Center is now being changed to SPECIAL MEETINGS INFORMATION CENTER, and with this issue of these *Notices* a section will be devoted to announcements of all special meetings: symposia, institutes, seminars, colloquia, special years. Such announcements will no longer be included in the section(s) devoted to News Items.

To enable the Special Meetings Information Center to maintain as complete a file as possible, organizers of these special meetings are requested to send in information early in the planning stages. A first announcement will be published in the *Notices* if it contains a call for papers, place, date, and subject, where applicable; a second announcement must contain reasonably complete details of the meeting in order for it to be published. Information on the pre-preliminary planning of a conference will be stored in the files and will be available to anyone desiring information on prospective conferences. The office will continue to notify organizers of meetings in cases where conflicts might occur.

Please send all communications on special meetings to the Special Meetings Information Center, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904.

RESEARCH PARTICIPATION CONFERENCE FOR COLLEGE TEACHERS

The State University of New York at Binghamton will hold a Research Participation Conference for College Teachers from June 14 to August 20, 1971. The subjects to be covered are PL topology of 3-manifolds, theory of mappings, graph theory, and functional analysis. The conference is open to postdoctoral mathematicians and to students ready to begin research for a thesis; it is primarily designed for research-oriented college staff from institutions without strong research programs. Information may be obtained by writing to Professor Louis F. McAuley, Department of Mathematics, State University of New York at Binghamton, Binghamton, New York 13901.

SEMINAR IN PROBABILITY AND STATISTICS

An Advanced Science Seminar in Probability and Statistics will be held at Clemson University from June 28 to August 16, 1971. Two courses will be offered: "Weak convergence with application to probability and statistics" given by P. Billingsley, and "Cluster analysis and pattern recognition" given by H. Chernoff. The seminar is intended for advanced graduate students and postdoctoral mathematicians. The deadline for submission of applications is April 1, 1971. Further information and application forms may be obtained by writing to either Professor K. T. Wallenius or Professor John Kenelly, Department of Mathematics, Clemson University, Clemson, South Carolina 29631.

INSTITUTE FOR ADVANCED MATHEMATICAL RESEARCH

La Recherche Coopérative has announced the Twelfth Meeting for Theoretical Physicists and Mathematicians to take place at the Institute for Advanced Mathematical Research in Strasbourg, France, May 24-29, 1971. Further information may be obtained by writing to the Secrétariat de la R. C. P. N° 25, Institute de Recherche Mathématique Avancée, 7, rue René Descartes, 67 Strasbourg, France.

INSTITUTE IN MANAGEMENT SCIENCE AND OPERATIONS RESEARCH

The University of Colorado, with the support of the National Science Foundation, will hold an Institute in Management Science and Operations Research on June 14 through July 15, 1971. The principal lecturers will include Professor Egon Balas, Carnegie-Mellon University; Professor Robert B. Fetter, Yale University; Professor Fred Glover, University of Colorado; Professor Ronald A. Howard, Stanford University; and Professor P. Rutenberg, Carnegie-Mellon University. The subjects to be covered are mathematical programming, simulation of stochastic processes, decision theory, network optimization, hospital and health administration, capital budgeting, and production scheduling and resource allocation. The institute is planned primarily for college professors in management science, applied mathematics, economics, health and hospital administration. Participants may expect to receive a stipend and travel expenses. Applications must be received before March 15, 1971. Application forms and further information may be obtained by writing to Dr. Donald R. Plane, Division of Management Science, Business Building, University of Colorado, Boulder, Colorado 80302.

SYMPOSIUM ON OPTIMIZING METHODS IN STATISTICS

The Division of Statistics of The Ohio State University will hold a symposium on Optimizing Methods in Statistics on June 14-16, 1971, with the support of the Air Force Office of Scientific Research. It is planned to have sessions on variational methods in statistics, optimum design of experiments, stochastic approximation procedures, mathematical programming in statistics, optimum seeking methods, and problems and applications. A session for contributed papers is also planned. The purpose of the symposium is to focus the attention of statisticians on this important area of research and application. Scholars from universities, industry, and government agencies have been invited to participate. A few travel grants for advanced graduate students are available. The proceedings of the symposium will be published. For further information, write to Professor Jagdish Rustagi, Division of Statistics, The Ohio State University, 231 West 18th Avenue, Columbus, Ohio 43210.

POINT SET TOPOLOGY CONFERENCE

The University of Houston has scheduled a Point Set Topology Conference on March 22-24, 1971. The conference will take place at the Shamrock Hilton Hotel in Houston, Texas. The speakers for the conference include A. Lelek, L. Mohler, C. Eberhart, W. Kuperberg, D. Bourgin, J. Fugate, W. Transue, K. R. Van Doren, E. Michael, J. Nagata, P. Zenor, A. Stone, H. Wicke, N. Stevenson, R. Hodel, S. Nadler, J. T. Rober, Jr., S. Williams, F. B. Jones, R. W. Heath, C. Riecke, S. Armentrout, D. O'Steen, J. Kropa, C. Bandy, and M. E. Rudin. Additional information and reservation forms for the Shamrock Hilton Hotel may be obtained by writing to Professor D. R. Traylor, Department of Mathematics, University of Houston, Houston, Texas 77004.

CONFERENCE ON THE THEORY OF ARITHMETIC FUNCTIONS

The Department of Mathematics at Western Michigan University will sponsor a conference on the Theory of Arithmetic Functions on April 29 through May 1, 1971, with the support of the National Science Foundation. The program will include one-hour invited addresses and a number of shorter contributed papers. The list of hour speakers includes Professors L. Carlitz, P. Erdős, E. Grosswald, D. J. Lewis, C. Ryavec, D. A. Smith, M. V. Subbarao, and H. P. F. Swinnerton-Dyer. The proceedings of the conference will be published. For information about travel and living accommodations, please write to Professor A. A. Gioia or Professor D. L. Goldsmith, Department of Mathematics, Western Michigan University, Kalamazoo, Michigan 49001.

ASSOCIATION FOR SYMBOLIC LOGIC

A meeting of the Association for Symbolic Logic will be held on Thursday and Friday, March 25-26, 1971, at the Beverly Hilton Hotel in Los Angeles, California, in conjunction with a meeting of the American Philosophical Association. Invited addresses will include the following: "A model theory for the λ-calculus" by Dana S. Scott, Princeton University, and "Any two elementarily equivalent models have isomorphic ultrapowers" by Saharon Shelah, University of California, Los Angeles. In addition, there will be several sessions for contributed papers. The program chairman for the meeting is Professor Richard M. Montague, Department of Philosophy, University of California, Los Angeles, California 90024.

CANADIAN MATHEMATICAL CONGRESS

The Canadian Mathematical Congress will meet at Lakehead University, Thunder Bay, Ontario, on June 16-18, 1971. The main topic of the meeting will be Computational Aspects of Combinatorics, Algebra, and Geometry. Professor Tutte of Waterloo will deliver the principal address entitled "The use of numerical computation in the enumerative theory of planar maps." All interested mathematicians are welcome to attend this meeting. Further information may be obtained by writing to The Secretary, Department of Mathematics, Lakehead University, Thunder Bay, Ontario, Canada.

SUMMER SCHOOL IN COMPLEX FUNCTION THEORY

A summer school in Complex Function Theory will be held at University College, Cork, Ireland, under the auspices of the Royal Irish Academy on July 19-24, 1971. The summer school will study functions of a single complex variable with special emphasis on the connection between the classical theory and functional analysis. Courses will be given by J. G. Clunie, Imperial College, London; F. R. Keough, University of Kentucky; P. R. Ahern, University of Wisconsin; and P. D. Barry, University College, Cork. Application forms and further information may be obtained from Dr. Finban Holland, Department of Mathematics, University College, Cork, Ireland.

SYMPOSIUM ON STATISTICAL MODELS AND TURBULENCE

A symposium on Statistical Models and Turbulence will be held at the University of California, San Diego (La Jolla), from July 15 to July 22, 1971. The meeting is sponsored by the International Association for the Application of Statistics in the Physical Sciences and supported by the National Science Foundation and the Office of Naval Research. One of the objects of the symposium is to provide an opportunity for specialists in probability and statistics and specialists in fluid mechanics with interests in turbulence and allied problems to meet, give talks, and discuss the current state of knowledge. Attendance will be limited. The list of participants, as yet incomplete, includes F. Busse, J. Dutton, C. Gibson, M. Kac, J. Lumley, B. Mandelbrot, W. Meedham, W. Munk, S. Orszag, E. Parzen, M. Rosenblatt, D. Ruelle, R. Stewart, and C. Van Atta. Members of the organizing committee are John Lumley, Pennsylvania State University; Murray Rosenblatt, University of California, San Diego; and Charles Van Atta, University of California, San Diego. Further information can be obtained by writing to Professor Murray Rosenblatt, Department of Mathematics, University of California, San Diego, P. O. Box 109, La Jolla, California 92037.

THIRD INTERNATIONAL CONGRESS FOR FOR THE THEORY OF MACHINES AND MECHANISMS

The Third International Congress for the Theory of Machines and Mechanisms will be held in Dubrovnik, Kupari, Yugoslavia, September 1-20, 1971. Those wishing to communicate their reports to the Congress should send the titles and brief summaries of their papers (up to five sentences) to the Organizing Committee by March 1, 1971. Complete papers, which will be published in the Congress Proceedings, must be sent to the committee before May 5, 1971. Papers may be written in English, Russian, German, French, or Serbo-Croatian and must be typewritten on white nontransparent paper with approximately thirty lines per page; diagrams, formulas, and so forth should be drawn in India ink. All those wishing to attend the Congress should send in a written application to the committee by March 15, 1971, accompanied by a registration fee of $60; those registering after March 15 will pay an $80 fee. Further information on room rates and travel arrangements within Yugoslavia may be obtained by writing to the Yugoslavia Committee on the Theory of Machines and Mechanisms, Mašinski fakultet, 27 mart 80, 11000 Beograd, Yugoslavia.

INTERNATIONAL CONFERENCE ON THE FUTURE OF APPLIED MATHEMATICS

To observe the twenty-fifth anniversary of the establishment of the Division of Applied Mathematics at Brown University, an International Conference on the Future of Applied Mathematics will be held on September 7-10, 1971, at Brown University in Providence, Rhode Island. The conference will be supported in part by the National Science Foundation. In addition to round table discussions on "Applied Mathematics: Its Position and Prospects" and "Science and Society," the program will include a number of lectures. The tentative list of lecturers consists of Professor William Prager, Brown University; Professor George Carrier, Harvard University; Dr. Hirsh Cohen, Thomas J. Watson Research Laboratory; Professor Peter Henrici, Eidgen Technische Hochschule, Zürich; Professor Mark Kac, Rockefeller Institute; Professor Jurgen Moser, New York University; Professor Chaim Pekeris, Weizmann Institute, Israel; Professor Alan Perlis, Carnegie-Mellon University; Professor John Tukey, Princeton University; and Professor Hans Ziegler, Eidgen Technische Hochschule, Zürich. Inquiries should be directed to 25th Anniversary Committee, Division of Applied Mathematics, Brown University, Providence, Rhode Island 02912.

NUMBER THEORY CONFERENCE

A conference in number theory will be held at Washington State University on March 24-27, 1971. The conference is sponsored by the Washington State University Graduate School, Washington State University Department of Mathematics, and Washington Alpha Chapter of Pi Mu Epsilon. Speakers who have agreed to participate are Professors Paul Erdős, Ron Graham, Ivan Niven, and John Selfridge. Those wishing to participate in the conference should write to Professor James H. Jordan, Department of Mathematics, Washington State University, Pullman, Washington 99163.

SYMPOSIUM ON SWITCHING AND AUTOMATA THEORY

The Twelfth Annual Symposium on Switching and Automata Theory, sponsored by the Switching and Automata Theory Committee of the IEEE Computer Group and the Department of Computer Science of Michigan State University, will be held in East Lansing, Michigan, on October 13-15, 1971. Papers describing original research in the general areas of switching theory, automata theory, and the theoretical aspects of computers, computation, and programming are being sought. Typical (but not exclusive) topics of interest include algorithms of theoretical interest, automata theory, cellular and iterative networks, computational complexity, formal languages, mathematical foundations for computation, program schemata, reliability and fault diagnosis, sequential machines and asynchronous circuits, theory of algorithms, theory of parallel computation, and theory of parsing and compiling. Authors are requested to send six copies of an extended abstract (no word limit) by May 3, 1971, to Professor Frederick C. Hennie, Project MAC, Massachusetts Institute of Technology, 545 Technology Square, Room 420A, Cambridge, Massachusetts 02139. The abstract must provide sufficient detail to allow the program committee to apply uniform criteria for acceptance, and should include appropriate references and comparisons with extant work. It is also helpful to include a brief interpretation of the major results and explanation of their significance. A total of five to eight typewritten pages is suggested. Authors will be notified of the acceptance or rejection of their papers by June 18. For inclusion in the Conference Record, a copy of each accepted paper, typed on special forms, will be due by August 2. Information on local arrangements may be obtained by writing to Professor M. A. Rahimi, Department of Computer Science, Michigan State University, East Lansing, Michigan 48823.

THIRTEENTH BIENNIAL SEMINAR OF THE CANADIAN MATHEMATICAL CONGRESS

The Thirteenth Biennial Seminar of the Canadian Mathematical Congress will be held on August 6 to September 3, 1971, at Dalhousie University, Halifax, Nova Scotia. The theme of the seminar will be Differential Geometry, Differential Topology, and Applications. The program of lectures is now being completed, and it is expected that the list will be published in February 1971, at which time a brochure describing the lecture program and facilities at Dalhousie University will be sent to universities and research centers. An application will form part of the descriptive brochure. It is hoped that 80 to 100 participants may attend the seminar, and that full or partial support may be available to some participants. Mathematicians interested in the seminar are invited to write to either of the following addresses: Professor J. R. Vanstone (Chairman of the Program Committee), Department of Mathematics, University of Toronto, Toronto 5, Canada; or Canadian Mathematical Congress, 985 Sherbrooke Street West, Montreal 110, Quebec, Canada.

SEMINAR IN COMBINATORIAL THEORY

A Seminar in Combinatorial Theory will be held at Bowdoin College from June 22 to August 12, 1971. The seminar will be supported under a grant from the National Science Foundation. Professor Gian-Carlo Rota of the Massachusetts Institute of Technology will give the main course entitled "Combinatorial theory and applications." In addition, there will be seminars at various levels on appropriate topics and a research colloquium. Graduate students who wish to attend should ask their chairmen or dissertation advisors to write for them; postdoctoral and senior research mathematicians should write directly to Professor Dan E. Christie, Department of Mathematics, Bowdoin College, Brunswick, Maine 04011. Complete applications for graduate students should be postmarked not later than March 8; postdoctoral and senior mathematicians should, if possible, file applications by February 8. Applicants will be notified of their acceptance not later than the week of March 29.

CONFERENCE ON COMPUTERS IN THE UNDERGRADUATE CURRICULA

A conference on Computers in the Undergraduate Curricula, supported by the National Science Foundation, will be held at Dartmouth College in Hanover, New Hampshire, on June 23-25, 1971. The conference has as its purpose the dissemination of actual experience and plans in the use of computers in undergraduate instruction. Refereed contributed papers will comprise the bulk of the parallel session conference, with invited papers, panel discussions, and demonstrations rounding out the meeting. The deadline for papers is March 1, 1971, and they should be mailed to Dr. Fred W. Weingarten, Director, Computer Services, Claremont Colleges, Claremont, California 91711. Further information on the conference can be obtained by writing to the Chairman of the Organizing Committee, Dr. Thomas Kurtz, Dartmouth College, Hanover, New Hampshire 03755.

II LATIN AMERICAN SCHOOL OF MATHEMATICS

The II Latin American School of Mathematics will take place in Mexico City on July 5-31, 1971, at the Department of Mathematics of the Centro de Investigación del IPN. The subject will be "Topology and differential structures." The School is sponsored by the Organization of American States, the National Science Foundation, and the Centro de Investigación del IPN. Seven intensive courses at the research level will be presented, each course consisting of eight lectures. The lecturers and subjects include the following: "Characteristic classes and foliations" by Raoul H. Bott, Harvard University; "Homotopy methods in differential topology" by William E. Browder, Princeton University; "Cobordismo y grupos formales" by Albrecht Dold, University of Heidelberg; "Operaciones de orden superior y osbtrucciones" by Samuel Gitler, Centro de Investigación del IPN; "Homotopy theory: problems old and new" by Ioan M. James, Oxford University; "Operadores differenciales" by J. J. Kohn, Princeton University; and a series of lectures by Louis Nirenberg, Courant Institute ofMathematical Sciences, the title to be announced. As indicated in the title, lectures will be presented in either English or Spanish. There will be a limited number of fellowships to cover travel and living expenses for citizens of the United States and Latin American countries. For further information, please write to Dr. Carlos Imaz, Centro de Investigación del IPN, Apartado Postal 14-740, México 14, D. F.

YEAR IN SEVERAL COMPLEX VARIABLES

The Department of Mathematics of the University of Washington is organizing a special emphasis year in Several Complex Variables during 1971-1972. The department welcomes mathematicians who are planning on sabbatical leaves for 1971-1972, and whose interests are in this field, to become Visiting Scholars for varying periods. There will also be available a few Visiting Lectureships; a Visiting Lecturer teaches two courses each quarter (6 hours a week). Application for Visiting Lecturer, which should include a brief résumé of the candidate's background and the names of three references, should be addressed to Professor Ross A. Beaumont, Department of Mathematics, University of Washington, Seattle, Washington 98105. Applications should reach Professor Beaumont by February 15, 1971. His telephone number is 206-543-1150.

THE MITTAG-LEFFLER INSTITUTE

During the academic year 1971-1972, the emphasis of the work at the Mittag-Leffler Institute will be on complex analysis, in particular Kleinean groups and quasi-conformal mappings. The work will be organized in collaboration with Finnish mathematicians; in particular, a summer school on the same topic will be held in Finland during the summer of 1971. Postgraduate students are accepted to the institute. The term starts about September 10, 1971. Further information may be obtained by writing to Professor Lennart Carleson, Institut Mittag-Leffler, Auravägen 17, S-182 62 Djursholm (Stockholm), Sweden.

SUMMER GRADUATE COURSES

The following is a list of graduate courses being offered in the mathematical sciences during the summer of 1971. Another list will appear in the April issue of these *Notices*.

ALABAMA

SAMFORD UNIVERSITY
Birmingham, Alabama 35209
Application deadline: Open
Information: W. D. Peeples, Jr., Head, Department of Mathematics

June 7 - August 7
Math 508 - Theory of Matrices

July 15 - August 20
Math 400G - Theory of Numbers

ARIZONA

NORTHERN ARIZONA UNIVERSITY
Flagstaff, Arizona 86001
Application deadline: May 14
Information: Richard D. Meyer, Chairman, Department of Mathematics, Box 5717

June 14 - July 17
Math 504 - Elements of Algebraic Systems
Math 507 - Modern Mathematics for Teachers
Math 511 - Introduction to Higher Algebra
Math 563 - Numerical Analysis
Math 630 - Theory of Functions of a Real Variable

July 19 - August 21
Math 505 - Elements of Analysis
Math 512 - Introduction to Higher Algebra
Math 531 - Advanced Calculus
Math 602 - Instructional Materials in Mathematics
Math 624 - Differential Geometry

ILLINOIS

NORTHEASTERN ILLINOIS STATE COLLEGE
Chicago, Illinois 60625
Application deadline: May 3
Information: Dean of the Graduate College

May - June (8 weeks)
Theory of Groups

July - August (8 weeks)
Theory of Fields

UNIVERSITY OF ILLINOIS
Urbana, Illinois 61801
Application deadline: May 15 (for degree candidates)
Information: Franz E. Hohn, Graduate Supervisor, Department of Mathematics, 263 Altgeld Hall

June 21 - August 14
Mathematics for Elementary Teachers
Theory of Sets and the Real Number System
Applied Modern Algebra
Topics in Geometry for Secondary Teachers
Selected Topics in Mathematics for Secondary Teachers
Linear Transformations and Matrices
Introduction to Higher Algebra I, II
Introduction to Set Theory and Topology
Advanced Calculus
Differential Equations and Orthogonal Functions
Complex Variables and Applications
Introduction to Higher Analysis - Real Variables
Introduction to Higher Analysis - Complex Variables
Elementary Theory of Numbers
Theory of Probability I
Advanced Statistics I
Mathematical Methods in Engineering and Science
Second Course in Abstract Algebra I, II
Group Theory
Logical Foundations of Mathematics I
General Topology I
Theory of Functions of a Complex Variable I
Real Analysis I
Partial Differential Equations
Mathematical Methods of Physics I, II
Reading Course
Thesis Research

KANSAS

FORT HAYS KANSAS STATE COLLEGE
Hays, Kansas 67601
Information: W. Toalson, Chairman, Department of Mathematics

June 7 - July 31
156 - Higher Algebra
159 - Matrices
171 - Theory of Numbers
291 - Teaching Techniques
325 - Functions of a Complex Variable
360 - Advanced Calculus II

KANSAS STATE COLLEGE OF PITTSBURG
Pittsburg, Kansas 66762
Application deadline: June 1
Information: Helen F. Kriegsman, Chairman, Department of Mathematics

June 1 - July 30
Basic Concepts of Linear Algebra
Basic Concepts of Geometry
Functions of Complex Variables

KENTUCKY

UNIVERSITY OF LOUISVILLE
Louisville, Kentucky 40208
Application deadline: June 10
Information: R. H. Geeslin, Chairman, Department of Mathematics

June 10 - July 28
420 - Elementary Linear Algebra (M.A.T. only)
421 - Theory of Numbers (M.A.T. only)
521 - Modern Algebra I
A552 - Geometry II
A633 - Fields, Rings, & Ideals

July 15 - August 17
595 - The Computer in Mathematics Teaching

MICHIGAN

EASTERN MICHIGAN UNIVERSITY
Ypsilanti, Michigan 48197
Information: Dean of the Graduate School

June 21 - July 30
Introduction to Mathematical Logic
Foundations of Mathematics
Elements of Set Theory
Real Number System
Introduction to Topology
Non-Euclidean Geometry
Geometry for High School Teachers
Special Topics
Independent Study

NEW YORK

CITY UNIVERSITY OF NEW YORK, LEHMAN COLLEGE
Bronx, New York 10468
Application deadline: April 28
Information: P. R. Meyer, Department of Mathematics

June 14 - July 22
Introduction to Geometries
Advanced Calculus
Numerical Methods
Topology

STATE UNIVERSITY OF NEW YORK AT BINGHAMTON
Binghamton, New York 13901
Application deadline: June 14
Information: Chairman, Department of Mathematics

June 14 - August 20
Topics in Graph Theory
Topics in Topology
Topics in Analysis
Independent Work

STATE UNIVERSITY OF NEW YORK AT STONY BROOK
Stony Brook, New York 11790
Application deadline: May 15
Information: Office of the Summer Session

June 21 - July 30
MSA 503 - Complex Analysis
MSA 506 - Finite Structures
MSM 504 - Homological Algebra
MSM 534 - Differential Geometry

NORTH CAROLINA

APPALACHIAN STATE UNIVERSITY
Boone, North Carolina 28607
Application deadline: May 7
Information: H. R. Durham, Chairman, Department of Mathematics

June 7 - July 9
Algebra
Analysis
Mathematics Education
Topology

July 12 - August 13
Algebra
Analysis
Mathematics Education
Topology

EAST CAROLINA UNIVERSITY
Greenville, North Carolina 27834
Application deadline: May 10
Information: Dean of Graduate School

June 7 - July 13
Math 365G - Theory of Numbers
Math 369G - The Historical Development of Mathematics
Math 470 - Modern Algebra I
Math 483 - Higher Plane Curves

July 15 - August 20
Math 371G - Theory of Equations
Math 420 - Elements of Probability
Math 431 - Foundations of Geometry
Math 471 - Modern Algebra II

PENNSYLVANIA

SHIPPENSBURG STATE COLLEGE
Shippensburg, Pennsylvania 17257
Information: James L. Sieber, Chairman, Department of Mathematics

First session
M.A. - Topology I
M.A. - Real Analysis I

Second session
M.A. - Topology II
M.A. - Real Analysis II

Third session
M.Ed. - Projective Geometry
M.Ed. - Foundations of Algebra

Fourth session
M.Ed. - Introduction to Analysis I
M.Ed. - Introduction to Logic I
M.Ed. - Abstract Vector Spaces
M.Ed. - Elements of Research

Fifth session
M.Ed. - Introduction to Analysis II

SOUTH CAROLINA

CLEMSON UNIVERSITY
Clemson, South Carolina 29631
Application deadline: May 1
Information: John Kenelly, Head, Department of Mathematics

May 17 - June 24
Math 601 - Statistical Methods I
Math 602 - Theory of Probability
Math 607 - Partial Differential Equations
Math 611 - Linear Algebra
Math 833 - Operational Mathematics
Math 854 - Graph Theory
Math 920 - Harmonic Analysis

June 28 - August 7
Math 601 - Statistical Methods I
Math 603 - Introduction to Mathematical Statistics
Math 604 - Introduction to Stochastic Processes
Math 605 - Statistical Methods II
Math 635 - Complex Variables
Math 839 - Integral Equations
Math 853 - Advanced Linear Algebra
Math 980 - Special Topics in Probability
Math 981 - Special Topics in Statistics
Math 982 - Special Topics in Analysis

TENNESSEE

EAST TENNESSEE STATE UNIVERSITY
Johnson City, Tennessee 37601
Application deadline: June 1
Information: Arthur H. DeRosier, Dean, Graduate School

June 7 - July 9
5510 - Functions of Complex Variable
5810 - Operations Research
5900 - Topics in Algebra

July 12 - August 11
5520 - Functions of Complex Variable
5820 - Operations Research
5901 - Topics in Algebra

TEXAS

STEPHEN F. AUSTIN STATE UNIVERSITY
Nacogdoches, Texas 75961
Application deadline: May 1
Information: W. I. Layton, Chairman, Department of Mathematics

May 31 - July 7
Probability Theory
Advanced Calculus I
Introduction of Higher Geometry

July 8 - August 13
College Geometry
Introduction of Higher Geometry

CANADA

UNIVERSITY OF WESTERN ONTARIO
London 72, Ontario, Canada
Application deadline:
Information: Department of Computer Science, Staging Building

May 18 - August 13*
Formal Languages and Automata

*The first two weeks are introductory.

PERSONAL ITEMS

Professor MICHAEL I. AISSEN of Fordham University has been appointed to a professorship and to the chairmanship of the Department of Mathematics at Rutgers University, Newark Campus.

Mr. KENNETH F. ANDERSON of the University of Toronto has been appointed to an assistant professorship at Royal Roads Military College.

Professor GEORGE E. ANDREWS of Pennsylvania State University has been appointed to a visiting professorship at the Massachusetts Institute of Technology.

Professor GREGORY F. BACHELIS of SUNY at Stony Brook has been appointed to a visiting associate professorship at Kansas State University.

Professor ALEX C. BACOPOULOS of Michigan State University has been appointed a visiting research associate at the University of Montreal.

Dr. ERWIN H. BAREISS of Argonne National Laboratory has been appointed to a professorship at Northwestern University and will also remain with the Argonne National Laboratory.

Dr. HORST E. H. BECKER of the University of Karlsruhe, Federal Republic of Germany, has been appointed to a professorship at the University of Trier, Kaiserslautern, Federal Republic of Germany.

Professor JONNIE B. BEDNAR of Drexel University has been appointed to an assistant professorship at the University of Tulsa.

Professor FREDERICK H. BELL of Clarion State College has been appointed to an assistant professorship at the University of Pittsburgh.

Professor ELIZABETH A. BERMAN of the University of Missouri, Kansas City, has been appointed to an assistant professorship at Rockhurst College.

Mr. BEN-AMI BRAUN of Purdue University has been appointed to an assistant professorship at the University of South Florida.

Mr. MARTIN G. BUNTINAS of the Illinois Institute of Technology has been appointed to an assistant professorship at Loyola University.

Professor PHYLLIS J. CASSIDY of Hunter College has been appointed to an assistant professorship at Fordham University.

Professor DAVID M. CLARK of Georgia Institute of Technology has been appointed to an assistant professorship at the State University of New York, College at New Paltz.

Professor WILLIAM A. CRABTREE, JR., of Austin Peay State University has been appointed to an associate professorship at Columbia State Community College.

Professor EDWARD B. CURTIS of the Massachusetts Institute of Technology has been appointed to an associate professorship at the University of Washington.

Professor RICHARD M. DAVITT of Lafayette College has been appointed to an assistant professorship at the University of Louisville.

Professor MARTIN H. DULL of Western Michigan University has been appointed to an assistant professorship at the University of Pittsburgh.

Professor LAWRENCE FEINER of SUNY at Stony Brook has been appointed to an assistant professorship at CUNY, Brooklyn College.

Professor ALESSANDRO FIGA-TÀLAMANCA of the University of Rome has been appointed to a professorship at the University of Genoa, Italy.

Professor GUNTHER H. FREI of the University of Notre Dame has been appointed to an adjoint professorship at Laval University, Quebec, Canada.

Professor RONALD C. FREIWALD of the University of Rochester has been appointed to an assistant professorship at Washington University, St. Louis, Missouri.

Professor JAY R. GOLDMAN of Harvard University has been appointed to an associate professorship at the University of Minnesota.

Mr. JAMES GUYKER of Lehigh University has been appointed to an assistant professorship at the State University of New York, College at Buffalo.

Professor MORRIS L. HAMILTON of St. Andrews Presbyterian College has been appointed to an assistant professorship at Pembroke State University.

Professor DONALD HARTIG of the University of California, Santa Barbara, has been appointed to an assistant professorship at Ohio University.

Professor ISRAEL N. HERSTEIN of the University of Chicago has been appointed to a professorship at the Weizmann Institute, Israel, and will henceforth have a joint appointment between the Weizmann Institute, where he will spend the winter and spring quarters, and the University of Chicago, where he will spend the summer and fall quarters.

Mr. JOHN M. HOWIE of the University of Stirling, Scotland, has been appointed to a professorship at the University of St. Andrews, Scotland.

Professor ROBERT W. HUNT of the Naval Postgraduate School and Southern Illinois University has been appointed to a professorship and named head of the Department of Mathematics at California State College, Bakersfield.

Mr. BING FUN IP of Wayne State University has been appointed to an assistant professorship at the University of Bridgeport.

Professor EUGENE ISAACSON of New York University-Courant Institute has been appointed to a visiting professorship and has been named acting chairman of the Department of Computer Sciences at CUNY, City College.

Professor LARRY S. JOHNSON of Western Illinois University has been appointed to an assistant professorship at Fort Lewis College.

Professor BEN J. JONES of the University of Oregon has been appointed to an assistant professorship at the University of New Mexico.

Professor LEE KAMINETZKY of New York University-Courant Institute has been appointed to an assistant professorship at CUNY, City College.

Mr. GORDON E. KELLER of the University of Minnesota has been appointed to an assistant professorship at the University of Virginia.

Professor KENNETH R. KIMBLE of Ohio State University has been appointed to an assistant professorship at the University of Tennessee.

Professor SHOSHICHI KOBAYASHI of the University of California, Berkeley, has been appointed to a visiting professorship at the Massachusetts Institute of Technology.

Professor EUGENE H. LEHMAN, JR., of Missouri Southern College has been appointed to a professorship at the University of Quebec at Trois-Rivieres.

Professor FRANK LEVIN of Rutgers University has been appointed to a visiting associate professorship at Carleton University.

Professor LAWRENCE G. LEWIS of the CUNY Graduate Center has been appointed to an assistant professorship at the University of Utah.

Professor EDWARD I. H. LEZAK of Indiana University has been appointed to an assistant professorship at Virginia Commonwealth University.

Dr. SHU-T'IEN LI has become a Professor Emeritus of Civil Engineering at South Dakota School of Mines and Technology. He has recently formed two consulting firms, one for Research and Development in the Denver area under the name of Li-Shih & Associates, and the other for Planning and Engineering in the Washington, D. C., area under the name of Li-Delyannis & Associates.

Professor ROBERT J. LINDAHL of Pennsylvania State University has been appointed to an associate professorship at Morehead State University.

Mr. FRANK H. LUEBBERT of St. Louis University has been appointed to an assistant professorship at Central Missouri State College.

Professor ROBERT H. MARTIN, JR., of Georgia Institute of Technology has been appointed to an assistant professorship at North Carolina State University.

Professor RUDOLPH M. NAJAR of St. Mary's College of California has been appointed to an assistant professorship at Wisconsin State University, Whitewater.

Professor JOHN D. NEFF has been appointed Acting Director of the School of Mathematics at Georgia Institute of Technology.

Mr. WILLIAM NELSON of Purdue University has been appointed to an assistant professorship at the University of Pittsburgh at Bradford.

Professor LOUIS NIRENBERG of New York University has been named director of the university's Courant Institute of Mathematical Sciences.

Mr. JAMES W. NOONAN of the University of Maryland has been appointed a postdoctoral resident research associate at the U.S. Naval Research Laboratory, Washington, D. C.

Professor ARTHUR E. OBROCK of Purdue University has been appointed to an associate professorship at Case Western Reserve University.

Mr. ROBERT H. OWENS of the University of Virginia has been appointed a temporary liaison scientist at the Office of Naval Research, London, England.

Professor NICHOLAS PASSELL of Carnegie-Mellon University has been appointed to an assistant professorship at the University of Florida.

Professor EDMUND PRIBITKIN of CUNY, Queens College, has been appointed to an associate professorship at Millersville State College.

Mr. KENNETH H. PRICE of the University of Texas at Austin has been appointed to an assistant professorship at Stephen F. Austin State University.

Professor X. B. REED, JR., of the University of Florida has been appointed to a visiting professorship at the University of New Brunswick.

Professor CHARLES S. REES of the University of Tennessee has been appointed to an assistant professorship at Louisiana State University, New Orleans.

Professor NATHANIEL R. RIESENBERG of Wisconsin State University, Whitewater, has been appointed to an assistant professorship at C. W. Post College.

Professor I. RICHARD SAVAGE of Florida State University has been elected to membership in the International Statistical Institute. Professor SAVAGE is currently at the Center for Advanced Study in the Behavioral Sciences in Stanford and will return to Florida State University in September 1971.

Dr. DAVID G. SCHAEFFER of Brandeis University has been appointed to an assistant professorship at the Massachusetts Institute of Technology.

Professor VICTOR P. SCHNEIDER of the University of Massachusetts has been appointed to an assistant professorship at the University of Southwestern Louisiana.

Professor MAXWELL E. SHAUCK, JR., of Duke University has been appointed to an associate professorship at North Carolina Central University.

Mr. BHAGAT SINGH of the University of Illinois, Urbana, has been appointed to an assistant professorship at the University of Wisconsin, Green Bay.

Mr. JAMES D. SMITH of Texas Christian University has been appointed to an assistant professorship at Southwest Texas State University.

Mr. GLENN A. STOOPS of Litton Industries, Inc., Fort Ord, California, has been appointed to an assistant professorship at the Naval Postgraduate School.

Professor JAMES J. TATTERSALL of the University of Oklahoma has been appointed to an assistant professorship at Providence College.

Mr. ERNEST THIELEKER of Argonne National Laboratory has been appointed to an assistant professorship at the University of South Florida.

Mr. BRIAN R. UMMEL of the University of Wisconsin, Madison, has been appointed a lecturer at the University of Wisconsin, Milwaukee.

Professor HENRY S. VALK of the University of Nebraska has been appointed Dean of the General College, Georgia Institute of Technology.

Mr. GARY I. WAKOFF of Harvard University has been appointed supervisor of financial analysis for the Management Sciences Division of the American Telephone and Telegraph Company.

Mr. MARTIN E. WALTER of the University of California, Irvine, has been appointed a visiting lecturer at the University of California, Los Angeles.

Mr. ROGER P. WARE of the University of California, Santa Barbara, has been appointed to an assistant professorship at Northwestern University.

Mr. DALLAS E. WEBSTER of the University of Wisconsin, Madison, has been appointed a research associate at the Institute for Advanced Study.

Professor I. JACOB WEINBERG of AVCO Systems Division has been appointed to an associate professorship at Lowell Technological Institute.

Dr. J. ERNEST WILKINS, JR., of Gulf General Atomic, Inc., has been appointed distinguished professor of applied mathematical physics at Howard University.

Mrs. ANN YASUHARA of New York University has been appointed a visiting member at the Institute for Advanced Study.

PROMOTIONS

To Professor. California State Polytechnic College, Pomona: TA LI; Massachusetts Institute of Technology: DANIEL J. KLEITMAN, ISADORE M. SINGER, W. GILBERT STRANG; Miami University, Oxford, Ohio: RICHARD G. LAATSCH; Salem College: ARLEY T. CURLEE; University of South Africa: HORST-SIEGFRIED GRASSER; University of Wisconsin, Madison: MEI-CHANG SHEN.

To Associate Professor. University of Arizona: RICHARD B. THOMPSON; Florida State University: FRED W. LEYSIEFFER; Fresno State College: MOSES E. COHEN; Miami University, Oxford, Ohio: STANLEY E. PAYNE; North Carolina State University: WILLIAM G. DOTSON, JR.; University of Southwestern Louisiana: HENRY E. HEATHERLY.

To Assistant Professor. Georgia Institute of Technology: STANLEY J. WERTHEIMER; Kingsborough Community College (CUNY): PHILIP J. GREENBERG; Monmouth College: JEFFREY M. LEVINE; University of Texas at Austin: ERNA H. PEARSON.

To Research Associate. Queen's University: CHARLES SMALL.

To Director of Communications Studies. Systems Applications, Inc., Beverly Hills, California: RICHARD N. LANE.

INSTRUCTORSHIPS

Massachusetts Institute of Technology: E. GRAHAM EVANS, EDWARD C. HOOK, PETER J. KIERNAN, JOHN M. MacINTYRE, RALPH C. REID, JR., RICHARD P. STANLEY; North Carolina State University: JAMES C. HALSEY, DANIEL W. KRIDER; Staten Island Community College: ELLENA K. SCHWEBER.

DEATHS

Mr. ALFREDO BANOS of Los Angeles, California, died on July 28, 1970, at the age of 29. He was a member of the Society for 4 years.

Professor ABRAM S. BESICOVITCH of Trinity College, Cambridge, England, died on November 2, 1970, at the age of 79. He was a member of the Society for 21 years.

Dr. FREDERICK W. BROWN of Rockville, Maryland, died on October 24, 1970, at the age of 62. He was a member of the Society for 32 years.

Sir EDWARD F. COLLINGWOOD of Northumberland, England, died on October 25, 1970, at the age of 70. He was a member of the Society for 40 years.

NEWS ITEMS AND ANNOUNCEMENTS

NATIONAL MEDAL OF SCIENCE WINNERS

The White House has announced the winners of the National Medal of Science for 1970. This award, which was established in 1959, is presented annually for distinguished achievement in science, mathematics, and engineering. Among the nine recipients of the award was Professor Richard D. Brauer, Harvard University, for his development of the theory of modular representations.

ASSOCIATION OF WOMEN MATHEMATICIANS

A new organization, the Association of Women Mathematicians, has recently been formed. Membership is open to all mathematicians, regardless of sex. Dues are two dollars annually. Further information may be obtained by writing to Professor Mary W. Gray, Department of Mathematics and Statistics, the American University, Washington, D. C. 20016.

ASSISTANTSHIPS AND FELLOWSHIPS IN MATHEMATICS IN 1971-1972

The Assistantships and Fellowships listed below are in addition to those listed on pages 1099-1206 of the December 1970 issue of these *Notices*.

TYPE *of financial assistance (with number anticipated in 1971-1972)*	STIPEND *amount in dollars*	*9 or 12 mo.*	TUITION *if not included in stipend (dollars)*	SERVICE REQUIRED *hours per week*	*type of service*

ALABAMA

Samford University

BIRMINGHAM, ALABAMA 35209
W. D. Peeples, Jr., Head
Department of Mathematics

NUMBER OF DEGREES AWARDED IN 1970
Baccalaureate degrees by institution 658
Baccalaureate degrees by department 9
Master's degrees by department 10

Teaching Assistantship (4)	1800	9	38.50/hr	6	
Scholarship (2)	1000	9	38.50/hr		

CALIFORNIA

San Fernando Valley State College

NORTHRIDGE, CALIFORNIA 91324
James C. Smith, Chairman
Department of Mathematics
Applications must be filed by July 1, 1971

NUMBER OF DEGREES AWARDED IN 1970
Baccalaureate degrees by institution 3360
Baccalaureate degrees by department 70
Master's degrees by department 5

Fellowship (8)	2300	9	may be waived	18	Tutoring, grading
Part-time Instructor (?)	285/unit				

CONNECTICUT

University of Hartford

WEST HARTFORD, CONNECTICUT 06117
Cecilia Welna, Chairman
Department of Mathematics
Applications must be filed by April 1, 1971

Teaching Assistantship (2)	2400	9	tuition abatement	6	Teaching

LOUISIANA

Nicholls State University

THIBODAUX, LOUISIANA 70301
Larry S. Haw, Head
Department of Mathematics
Applications must be filed by March 15, 1971

NUMBER OF DEGREES AWARDED IN 1970
Baccalaureate degrees by institution 491
Baccalaureate degrees by department 11
Master's degrees by department 3

Teaching Assistantship (6)	2500	9		6	Teaching

MICHIGAN

University of Michigan

ANN ARBOR, MICHIGAN 48104
Arthur W. Burks, Chairman
Department of Computer and Communication Sciences
Applications must be filed by February 1, 1971

NUMBER OF DEGREES AWARDED IN 1970
Baccalaureate degrees by institution 4966
Baccalaureate degrees by department 26
Master's degrees by department 9

Ph.D. degrees awarded during last three years by field of specialization: computer science and numerical analysis (27). TOTAL 27.

Fellowship (25)	2500	9 or 12			
Teaching Fellowship (23)	1500-3250	9	*	2-4	Teaching
Teaching Assistantship (6)	1000-1625	12	**	20	
Research Assistantship (20)	3000-7500	12	**	Half-full time	Research

*In-state.
**Tuition is not included in stipend.

NEW YORK

C. W. Post College

BROOKVILLE, NEW YORK 11548
Bernard Seckler, Chairman
Department of Mathematics
Applications must be filed by March 15, 1971

NUMBER OF DEGREES AWARDED IN 1970
Baccalaureate degrees by institution 850
Baccalaureate degrees by department 20
Master's degrees by department 15

Graduate Assistantship (5)	1700-2500	9		18	Grading, Teaching on occassion, Tutoring

OHIO

Case Western Reserve University

CLEVELAND, OHIO 44106
A. J. Lohwater, Chairman
Department of Mathematics and Statistics
Applications must be filed by March 1, 1971

NUMBER OF DEGREES AWARDED IN 1970
Baccalaureate degrees by institution 1048
Baccalaureate degrees by department 20
Master's degrees by department 7

Ph.D. degrees awarded during last three years by field of specialization: algebra and number theory (2); geometry and topology (2); logic (2); analysis and functional analysis (10); probability and statistics (11); computer science and numerical analysis (4); applied mathematics (1). TOTAL 32.

Fellowship (20)	2300-3000	9			
Teaching Fellowship (20)	2800-3200	9		3-4	Teaching
Research Assistantship (5)	2300-2800	9			

PENNSYLVANIA

Carnegie-Mellon University

PITTSBURGH, PENNSYLVANIA 15213
Alan J. Perlis, Head
Department of Computer Science
Applications must be filed by March 15, 1971

NUMBER OF DEGREES AWARDED IN 1970
Baccalaureate degrees by institution 728
Master's degrees by department 1

Ph.D. degrees awarded during last three years by field of specialization: computer science and numerical analysis (13). TOTAL 13.

Fellowship (10)	1800-4800	9-12			
Research Assistantship (44)	2200-2200	9		20	

WASHINGTON

Eastern Washington State College

CHENEY, WASHINGTON 99004
Hugh Sullivan, Chairman
Department of Mathematics
Applications must be filed by March 15, 1971

NUMBER OF DEGREES AWARDED IN 1970
Baccalaureate degrees by institution 1331
Baccalaureate degrees by department 53
Master's degrees by department 7

Teaching Assistantship (8)	2700	9	120/qtr (currently)	5	Teaching
				10-13	Miscellaneous

WISCONSIN

University of Wisconsin

MADISON, WISCONSIN 53706
Norman R. Draper, Chairman
Department of Statistics
Applications must be filed by January 15, 1971

NUMBER OF DEGREES AWARDED IN 1970
Baccalaureate degrees by institution 5131
Master's degrees by department 13

Ph.D. degrees awarded during last three years by field of specialization: probability and statistics (10). TOTAL 10.

Fellowship (4)	3100-3240	9	payable		
Teaching Assistantship (8)	3896	9		20	Teaching
Research Assistantship (12)	2985	9		20	Research

ERRATA

Brown University

PROVIDENCE, RHODE ISLAND 02912
J. P. LaSalle, Chairman
Division of Applied Mathematics

Ph.D. degrees awarded during last three years by field of specialization: analysis and functional analysis (10); probability and statistics (2); computer science and numerical analysis (6); applied mathematics (16). TOTAL 34.

The figures listed in the December 1970 issue were for one year only. All other information remains the same.

Washington State University

PULLMAN, WASHINGTON 99163
Calvin T. Long, Chairman
Department of Mathematics
Applications must be filed by March 1, 1971

Fellowship (3)	2600*	12	216/sem.		
Teaching Assistantship (34)	2600-2800*	9	216/sem.	20	
Scholarship, dept. (3)	155		216/sem.		

*Dependency allowances usually bring the figure to $3600 or $3800.

The stipends listed above vary slightly from those listed in the December 1970 issue.

A RATING OF GRADUATE PROGRAMS

The American Council on Education (ACE) has released a new report entitled "A Rating of Graduate Programs" by Kenneth D. Roose and Charles J. Anderson. This report is based on a study made in 1969, and is the third study on graduate programs in the United States. The first, in 1957, was an educational survey of the University of Pennsylvania by Hayward Keniston*; the second was done in 1964 by Allan M. Cartter**, then vice-president of the American Council of Education. The tables from the latter report were reproduced in the December 1966 *NOTICES*. This present report by Roose and Anderson fulfills the intention of Cartter to repeat the 1964 study within five years "to avoid 'freezing' the reputations of various universities." For the Roose-Anderson report, 65 departments of mathematics, out of 102 departments rated, qualified for a listing in the study, This number includes approximately one-half of the Ph.D. granting departments. The accompanying tables compare the present ratings with those of the 1957 and 1964 studies, insofar as information was available in those studies.

Copies of "A Rating of Graduate Programs" are available at $4 from the American Council on Education, One Dupont Circle, Washington, D. C. 20036.

Leading Institutions, by Rated Effectiveness of Doctoral Program

Institutions with "Effectiveness of Doctoral Program" Scores of:			
2.0—3.0[a]	1.5—1.9[a]	.8—1.4[b]	
Harvard •	Brandeis •	Arizona	Michigan State
Princeton †	Cal. Tech. †	California, Riverside	North Carolina
M.I.T.	Cornell †	Carnegie-Mellon	Notre Dame
California, Berkeley	Brown •	Case Western Reserve	Ohio State
Chicago •	California, Los Angeles †	Colorado	Oregon
Stanford †	Illinois †	Duke	Oregon State
Yale	Washington (Seattle) †	Florida	Penn State
Wisconsin	Columbia	Florida State	Purdue
Michigan •	Pennsylvania •	Georgia	Rensselaer
N.Y.U. †	Rockefeller †	Indiana	Rice
	Virginia †	Iowa (Iowa City)	Rochester
	California, San Diego •	Iowa State (Ames)	Rutgers
	Minnesota †	Johns Hopkins	Southern California
	Northwestern †	Kansas	Syracuse
	Tulane †	Lehigh	Texas
		Louisana State	Utah
		Maryland	Washington (St. Louis)
		Massachusetts	Yeshiva

• Score and rank are shared with the institution immediately below.
† Score and rank are shared with the institution immediately above.
a. Institutions are listed in rank order.
b. Institutions are listed in alphabetical order.

* Hayward Keniston, Graduate Study and Research in the Arts and Sciences at the University of Pennsylvania, University of Pennsylvania Press, 1959.

** Allan M. Cartter, An Assessment of Quality in Graduate Education, American Council on Education, 1966.

Leading Institutions, by Rated Quality of Graduate Faculty

Rankings				Percentage[a] of Raters Who Indicate: "Quality of Graduate Faculty" Is:			
1957	1964	1969	Institution	Distinguished and strong	Good and adequate	All other	Insufficient information
			Twenty-seven institutions with scores in the 3.0—5.0 range, in rank order				
4	2*	1*	California, Berkeley	91	–	–	8
1	1	1*	Harvard	90	2	–	8
3	2*	3	Princeton	91	1	–	8
2	4	4	Chicago	86	2	–	13
‡	5	5	M.I.T.	85	2	–	13
9	6	6	Stanford	79	5	–	17
6	7*	7	Yale	77	5	–	18
8	7*	8	N.Y.U.	67	9	–	24
11	10*	9	Wisconsin	78	10	–	12
7	9	10*	Columbia	67	12	–	19
5	10*	10*	Michigan	68	15	–	17
10	12*	12*	Cornell	59	18	–	23
12	12*	12*	Illinois	61	20	–	19
‡	15*	14	California, Los Angeles	54	24	–	22
‡	19*	15*	Brandeis	42	27	1	29
‡	17*	15*	Brown	42	33	–	25
‡	12*	15*	Cal.Tech.	41	32	–	26
14	15*	18*	Minnesota	35	33	–	31
15*	22*	18*	Pennsylvania	36	32	–	32
‡	17*	18*	Washington (Seattle)	40	33	1	26
‡	22*	21*	Purdue	29	40	–	31
‡	‡	21*	*Rockefeller*	22	25	2	50
‡	19*	23*	Johns Hopkins	26	39	–	35
‡	19*	23*	Northwestern	23	42	–	35
‡	22*	23*	Virginia	21	41	1	38
‡	‡	26*	*California, San Diego*	16	31	2	50
‡	25	26*	Indiana	17	48	–	35

Seventeen institutions with scores in the 2.5—2.9 range, in alphabetical order

Carnegie-Mellon † [b]
Colorado †
Duke
Maryland
Michigan State †
North Carolina
Notre Dame †
Ohio State
Oregon †
Penn State †
Rice
Rochester
Rutgers †
Southern California †
Tulane
Washington (St. Louis) †
Yeshiva †

Twenty-one institutions with scores in the 2.0—2.4 range, in alphabetical order

Arizona †
Buffalo †
California, Riverside
Case Western Reserve † [c]
Florida †
Florida State †
Georgia
Ill. Inst. of Tech. †
Iowa (Iowa City)
Iowa State (Ames) †
Kansas
Kentucky †
Lehigh †
Louisiana State †
Massachusetts
Oregon State †
Rensselaer †
Syracuse
Texas
Utah †
Wayne State

Italicized institutions were not included in the 1964 survey of this discipline.
* Score and rank are shared with another institution.
† Institution's 1969 score is in a higher range than its 1964 score.
‡ Not ranked.
a. Percentages add across; the sum may not total 100 because of rounding.
b. Institution rated in 1964 was Carnegie Institute of Technology.
c. Both Case Institute of Technology and Western Reserve University were rated in 1964.

Estimated Change in the Last Five Years

Institution	Percentage[a] of Raters Who Indicate: Quality of Graduate Education Is: Better than 5 years ago	Little changed in last 5 years	Worse than 5 years ago	Insufficient information	Change in "Quality of Graduate Faculty" Score from 1964 to 1969[b]
Twenty-seven institutions with "Quality of Graduate Faculty" scores in the 3.0—5.0 range, in rank order					
California, Berkeley	12	54	11	23	+.1
Harvard	6	65	3	26	=
Princeton	10	53	12	25	=
Chicago	7	57	8	29	=
M.I.T.	23	46	2	30	+.1
Stanford	15	44	6	35	+.1
Yale	12	48	5	35	+.1
N.Y.U.	9	47	5	40	=
Wisconsin	17	54	1	28	+.1
Columbia	6	44	16	35	−.1
Michigan	5	49	11	35	=
Cornell	12	45	6	37	+.1
Illinois	11	50	2	38	+.1
California, Los Angeles	23	35	1	42	+.2
Brandeis	26	26	1	47	+.4
Brown	13	34	7	46	+.2
Cal. Tech.	7	43	6	44	−.1
Minnesota	8	38	6	47	=
Pennsylvania	17	29	8	46	+.4
Washington (Seattle)	16	40	2	42	+.1
Purdue	15	36	3	47	+.2
Rockefeller	21	16	2	61	tt
Johns Hopkins	2	33	15	50	=
Northwestern	9	33	4	53	=
Virginia	16	32	5	48	+.1
California, San Diego	35	6	1	58	tt
Indiana	20	25	8	48	+.1
Six selected[c] institutions with "Quality of Graduate Faculty" scores in the 2.5—2.9 range, in alphabetical order					
Colorado	23	24	2	51	t
Maryland	23	27	2	48	s
Michigan State	23	30	3	44	t
Oregon	23	26	–	51	t
Rice	20	20	10	51	s
Rutgers	31	19	2	48	t
Seven selected[c] institutions with "Quality of Graduate Faculty" scores in the 2.0—2.4 range, in alphabetical order					
Arizona	21	14	2	63	t
Buffalo	29	13	3	55	t
California, Riverside	20	13	5	63	tt
Case Western Reserve [d]	28	15	5	53	t
Kentucky	26	14	2	58	t
Massachusetts	38	7	–	55	tt
Utah	20	15	1	64	t

a. Percentages add across; the sum may not total 100 because of rounding.
b. Legend: +: institution's 1969 score is higher than its 1964 score by the amount shown.
—: institution's 1969 score is lower than its 1964 score by the amount shown.
=: institution's 1969 score is equal to its 1964 score.
t: institution's 1969 score is in a higher range than its 1964 score.
s: institution's 1969 score is in the same range as its 1964 score.
tt: institution was not included in the 1964 survey of this discipline.
c. Institutions with ratings of "Better than 5 years ago" from 20 percent or more of the raters.
d. Both Case Institute of Technology and Western Reserve University were rated in 1964.

LETTERS TO THE EDITOR

Editor, the *Notices*

Among the advertisers in the November 1970 issue of the *Notices* is the University of the Witwatersrand of Johannesburg, South Africa, which is seeking to hire two people for its Mathematics Department. In view of the racial policies of the government of South Africa, I wonder which races are to be automatically excluded from these positions.

The editors may not wish to include or exclude advertisers of another country based solely upon approval or disapproval of the government of that country. A university's hiring policies, however, should be one of the concerns of the *Notices*. The editors ought to determine if these policies are generally in compliance with the minimum legal standards of our country (which forbid, of course, any racial test for employment) and accept advertising only from institutions which meet such standards.

Susan Schindler

Editor, the *Notices*

Members of the American Mathematical Society may wish to consider joining a political action arm of science. The Federation of American Scientists, now chaired by Herbert F. York, is expanding its 25 year old program on Capitol Hill. Unlike tax-exempt professional associations, the Federation is licensed to lobby and has become the largest lobbying arm of science.

Congressional Quarterly listed FAS as one of two outside organizations most effective in the debate over the ABM, and FAS has played a significant role also in the SST debate. Members of the Federation receive its newsletter and may participate in local chapters. Calender year dues are $15.00.

Today, professional obligations involve more than support of professional societies only. Let us keep you informed of the Capitol Hill perspective on problems of science and society. Write 203 C Street, N. E., Washington, D. C. 20002.

Jeremy J. Stone
Director, FAS

Editor, the *Notices*

The woman mathematician today is better off than Emmy Noether, who taught without pay. But there is still much discrimination. See the article in the September 1970 AAUP Bulletin "On women in academe". The following quotation is from page 6 of "Finding employment in the mathematical sciences," published by the Mathematical Sciences Employment Register: "Women find the competive situation in the government somewhat more advantageous to them, since it is relatively hard to secure a well-qualified mathematician for many higher-level government jobs. In many such cases women are welcomed if their qualifications are better than those of the available men." The writer seems to imply that outside the government a woman has trouble getting a job no matter how well-qualified she is. I hope that the law prohibiting discrimination by sex will cause revision of this rather patronizing quotation. Mathematicians are trained in rational thought. They should recognize that a time-honored custom deserves dishonor if its effect is the unemployment of able mathematicians.

Elizabeth Berman

Editor, the *Notices*

One of the important communication chains in science is the distribution of reprints by the author of a published journal article. Authors differ widely in their habits of reprint distribution: some send out great quantities wholesale, while others modestly send out just a few or even none.

We write to plead that authors at least send reprints to living authors of works in the bibliography of the reprinted article. Both of us have had the surprising experience of discovering much later that one of our publications had been cited in an article that we would have wished to study earlier.

The reasons for sending reprints to cited authors are, as we see them, first courtesy, and second (more important) using a bibliographical connection to help move information along the network of scientific connections. Distribution of reprints to previous authors increases the opportunities for serious criticism and communication of current developments. The earlier author will have often become an information center for his specialized subject.

Copies of this letter have been sent to Science, Nature, Notices of the American Mathematical Society, and the American Statistician.

William Kruskal
I. Richard Savage

Editor, the *Notices*

I entirely agree with the tenor of the letter by Saunders Mac Lane in the issue of the *Notices* of January 1971. I also had serious misgivings about the emphasis on logic and formalism in the "Elements of Mathematics" program of CSMP. I, myself, attended the conference on the teaching of mathematics to which Saunders Mac Lane was also invited and many of us there deeply regretted his absence. I would like to reassure him that in fact an entire session was set aside by the organizers of the conference for a discussion of this issue and, in fact, extremely frank views were expressed on both sides. It is my understanding that a record of this discussion will appear in the Proceedings of the conference.

I think it also fair to point out that the EM program forms only a very small part of CSMP activities, so that one should not indict the entire operation simply on the basis of certain idiosyncratic features of that program. The main thrust of CSMP is in the direction of individualized instruction throughout the student's entire pre-college experience from kindergarten onwards. I believe that we can all approve of this objective.

Peter J. Hilton

Editor, the *Notices*

Judging by recent articles in the New York Times and Science magazine, we face a fundamentally new and disastrous financial situation that will last into the 1980's. Of course, there is no point to extreme recriminations about the folly of past policies, but there are some lessons to be learned.

In my opinion, one of these lessons is that it was not wise to encourage the complete separation between mathematics and its applications in the sciences, as many eminent mathematicians have done. In the past, many of the best "applied" mathematicians were those trained in pure mathematics who, after their Ph. D., but while they were still young, and not completely set in their ways, became interested in the applications. Presumably some of these men first became interested in applied work because of the lack of career opportunities in pure mathematics, but then found that they liked the stimulus of the applications. Of course, ideally scientific work should not be distorted by such materialistic pressures, but the historical facts are that, at least in theoretical physics and mathematics, such pressures have often been beneficial in the long run.

Thus, I would urge two modest steps that could readily be incorporated within our existing institutions. First, training students of pure mathematics in the mathematical structure of fields of science and engineering that do make extensive use of mathematics. I have in mind not the traditional professional courses in the techniques of application of mathematics, but "cultural" courses that teach students what the sciences look like in the subtle language of modern mathematics. Second, some program of postdoctoral training and research for young mathematics Ph.D.'s who are highly motivated to apply their training and want to invest their time in learning how to do it.

Judging from what I have seen, I do not believe, however, that our existing institutions are sufficiently flexible and imaginative to respond in such a constructive way. More likely, everyone will just withdraw into their institutional shell and carry on as far as possible in the current directions. However, some grass-roots pressure may be beneficial; in writing this letter, I hope to help stir it up.

Robert Hermann

Editor, the *Notices*

This fall it came to my attention that University Microfilms have raised (without notification) their minimum price for xerographic copies of dissertations from \$3.00 to \$6.00, and that they have also significantly reduced the time needed to process an order. The time reduction is certainly worth something, but one wonders if a doubling of price is necessary. Of course, if one needs a thesis and University Microfilms is the only place to get it, it is worth the price. The advantages of having most of the dissertations written in this country available from a single source are obvious. But at some time it might be worthwhile for the academic community to consider the possible benefits of a little competition in this area.

William P. Wardlaw

Editor, the *Notices*

In his recent letter Oscar Zariski raises questions which must be fully explored by members of the Society. He states that domestic political matters and matters of national foreign policy should not be matters of professional concern to members of the Society.

I must disagree very strongly with Professor Zariski, for this view would limit our professional concerns to only the creation and teaching of mathematics but not to the uses to which mathematics is put in our society. As I understand Professor Zariski, we can be professionally pleased at the expansion of mathematical activity due to the influx of large sums of money from Washington, for this increases mathematical discovery and mathematics departments. But when our mathematical work has applications to war technology or when our students work in "defense" industries, we are to take no professional notice. Such a view of professionalism makes the mathematical profession a willing tool for any policy decreed in Washington.

The present war is an example. One of the main reasons decision-makers in Washington thought they could win where the French had lost was our superior technology, which developed in good measure in the post-sputnik expansion of scientific education and personnel. The government-financed expansion of mathematical creation and education, in which many members of the Society have so joyously participated (myself included), has been a vital contribution to the preparation for and the prosecution of the barbarous war in Southeast Asia. Thus a statement by the Society condemning the war can and should be a matter of professional concern for all members.

Statements on conscience by mathematicians speaking as individuals are of little influence and invite reprisals by the government. To refuse as a profession to address ourselves to the uses of our professional work while expanding such work on the basis of funds provided by Washington is an abdication of professional responsibility most charitably described as narrowly self-serving. If a discussion of the uses of mathematics causes divisions within the Society, it simply means we do not agree on our professional responsibilities, but that can hardly be a reason for avoiding such discussions. If the entire mathematical profession remains silent, it will signify our acquiescence in the prostitution of our work in the continuing "technologically sophisticated" atrocity in Southeast Asia and will tell decision-makers in Washington that we will acquiesce in future atrocities.

Robert D.M. Accola

Editor, the *Notices*

I support the position so excellently expressed by our president in the October *Notices*.

James E. Keisler

Editor, the *Notices*

The letter of M. C. Goodall in the *Notices* of January 1971 constitutes a superb example of misinterpretation of a perfectly straightforward statement; his response to the Zariski letter is so distorted that I find it difficult to believe that his action was not deliberate.

I do not believe that Professor Zariski is asserting that we should behave like the proverbial ostrich; rather, I think he is simply saying that mathematicians (or chemists or musicologists) should act on their political convictions as thinking members of their communities, not through their professional societies.

I happen to believe that the American intervention in Vietnam is a colossal strategic blunder as well as a moral atrocity. (On the other hand, I know several members of the AMS, men of decency, who, while certainly not relishing the American involvement, feel that it was necessary in order to fend off an even greater calamity.) I also happen to be worried about (among other things) dangerous conditions in our coal mines, deterioration of public transportation, dishonest advertising, the Middle East situation, and the unhappy condition of Spanish Basques, Soviet Jews who want to emigrate to Israel, and independent-minded Soviet writers. But the AMS is not the proper milieu in which to agitate about these matters. From such a policy it would be a logical and short step to requiring a "loyalty oath" of present and prospective members.

Bernard Epstein

NEWS ITEMS AND ANNOUNCEMENTS

NATO POSTDOCTORAL FELLOWSHIPS IN SCIENCE

The National Science Foundation and the Department of State have announced the award of 45 North Atlantic Treaty Organization (NATO) Postdoctoral Fellowships in Science. The 45 recipients of these Fellowships were selected from among 432 applicants. Among those receiving awards were the following mathematicians: Kenneth L. Fields, University of Chicago; George S. Sacerdote, University of Illinois, Urbana; Erik A. Lippa, University of Michigan.

NSF SENIOR POSTDOCTORAL AND SCIENCE FACULTY FELLOWSHIP AWARDS

The National Science Foundation has announced the award of 54 Senior Postdoctoral Fellowships and 213 Science Faculty Fellowships for 1971. Senior Postdoctoral Fellowships permit scientists to pursue further research and advanced training in their particular fields. Science Faculty Fellowships help college, university, and junior college science teachers to enhance their effectiveness as teachers. Among those receiving Senior Postdoctoral Fellowships were two mathematicians: Professor Wu-chung Hsiang of Yale University and Professor George I. Glauberman of the University of Chicago. Fifty-five mathematicians were awarded Science Faculty Fellowships.

RESEARCH PAPERS

Traditionally the institution listed at the end of a research paper published by the American Mathematical Society has been that of the university at which the research was done, and during the past few years has been the institution requested to support the publication costs of the article. In many cases authors have not understood the policy and have asked to have the address changed in proof. This, of course, has not been possible when page charge commitments had already been accepted. In the spring of 1971, at the request of the Committee to Monitor Problems in Communication, both the mailing address of the author and the name of institutional sponsor will be included with papers published in AMS publications. When submitting papers to the editors, it will be appreciated if authors will include both addresses if they are different. This will enable readers to communicate more easily with authors of research papers.

MEMORANDA TO MEMBERS

MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

Applicant qualification forms and position description forms are now available for those persons who wish to list in the May issue of the Mathematical Sciences Employment Register. The deadline for receipt of the completed forms is April 1, 1971. There is no charge for listing in the published list except when the late listing charge of $5 is applicable. Provision may be made for anonymity of applicants upon payment of $5 to defray the cost involved in handling such a listing; this fee must be submitted with the applicant qualification form. Forms may be obtained from the Mathematical Sciences Employment Register, Post Office Box 6248, Providence, Rhode Island 02904.

An open Register will be maintained at the meeting of the American Mathematical Society to be held on April 7-10, 1971. The Register will be open from 9:00 a.m. to 5:00 p.m. on Thursday and Friday, April 8-9, in the Jade Room of the Waldorf-Astoria in New York City. A complete announcement of this meeting appears in this issue of these *Notices*. As this is not a joint meeting with the other sponsoring organizations, the costs of this Register will be borne entirely by the Society. To assist the Register personnel in planning for this open Register, it is requested that those persons who expect to participate send a note to the Providence office. Employers are requested to indicate the number of positions available and to specify which day or days they will be available for interviewing.

The May issue of the Register will be mailed from Providence in late April. A subscription to the lists, which includes three issues (May, August, and January) of both the applicants list and the positions list, is available for $30 a year; the individual issues of both lists may be purchased in May, August, and January for $15. A subscription to the applicants list alone or single copies of that list are not available. Copies of the positions list only may be purchased for $5. A subscription to the list of positions, which also includes three issues (May, August, and January), is available for $12 a year.

It should be noted that the lists are mailed "Book Rate" (delivery time from Providence to most locations in the U.S. or Canada is approximately 14 days or longer) unless the purchaser either indicates a willingness in advance to pay the "First Class" or "Air Mail" charges, or includes the fee for this service when prepayment is made. The applicable postage charges, determined by the location of the purchaser, will be furnished on request to those persons who would like to take advantage of this service.

CORPORATE MEMBERS AND INSTITUTIONAL ASSOCIATES

The Society is pleased to announce that, as of December 31, 1970, the following companies and corporations are supporting the Society through Corporate Memberships:

Academic Press, Incorporated
Bell Telephone Laboratories, Incorporated
The Boeing Company
Eastman Kodak Company
Ford Motor Company
General Motors Corporation
Hughes Aircraft Company
International Business Machines Corporation
Mobil Oil Corporation
North American Rockwell Corporation
RCA Corporation
Texaco, Incorporated
TRW, Incorporated

The following corporations and companies are supporting the Society as Institutional Associates:

Addison-Wesley Publishing Company
Chelsea Publishing Company
Chemical Rubber Company
Dover Publications
Plenum Publishing Corporation
Princeton University Press
Prindle, Weber & Schmidt, Incorporated

Shell Development Company
Springer-Verlag
Union Oil Company of California

FRIENDS OF MATHEMATICS FUND

The Society recently received an anonymous gift of forty shares of stock in the Lincoln National Corporation, and the Society is deeply grateful to this anonymous donor. With this gift the Society created a special fund, the first of its kind, to be called the Friends of Mathematics Fund. Future unrestricted donations, anonymous or otherwise, will be included in this special fund, the proceeds of which will be included in the invested assets of the Society. The names of those contributing to the Friends of Mathematics Fund, but not the amounts contributed, will be listed yearly in the *Notices* unless, of course, a contributor desires his name to be withheld.

NEWS ITEMS AND ANNOUNCEMENTS

GRANTS AND PROPOSALS

The Council of the Society has approved the publication of a booklet entitled "Grants and Proposals" that will contain information on obtaining research funds and how to prepare a proposal. Specifically, the booklet will contain a list of source material for the mathematician or administrator of a department of mathematics to refer to in looking for support for research; a list of government agencies and private foundations with the names and telephone numbers of key personnel; and a section devoted to the preparation of a proposal to a government agency. The Council has further authorized that every department chairman be given one free copy, and these will be mailed as soon as possible after the booklet is printed. The booklet will be ready for distribution in early March, and the price per copy will be one dollar ($1). Orders, accompanied by check or money order, should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904.

CENTER FOR APPLIED MATHEMATICS, ILLINOIS INSTITUTE OF TECHNOLOGY

The Illinois Institute of Technology has announced the establishment of the Center for Applied Mathematics which will enable interested students to apply theoretical mathematics to scientific and engineering problems and to receive an advanced degree as well. The Center will bring together faculty and students from various departments of the university who are involved in study and research belonging generally to the area of applied mathematics. Professor Barry Bernstein has been appointed acting director. Anyone with a baccalaureate degree and an adequate background in a field which involves mathematics may apply for admission to the Center.

NEW AMS PUBLICATIONS

PROCEEDINGS OF THE STEKLOV INSTITUTE

Number 98

THE CALCULI OF SYMBOLIC LOGIC. I, Edited by V. P. Orevkov

304 pages; List Price $17.90; Member Price $13.43

This collection of seven papers is from the Mathematical Logic Seminar of the Leningrad Branch of the Steklov Institute of Mathematics, Academy of Sciences of the USSR, and has as its subject the theory of logical deduction and its applications to the construction of automatic deduction search algorithms. Interest in this field of mathematical logic has been growing recently in connection with the development of mathematical cybernetics, especially of those of its branches which involve computer modelling of complex forms of human mental activity. The papers concern the calculi of both constructive and classical logic. The calculi of classical logic are nowhere employed as the logical foundation of any investigation, however; they are considered only as constructively defined objects of study and are investigated on the basis of the principles of the constructive trend in mathematics. The present collection may therefore be regarded as belonging to the constructive trend in mathematics. All of the papers were prepared and presented at the seminar in 1964-1966.

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

Number 106
NUMBERS WITH SMALL PRIME FACTORS, AND THE LEAST k^{th} POWER NON-RESIDUE
By Karl K. Norton

108 pages; List Price $2.10; Member Price $1.58

This paper has two main objectives. The first is to discuss in some detail the asymptotic distribution of positive integers which have only relatively small prime factors and which lie in an arithmetic progression or are relatively prime to a given number. The second objective is to give various upper bounds for the least positive k^{th} power non-residue modulo n which is relatively prime to n, where n is a positive integer. The bounds obtained for this non-residue are of two types: for arbitrary n, O-estimates are found by applying a theorem on numbers with small prime factors (the results generalize classical work in which n was always assumed to be prime); for prime n, several estimates with specific constants are found by different methods. There is an expository section which gives a comprehensive review of previous work on the distribution of numbers with small prime factors. Finally, there is a short derivation of an explicit upper bound for the number of distinct prime factors of n.

SELECTED TRANSLATIONS IN MATHEMATICAL STATISTICS AND PROBABILITY

Volume 9

324 pages; List Price $16.20; Member Price $12.15

Volume 9 of the Selected Translations in Mathematical Statistics and Probability contains the following papers: "Certain occupancy problems connected with the game of lotto. I and II" by András Békéssy; "Strictly equivalent Gaussian measures" by J. Hájek; "A remark on conditional expectation" by Yang Tsung-p'an; "Limit theorems for some classes of random functions" by N. N. Čencov; "A local limit theorem for recurrent events" by A. Aleškevičienė; "Limit theorems for k-sequences of independent random variables" by V. V. Petrov; "On the maximum of the density of the sum of random variables with a unimodal distribution" by B. A. Rogozin; "On the class of limit distributions for thinning streams of homogeneous events" by I. N. Kovalenko; "On the central limit theorem for m-independent variables" by V. V. Petrov; "Sequences of functions and the central limit theorem" by V. F. Gapoškin; "The central limit problem for sums of random variables defined on a Markov chain" by G. Aleškevičius; "On the decomposition of infinitely divisible lattice laws" by I. V. Ostrovskiĭ; "On a power series" by W. K. Hayman; "Application of a theorem of W. K. Hayman to a question in the theory of decomposition of probability laws" by A. A. Gol'dberg and I. V. Ostrovskiĭ; "On a new viewpoint of limit theorems taking into account large deviations" by V. M. Zolotarev; "Limiting distributions for additive functionals of a sequence of sums of independent identically distributed lattice random variables" by A. V. Skorohod and N. P. Slobodenjuk; "Large deviations of sums of a random number of random variables" by A. Aksomaĭtis; "On the distribution of a variable of first transition" by G. I. Prizva and S. N. Simonova; "Estimation of the spectral density of a class of stationary random processes" by M. F. Romanov; "Random walk described by a homogeneous process with independent increments, and asymptotic analysis of its characteristics" by D. V. Gusak; "On the asymptotic behavior of certain functionals of a Brownian motion process" by A. V. Skorohod and N. P. Slobodenjuk; "One-dimensional diffusion processes in variable regions and their equivalence to locally Wiener processes" by I. D. Čerkasov; "Line Markov processes and their application to problems of the theory of reliability" by Ju. K. Beljaev; "Convergence of sums of Markov renewal processes to a Poisson process" by J. Sapagovas; "Large deviations for homogeneous Markov chains" by A. Aleškevičienė; "Examples of optimal nonlinear extrapolation of stationary random processes" by A. M. Jaglom; "Integro-differential equations for probability distributions of additive functionals of diffusion processes" by N. I. Portenko; "On linear and plane searches" by E. Gečiauskas.

NEWS ITEMS AND ANNOUNCEMENTS

TRANSACTIONS OF THE NEW YORK ACADEMY OF SCIENCES

The Transactions of The New York Academy of Sciences, published by the Academy since 1881, is being changed to an interdisciplinary journal of reviewed papers beginning with the January 1971 issue. In the past, the Transactions has confined itself to the publication of papers presented at section meetings held at the Academy. Under the new editorial policy, papers will be solicited from outside the Academy membership. Each paper will be reviewed by two, or in case of a tie, three competent scientists in the author's field, these referees being chosen by members of an Editorial Advisory Board. The Board will consist of the nineteen Scientific Section Chairmen of the Academy and others chosen from time to time. The aims of the new Transactions will be to publish results of outstanding research in all scientific fields, in order to disseminate new information as quickly as possible; and to reflect accurately the state of science, both in its increasing narrowness in terms of specialization and, paradoxically, a concurrent growth in interdisciplinary efforts and interests. The Transactions is published eight times during the academic year, and the subscription price is $20 per year. Further information may be obtained by writing to The New York Academy of Sciences, 2 East 63rd Street, New York, New York 10021.

NATIONAL TRANSLATIONS CENTER

The National Translations Center at the John Crerar Library is the principal U. S. depository and information center for unpublished translations into English from the world literature of the natural, physical, medical, and social sciences. The Center, a recipient of support from the National Science Foundation, is a cooperative, nonprofit enterprise. Scientific and professional societies, industrial and other special libraries, governmental agencies, colleges, universities, and other institutions in the United States and abroad deposit in the Center translations prepared by them. Among its services, the Center will make a search of its files to determine if and where an English translation of a particular bibliographic citation is available, for which there is no charge; will provide either paper or microfilm copies of translations in its collection for a small fee; and will conduct literature searches in its files for all translated works of a given author or all translations from a given journal (excluding those included in cover-to-cover translations). The Advisory Board of the Center is comprised of representatives of professional associations concerned with translation activities and access to information; Professor Ralph P. Boas is the representative of the Society on the Board. Information about contributions to the Center, translation searches, services, and prices may be obtained by writing to the National Translations Center, John Crerar Library, 35 West 33rd Street, Chicago, Illnois 60616. The telephone number is 312-225-2526.

AUDIO RECORDINGS OF MATHEMATICAL LECTURES

Several additional sets of taped lectures are now ready for distribution. (20) "The concept of torsion and Gorenstein rings" by Maurice Auslander of Brandeis University; (23) "On finite groups generated by transvections" by Jack E. McLaughlin of the University of Michigan; (24) "Geometry of numbers of convex bodies" by Kurt Mahler of Ohio State University; (25) "Bounded convergence of analytic functions" by Lee A. Rubel of the University of Illinois at Urbana; (26) "Finite-dimensional H-spaces" by Morton L. Curtis of Rice University and the University of California, Berkeley; (27) "Ordinal solution of G_δ games" by David Blackwell of the University of California, Berkeley; (28) "Aspects of the theory of spherical functions on symmetric spaces" by Ramesh Gangolli of the University of Washington; (29) "Some probability results connected with diophantine approximation" by Patrick

P. Billingsley of the University of Chicago; (30) "Differential algebraic geometry" by James B. Ax of the State University of New York at Stony Brook; (31) "How functional analysis and approximation theory mix today with numerical analysis" by Richard S. Varga of Kent State University; and (32) "Representations of algebras by continuous sections" by Karl H. Hofmann of Tulane University.

The Audio Recordings of Mathematical Lectures may be purchased for \$6 each with the exception of the Colloquium Lectures (Nos. 5 and 6) which are \$10 for each set of two tapes. Additional copies of the manual that accompanies the tapes may be purchased for \$0.30 each. Standing orders for the entire series of lectures may be placed. Orders should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904.

EDWARD T. KOBAYASHI MEMORIAL FUND

Colleagues of the late Professor Edward T. Kobayashi have created a memorial fund for his children, Bertram and Niels. Professor Kobayashi died on January 3, 1971, following a period of ill health. Assets of the fund will be used at the discretion of Mrs. Kobayashi. Contributions may be sent to the Edward T. Kobayashi Memorial Fund, Farmers and Merchants Bank, Las Cruces, New Mexico 88001. Questions about the fund should be addressed to Professor David Arnold, Department of Mathematical Sciences, New Mexico State University, Las Cruces, New Mexico 88001.

CBMS REGIONAL CONFERENCE SERIES IN APPLIED MATHEMATICS

With the support of the National Science Foundation, the Conference Board of the Mathematical Sciences sponsors a series of Regional Conferences featuring topics of current research interest in the mathematical sciences. The Society for Industrial and Applied Mathematics has been selected to publish certain papers which are essentially in applied mathematics. These monographs will be entitled CBMS Regional Conference Series in Applied Mathematics. The first, to appear in February, is "Numerical solution of elliptic equations" by Garrett Birkhoff of Harvard University; the second, to appear in March, is "Bayesian statistics - a review" by D. V. Lindley of University College, London. Further information may be obtained by writing to SIAM, 33 South 17th Street, Philadelphia, Pennsylvania 19103.

DOCTORATES GRANTED

The following mathematicians received their Ph. D.'s in computer science from Northwestern University in June 1970: James Boyle and Alan Tucker. Both have accepted positions. This information did not reach the Society in time to be included in the List of Ph.D.'s in Mathematics published in the October and November *Notices*.

BATTELLE MEMORIAL INSTITUTE RESEARCH INSTRUCTORSHIP

The Battelle Memorial Institute through its Research Center in Seattle is sponsoring a research instructorship in mathematics at the University of Washington for the academic year 1971-1972. This instructorship is open to young mathematicians who have recently received, or who expect to receive soon, their Ph. D. in mathematics, and who show a strong potential for research. Appointments are for one year but are generally extended for a second year by an appointment as a regular instructor in the Department of Mathematics. The teaching duty for the initial appointment is one course each quarter. The appointee also receives office and visiting scholar privileges at the nearby Battelle Seattle Research Center. The academic year salary for 1971-1972 will be \$1,000. During the academic year of 1971-1972 the Department of Mathematics is conducting a special emphasis year in Several Complex Variables in collaboration with Battelle, so candidates in this field will receive special consideration. Inquiries or requests for application forms should be addressed to Professor Ross A. Beaumont, Department of Mathematics, University of Washington, Seattle, Washington 98105. Such correspondence should reach Professor Beaumont by March 20, 1971.

BACKLOG OF MATHEMATICS RESEARCH JOURNALS

Information on the backlog of papers for research journals is published in the February and August issues of these *Notices* with the cooperation of the respective editorial boards. Since all columns in the table are not self-explanatory, we include further details on their meaning.

Column 3. This is an estimate of the number of printed pages which have been accepted but are not necessary to maintain copy editing and printing schedules.

Column 5. The first (Q_1) and third (Q_3) quartiles are presented to give a measure of normal dispersion. They do not include misleading extremes, the result of unusual circumstances arising in part from the refereeing system.

The observations are made from the latest issue of each journal received at the Headquarters Offices before the deadline for the appropriate issue of these *Notices*. Waiting times are measured in months from receipt of manuscript in final form to receipt of final publication at the Headquarters Offices. When a paper is revised, the waiting time between an editor's receipt of the final revision and its publication may be much shorter than is the case otherwise, so these figures are low to that extent.

	1	2	3		4	5		
JOURNAL	No. issues per year	Approx. No. pages per year	BACKLOG 12/31/70	6/30/70	Est. time for paper submitted currently to be published (in months)	Observed waiting time in latest published issue (in months) Q_1	Med.	Q_3
American J. of Math.	4	1050	0	NR*	6	###	###	###
Annals of Math. Stat.	6	2250	0	NR*	10	10	11	15
Annals of Math.	6	1200	NR*	NR*	12	10	13	18
Canad. J. of Math.	6	1292	680	NR*	9-10	10	12	15
Duke Math. J.	4	800	900	1040	15-18	25	26	27
Illinois J. of Math.	4	700	1700	1700	28	28	29	29
Indiana Univ. Math. J. **	12	1200	500	1000	5	12	12	13
J. Amer. Stat. Assoc.	NR*	NR*	NR*	500	NR*	##	##	##
J. Assoc. for Comp. Mach.	4	700	0	40	12	6	7	11
J. Diff. Geometry	4	530	300	600	10-12	15	16	19
J. Math. Physics	12	3500	NR*	950	8	7	8	9
J. Symbolic Logic	4	800	0	NR*	10	15	19	23
Linear Algebra and Appl.	4	420	80	6	4***	9	10	13
Math. Biosciences	6	1200	100	0	4***	##	##	##
Math. of Comp.	4	1000	0	0	8	14	15	17
Michigan Math. J.	4	400	150	200	11	10	11	14
Operations Research	6	1650	700	200	14	13	16	22
Pacific J. Math.	NR*	NR*	NR*	900	NR*	10	12	15
Proceedings of AMS	12	3250	50	200	10	10	11	14
Proc. Nat'l. Acad. Sci.	12	500#	0	0	2	3	4	5
Quarterly of Appl. Math.	4	600	600	560	12	12	13	16
SIAM J. of Appl. Math.	8	1700	0	NR*	6-8	10	10	12
SIAM J. on Control	4	650	0	NR*	4-6	9	10	11
SIAM J. on Math. Anal.	4	600	0		4-6	9	10	12
SIAM J. on Numer. Anal.	4	700	0	NR*	6-8	10	11	11
SIAM Review	4	700	0	NR*	3-5	9	9	9
Transactions of AMS	12	5000	50	700	11	11	14	17

*NR means that no response was received to a request for information

**Formerly the Journal of Mathematics and Mechanics.

***After acceptance.

#In mathematics.

##Date of receipt of manuscript not indicated in this journal.

###Not computable for this journal.

NATIONAL SCIENCE FOUNDATION BUDGET FOR 1972

Dr. William D. McElroy, Director of the National Science Foundation, has recently released a statement on the Administration's proposal to Congress for the 1972 budget for NSF. The report contains details of the proposed budget and some ten pages of text devoted to the "program summary," in which mathematics is not mentioned once. The budget shows a 23% increase over the 1971 estimated budget with many changes in programs. A large proportion of the additional funds requested by the Foundation will be used to support research activities now in the province of the Department of Defense and the National Aeronautics and Space Administration. A great deal of emphasis is being placed on research in social problems: ecology, population, transportation, and urban studies. For instance, the program begun in 1970 called Interdisciplinary Research Relevant to the Problems of Society will be expanded into a division to be known as the Division of Research Applicable to National Needs with an increase of funds of 138%.

The total budget shows an increase of $116,000,000. Both the substantial increases and decreases are in areas that affect mathematics negatively. Major changes involve increases in the support of laboratories and programs that have very little to do with mathematics; actually $107,000,000. These include such programs as National and Special Research, National Research Centers, Program Development and Management, and so forth. The only part of the increase in which mathematics would share substantially is in the Research Support and Computing Activities which are discussed further on in this report. The budget calls for decreases for major programs in the amount of $53,700,000; and all of the decreases, with the exception of that in the National Sea Grant Program, affect mathematics adversely in one way or another. Programs in which decreases are proposed include Science Development, Institutional Grants for Science, Student Development, Instructional Personnel Development.

The budget includes $257,800,000 for Scientific Research Project Support (an increase over 1971 of 46.6%), and funds in the amount of $40,000,000 are to be allocated from this project to support research no longer supported by other Federal agencies (DoD and NASA). The program summary states that "within the subactivities of this program, preferential emphasis has been given to increasing fundamental research in the biological sciences, engineering, chemistry, oceanography, and the social sciences." The budget for engineering will be up 55.9%; social sciences up 58%; physics up 39.6%; chemistry up 42%; and mathematics up 18.7%. Mathematics is the only discipline for which the increase was less than the overall increase of 23% in the NSF budget for 1972. The appropriation to mathematics from the budget for the Scientific Research Project has been steadily decreasing. It was 7.85% in 1970; 7.62% in 1971; and 6.17% in 1972.

For Computing Activities in Education and Research, $17,500,000 is requested, and it is stated that "increased emphasis will be given to computer innovation in education." For this program the budget shows a slight increase over 1971, but this is still only a fraction above the actual expenditures for 1970. Computer Applications in Research was $6,900,000 in 1970, and a decrease to $4,100,000 is proposed for 1972.

Several programs have been terminated under the item of Science Education Support. Science Education for Students, Postdoctoral and Science Faculty Fellowships, Senior Foreign Scientist Fellowships have all been terminated. Support will still be provided for Graduate Fellowships and Traineeships at a reduced level, from $28,300,000 in 1971 to $20,000,000 in 1972; in 1970, $37,700,000 was allocated to this program. These changes in programs are particularly difficult in mathematics where it is imperative to provide support for young researchers. From the standpoint of mathematics, the termination of the Postdoctoral and Science Faculty Fellowships, coupled with the reduction in Graduate Fellowships and Traineeships, is very disturbing.

The actual dollar-reductions in some of the programs may not appear to be great, but the consequences to mathematics may well be catastrophic. Of concern particularly to mathematicians is the statement of the Administration that the most pressing need now "is the application of what we already know." As mathematicians are more concerned with what is not known, but should be, than with what is already known, this philosophy could be extremely detrimental to the future of mathematical research.

This report has discussed only the proposed changes in the budget for the National Science Foundation, not changes in budgets of other Federal agencies. New budgets of other agencies involve changes that may be detrimental to mathematical research. While all of the agency-proposals are now included in the Budget of the United States for Fiscal Year 1972, Congress will make some changes, and it remains to be seen what the actual appropriation to each agency will be.

ABSTRACTS OF CONTRIBUTED PAPERS

The November Meeting in Urbana, Illinois
November 28, 1970

681-A31. JANOS GALAMBOS, Temple University, Philadelphia, Pennsylvania 19122. Some results and problems concerning the Oppenheim series.

A general algorithm was recommended by A. Oppenheim to expand real numbers into infinite series of rationals whose expansion includes the classical series expansions of Engel, Sylvester and Lüroth and the product expansion of Cantor. Putting p_n/q_n for the first n terms in the expansion of x, asymptotic formulas are given for $\log(x - p_n/q_n)$ for a subclass of these series. Out of the many unsolved problems it is worthwhile to point out that, except the Lüroth series, nothing is known concerning the Hausdorff dimension of sets related to the Oppenheim expansion and which sets turn out to have Lebesgue measure zero. (Received December 14, 1970.)

The March Meeting in Chicago
March 26-27, 1971

Algebra & Theory of Numbers

683-A1. JIMMY T. ARNOLD, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061 and JAMES W. BREWER, University of Kansas, Lawrence, Kansas 66044. Essential valuation overrings of $D[[X]]$. Preliminary report.

Let D be an integral domain with identity, let P be a prime ideal of D, and let $P[[X]]$ denote that prime ideal of $D[[X]]$ consisting of those formal power series each of whose coefficients belongs to P. Theorem 1. If $(D[[X]])_{P[[X]]}$ is a valuation ring, then $(D[[X]])_{P[[X]]}$ and hence D_P are Noetherian valuation rings. Example. If D is a countable almost Dedekind domain which is not Dedekind and if M is a maximal ideal of D which is not finitely generated, then $(D[[X]])_{M[[X]]}$ is not a valuation ring. Theorem 2. If D_P is a Noetherian valuation ring and if $PD[[X]] = P[[X]]$, then $(D[[X]])_{P[[X]]}$ is a valuation ring. (Received July 28, 1970.)

683-A2. HENRY W. LEVINSON, Rutgers University, New Brunswick, New Jersey 08903. On the genera of graphs of group presentations. III.

Every connected graph is isomorphic to a subgraph of a Cayley diagram of a (usually not free) presentation of a free group. In particular, each finite connected planar graph appears as a subgraph of a planar Cayley diagram of a (not necessarily freely presented) presentation of a free group. Criteria for the forms of presentations of free groups giving rise to Cayley diagrams containing previously chosen finite connected planar graphs as subgraphs are derived. (Received December 15, 1970.)

683-A3. K. DEMYS, 844 San Ysidro Lane, Santa Barbara, California 93103. Meta-Cayley hypercomplex algebras must include at least one hypercomplex square root of positive unity.

Based on the number-geometry investigations of E. Hopf, J. F. Adams, M. Kervaire and J. Milnor, enunciated theorems showing that if one tried to form algebras based on $\sqrt{-1}$ beyond the 8-base Cayley-Graves-Dickson algebra, there were in general no inverses, whereby was meant finite multiplicative inverses; the 8-fold basis (which includes the quaternion basis $1, i_1, i_2, i_3$) being $1, i_1, i_2, i_3, i_4, i_5, i_6, i_7$, where $i_n^2 = -1$ and $i_a i_b = -i_b i_a$ if $a \neq b$. It can be shown further, however, exactly what does happen: the meta-Cayley product $i_4 i_8$ does not yield i_{12} but rather the hyperbolic or countercomplex unit ϵ_1; where $\epsilon_n^2 = +1$, $\epsilon_n \neq \pm 1$; and $\epsilon_a \epsilon_b = -\epsilon_b \epsilon_a$ for $a \neq b$. Moreover $i_n \epsilon_n = \epsilon_n i_n = -i_0$, $i_0 i_n = i_n i_0 = \epsilon_n$, and $i_0 \epsilon_n = \epsilon_n i_0 = -i_n$, where $i^2 = -1$. Thus i_0 is a commutative square root of negative unity, its simplest matrix form being $\begin{pmatrix} i_1 & 0 \\ 0 & i_1 \end{pmatrix}$ where $i_1 \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ but $\epsilon_1 \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. Now $(1 \pm \epsilon_n)$ are divisors of zero and have no reciprocals in any finite neighborhood of zero, which is the reason the theorems first stated are true. Thus the theorem $i_4 i_8 = \epsilon_1$ is a deeper and more accurate theorem than those. (Received January 8, 1971.)

683-A4. STEPHEN J. McADAM, University of Texas, Austin, Texas 78712. Ideals, inverses and conductors.

Let R be a commutative domain with one. Call the ideal I an FFI ideal if both I and I^{-1} are finitely generated. Any of the following conditions on R imply that each FFI ideal is invertible. (i) Each maximal ideal is invertible. (ii) R is integrally closed and each maximal ideal is minimal over an invertible ideal. (iii) R is integrally closed with only finitely many primes. Define R to be an FC domain if each conductor ideal of R is finitely generated. Being an FC domain is equivalent to saying I^{-1} is finitely generated for each ideal I generated by two elements. If an FC domain satisfies any of the three above conditions, it is Prufer. Finally R is Prufer if and only if (a) R is FC, (b) R is integrally closed, and (c) for each maximal ideal M, the primes of R_M are linearly ordered by inclusion. (Received January 11, 1971.)

683-A5. HASSO C. BHATIA, Michigan State University, East Lansing, Michigan 48823. A generalized Frattini subgroup of a finite group.

W. Gashütz and J. Rose have considered the family of nonnormal maximal subgroups of a finite group. In this paper the family $F(G)$ of maximal subgroups with nonprime index is considered. Let $L(G)$ be the intersection of the members of $F(G)$. (i) $L(G)$ is supersolvable. (ii) If G is solvable, then $L(G) \cap G'$ is nilpotent. (iii) $L(G)$ contains the largest supersolvably embedded subgroup of G. (iv) If every maximal subgroup with nonprime index is nilpotent, then G is solvable. Definition a. Let π be the set of primes dividing $|G|$. Let $\pi_1 \subseteq \pi$ such that for all elements p and q of π_1 and $\pi \sim \pi_1$ respectively, $p > q$. Then π_1 is an upper set (UP-set) for G. Definition b. A proper normal subgroup H is a special L-subgroup of G if for every $N \trianglelefteq G$

and A a Hall π-subgroup for π an arbitrary UP-set for N, $G = HN_G(A)$ implies $G = N_G(A)$. Definition c. A proper normal subgroup H has property (P) in G, if for every $N \trianglelefteq G$ with $H \leq N$, N/H π-closed implies N π-closed, π a UP-set for N. Then (1) L(G) has property (P) in G; (2) if H is a sp-L-subgroup of G, then G/H has the Sylow tower property iff G has that property; (3) for a nilpotent normal subgroup H the definitions (b) and (c) are equivalent. (Received January 11, 1971.)

683-A6. JOEL A. WINTHROP, University of California, Davis, California 95616. A natural transformation on $Ext_{\mathcal{P}}$.

Let $\mathcal{A}$ be an abelian category and $\mathcal{P}$ be a proper class of short exact sequences in $\mathcal{A}$ (see S. Mac Lane, "Homology"). Let T be a $\mathcal{P}$-exact, additive functor from $\mathcal{A}$ to $\mathcal{A}$. Theorem. The map t defined by $t((\varkappa, \sigma)) = (T(\varkappa), T(\sigma))$ gives a natural transformation between the bifunctors, $Ext^1_{\mathcal{P}}(\_,\_)$ and $Ext^1(T(\_), T(\_))$. Corollary. Proposition 2, R. J. Nunke, "On the structure of Tor, II." (Received January 12, 1971.)

683-A7. JACK E. OHM and DAVID EUGENE RUSH, Louisiana State University, Baton Rouge, Louisiana 70803. The finiteness of I when D[X]/I is flat. Preliminary report.

Let D be an integral domain, X an indeterminate, and I a nonzero ideal of D[X]. Let c(I) denote the ideal of D generated by the coefficients of the elements of I. Theorem 1. If D[X]/I is a flat D-module, then I is a finitely generated ideal. This is false for an arbitrary commutative ring R with identity. However, Theorem 2. These are equivalent. (i) I is R[X]-projective and c(I) is generated by an idempotent. (ii) I is finitely generated and R[X]/I is R-flat. From (1) and (2), we obtain Theorem 3. D[X]/I is D-flat iff I is an invertible ideal of D[X] and c(I) = D. Further conditions are studied which insure that $ID_P[X]$ be principal for each prime ideal P of D. For instance, Theorem 4. Let ξ be the image of X in D[X]/I. These are equivalent. (i) $D + \xi D + \ldots + \xi^t D$ is D-flat for each $t \geqq 0$. (ii) c(I) = D and $ID_P[X]$ is principal for each prime P of D. (iii) D[X]/I is a torsion free D-module and c(f) is invertible for each f of minimal degree in I. If D is integrally closed, then (i) - (iii) are also equivalent to: (iv) D[X]/I is D-flat. (v) I is invertible and c(I) = D. If ξ is a regular element of D[X]/I then (i) - (iii) are also equivalent to: (vi) $D[\xi]$ and $D[1/\xi]$ are D-flat. (Received January 14, 1971.)

683-A8. FRANCIS A. ROACH, University of Houston, Houston, Texas 77004. Diophantine approximations for quaternions and Cayley numbers. Preliminary report.

Ford [Trans. Amer. Math. Soc. 26(1925), 146-154] used certain geometrical constructions in E^3 to obtain diophantine approximations for complex numbers. Utilizing the techniques of Ford, together with a type of generalized continued fraction [F. A. Roach, Proc. Amer. Math. Soc. 24(1970), 576-582], results concerning diophantine approximations for quaternions and Cayley numbers are obtained. (Received January 25, 1971.)

683-A9. MARK BENARD, Tulane University, New Orleans, Louisiana 70118. Primary elements in the Schur subgroup.

The Schur subgroup $S(k)$ of the Brauer group $B(k)$ of a field k consists of the algebra classes which contain a simple component of the rational group algebra QG for some finite group G. Since each simple component of QG corresponds to an irreducible character χ of G and has center $Q(\chi)$, $S(k)$ is trivial unless k is a subfield of a cyclotomic extension of Q. For any such k, $S(k)$ is known to have infinitely many elements of order 2. Theorem. Let p be a prime such that $p \nmid |k:Q|$. If k does not contain a primitive pth root of unity, then $S(k)$ contains no element of order p. Corollary. If $|k:Q| = 2$ and $k \neq Q(\sqrt{-3})$, then $S(k)$ contains no element of odd order. If $k = Q(\sqrt{-3})$, then an element of $S(k)$ has order $2^a 3^b$. (Received January 18, 1971.)

683-A10. DANIEL A. ROBINSON, Georgia Institute of Technology, Atlanta, Georgia 30332. An "extra" characterization of Moufang loops. Preliminary report.

F. Fenyves ["Extra loops. I," Publ. Math. Debrecen 15(1968), 235-238] calls loops $(G, \cdot)$ with the property that $(xy \cdot z)x = x(y \cdot zx)$, for all $x, y, z \in G$, extra loops and proves that extra loops are Moufang. (Clearly there are Moufang loops which are not extra.) The following result generalizes this and at the same time gives a characterization of Moufang loops. Theorem. A loop $(G, \cdot)$ is Moufang if and only if there is a mapping α of G into G so that $(xy \cdot z)x\alpha = x(y(z \cdot x\alpha))$ for all $x, y, z \in G$. Partial motivation for such considerations is provided by recent investigations of H. Pflugfelder ["A special class of Moufang loops," Proc. Amer. Math. Soc. 26(1970), 583-586]. (Received January 20, 1971.)

683-A11. ROBERT S. SMITH, Miami University, Oxford, Ohio 45056. A note on distributive filter lattices. Preliminary report.

In this note we continue the study of the lattice filters of a groupoid initiated by Frink and Smith (Abstract 663-393, these *Notices* 16(1969), 199) and generalize some results of Green (Abstracts 69T-A182, these *Notices* 16(1969), 963 and 70T-A34, these *Notices* 17(1970), 425). The author uses the terminology of Frink and Smith. A filter $F \subseteq G$ is prime (irreducible) if it is a meet-prime (meet-irreducible) element of $\mathfrak{F}(G)$. Theorem. The following conditions are equivalent: (1) $\mathfrak{F}(G)$ is distributive. (2) $\mathfrak{F}(G^1)$ is distributive. (3) If $x, y, z \in G$ and $x \subset yz$ but $x \not\subset y$ and $x \subset z$ then there exists $y' \subset y$ and $z' \subset z$ such that $x \sim y'z'$. (4) If $x \subset yz$ then $F_x \subseteq (F_x \wedge F_y) \vee (F_x \wedge F_z)$. (5) Every irreducible filter is prime. (6) Every filter is the intersection of prime filters. (7) For every $F \in \mathfrak{F}(G)$ and for every $x \notin F$, there is a prime filter Q such that $F \subseteq Q$ and $x \notin Q$. (8) For every $F, H \in \mathfrak{F}(G)$ and for every $x \in F \cup H$ there is $\{b_i \mid 1 \leq i \leq n\} \subseteq F \cup H$ such that $x \sim P(b_1, \ldots, b_n)$, where $P(b_1, \ldots, b_n)$ is some product involving the elements $b_1, \ldots, b_n$. (Received January 20, 1971.)

683-A12. ROBERT M. FOSSUM and PHILLIP A. GRIFFITH, University of Illinois, Urbana, Illinois 61801. The category of maps; coherence and Gorenstein properties.

Let A and $\mathbb{B}$ be abelian categories, $F: A \to \mathbb{B}, G: \mathbb{B} \to A$ functors. Let $\mathbb{F}$ (resp. $\mathbb{G}$) denote the category whose objects are maps $FA \to B$ (resp. $A \to GB$) and whose maps are commutative squares. $\mathbb{F}$ ($\mathbb{G}$) is abelian iff F (G) is right (left) exact. Proposition. $f \in \mathbb{F}$ is projective iff $f = id_{FP} \oplus (F0 \to Q)$ where P (Q) is projective in A ($\mathbb{B}$). (The dual statement holds for injectives in $\mathbb{G}$.) The homological dimensions of $\mathbb{F}$ (and $\mathbb{G}$) can then be computed. (If there are not enough projectives, one can use the Baer-Yoneda construction of Ext.) Theorem. $\mathrm{Max}(DA, DB) \leqq D\mathbb{F} \leqq 1 + DA + DB$, where D is either finitistic or of global dimension. The same hold for $\mathbb{G}$. As applications we get Proposition. Let T be the 2×2 triangular matrices over the ring R. Let E_j denote the jth term in a minimal injective resolution. Then flat dim $E_j(R) \leqq j$ for $0 \leqq j \leqq k$ iff the same holds for T. (This says that T is k-Gorenstein iff R is k-Gorenstein when R is coherent.) Proposition. Let T be a ring, e an idempotent in T such that one of $eT(1-e)$ or $(1-e)Te$ is zero. Let D be one of the dimensions in the Theorem above. Then DT is finite iff both $D(eTe)$ and $D((1-e)T(1-e))$ are finite. And the same bounds hold. We also consider coherence properties of the categories. (Received January 19, 1971.)

683-A13. ROGER P. WARE, Northwestern University, Evanston, Illinois 60201. On the structure of Witt rings of forms.

In 1937 Witt defined a commutative ring $W(F)$ whose elements are equivalence classes of anisotropic quadratic forms over a field F of characteristic not 2. Other similar rings have since been defined; for example the Witt-Grothendieck ring $WG(F)$ of nondegenerate quadratic forms over F, the Witt-Grothendieck ring $W(G)$ of a profinite group G [Scharlau, Invent. Math. 4(1967), 238-264], and the Witt ring $W(C)$ of symmetric bilinear forms over a connected semilocal ring C [Knebusch, Sitzber. Heidelberg Akad. Wiss., 1969/1970, 93-157]. All these rings have the form $Z[G]/K$ where G is an abelian group of exponent 2 and K is an ideal which under every homomorphism of $Z[G]$ to Z is mapped onto 0 or $2^n Z$. Using his theory of multiplicative forms, Pfister in 1966 proved a number of structure theorems for $W(F)$. This paper will show that his results depend only on the fact that $W(F) \cong Z[G]/K$ with G, K as above. Thus unified proofs for all the Witt and Witt-Grothendieck rings mentioned are obtained. This is joint work with M. Knebusch and A. Rosenberg. (Received January 19, 1971.)

683-A14. DAVID L. SHANNON, Division of Mathematical Sciences, Purdue University, Lafayette, Indiana 47907. On monoidal transforms of regular local rings.

By (R,S) we denote a pair of regular local rings with the same quotient field such that $\dim R = \dim S$ and S dominates R. By a normal (resp. regular) factorization of (R,S) we mean a finite sequence of normal (resp. regular) local rings $R_1, \ldots, R_t$ with the property that $R_1 = R$, $R_t = S$, and R_{i+1} dominates R_i for all i. A regular factorization (R_i) of (R,S) is called a monoidal factorization if R_{i+1} is an immediate monoidal transform of R_i for all i. It is well known that every pair (R,S) with $\dim R = \dim S = 2$ has a monoidal factorization; in particular it follows for this case that the lengths of all regular factorizations of (R,S) are bounded. An example is given in which the lengths of normal factorizations are not bounded. For a certain class of pairs (R, S) with $\dim R = \dim S = 3$, a necessary condition is given in order that there exist a monoidal factorization. Using this condition, an example (R, S) is constructed where $\dim R = \dim S = 3$ and (R, S) has no monoidal factorization. (Received January 4, 1971.)

683-A15. SHREERAM S. ABHYANKAR, Purdue University, Lafayette, Indiana 47907. Epimorphisms of polynomial rings.

This is a preliminary report on the following conjecture. It represents joint work with T. T. Moh. Let x, y, t be indeterminates over an algebraically closed field k of characteristic zero. Abstract conjecture. Given any epimorphism u of the polynomial ring $k[x,y]$ onto the polynomial ring $k[t]$, there exists an automorphism v of $k[x,y]$ such that $u(v(x)) = t$. The above conjecture can be shown to be equivalent to: Concrete conjecture. Let m and n be integers such that $1 < m < n$ and n is not divisible by m. If $f(t)$ and $g(t)$ are elements in $k[t]$ of degree m and n respectively, then there does not exist any $h(x,y) \in k[x,y]$ such that $h(f(t), g(t)) = t$. The following cases of this have been settled affirmatively; (here $(q_1, \ldots, q_s)$ denotes the greatest common divisor of $q_1, \ldots, q_s$): (1) $(m,n) = 1$. (2) $(m,n) \neq 1$ and $(m-1)(n-1)$ is not divisible by $(p_1 - 1, \ldots, p_r - 1)$ where $p_1, \ldots, p_r$ are the prime factors of (m,n). (3) $m = 4$ and n arbitrary. (4) $m = 6$ and $n = 9$. (Received January 19, 1971.)

683-A16. SHREERAM S. ABHYANKAR and WILLIAM J. HEINZER, Purdue University, Lafayette, Indiana 47907 and PAUL EAKIN, University of Kentucky, Lexington, Kentucky 40506. Uniqueness of the coefficient ring in a polynomial ring.

The title refers to the following question. If A and B are commutative rings with identity and the polynomial rings $A[X_1, \ldots, X_n]$ and $B[Y_1, \ldots, Y_n]$ are isomorphic does it follow that A and B are isomorphic? Theorem. If A is a Dedekind domain containing a field of characteristic zero and $\varphi : A[X_1, \ldots, X_n] \to B[Y_1, \ldots, Y_n]$ is an isomorphism, then A and B are isomorphic. Moreover, if A is not a polynomial ring over a field, then $\varphi(A) = B$. If R is an integral domain, let R_u denote the subring of R generated by the units of R and let R_c denote the algebraic closure of R_u in R. Theorem. Let R be a unique factorization domain such that $R = R_c$. If the polynomial rings $R[X_1, \ldots, X_n]$ and $B[Y_1, \ldots Y_{n-1}]$ are isomorphic, then B is isomorphic to $R[X_1]$. These are results from a continuation of the study reported in Abstract 677-13-2 these *Notices* 17(1970), 763. (Received January 19, 1971.)

683-A17. MICHAEL R. STEIN, Northwestern University, Evanston, Illinois 60201. Relativizing functors on rings and algebraic K-theory.

Let F be a functor from rings to groups and suppose q is a 2-sided ideal in the ring A. In many situations, one would like to define a "relative" group $F(A,q)$ so that the sequence $F(A,q) \to F(A) \to F(A/q)$ is exact. Moreover it is clear from the anticipated applications that $F(A,q)$ should depend functorially on the pair (A,q) and that the left hand arrow should not, in general, be injective. Such relative groups have been defined, ad hoc, for many such functors F. A uniform way to define such relative groups is described, which has the pleasant property of making certain theorems about these groups formal consequences of the corresponding theorems for the case $q = A$-- the "absolute" case. It is then shown that this construction yields the relative groups as previously defined for a variety of functors in algebraic K-theory. Examples are given of theorems of algebraic K-theory whose relative versions thus follow formally from their absolute versions. (Received January 19, 1971.)

683-A18. HARALD G. NIEDERREITER, Southern Illinois University, Carbondale, Illinois 62901. Sums of polynomials as permutation polynomials.

This paper is a continuation of the work of the author on permutation polynomials in several variables [Proc. Japan Acad., to appear], and was initiated by results of R. Lidl in the same direction. All polynomials considered here have coefficients in a given Galois field GF(q). Theorem 1. The polynomial $f(x_1, \ldots, x_k) + g(x_{k+1}, \ldots, x_m)$ is a permutation polynomial over GF(q), q prime, iff at least one of f and g is a permutation polynomial. This is best possible in the following sense. Theorem 2. In GF(q), q not prime, there exist polynomials $f(x_1, \ldots, x_k)$ and $g(x_{k+1}, \ldots, x_m)$ such that $f + g$, but neither f nor g, are permutation polynomials. Theorem 3. Suppose $f(x_1, \ldots, x_{n-1})$ has k zeroes in GF(q) with $q \nmid k$, let $g(x_1, \ldots, x_{n-1})$ be arbitrary and let $p(x_1, \ldots, x_n)$ be a polynomial such that $p(b_1, \ldots, b_{n-1}, x_n)$ is a permutation polynomial in x_n for all $b_1, \ldots, b_{n-1} \in GF(q)$. Then $h(x_1, \ldots, x_n) = p(x_1, \ldots, x_n) \cdot f(x_1, \ldots, x_{n-1}) + g(x_1, \ldots, x_{n-1})$ is not a permutation polynomial. This is best possible in the following sense. Theorem 4. Suppose $f(x_1, \ldots, x_{n-1})$ has k zeroes in GF(q) with q/k, and take $p(x_1, \ldots, x_n)$ as in Theorem 3. Then there exists $g(x_1, \ldots, x_{n-1})$ such that $h(x_1, \ldots, x_n) = p(x_1, \ldots, x_n) \cdot f(x_1, \ldots, x_{n-1}) + g(x_1, \ldots, x_{n-1})$ is a permutation polynomial. (Received January 21, 1971.)

683-A19. WAYNE BISHOP, Western Michigan University, Kalamazoo, Michigan 49001. A theory of multiplicity for multiplicative filtrations.

Let A be a noetherian commutative ring with identity, $f = \{\mathfrak{a}_n\}$ a multiplicative filtration on A with $\dim(f) = \dim(\mathfrak{a}_n) = 0$ and $\mathrm{alt}(f) = \mathrm{alt}(\mathfrak{a}_n) = s$, and M a finitely generated A-module. Definition. The multiplicity of M with respect to f is $\mu(f, M) = s! \lim_{n \to \infty} L_A(M/\mathfrak{a}_n M)/n^s$ provided this limit exists. In the classical case of $f = \{\mathfrak{a}^n\}$ for some ideal $\mathfrak{a}$, $\mu(f, M) = \mu(\mathfrak{a}, M)$ is known to be an integer. Theorem 1. For f as above, $\lim_{n \to \infty} n^{-s} \mu(\mathfrak{a}_n, M)$ exists and is greater than or equal to $\lim \sup\{s! \, L_A(M/\mathfrak{a}_n M)/n^s\}$. Remark. If $\mu(f, M)$ exists and is the upper bound of Theorem 1 for each finitely generated module M, the function $\mu(f, \_\_\_)$ inherits many of the properties of the usual multiplicity function. Theorem 2. If f has the property that $\forall\, j \; \exists\, k_j \in \mathbb{N} \ni \mathfrak{a}_{k_j n} \subseteq \mathfrak{a}_j^n$ for all n and $k_j/j \to 1$, then $\mu(f, M)$ exists and is the upper bound of Theorem 1 for every finitely generated A-module M. Remark. This condition is satisfied by some nonnoetherian filtrations as well as many noetherian ones which do not arise as the powers of a fixed ideal. (Received January 22, 1971.)

683-A20. NAOKI KIMURA, University of Arkansas, Fayetteville, Arkansas 72701. On Schein semigroups.

A partial answer to the problem "Describe structure of semigroup S such that, for every $a, b, c \in S$, abc equals ab or ac or bc" proposed by B. M. Schein (Semigroup Forum, vol. 1, no. 1, p. 91), will be given.

Let S be a semigroup satisfying the condition and E the set of idempotents of S. Then E is a subsemigroup of S. For each $e \in E$ let S(e) be the set of all elements x such that $x^2 = e$. Then S(e) is a subsemigroup of S such that $S(e) \cap S(f) = \emptyset$ if $e \neq f$, $S(e)S(f) = S(ef)$, and S is the disjoint union of S(e)'s. The subsemigroup S(e) has only one idempotent e, which turns out to be the zero. The structure of S(e), though not complete, will be presented in terms of the ternary relation (a,b,c) defined by $abc \neq 0$. (Received November 9, 1970.)

683-A21. HELEN SKALA, 1927 S. Racine, Chicago, Illinois 60608 and University of Massachusetts, Boston, Massachusetts 02116. A variation of the friendship problem.

Let S be a finite set of elements with a symmetric nonreflexive binary relation $\circ$, called on. S is called a friendship set if (1) for any two elements a and b such that a is not on b, there exists a unique element c such that $c \circ a$ and $c \circ b$; (2) if a is on b there exists at most one element c such that $c \circ a$, $c \circ b$; (3) there is no element which is on every other element of S. Theorem. Let S be a friendship set and m the maximum integer such that there exists an element on m distinct elements. Then every other element is either on m or $m - 1$ elements; the number of elements of S is either $m^2 + 1$, in which case every element is on m elements, or $m^2 - m + 1$, in which case there is at least one element on exactly $m - 1$ elements. (Received January 27, 1971.)

683-A22. TIMOTHY V. FOSSUM, University of Utah, Salt Lake City, Utah 84112. Lattice isomorphisms between Morita-related modules. Preliminary report.

Let R be a ring (associative, with identity), P_R a finitely generated projective right R-module. Let $S = \mathrm{End}_R(P)$, so that P is a left-right (S,R)-bimodule. Then $P \otimes_R$ ___ defines a functor from ${}_R\mathfrak{M}$ to ${}_S\mathfrak{M}$, the categories of left R- and left S-modules, respectively (which is the "classical Morita equivalence" if P_R is also a generator). Let $T = T_R(P) = \Sigma\{f(P) : f \in P^*\}$ be the trace ideal of P in R, where $P^* = \mathrm{Hom}_R(P,R)$. Theorem. Let M be a left R-module. For left R-submodules M_1 and M_2 of M, write $M_1 \underset{T}{\sim} M_2$ if $TM_1 = TM_2$. Then $\underset{T}{\sim}$ is a congruence relation on the lattice of left R-submodules of M. Moreover, there is a lattice isomorphism between the "factor lattice" of left R-submodules of M modulo $\underset{T}{\sim}$ and the lattice of left S-submodules of $P \otimes_R M$. Letting $M = P^*$ gives a characterization of the lattice of left ideals of $S \cong P \otimes_R P^*$.
Corollary. Let M be a left R-module such that $TM = M$, and $m = 0$ whenever $Tm = 0$. Then M is a simple left R-module if and only if $P \otimes_R M$ is a simple left S-module. (Received January 27, 1971.)

683-A23. JOHN A. EAGON, University of Minnesota, Minneapolis, Minnesota 55455. Cohen-Macaulay rings and resolutions of perfect ideals.

Let R be a Cohen-Macaulay (C-M) ring. It is known that if I is a perfect ideal, i.e. the grade of I is the same as the homological dimension of R/I over R, then R/I is again C-M. Thus the study of new classes of perfect ideals leads to new classes of C-M rings. It is sometimes possible to consider large classes of ideals which fit together in certain ways and prove them all perfect by an inductive proof. Such is the case with the following Theorem. Let R be Noetherian. Let $M = (c_{ij})$ be an r by s matrix with entries in R. Let $H = (s_0, \ldots, s_m)$ be a strictly increasing sequence of nonnegative integers such that $s_0 = 0$, $s_m = s$, and $m < r$. Let n be an integer, $0 \leq n \leq s$. Let $I = I_{H,n} = I_{H,n}(M)$ be the ideal of R generated by the t+1 by t+1 minors of the first s_t columns of M together with $c_{11}, \ldots, c_{1n}$. Let h be the least integer such that $s_h \geqq n$. Suppose that the grade of I is as large as possible, a number that turns out to be a function of H and n. Then $I_{H,n}$ is perfect. When $H = (0, 1, 2, \ldots, m-1, s)$ and $n = 0$, $I_{H,n}$ is the ideal of $m + 1$ by $m + 1$ minors of M and the generic perfection of determinantal ideals is obtained as a corollary. It seems difficult to prove this corollary without considering the larger more complex class of ideals $I_{H,n}$. It is possible to do this for other classes of rings as shown by Hochster. (Received January 22, 1971.)

683-A24. TSUONG-TSIENG MOH, Purdue University, Lafayette, Indiana 47907. On analytic independence.

Let $k[[y]]$ be a power series ring, $z_1, \ldots, z_n \in k[[x,y]]$ with z_i nonunit. Then the following theorems are proved in our joint work with S. S. Abhyankar. Theorem 1. Let $z_1 = \sum_{i>1}^{\infty} f_i(y)x^i$. If $[k(y)\{f_i(y)\}_{i=1}^{\infty} : k(y)] = \infty$ then z_1 is analytically independent over $k[[x, xy]]$. Theorem 2. Let $z_1 = \sum_{i>1}^{\infty} f_i(y)x^i$ with $f_i(y)$ algebraic over $k(y)$. If the set of poles of $\{f_i(y)\}$ is an infinite set, then z_1 is analytically independent over $k[[x, xy]]$. Theorem 3. Let $z_j = \sum_{i\geq 1}^{\infty} f_{ji}(y)x^i$ with $f_{jj'}(y)$ the first coeff which is transcendental over $k(y)$. If $f_{11'}, f_{22'}, \ldots, f_{nn'}$ are algebraically independent over $k(y)$, then $z_1, \ldots, z_n$ are analytically independent over $k[[x, xy]]$. Theorem 4. A "gap" theorem for analytical independence. Several other facts about analytic independence can be pointed out. (Received January 19, 1971.)

683-A25. EBEN MATLIS, Northwestern University, Evanston, Illinois 60201. Rings of type I.

We make the following definitions: Definition. A ring of type I is an integral domain R with the following properties: (1) R has exactly two distinct maximal ideals M_1 and M_2. (2) $M_1 \cap M_2$ does not contain a nonzero prime ideal of R. (3) R_{M_1} and R_{M_2} are maximal valuation rings. Definition. An integral domain is said to have property D if every torsion-free R-module of finite rank is a direct sum of R-modules of rank one. Definition. An integral domain R is said to have a remote quotient field Q if there exists an R-module S such that $S \subset Q$, $S \neq Q$, and $S^{-1} = 0$, where $S^{-1} = \{q \in Q \mid qS \subset R\}$. We then have the following: Theorem. R is a ring of type I if and only if R has property D and a remote quotient field. (Received January 27, 1971.)

Analysis

683-B1. ROBERT E. FENNELL and THOMAS G. PROCTOR, Clemson University, Clemson, South Carolina 29631. On asymptotic behavior of perturbed nonlinear systems.

A version of the variation of constants formula for nonlinear systems is used to study the comparative asymptotic behavior of the systems (1) $x' = f(t,x)$ and (2) $y' = f(t,y) + g(t,y)$. Assume f, f_x, g: $[\alpha, \infty) \times \Omega \to R^n$ are continuous, Ω a region in R^n. Let Ω be open with $\bar{\Omega} \subset \Omega$ and assume for $\gamma \in \bar{\Omega}_1$, $\alpha \leqq \tau$ the solution $x(t, \tau, \gamma)$ of (1) exists for $t \geqq \alpha$. Let $w: [\alpha, \infty) \times R^+ \to R$ be a continuous nonnegative function with $w(t, \gamma_1) \leqq w(t, \gamma_2)$ for $\gamma_1 \leqq \gamma_2$ and assume for some $k > 0$ there is a unique solution of $r' = w(t, r)$, $r(\alpha) = k$ and $r(t) \to r^{\infty}$ as $t \to \infty$. Theorem. Let D(t) be a continuous nonsingular $n \times n$ matrix for $t \geqq \alpha$ and $\Omega_2 \subset \Omega_1$ be such that (a) $t \geqq \alpha$, $|D(t)\gamma| \leqq r^{\infty}$ implies $\gamma \in \Omega_1$; (b) $|D(t)\Phi(t, \tau, \gamma) g(\tau, \gamma)| \leqq w(\tau, |D(\tau)\gamma|)$ for t,

$\tau \in [\alpha, \infty)$, $\gamma \in \Omega_1$; (c) $|D(t)x(t, \alpha, \gamma)| \leq k$ for $t \geq \alpha$, $\gamma \in \Omega_2$. Then for $\gamma \in \Omega_2$ there is a solution $y(t)$, $t \geq \alpha$, of (2) with $y(\alpha) = \gamma$; and for each such solution there corresponds a solution $x^*(t)$, $\alpha \leq t$, of (1) such that $|D(t)[y(t) - x^*(t)]| \to 0$ as $t \to \infty$. The converse problem is also considered. (Received December 21, 1970.)

683-B2. KENNETH D. SHERE, U. S. Naval Ordnance Laboratory, Mathematical Analysis Division, Silver Spring, Maryland 20910. Multiple asymptotic expansions and singular problems.

Applications of multiple asymptotic expansions to singular differential equations have been investigated by means of four heuristic examples. The first example is an equation with an essential singularity in the leading coefficient. The results are then compared to the results obtained by the techniques of H. Schmidt. Then two linear problems of singular perturbation are investigated. For a boundary value problem it is shown that the technique of multiple asymptotic expansions yields the same result as the two-variable technique and for an initial value problem it is shown that this technique improves upon the two-variable technique. Finally a nonlinear boundary value problem of singular perturbation is considered. It has been shown that, whenever the calculations are not too onerous, the technique of multiple asymptotic expansions can yield new insights on the nature of the solution. (Received December 29, 1970.)

683-B3. PHILIP W. SCHAEFER, University of Tennessee, Knoxville, Tennessee 37916. On the Cauchy problem for the nonlinear biharmonic equation.

Consider the Cauchy problem for the nonlinear biharmonic equation $\Delta^2 v = h(x, v, v_{,i}, \Delta v, \Delta v_{,i})$, where Δ is the Laplace operator in n-dimensions, comma notation denotes partial differentiation with respect to x_i, h is assumed to satisfy a uniform Lipschitz condition in its latter four arguments, and v is a C^4 function which is assumed to be uniformly bounded in some domain D. In addition, upper bounds for the error in measurement of the Cauchy data on the initial surface are prescribed. By means of the logarithmic convexity of a suitable functional, an a priori inequality is obtained for the integral of V^2, where $V = v - \psi$ for ψ a sufficiently smooth approximating function. Uniqueness and continuous dependence on the Cauchy data results follow from this inequality and we may further utilize it to obtain pointwise bounds for the solution and its derivatives. (Received December 30, 1970.)

683-B4. HERBERT W. HETHCOTE, University of Iowa, Iowa City, Iowa 52240. Analogues of the Schwarz lemma for elliptic partial differential equations.

If the value at an interior point and the maximum absolute value of the boundary data for the Dirichlet problem are known, then maximum growth rates between the point and the boundary are given. For example, if u satisfies Laplace's equation on the unit disk with piecewise continuous boundary data $f(\theta)$ such that $u(0, 0) = u_0$ and $|f(\theta)| \leq M$, then $|u(r, \theta) - u_0(1-r^2)/(1+r^2)| \leq (2M/\pi) \arctan [2r/(1-r^2)]$. This inequality is sharp. If $L[u]$ is an n dimensional second order uniformly elliptic operator with bounded coefficients and u is a nonconstant solution of $L[u] = 0$ in the unit sphere with $u(0) = u_0$ and boundary data $f \ni |f| \leq M$, then there exists an increasing function $K(r)$ such that $K(0) = 0$, $K(1) = 1$, and $|u(x) - u_0(1-K(|x|))| \leq K(|x|)M$. Similar theorems are obtained for other interior points, other domains, and other equations. For solutions of Laplace's equation, growth rates on an L_2 norm are also obtained. (Received January 4, 1971.)

683-B5. DAVID F. DAWSON, North Texas State University, Denton, Texas 76203. A generalization of the Silverman-Toeplitz theorem.

The following result is proved. Theorem. The matrix $A = (a_{pq})$ sums every sequence which is (C, k)-summable iff the following four conditions hold: (1) A has convergent columns, (2) $\{\sum_{q=1}^{\infty} a_{pq}\}_{p=1}^{\infty}$ converges, (3) $\sup_p \sum_{q=1}^{\infty} \binom{q+k-1}{k} |\Delta_2^k a_{pq}| < \infty$, (4) $\{\binom{q+k-1}{k} a_{pq}\}_{q=1}^{\infty}$ is bounded, $p = 1, 2, 3, \ldots$, where $\Delta_2^0 a_{pq} = a_{pq}$, $\Delta_2^1 a_{pq} = a_{pq} - a_{p,q+1}$, $\Delta_2^2 a_{pq} = \Delta_2^1 a_{pq} - \Delta_2^1 a_{p,q+1}$, etc. For $k = 0$ or 1, (4) can be dropped. Hence for $k = 0$ (with $(C, 0)$-summability as ordinary convergence) the theorem reduces to the classical Silverman-Toeplitz theorem. (Received January 4, 1971.)

683-B6. STEPHEN R. BERNFELD, University of Missouri, Columbia, Missouri 65201 and A. LASOTA, Jagiellonian University, Krakow, Reymonta 4, Poland. Quickly oscillating solutions of nonlinear ordinary differential equations. Preliminary report.

A solution $x(t)$ of (E) $\dot{x} = F(x)$, where $F: R^n \to R^n$, is quickly oscillating if for each component x_j of x there exists a sequence of points $\{t_i\}$ such that $x_j(t_i) = 0$, $t_{i+1} > t_i$, $\lim_{i\to\infty} t_i = \infty$, and $\lim_{i\to\infty}(t_{i+1} - t_i) = 0$. We define $\|x\| = \sum_{i=1}^{n} |x_i|$. Theorem 1. If x is a quickly oscillating solution of (E) then either (1) $\lim_{t\to\infty}\|x(t)\| = 0$ or (2) $\lim_{t\to\infty}\|x(t)\| = \infty$. Theorem 2. If F satisfies a linear growth condition $\|F(x)\| \leq C(1+\|x\|)$ for some $C > 0$ then every quickly oscillating solution of (E) satisfies (1). Theorem 3. If there exists a nontrivial bounded quickly oscillating solution of system (E) then F is not Lipschitz at the origin. (Received January 11, 1971.)

683-B7. AARON STRAUSS, University of Maryland, College Park, Maryland 20742 and A. LASOTA, Jagiellonian University, Krakow, Reymonta 4, Poland. Local linearization without differentiability.

The following result is well known. Theorem 1. Let f be C^1 on R^n. If all solutions of (1) $u' = f_x(0)u$ approach zero as $t \to \infty$, then there exist positive constants δ, K, and σ such that all solutions $x(\cdot)$ of (2) $x' = f(x)$ with $|x(0)| < \delta$ satisfy $|x(t)| \leq K|x(0)|e^{-\sigma t}$ for $t \geq 0$. Now we assume that f is merely continuous on R^n and Lipschitz at $x = 0$. Then we consider, rather than the Frechet differential of f, the "multi-valued differential" D_f of f. This is, roughly speaking, the closed convex hull of all possible differential quotients of f in the direction of x. Then Theorem 1 remains true for such f if we replace (1) by the multi-valued differential equation (3) $u' \in D_f(u)$. In the C^1 case the standard proof uses the variation of constants formula, Gronwall's inequality, and $f(u) = f_x(0)u + o(|u|)$. None of these three tools is available in the general case so that the proof becomes more difficult. (Received November 23, 1970.)

683-B8. WITHDRAWN.

683-B9. MAURICE MACHOVER, St John's University, Jamaica, New York 11432. On the inverse of a measurable function.

There is hardly any discussion in the literature or in texts on real variables of the measurability of the inverse of a one-to-one measurable function. Here is constructed (by suitably modifying the Cantor ternary function) a simple illustration of a one-to-one Lebesgue measurable function from R onto R with nonmeasurable inverse. In contrast with this a one-to-one continuous function on R has a continuous inverse. (Received January 15, 1971.)

683-B10. WILLIAM R. DERRICK, University of Utah, Salt Lake City, Utah 84112. Weighted convergence in length. Preliminary report.

Let H^1 be the one-dimensional Hausdorff measure in E^m. Theorem. Let $\{r_n\}$ be a sequence of rectifiable curves in E^m of length exceeding $L > 0$ parametrized by arc-length. Suppose $\gamma_n(0) \to \gamma_0$ and γ_S is an accumulation point of the set $\{\gamma_n(S)\}$, $0 < S \leq L$. Then some subsequence $\{\gamma_j\}$ converges uniformly on $[0, S]$ to a curve γ containing γ_0 and γ_S such that for every nonnegative lower semicontinuous function $f: E^m \to E^1$, $\liminf_{j\to\infty} \int_{\gamma_j[0,S]} f\, dH^1 \geq \int_\gamma f\, dH^1$. Moreover, the condition $\lim_{j\to\infty} \int_0^S |\nabla\gamma_j - \nabla\gamma|\, dt = 0$, holds if and only if $\lim_{j\to\infty} \int_{\gamma_j[0,S]} f\, dH^1 = \int_\gamma f\, dH^1$, for all continuous functions f. (Received January 18, 1971.)

683-B11. SEYMOUR SINGER, University of Notre Dame, Notre Dame, Indiana 46556. The singular Cauchy problem for a quasilinear hyperbolic equation.

Consider the quasilinear partial differential equation $r(x,y)^2 u^{2\beta} u_x^{2\gamma} u_{xx} - u_{yy} + f(x,y,u,u_x,u_y) = 0$ with initial data $u(x,0) = 0$, $u_y(x,0) = \varphi(x)$ prescribed on a bounded segment I of the x-axis. The exponents β, γ are nonnegative real constants with $\beta + \gamma > 0$. $r(x,y)$ and $f(x,y,u,p,q)$ are smooth functions and r is bounded away from zero. Suppose there exist constants a_0, A_0, m, n such that $0 < a_0 < m \leq \varphi(x) \leq n < A_0$ for all $x \in I$. Then the equation is of hyperbolic type for every twice-differentiable solution when $y > 0$ and degenerates to a parabolic equation on the initial segment I. Assuming the conditions $f_p(x,y,u,p,q) = o(y^{\beta+\gamma-1})$ as $y \to 0$ and $\beta A_0/a_0 < (\beta+\gamma+1)/(\gamma+1)$, there exists a smooth solution to the Cauchy problem in a strip $0 < y < \delta$ bounded by characteristics through the end points of I. The Cauchy problem is replaced by an equivalent system of integral equations which induces a mapping of an appropriate Banach space into itself. The proof consists in showing that this associated mapping satisfies the hypotheses of Schauder's fixed point theorem. (Received January 18, 1971.)

683-B12. PETRU MOCANU, Babès-Bolyai University, Cluj, Romania and MAXWELL O. READE, University of Michigan, Ann Arbor, Michigan 48104. On generalized convexity in conformal mappings. Preliminary report.

The purpose of this note is to give an analytic proof of the following result. Theorem. Let $f(z) = z + \ldots$ be analytic in the unit disc Δ, with $f(z)f'(z)/z \neq 0$ there, and let α be a real constant, $0 \leq \alpha \leq 1$. If the real part of $[(1-\alpha)zf'(z)/f(z) + \alpha(1 + zf''(z)/f'(z))]$ is nonnegative in Δ, then f(z) is a univalent and star-like function in Δ. This result is due to Mocanu [Mathematica 11(1969), 127-133] who gave a geometric proof of that theorem. (Received January 20, 1971.)

683-B13. JOHN W. HEIDEL and DON HINTON, University of Tennessee, Knoxville, Tennessee 37916. The existence of oscillatory solutions for a nonlinear differential equation.

Consider the differential equation (1) $\ddot{y} + q(t)y^{\gamma} = 0$ where $q(t) \geqq 0$ and continuous on $(0, \infty)$ and γ is the quotient of odd, positive integers. A nontrivial solution of (1) is called oscillatory if it has no last zero, i.e., if $y(t_1) = 0$ then there is a $t_2 > t_1$ such that $y(t_2) = 0$. Theorem. Suppose $1 < \gamma$. If $q(t)t^{(\gamma+3)/2} > 0$ and $d/dt\, (q(t)t^{(\gamma+3)/2}) \geqq 0$ for $t > 0$, then every solution of (1) with a zero at $t_0 > 0$ is oscillatory. Theorem. Suppose $0 < \gamma < 1$. If $q(t)t^{(\gamma+3)/2} > 0$ and $d/dt\, (q(t)t^{(\gamma+3)/2}) \geqq 0$ for $t > 0$ and $\lim_{t\to\infty} t\, d/dt\, (q(t)t^{(\gamma+3)/2}) = \infty$, then every solution $y(t)$ of (1) such that $y(t_0) = 0$, $t_0 > 0$, and $|y'(t_0)|$ is sufficiently small is oscillatory. Theorem. Suppose $0 < \gamma < 1$. If $q(t)t^{(\gamma+3)/2} > 0$, $d/dt\, (q(t)t^{(\gamma+3)/2}) \geqq 0$, and $q(t)t^{(\gamma+3)/2} \leqq k < \infty$ for $t > 0$, then every solution $y(t)$ of (1) such that $y(t_0) = 0$ and $|y'(t_0)|$ is sufficiently small is oscillatory. Theorem. Suppose $0 < \gamma < 1$ and $r, q \in C^{(2)}[a, \infty)$. Let $\eta(t) = [r(t)q(t)]^{-1/(\gamma+3)}$. If $\int_a^{\infty} |\eta(r\dot{\eta})^{\cdot}|dt < \infty$ and $\int_a^{\infty} (1/r\eta^2)dt = \infty$ then every solution of $(r(t)\dot{y})^{\cdot} + q(t)y^{\gamma} = 0$ with sufficiently small initial conditions is oscillatory. The first three theorems give sharp results for the Emden-Fowler equation $r(t) \equiv 1$, $q(t) = t^{\sigma}$. An asymptotic distribution of zeros is given in the fourth theorem. (Received January 21, 1971.)

683-B14. JOAO B. PROLLA, University of Rochester, Rochester, New York 14627. Weighted spaces of vector-valued continuous functions.

Let X be a completely regular Hausdorff space and E a locally convex Hausdorff TVS. Let $CV_{\infty}(X, E)$ and $CW_{\infty}(X, E)$ be two weighted spaces (Nachbin, Machado and Prolla, J. Math. Pures Appl. 49(1970)) such that for every weight $v \in V$ there is a $w \in W$ such that $v \leqq w$ pointwise. Theorem. Suppose E is complete and that for each $x \in X$ there is a $w \in W$ such that $w(x) > 0$. Then $CV_{\infty}(X, E)$ complete (resp. quasi-complete) implies $CW_{\infty}(X, E)$ complete (resp. quasi-complete). If X is locally compact and E' is endowed with the weak-star topology, let $M_b(X, E')$ be the set of all E'-valued bounded Radon measures m on X for which there exists some continuous seminorm p on E such that m is p-dominated. Theorem. The dual space $CV_{\infty}(X, E)'$ is isomorphic to $V \cdot M_b(X, E')$. (Received January 21, 1971.)

683-B15. ROBERT S. BORDEN, Knox College, Galesburg, Illinois 61401. The approximation problem.

A locally convex topological vector space is said to have the approximation property if on all compact sets the identity operator can be uniformly approximated by continuous linear operators of finite rank. We consider the question whether or not all Banach spaces enjoy this property, and we conclude that they do. We accomplish the proof by showing the existence of a suitable approximating operator, given a space B with a compact subset K. First, B is embedded in a space E (its injective envelope) which does have the property; then an approximating operator $T: E \to E$ can be found. Finally, we compose T with a linear map τ^{-1} which sends the range of T back into B. The map $\tau^{-1}T: B \to B$ turns out to be a suitable approximating operator. (Received January 21, 1971.)

683-B16. DAN J. EUSTICE, Ohio State University, Columbus, Ohio 43210. Holomorphic idempotents and common fixed points on the 2-disk.

We have the following result: Theorem. If f and g are commuting continuous mappings of the closed 2-disk into itself which are holomorphic on the open 2-disk, then there is a common fixed point for f and g. This generalizes a result of Shields [Proc. Amer. Math. Soc. 15(1964), 703-706]. To prove this result we need to classify the holomorphic idempotents mapping the open 2-disk into itself. Such mappings are conjugate [$L^{-1}FL$, where L is a holomorphic homeomorphism of the 2-disk] to one of the following: (i) the identity mapping, (ii) the constant zero mapping, or (iii) the mapping whose image is $\{(z, g(z)) : z$ in the unit disk$\}$ [or $\{(g(z), z)\}$], where g is a holomorphic function on the unit disk with $|g| < 1$. (Received January 21, 1971.)

683-B17. MAYNARD G. ARSOVE and HEINZ LEUTWILER, University of Washington, Seattle, Washington 98105. Quasibounded and singular functions.

A general concept of quasibounded and singular functions can be formulated as follows. On a plane region Ω let $\mathfrak{M}$ be the class of all nonnegative functions admitting superharmonic majorants. For each $\lambda > 0$ let $I_\lambda u$ be the reduced function of $(u - \lambda)^+$, i.e. the infimum of the class of its superharmonic majorants. Then $S_\lambda u$ and Su are the superharmonic functions defined, respectively, as the lower regularizations of $I_\lambda u$ and of the function $\inf_{\lambda > 0} S_\lambda u$. In terms of the operator S, a function u in $\mathfrak{M}$ is called quasibounded if $Su = 0$ and singular if $Su = u$. These definitions are shown to conform to the classical definitions in the case when u is harmonic. In studying the operator S, mapping $\mathfrak{M}$ into the class of its superharmonic elements, use is made of the positive homogeneity, subadditivity, and monotoneity of S. A number of classical theorems on quasibounded and singular functions carry over to the present setting, in particular the following Theorem. A function u in $\mathfrak{M}$ is quasibounded if and only if there exists a nonnegative, increasing, convex function φ on $(0, +\infty)$ such that $\lim_{x\to\infty} \varphi(x)/x = +\infty$ and $\varphi \circ u$ is in $\mathfrak{M}$. This generalizes results given by M. H. Heins in "Hardy classes on Riemann surfaces" (Lecture Notes in Mathematics, No. 98, Springer-Verlag, Berlin - New York, 1969). (Received January 22, 1971.)

683-B18. AHMED N. CURRIM, Western Carolina University, Cullowhee, North Carolina 28723. Study of the transport operator for the infinite slab. Preliminary report.

We study properties of the operator $T : Tf = -\mu\, \partial f/\partial x + (c(x)/2) \int_{-1}^{1} f(x, \nu)\, d\nu$ on the Hilbert Space L_2 $\{-\infty < x < \infty; -1 \leq \mu \leq +1\}$. T appears in the study of transport of neutrons in an infinite slab extending in the x direction from -a to +a; μ is the cosine of the direction of neutrons. Using previous work of Lehner and Wing we obtain bounds on the resolvent, $(T-\lambda)^{-1}$, in the right half and left half of the complex λ plane. We then apply a theorem of Dolph and Penzlin to obtain a weak representation theorem of the resolvent in the left half plane. Finally we define certain operators that are contour integrals of the resolvent times certain converging factors around the imaginary axis, and obtain relations among them. (Received January 22, 1971.)

683-B19. JUAN A. GATICA and WILLIAM A. KIRK, University of Iowa, Iowa City, Iowa 52240. Fixed point theorems for lipschitzian pseudo-contractive mappings.

Let X be a Banach space and $D \subset X$. A mapping $U: D \to X$ is said to be pseudo-contractive if for all $u, v \in D$ and all $r > 0$, $\|u - v\| \leq \|(1+r)(u-v) - r(U(u) - U(v))\|$. F. Browder has shown that these are precisely the mappings U such that I - U is accretive. Theorem 1. Let G be an open bounded subset of X with $0 \in G$, and let $U: \overline{G} \to X$ be a lipschitzian pseudo-contractive mapping satisfying: (i) $U(x) \neq \lambda x$ if $x \in \partial G$ and $\lambda > 1$, (ii) $(I - U)(\overline{G})$ is closed. Then U has a fixed point in $\overline{G}$. Theorem 2. If X and X^* are uniformly convex with $G \subset X$ as in Theorem 1, and if $U: X \to X$ is a lipschitzian pseudo-contractive mapping satisfying (i) on ∂G, then U has a fixed point in G. We say that a mapping $U: X \to X$ is strongly pseudo-contractive relative to $D \subset X$ if for each $x \in X$, $r > 0$, there exists a number $\alpha_r(x) < 1$ such that $\|x - y\| \leq \alpha_r(x) \|(1+r)(x-y) - r(U(x) - U(y))\|$, $y \in D$. Theorem 3. Let X be a reflexive Banach space, G be a bounded convex open subset of X with $0 \in G$, and suppose $U: X \to X$ is a lipschitzian strongly pseudo-contractive mapping relative to $\overline{G}$ satisfying (i) on ∂G. Then U has a fixed point in $\overline{G}$. (Received January 22, 1971.)

683-B20. DAVID L. ROD, 2505 Monterey Drive, No. 3, Madison, Wisconsin 53704 and University of Calgary, Calgary 44, Alberta, Canada. Invariant sets in the Monkey saddle.

Our analysis concerns the set of solutions which lie in a neighborhood of the (degenerate) critical point of a Hamiltonian system. Specifically the Hamiltonian function is $H(x_1, x_2, y_1, y_2) = \frac{1}{2}(y_1^2 + y_2^2) + \frac{1}{3}x_1^3 - x_1 x_2^2$ and the differential equations can be interpreted as describing the motion of a point mass sliding in a Monkey saddle under gravity. We show that to any bi-infinite sequence on three symbols (in which no symbol succeeds itself) corresponding to the three "legs" of the saddle there exists an uncountable number of orbits running through the legs in sequence. Given a periodic sequence the corresponding set of orbits contains subsets analogous to the "Cantor cylinder" of orbits passing near a nondegenerate homoclinic point. These results, which confirm some conjectures of C. Conley (Proc. U. S.-Japan Seminar on Differential and Functional Equations, (Minneapolis, Minn., 1967), Benjamin, New York, 1967, pp. 443-447), are obtained by constructing a periodic orbit in each leg and showing the existence of (topologically) nondegenerate heteroclinic orbits. (Received January 14, 1971.)

683-B21. HAROLD E. BENZINGER, University of Illinois, Urbana, Illinois 61801. The L^2 behavior of eigenfunction expansions. Preliminary report.

Let τ be the differential expression defined by $\tau u = u^{(2)} + p_1(x)u^{(1)} + p_0(x)u$, where p_j is in $L^1[0,1]$, $j = 0, 1$. By requiring u to satisfy a two point boundary condition, we define a dense subspace of $L^2[0,1]$, and thus τ generates a linear operator L of $L^2[0,1]$ into itself. We are concerned with the convergence, in the norm of $L^2[0,1]$, of the eigenfunction expansions of functions f in $L^2[0,1]$. Let $G(x, t, \rho)$ be the Green's function of L. By examining the growth of G as $|\rho| \to \infty$, we obtain an integer $\nu \geq 0$. Assume $p_1^{(\nu+1)}$ is in $L^1[0,1]$ and $p_0^{(\nu)}$ is in $L^1[0,1]$. Assume f is such that $f^{(\nu)}$ is in $L^2[0,1]$, and if $\nu > 0$, that $f^{(k)}(0) = f^{(k)}(1) = 0$ for $0 \leq k \leq \nu - 1$. Then the eigenfunction expansion of f converges in the mean to f. If the boundary condition defining L is Birkhoff regular, then $\nu = 0$ and all f in $L^2[0,1]$ can be expanded. For such problems, the operators L are known to be spectral operators, and the expansion theorem can be obtained using the theory of spectral operators. For $\nu \geqq 1$, there are examples of nonspectral operators. (Received January 25, 1971.)

683-B22. DONALD A. LUTZ, University of Wisconsin, Milwaukee, Wisconsin 53201. On the reduction of rank of linear differential systems.

The rank of a linear differential system in the neighborhood of a pole of the system is defined to be one less than the order of the pole of the coefficient matrix of the system. H. L. Turrittin (Duke Math. J. 30(1963), 271-274) has shown that arbitrary rank can be reduced to rank one at the expense of increasing the dimension of the system in proportion to the amount of reduction. We ask whether this procedure can lead to extraneous solutions of the rank-reduced system which differ in behavior from solutions of the given system. This question is answered by a transformation of the rank-reduced system to a block-diagonal form, exhibiting the precise relation between solutions of the two systems. In particular, if the original system has a regular singularity at the pole in question, then so does the rank-reduced system. An application of this gives some new necessary conditions for the regular singularity of linear differential systems (§3). (Received January 6, 1971.)

683-B23. PO-FANG HSIEH, Naval Research Laboratory, Mathematics Research Center, Washington, D.C. 20390 and Western Michigan University, Kalamazoo, Michigan 49001. General solution of a system of nonlinear equations at an irregular type singularity.

Given an (m+n)-system (E) $x^{\sigma+1}y' = f(x,y,z)$, $xz' = g(x,y,z)$, where the nonsingular $f_y(0,0,0)$ has eigenvalues of high multiplicity, $g_z(0,0,0) = \mathrm{diag}(\mu_1, \ldots, \mu_n)$ with Re $\mu_k > 0$ $(1 \leqq k \leqq n)$. By the results in Abstract 682-34-14, these *Notices* 18(1971), 161, $f_y(x,0,z)$ can be assumed block-diagonalized. Under suitable assumptions on the coefficients of $f_y(x,0,z)$, up to that of x^{σ}, in the expansion of $f_y(x,0,z)$ in powers of x, and also on relations among μ_k, a general solution of (E) is found. (Received January 25, 1971.)

683-B24. WILLIAM A. HARRIS, JR., University of Southern California, Los Angeles, California 90007 and YASUTAKA SIBUYA, University of Minnesota, Minneapolis, Minnesota 55455. Asymptotic distribution of eigenvalues for a boundary value problem.

We are concerned with the eigenvalue problem $y''(x) - \lambda^2 p(x)\, y(x) = 0$, $\int_a^b |y(x)|^2 dx < \infty$, and in particular with large positive eigenvalues for this physically motivated problem. We assume that p is a real valued function for $x \in (-\infty, +\infty)$ with an even, finite number of simple real zeros, with polynomial type growth as $|x| \to \infty$. Utilizing asymptotic expansions, we discuss this problem for the cases, p(x) a polynomial, p(x) a real analytic function, and p(x) of class C^k $(k \geqq 3)$; and determine the asymptotic distribution of positive eigenvalues. Our work complements recent work on the same problem by M. A. Evgrafov and M. V. Fedoryuk, Y. Sibuya, and L. Weinberg. (Received January 25, 1971.)

683-B25. THOMAS L. SHERMAN and JERRY R. RIDENHOUR, Arizona State University, Tempe, Arizona 85281. Conjugate points for higher order equations.

Let $r_{i_1 i_2 \ldots i_k} = \infty$ mean that no nontrivial solution of $y^{(n)} + \sum_{i=0}^{n-1} p_i(x) y^{(i)} = 0$ has an $i_1 - i_2 - \ldots - i_k$ distribution of zeros. Theorem 1. Suppose for $n=4$, $r_{121} = \infty$. Then $r_{13} = r_{31} = \infty$, and $\eta_k(t)$, the kth right conjugate point of t, is achieved by an extremal solution with double zeros at t and $\eta_k(t)$ and only simple zeros in $(t, \eta_k(t))$. Further, it can be shown by example that there is an equation for which $r_{13} = r_{31} = \infty$, but it is not true that $r_{121} = \infty$, and $\eta_k(t)$ for some $k > 1$ is achieved by an extremal solution with more than two double zeros. For the nth order equation $(n > 2)$ the following results hold. Theorem 2. Assume that if $\sum_{i=1}^{m} \alpha_1 = n$ and $\alpha_i > 1$ for $i = 1, \ldots, m$, then $r_{\alpha_1 \alpha_2 \ldots \alpha_m} = \infty$. Then any extremal solution for $\eta_k(t)$ has only one multiple zero in $[t, \eta_k(t)]$ and the multiplicity of this zero is $n-1$. Theorem 3. Assume that there always exists an extremal solution for $\eta_k(t)$ having at most two multiple zeros. Assume also that no nontrivial solution has $n+1$ zeros at two points. Then $\eta_k(t)$ is an increasing continuous function of t. Theorem 4. Assume the hypothesis of Theorem 2. Assume also that the equation is selfadjoint. Then the equation is oscillatory iff every solution having a zero of order $n-1$ is oscillatory. (Received January 25, 1971.)

683-B26. ZEEV NEHARI, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. Oscillation criteria for systems of linear differential equations.

Let A be a (real or complex) continuous $n \times n$ matrix on a real interval I, or an analytic matrix on a smoothly bounded domain D in the complex plane. The system of linear equations $y' = Ay$ is said to be nonoscillatory on I (or D) if any nontrivial solution vector y has at least one component which does not vanish on I (or D). The system is said to be suborthogonal if all nontrivial solution vectors y have the property that $\mathrm{Re}\{y(z)\overline{y(\zeta)}\} > 0$ for any z, ζ in I (or D). A number of nonoscillation and suborthogonality criteria are obtained in both the continuous and analytic cases. Sample. If A is analytic in D and $\int_{\partial D} \|A\| \, |dz| < \Pi$, then $y' = Ay$ is both nonoscillatory and suborthogonal in D. The constant Π is the best possible. In the case of nonoscillation, this improves a result of W. J. Kim [J. Math. Anal. Appl. 26(1969), 9-19] who obtained the value 1 for the constant in question. (Received January 25, 1971.)

683-B27. WALTER C. STRODT, St. Lawrence University, Canton, New York 13617. Asymptotic expansions and structure theorems. Preliminary report.

Let $u_0, u_1, \ldots, u_n, \ldots$ be a sequence of functions, meromorphic in a sector, such that u_n asymptotically dominates u_{n+1} (written $u_n \succ u_{n+1}$). For the differential polynomial P(Y) to have a solution with asymptotic expansion $u_0 + u_1 + \ldots + u_n + \ldots$, it obviously is necessary that P(Y) be unstable at u_0(that is, that $y \sim u_0$ not imply $P(y) \sim P(u_0)$) and that $P(u_0 + u_1 + \ldots + u_n + Y)$ be unstable at u_{n+1}, for all n. For a general class of first order differential polynomials and a general class of functions u_n these necessary conditions are shown to be sufficient. The proof of this theorem is interwoven with the proof of a related structure theorem, to the effect that, if $v_0 \succ v_1 \succ \ldots \succ v_n \succ \ldots$, then P(Y) can not have all the functions $v_0, v_0 + v_1, \ldots, v_0 + v_1 + \ldots + v_n, \ldots$ as solutions. In this work, frequent use is made of a general theorem of Robert K. Wright and the author, which asserts the existence, for first-order differential polynomials with coefficients in asymptotically well-behaved fields, of solutions lying in asymptotically well-behaved overfields. (Received January 25, 1971.)

683-B28. ROBERT K. WRIGHT, University of Vermont, Burlington, Vermont 05401. Asymptotic compatibility of second-order solutions with coefficient fields of logarithmic type.

A constrained field F is a differential field (of functions) whose elements are $\sim$ (asymptotically equivalent in the sense of Strodt) to logarithmic monomials of bounded rank. We show that if $y \notin F$ is a solution of a second-order algebraic differential equation with coefficients in F and if F satisfies a certain closure condition with respect to first-order solutions, then $F(y, Dy, D^2y)$ is constrained if and only if $y \sim$ a logarithmic monomial in F and $y - f \sim$ a logarithmic monomial in F for each $f \in F$ and $\sim y$. We construct fields satisfying the closure condition, and discuss problems involved in embedding a given constrained field in one satisfying the closure condition. Our work uses and generalizes results to appear in an American Mathematical Society Memoir by W. Strodt and the author. (Received January 25, 1971.)

683-B29. CHUNG-CHUN YANG, Naval Research Laboratory, Washington, D. C. 20390. On meromorphic solutions of generalized algebraic differential equations. Preliminary report.

The rate of growth of meromorphic functions f, which are solutions of algebraic differential equations whose coefficients a(z) are arbitrary meromorphic functions is investigated. By a method based on Nevanlinna's theory of meromorphic functions, it has been shown that if f has 0 and ∞ as its Nevanlinna exceptional values, then the ratio $T(r, f'/f)/T(r, a(z))$, as $r \to \infty$ outside a set of r values of finite measure, is bounded for at least one of the coefficients a(z). (Received January 25, 1971.)

683-B30. GILBERT STENGLE, Lehigh University, Bethlehem, Pennsylvania 18015. Asymptotics of some random second order differential equations.

The second order system of stochastic differential equations, $du = v\,dt$, $dv = e(u,v)db(t) + f(u,v)dt$, is analyzed by the following method ($b(t)$ is the Brownian motion). Let $P(u,v)$ be a solution of $(1/2)e^2P_{vv} + fP_v + vP_u = 0$ for which $P_v \neq 0$. Then the solutions of the system have the same probability law as the solutions of the ordinary plane autonomous system defined implicitly by $P(u,v) = P_0 + b(T(t))$, $\dot{u} = v$, $\dot{T} = e^2P_v^2$. The long-time behavior of the original system is analyzed in some cases for which the derived system is tractable. These cases include certain second-order linear homogeneous differential equations with white noise in their coefficients. (Received January 25, 1971.)

683-B31. STEVEN B. BANK, University of Illinois, Urbana, Illinois 61801. A representation theorem for large and small solutions of algebraic differential equations in sectors.

We treat first order algebraic differential equations whose coefficients belong to a certain type of function field which was introduced and investigated by W. Strodt [Trans. Amer. Math. Soc. 105(1962)]. These are fields of functions, each analytic in a sectorial neighborhood of ∞ (which is a union of sectors), such that for each element f in the field except 0, there is a logarithmic monomial M of rank $\leqq$ p (i.e. a function of the form $cz^{a_0}(\log z)^{a_1}(\log\log z)^{a_2}\ldots(\log_p z)^{a_p}$, for $c \neq 0$ and real a_j), such that $f/M \to 1$ as $z \to \infty$ over a filter base F consisting of sectorial neighborhoods of ∞. (Such a field is said to be of rank p, and the case $p=0$ includes the case of rational functions.) The main result states that there exists a positive constant N, depending only on the equation and the angle-opening of the elements of F, such that any solution h, which is meromorphic in an element of F and which satisfies $z^{-N}h(z) \to \infty$ as $z \to \infty$ over F, must be of the form $\exp\int W$, where W is analytic in an element of F and where for some logarithmic monomial M of rank $\leqq$ p, $W/M \to 1$ as $z \to \infty$ over F. A similar representation holds for nonidentically zero, analytic solutions g which satisfy a condition $z^K g(z) \to 0$ over F, where again K depends only on the equation and the angle-opening. (Received January 25, 1971.)

683-B32. WILLIAM T. REID, University of Oklahoma, Norman, Oklahoma 73069. Variational aspects of oscillation phenomena for higher order differential equations.

For higher order linear vector differential equations there are derived criteria of oscillation and disconjugacy, largely as consequences of general results for selfadjoint Hamiltonian systems established earlier by the author, [see, in particular, Trans. Amer. Math. Soc. 101(1961), 91-106; Proc. U.S.-Japan Seminar on Differential and Functional Equations, (Minneapolis, Minn., 1967), Benjamin, New York, 1967, pp. 267-299]. Particular attention is devoted to quasi-differential equations of the canonical form considered by the author in Trans. Amer. Math. Soc. 85(1957), 446-461. (Received January 25, 1971.)

683-B33. WILLIAM C. BENNEWITZ, Southern Illinois University, Edwardsville, Illinois 62025. Necessary and sufficient conditions that an operator be a limit.

We use the limit concept in McShane and Botts, "Real analysis," The University Series in Undergraduate Mathematics, Van Nostrand, Princeton, N. J., 1959. Consider any nonempty collection F of real-valued functions which is closed under addition and squaring and any nonconstant mapping $L: F \to R$ of F into the reals. Then our main result is: L is a limit if and only if for every f and g in F the following hold: $L(f+g) = Lf + Lg$, $L(f^2) = (Lf)^2$, and $Lf \leqq Lg$ provided $f(x) \leqq g(x)$ for every x which lies both in the domain of f and in the domain of g. Let F be the collection of all real-valued functions having a limit along a direction p. As one application of our main result, we show that the limit is the only nontrivial homomorphism of F into the reals. (Received January 25, 1971.)

683-B34. PHILIP BRENNER, Case Western Reserve University, Cleveland, Ohio 44118. Initial value problems in L_p for systems with variable coefficients. Preliminary report.

Let $P(x, D)$ be a system of pseudo-differential operators, with principal part $P_d(x, D)$. The symbol of $P_d(x, D)$ is denoted $P_d(x, y)$. Then the main result is the following: If the Cauchy problem for $\partial/\partial t - P(x, D)$ is well posed in L_p, then $\exp(P_d(x_0, y))$ is a multiplier on FL_p^N, for all $x_0 \in R^n$. Here FL_p^N is the set of Fourier transforms of N-vectors with components in L_p. For $p = 2$ and differential operators $P(x, D)$, this was proved by other methods by G. Strang (J. Differential Equations 2(1966)). Some consequences of the main result are stated. (Received January 25, 1971.)

683-B35. ALAN R. ELCRAT, Wichita State University, Wichita, Kansas 67208. An a priori estimate for Poisson's equation.

In investigating generalized solutions of the equation $\Delta u = f$ it may be useful to put Δ in the role of a linear operator mapping $W^2_{2,0}(D)$ into $L_2(D)$. In this context an inequality $\|u\|_2 \leqq \text{const.} \|\Delta u\|_0$ (where $\| \;\|_2$, $\| \;\|_0$ denote the norms in $W^2_{2,0}(D)$, $L_2(D)$, respectively) is desirable. It can be shown that for domains D such that ∂D has nonnegative mean curvature an explicit value for this constant can be given in terms of the first eigenvalue of Δ in D. It is shown here that this value is sharp for certain plane domains. (Received January 25, 1971.)

683-B36. ALAN J. HECKENBACH, Iowa State University, Ames, Iowa 50010. Pointwise recurrent solutions of nonautonomous differential equations.

Let W be an open set in R^n and f a continuous function from $R \times W$ to R^n such that for some sequence $\{t_k\}$, $t_k \to \infty$, $f(t+t_k, x)$ converges uniformly to $f(t, x)$ on sets $R \times W$ where G is any compact subset of W. Assume the differential equation $x' = f(t, x)$ has a unique solution $\varphi(t; t_0, x_0)$ such that $\varphi(t_0; t_0, x_0) = x_0$ for each $(t_0, x_0) \in R \times W$. A solution $\varphi(t; t_0, x_0)$ is called pointwise recurrent if for each $t \in R$ and each pair ϵ, T of positive numbers, there exists a number $\tau > T$ such that $\|\varphi(t+\tau; t_0, x_0) - \varphi(t; t_0, x_0)\| < \epsilon$. If $\psi(t; t_0, x_0)$ is a positively compact solution, then there is a pointwise recurrent solution contained in the positive limit set of ψ. (Received January 25, 1971.)

683-B37. THOMAS G. PROCTOR, Clemson University, Clemson, South Carolina 29631. Periodic solutions for perturbed nonlinear differential equations. II.

The existence of periodic solutions of the periodic system of differential equations $\dot{x} = \epsilon g(t, x, y, \epsilon)$, $\dot{y} = f(t, y) + \epsilon h(t, x, y, \epsilon)$ is proved in two cases when ϵ is sufficiently small. Here it is assumed that x and y are vectors, the solution $\varphi(t, \tau, \gamma)$ of the initial value problem $\dot{y} = f(t, y)$, $y(t) = \gamma$ is known, and the functions φ, g, h satisfy some algebraic and smoothness conditions. The results generalize corresponding known theorems for the case $f(t, y) = Ay$. (Received January 25, 1971.)

683-B38. L. DAVID SABBAGH, Bowling Green State University, Bowling Green, Ohio 43403. Global weak asymptotic stability for dynamical polysystems.

Let $\lambda: \Omega \times X \times R \times R \to X \times R$, where X is a metric space, Ω a topological space, R the real numbers, be a dynamical polysystem as defined by Bushaw (Contributions to Differential Equations 2(1963), 351-365). Let $\Psi_u : X \times R \times R \to X$ be defined by $\Psi_u(x,s,t) = \pi \circ \lambda(u,x,s,t-s)$ where π is the projection of $X \times R \to X$. (This was defined as a dynamical polysystem of type S by Lovingood in [J. Differential Equations 6(1969), 326-336].) The mappings ψ_u can be thought of as the trajectories of a control system. The two theorems below are Liapunov-type theorems for boundedness and global weak asymptotic stability. Theorem 1. Let $v(x,t)$ defined on $X \times R$ satisfy: (1) v is positive definite on $A \subset X$, (2) v is radially unbounded, (3) v is lower semicontinuous, and (4) $D^+v(x,t) \leq 0$. Then every trajectory ψ_u of the polysystem is bounded. Theorem 2. Let $v(x,t)$ defined on $X \times R$ satisfy conditions (1)-(4) of Theorem 1 and also satisfy (5) v is decrescent wrt A, and (6) $v(\psi_u(x_0,t_0,t),t) \to 0$ as $t \to \infty$ for some $u \in \Omega$. Then the set A is globally weakly asymptotically stable. (Received January 25, 1971.)

683-B39. PHILIP C. TONNE, Emory University, Atlanta, Georgia 30322. Linear transformations on the power-series convergent on the unit disc which have matrix representations. Preliminary report.

Let S be the space of all sequences A such that if z is a complex number and $|z| < 1$ then $\sum A_n z^n$ converges. In a paper to appear in the London Journal (see Abstract 672-176, these *Notices* 17(1970), 133), the matrix transformations from S to S are characterized. Here those linear transformations from S to S which have matrix representations are characterized. Theorem. Suppose that L is a linear transformation from S to S. These statements are equivalent: (1) L has a matrix representation. (2) L is continuous with respect to the topology induced by uniform convergence of corresponding functions on closed subsets of the unit disc. (3) If $0 < R < 1$ then there is a number r between 0 and 1 such that L is a continuous linear transformation from the Banach space $\{S, N_r\}$ to $\{S, N_R\}$. (For A in S and $0 < s < 1$, $N_s(A) = \sum_0^\infty |A_n| s^n$.) (4) There are numbers r and R between 0 and 1 such that L is a continuous linear transformation from $\{S, N_r\}$ to $\{S, N_R\}$. (Received January 25, 1971.)

683-B40. JAMES D. BAKER, Honeywell Corporate Research Center, 500 Washington Avenue South, Hopkins, Minnesota 55343. On an inequality for Stieltjes integrals.

The integral, $\int_a^b h\,dg$, is defined to be the refinement limit of sums $\sum_{i=1}^n \alpha_h(x_{i-1},x_i)[g(x_i) - g(x_{i-1})]$, where $P = \{a = x_0 < x_1 < \ldots < x_n = b\}$, h and g are real-valued functions on $[a,b]$, and α is an interval function with the property that $\alpha_h(p,q) \geq 0$ if $h(x) \geq 0$ for $x \in [p,q]$. The number S_g denotes $\sup\{\int_\alpha^\beta dg | a \leq \alpha < \beta \leq b\}$. Theorem. Suppose h and g are real-valued functions on $[a,b]$, $h \in BV$ and $\inf h = 0$, integration-by-parts holds for each subinterval of $[a,b]$, and $\int_a^b g\,dh$ exists. Then $\int_a^b h\,dg \leq S_g V_a^b(h)$. This is an extension of a recent result of Darst and Pollard (Proc. Amer. Math. Soc. 25(1970), 912-913) in terms of both the class of functions for g and the types of Stieltjes integrals for which the inequality holds. (Received January 26, 1971.)

683-B41. NAM P. BHATIA, Institute for Fluid Dynamics, University of Maryland, College Park, Maryland 20742 and SHUI-NEE CHOW, Michigan State University, East Lansing, Michigan 48823. Weak attraction, minimality, recurrence and almost periodicity in semiflows.

A semiflow is a map $\pi : X \times G^+ \to X$ (X: metric space with metric ρ, G^+: nonnegative reals or nonnegative integers) satisfying (i) $\pi(x,0) = x$, (ii) $\pi(\pi(x,t),s) = \pi(x,t+s)$. Important examples are functional differential equations (Bhatia-Hajek, "Local semi-dynamical systems," Springer-Verlag, New York, 1969) and iterates of a continuous function $f : X \to X$. A solution is a map $\sigma : I \to X$ (I: interval in G; G: reals or integers) satisfying $\sigma(t) = \pi(\sigma(s), t-s)$ for $s \leqq t$, $s,t \in I$. $\sigma_x : G^+ \to X$ where $\sigma_x(t) = \pi(x,t)$ is the positive solution through x. A solution p_x with $p_x(0) = x$ and domain $p_x = G$ is called a principal solution through x. σ_x is asymptotically almost periodic if given $\epsilon > 0$ there is relatively dense set D in G^+ with $\rho(\sigma_x(t+\tau_1), \sigma_x(t+\tau_2)) < \epsilon$ for $t \in G^+$ and $\tau_1, \tau_2 \in D$. Let X be complete. Theorem 1. σ_x is asymptotically almost periodic iff range σ_x has compact closure and the positive limit set $L^+(x)$ is a positively minimal set with σ_y positively almost periodic for each $y \in L^+(x)$. Theorem 2. Let σ_x be asymptotically almost periodic. Then for each $y \in L^+(x)$ there is a unique principal solution p_y with range p_y $L^+(x)$ and the map $\pi^* : L^+(x) \times G \to L^+(x)$ given by $\pi^*(y,t) = p_y(t)$ defines a dynamical system on $L^+(x)$ (Bhatia-Szegö, "Stability theory of dynamical systems," Springer-Verlag, New York, 1970). (Received January 25, 1971.)

683-B42. PAUL O. FREDERICKSON, Lakehead University, Thunder Bay, Ontario, Canada. Triangular spline interpolation in the plane.

Denote by $S_h^{n,q}$ the class of q-times differentiable functions on the plane which are piecewise polynomials of degree n with respect to a regular triangular partition of mesh h. Hermite interpolation in various of the spaces $S_h^{n,q}$ has been developed rather recently. In the present paper invariant spline interpolation operators are constructed and error estimates developed. Refer to a spline K in $S_h^{n,q}$ as a k-exact basic spline if it has compact support and linear combinations of translates interpolate polynomials of degree k exactly. If the Fourier transform of its lattice values does not vanish, then the interpolation operator $\mathscr{I}_K$ derived from K satisfies $\| \mathscr{D}^{\alpha}(f(x) - (\mathscr{I}_K f_h)(x)) \|_{\infty} = o(h^{k-|\alpha|})$ for suitably smooth and damped f. Weaker estimates apply when the transform of K vanishes; in some cases f is restricted to be periodic. Particular operators, of degree h^3 and h^2 respectively, are constructed in $S_h^{4,2}$ and $S_h^{3,1}$ respectively. Application of these results to the Raleigh-Ritz-Galerkin solution of elliptic equations is discussed. (Received September 11, 1970.)

683-B43. WITHDRAWN.

683-B44. JAMES R. CHOIKE, Department of Mathematics and Statistics, Oklahoma State University, Stillwater, Oklahoma 74074. One-sided boundary behavior for analytic and bounded functions. Preliminary report.

Let f(z) be analytic and bounded, $|f(z)| < 1$, in $|z| < 1$. Let $f^*(e^{i\theta})$ be the radial limit value of f(z) at $e^{i\theta}$. We shall say that f(z) has a right-sided (left-sided) limit at $e^{i\theta_0}$ if there exists $\alpha > 0$ such that $f^*(e^{i\theta})$

exists and is continuous for all θ, $\theta_0 - \alpha \leqq \theta \leqq \theta_0$ ($\theta_0 \leqq \theta \leqq \theta_0 + \alpha$). Theorem. If $f^*(e^{i\theta})$ exists and is of modulus 1 a. e. on an arc $a < \theta < b$, and if $P = e^{i\theta_0}$, $a < \theta_0 < b$, is a singular point for $f(z)$, then either (i) the values $f^*(e^{i\theta})$, $a < \theta < \theta_0$, cover $|w| = 1$ infinitely many times and $f(z)$ has a left-sided limit at $e^{i\theta_0}$ of modulus 1, or (ii) the values $f^*(e^{i\theta})$, $\theta_0 < \theta < b$, cover $|w| = 1$ infinitely many times and $f(z)$ has a right-sided limit at $e^{i\theta_0}$ of modulus 1, or (iii) the values $f^*(e^{i\theta})$ cover $|w| = 1$ infinitely many times for each arc $a < \theta < \theta_0$ and $\theta_0 < \theta < b$. This extends a result by Calderón, González-Domínguez, and Zygmund (Rev. Un. Mat. Argentina 14(1949), 16-19). We also show that the above result is sharp by giving a necessary and sufficient condition for a Blaschke product to have a right-sided limit but not a left-sided limit at $e^{i\theta_0}$.
(Received January 27, 1971.)

683-B45. PETER HESS, University of Chicago, Chicago, Illinois 60637. On nonlinear equations of Hammerstein type in Banach spaces.

The following result is presented: Theorem. Let N be a hemicontinuous monotone (nonlinear) mapping of the real reflexive Banach space X into its conjugate space X^*, and let the monotone linear operator $T: X^* \to X$ satisfy the condition: (*) There exists a constant $c > 0$ such that $(v, Tv) \geqq c\|Tv\|^2$ for all $v \in X^*$. Then the Hammerstein equation $u + TNu = f$ admits a unique solution for each f in X. It is shown that condition (*) is a proper weakening of the concept of angle-boundedness of a bounded linear operator. The Theorem thus sharpens related assertions by Amann ("Ein Existenz- und Eindeutigkeitssatz für die Hammersteinsche Gleichung in Banachräumen", Math. Z. 111(1969), 175-190) and by Browder-Gupta ("Monotone operators and nonlinear integral equations of Hammerstein type", Bull. Amer. Math. Soc. 75(1969), 1347-1353). (Received January 26, 1971.)

Applied Mathematics

683-C1. GILBERT STRANG, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. The finite element method.

We consider Ritz-Galerkin approximations to the solution of elliptic problems, using piecewise polynomials as trial functions -- in other words, the finite element method. We shall discuss boundary conditions, error estimates, and the condition number of the discrete approximating problem for an irregular mesh, that is, for a domain divided into elements of unequal size. In the regular case, when the elements are defined by a mesh of width h and the space of trial functions includes a complete polynomial of degree p, we prove that the error (in displacement) behaves asymptotically either like $h^{p+1}\omega(x/h)$ or like $h^{2(p+1-m)}\psi(x)$, depending which exponent of h is the smaller. Here ω is a 1-periodic function, and ω and ψ are related in a simple way to the piecewise polynomials and the solution u. (Received January 6, 1971.)

683-C2. HENRY H. RACHFORD, JR., Rice University, Houston, Texas 77001. Application of variational methods to approximate the transient response of gas transmission systems.

The flow of gas (ideal or not) in a pipeline is described by a nonlinear first order hyperbolic differential system in one space variable and time involving two dependent variables, usually with given initial values and a linear time-varying specification of one of the variables at each end. The interconnection of n pipeline sections into a transmission system yields a set of n such first order hyperbolic systems with interdependent boundary data, which must necessarily be considered simultaneously. A discrete, second-order-correct-in-time Galerkin procedure has been sucessfully applied to treat such systems with n restricted only linearly by the resources of the computer. Comparisons with the standard characteristics method for solving the problem shows that gas transmission system simulation by the Galerkin procedure reduces the computing required of the order of 10-fold. (Received January 11, 1971.)

683-C3. MARY FANETT WHEELER, Rice University, Houston, Texas 77001. A priori L_2-error estimates for Galerkin approximations to parabolic partial differential equations.

L_2-error estimates for the continuous time and several discrete time Galerkin approximations to solutions of some second order nonlinear parabolic partial differential equation are derived. Both Neumann and Dirichlet boundary conditions are considered. These error estimates are obtained by relating the error for the nonlinear parabolic problem to known L_2-error estimates for a linear elliptic problem. With additional restrictions on basis functions and region L_∞-error estimates are derived. Possible extensions to other discrete time Galerkin schemes and to higher order parabolic equations and systems of parabolic equations are suggested. (Received January 11, 1971.)

683-C4. GERALD W. HEDSTROM, Case Western Reserve University, Cleveland, Ohio 44106. A new algorithm for the nonlinear wave equation.

This algorithm is for approximating solutions of the system, $u_t - v_x = 0$, $v_t + (f(u))_x = 0$, where $u(x, 0)$ and $v(x, 0)$ are given functions and $f' < 0$, $f'' > 0$. One solves instead the system, $u_t - v_x = 0$, $v_t + (f_h(u))_x = 0$, where $u(x, 0)$ and $v(x, 0)$ are piecewise-constant functions and f_h is the piecewise-linear function such that $f_h(\bar{u}_j) = f(\bar{u}_j)$ for some set $\{\bar{u}_j\}$. This new problem can be solved exactly. The solution consists of piecewise-constant states, separated by lines. For initial data consisting of 3 constant states experimental computations with $f(u) = 1/u$ show that this method gives a very good approximation to the solution of the original problem. For Riemann problems with shocks in the solution this method gives the shock speed, correct to $O(h^2/|[u]|)$, where $h = \max(\bar{u}_{j+1} - \bar{u}_j)$ and $[u]$ is the jump in u across the shock. (Received January 25, 1971.)

683-C5. THOMAS R. LUCAS, University of North Carolina, Charlotte, North Carolina 28213 and GEORGE W. REDDIEN, Georgia Institute of Technology, Atlanta, Georgia 30332. Some projection schemes for second order nonlinear boundary value problems.

Approximations to solutions of two-point nonlinear boundary value problems of the form $D^2u = f(x,u,u')$, $0 < x < 1$, with boundary conditions $u(0) = u(1) = 0$ are developed in spaces of cubic splines using a weighted subdomain method and Galerkin's method. Proofs are based on the general theory of approximation methods rather than, for example, a variational or monotone operator theory approach. As a result, both methods are shown to be applicable to wider classes of problems than apparently was previously known. Convergence rates are given and computational aspects of the weighted subdomain method are discussed including results of numerical experiments. (Received January 25, 1971.)

683-C6. THOMAS R. LUCAS, University of North Carolina, Charlotte, North Carolina 28213 and GEORGE W. REDDIEN, Georgia Institute of Technology, Atlanta, Georgia 30332. Two higher-order projection schemes for second order nonlinear boundary value problems.

Approximations to solutions of two-point nonlinear boundary value problems of the form $D^2u = f(x,u,u')$, $0 < x < 1$, with boundary conditions $u(0) = u(1) = 0$ are developed by two schemes. The first uses cubic splines as approximating functions and the orthogonal projection operator into linear splines. If $f(x,u,u') = f(x,u) + e(x)u'$, a solution u_0 exists, f, f_u are continuous, and $D^2u - f_u(x,u_0)u - e(x)u' = 0$ and $u(0) = u(1) = 0$ has only the zero solution, then convergence is shown to be fourth order if u_0 is in C^4. Previous results found in the literature for this method discuss only linear problems and establish only third order convergence. The second method used is a weighted subdomain projection using quintic splines, which gives sixth order convergence if u_0 is in C^6. Computational aspects of both methods are discussed and some numerical results mentioned. (Received January 25, 1971.)

683-C7. CARL P. SIMON, University of California, Berkeley, California 94720. Periodic and almost periodic orbits in a central force problem.

The equations of motion of a particle with unit mass in a central force problem can be written as $\ddot{r} = (\partial/\partial r)V_p(r)$ where $V_p(r) = V(r) + \frac{1}{2}p^2/r^2$, $V: \mathbb{R}^+ \to \mathbb{R}$ is the potential energy and p is the angular momentum constant. If the energy integral c lies in a "potential well" on the graph of V_p, motion is bounded for the integrals (c,p). Then, we can define the rotation number $\rho_V(c,p)$ of the motion, which is irrational if the (bounded) orbits are almost periodic and rational if the orbits are periodic. Using elementary integration techniques, one obtains: Theorem 1. $\rho_V(c,p)$ is a C^∞ function of (c,p). Let $A = \{(V,p) \in C^\infty(\mathbb{R}^+,\mathbb{R}) \times \mathbb{R}: V_p$ has a relative minimum$\}$. Inverting integral equations, one shows Theorem 2. The following subset B is Baire in A: $(V,p) \in B$ iff $\rho_V(p,c)$ as a function of c has no degenerate critical points. Answering a problem of Smale, this shows that generically for V and p, bounded orbits alternate constantly between periodic orbits and orbits dense in some annulus as energy c varies. (Received January 25, 1971.)

683-C8. M. Z. v. KRZYWOBLOCKI, Department of Mechanical Engineering, Michigan State University, East Lansing, Michigan 48823. Association of nonlinear partial differential equation with a linear system of Schroedinger equations.

In the wave mechanics theory of turbulence based upon the principles of quantum theoretical methods of statistical physics there appears a necessity for the association of the highly nonlinear Navier-Stokes partial differential equation (deterministic and causal law of the conservation of momentum) with linear partial differential Schroedinger wave equation, known so well from the quantum field theory. The terms in the Navier-Stokes equations are grouped together and expressed in terms of a wave function, which is inserted into the Schroedinger equation, decomposed into two parts, real and imaginary, each of which is separately equal to zero. Briefly, a solution of the single nonlinear differential equation is sought in terms of solutions of a (finite) system of linear partial differential (Schroedinger) equations superimposed one upon the other with the appropriate initial (time) and two point (space) boundary conditions. Neglecting (for the time being) the uniqueness proof (very difficult, if, in general, possible), the existence proof is not available, as yet. The method seems to offer some strong potential towards solutions of some nonlinear partial differential equations of a special form. Solution may be achieved by means of numerical techniques. (Received December 3, 1970.)

683-C9. FRED H. BRINK and ARTHUR O. GARDER, Southern Illinois University, Edwardsville, Illinois 62025. Numerical solutions of a hyperbolic problem. Preliminary report.

Three numerical approximations have been studied to the hyperbolic problem $u_{tt} = C^2(x)\, u_{xx}$ $-x_0 < x < x_0$, where $C(x) = C_1$, $-x_0 < x < 0$ and $C(x) = C_2$, $0 < x < x_0$, $C_1 \neq C_2$. Initially u and u_t are 0 for $x > 0$. For $x < 0$, $u(x, 0) = F(X)$, $u_t(x, 0) = -C_1 F'(X)$. If x_0 is finite, $u(x_0, t) = u(-x_0, t) = 0$. At $x = 0$, require that (1) $u(0, t) \in C$ and (2) $u_x(0, t) \in C$. In case $x_0 = \infty$, an analytic solution is known. First and second order correct implicit difference approximations have been proved to be stable. For the case $x_0 = \infty$, an adaptation is proposed of the normal form in the Courant-Isaacson Rees procedure. Computer tests have been made of the three procedures with $C_1 = 1$, $C_2 = 2$ and results have been compared with the analytic solution. (Received December 31, 1970.)

683-C10. IVO BABUŠKA, Institute for Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland 20742. Approximation by hill functions. Preliminary report.

The approximation in Sobolev spaces by hill functions will be studied. The simultaneous approximation of different orders on different manifolds will be analyzed. Some best possible values for error bound will be given. (Recieved January 14, 1971.)

683-C11. JIM DOUGLAS, JR., University of Chicago, Chicago, Illinois 60637. Galerkin methods for nonlinear parabolic equations with nonlinear Neumann boundary conditions.

This work, done jointly with Todd Dupont, describes error bounds for Galerkin-type approximations to solutions of initial-boundary problems for certain nonlinear parabolic and degenerate-parabolic systems. The boundary conditions, which are weakly imposed, give the conormal derivative of the solution as a function of its value. Some of the procedures considered are computationally linear even though the equation and boundary condition are nonlinear. Energy methods are used to derive these error bounds with trace theorems used to estimate the boundary terms. (Received January 27, 1971.)

683-C12. TODD DUPONT, University of Chicago, Chicago, Illinois 60615. The effect of interpolating coefficients in nonlinear parabolic Galerkin methods.

This is joint work with Jim Douglas, Jr. In order to use Galerkin methods for nonlinear parabolic problems it is necessary to compute large numbers of integrals involving the coefficients in the differential operator. An efficient (and practically successful) method of evaluating these integrals is to interpolate the coefficients and compute the resultant integrals by formula. It is possible to show that for certain local interpolation schemes there is no loss in order of convergence. For example, suppose that the Hermite cubics on uniform node spacing h are used to represent the solution and to interpolate the coefficient in the equation $u_t - (a(x,u)u_x)_x = 0$. Suppose also that a and u are sufficiently smooth and that a predictor-corrector version of Crank-Nicolson is used to solve the system of ordinary differential equations in t. Then the L^2-norm of the error at each step is $O(h^4+(\Delta t)^2)$. Analogous results hold for other Hermite spaces, etc. (Received January 27, 1971.)

683-C13. GEORGE J. FIX, Harvard University, Cambridge, Massachusetts 02138. Finite element approximations to parabolic problems.

The use of spaces of piecewise polynomials of degree $k - 1$ with Galerkin methods for approximating solutions to parabolic problems is analysed. It is shown that xth derivative has an error of magnitude $O(h^{k-x})$ where h is the maximum mesh length. In addition, it is shown that if suitable difference quotients of the Galerkin solution are used, then approximations to the xth derivative of the solution are obtained which have an error of $O(h^k)$. (Received January 27, 1971.)

683-C14. MARCO RAUPP, University of Chicago, Chicago, Illinois 60637. Galerkin methods for the unsteady 2-dimensional flow of an inviscid incompressible fluid.

Galerkin type methods are proposed to approximate the unsteady 2 dimensional flow of an inviscid incompressible fluid. Two cases are considered: flow in a region with impermeable boundary and flow crossing the boundary but known there. The equations defining the approximations are formally based on an equivalent formulation of the flow problems in terms of an evolution equation for the vorticity and a family of elliptic problems for some stream functions. L^2-bounds for the error in the vorticity and velocity field are obtained which give convergence of the methods when combined with results from approximation theory. (Received January 27, 1971.)

683-C15. MARTIN H. SCHULTZ, Yale University, New Haven, Connecticut 06520. Quadrature-Galerkin approximations to solutions of elliptic differential equations.

In practice the Galerkin method for solving elliptic partial differential equations yields equations involving certain integrals which cannot be evaluated analytically. Instead these integrals are approximated numerically and the resulting equations are solved to give "quadrature-Galerkin approximations" to the solution of the differential equation. Using a technique of J. Nitsche, L^2 a priori error bounds are obtained for the difference between the solution of the differential equation and a class of quadrature-Galerkin approximations. (Received January 27, 1971.)

683-C16. JAMES H. BRAMBLE, Cornell University, Ithaca, New York 14850. Some new projection methods for the approximation of solutions of elliptic and parabolic problems.

Some new results on a variant of the least squares method are presented. For the parabolic case some difference-projection methods are discussed and for the elliptic case a new variant of the least squares method is introduced and its properties analyzed. (Received January 27, 1971.)

Geometry

683-D1. ROBERT ALLEN LIEBLER, Dartmouth College, Hanover, New Hampshire 03755. A characterization of the Lüneburg planes.

Let π be a finite affine plane and suppose G is a group of collineations that acts as a rank 3 group on the points of π and as a rank 2 group on the ideal points of π. Then π is a Lüneburg plane. This theorem answers affirmatively a conjecture of D. G. Higman [Math. Z. 102(1968), 147-149]. (Received January 13, 1971.)

Logic and Foundations

683-E1. ROBERT I. SOARE, University of Illinois, Chicago, Illinois 60680. The Friedberg-Muchnik theorem re-examined.

In the well-known solution to Post's problem Friedberg and Muchnik each constructed a pair of incomparable r.e. degrees $\underset{\sim}{a}$ and $\underset{\sim}{b}$. Subsequently, Sacks constructed r.e. degrees $\underset{\sim}{c}$ and $\underset{\sim}{d}$ such that $\underset{\sim}{c} \cup \underset{\sim}{d} = \underset{\sim}{0}'$ and $\underset{\sim}{c}' = \underset{\sim}{d}' = \underset{\sim}{0}'$. Lachlan showed that such degrees $\underset{\sim}{c}$, $\underset{\sim}{d}$ could have no greatest lower bound in the upper semilattice of r.e. degrees. Theorem 1. The original Friedberg-Muchnik degrees $\underset{\sim}{a}$, $\underset{\sim}{b}$ automatically satisfy Sacks' conditions and hence witness that the upper semilattice of r.e. degrees is not a lattice. Theorem 2. Let $\underset{\sim}{c}$ be any nonzero r.e. degree. Let $\underset{\sim}{a}, \underset{\sim}{b} \leq \underset{\sim}{c}$ be the incomparable r.e. degrees below $\underset{\sim}{c}$ constructed by combining the Friedberg-Muchnik technique with the natural "permitting" argument. Then $\underset{\sim}{a} \cup \underset{\sim}{b} = \underset{\sim}{c}$, and $\underset{\sim}{a}' = \underset{\sim}{b}' = \underset{\sim}{0}'$. Martin and Lachlan showed that certain constructions of r.e. degrees $\underset{\sim}{a}$ automatically insure $\underset{\sim}{a} = \underset{\sim}{0}'$. Jockusch and the author observed when $\underset{\sim}{a}$ is constructed below a fixed r.e. $\underset{\sim}{c} \neq \underset{\sim}{0}$, frequently $\underset{\sim}{a} = \underset{\sim}{c}$ automatically. Theorems 1 and 2 assert a similar "maximum degree principle" for *pairs* of r.e. degrees, as suggested to us by Lerman. (Received January 13, 1971.)

Topology

683-G1. NADIM A. ASSAD, University of Iowa, Iowa City, Iowa 52240. *Fixed point theorems for set valued transformations on compact sets.*

Let M be a complete, metrically convex, metric space, $\mathcal{J}(M)$ the family of nonempty bounded closed subsets of M, and let $K \subset M$. A mapping $\varphi: K \to \mathcal{J}(M)$ is said to be a contractive mapping if for all x, y in K and $x \neq y$, $D(\varphi(x), \varphi(y)) < d(x, y)$ where D denotes Hausdorff distance in $\mathcal{J}(M)$. *Theorem* 1. If K is a nonempty compact subset of M and if $\varphi: K \to \mathcal{J}(M)$ is a contractive mapping for which $\varphi(x) \subset K$ for all x in the boundary of K, then there exists $x_0 \in K$ such that $x_0 \in \varphi(x_0)$. This result has the following application in Banach spaces. *Theorem* 2. Let K be a closed bounded subset of H where H is a closed convex subset of a finite dimensional Banach space X. Suppose T is a contractive set valued mapping defined on K whose values are in $\mathcal{J}(H)$. If $Tx \subset K$ whenever x is in the relative boundary of K in H, then there exists $x_0 \in K$ such that $x_0 \in Tx_0$. (Received December 7, 1970.) (Author introduced by Professor William A. Kirk.)

683-G2. J. M. BOYTE and ERNEST P. LANE, Appalachian State University, Boone, North Carolina 28607. *Subnormal and normal spaces.* Preliminary report.

Definition. A space X is *subnormal* if each open cover of X has a subcover C such that if F is a closed set contained in some member of C, then there exists a countable subset C' of C and there exists an open set W with F contained in W such that $\overline{W}$ is contained in $\bigcup C'$. *Remark*. It is clear that every normal space is subnormal and that every Lindelöf space is subnormal. *Theorem* 1. A regular subnormal space is normal. *Corollary* 1A. Every regular Lindelöf space is normal. *Corollary* 1B. A space is T_4 iff it is T_3 and subnormal. *Theorem* 2. The following are equivalent: (1) The space X is normal. (2) For each closed subset F of X, and for each open set V(F) that contains F, there exists a countable collection of open sets $\{W_i(F) \mid i = 1, 2, \dots\}$ such that each $W_i(F)$ contains F and $\bigcap_{i=1}^{\infty} \overline{W_i(F)} \subset V(F)$. (3) If A and B are disjoint closed subsets of X, then there exists a regular G_δ set that contains B and is disjoint from A. *Corollary* 2A. The following are equivalent: (1) The space X is perfectly normal. (2) Each closed subset of X is a regular G_δ set. (3) Each open subset of X is a countable union of regular closed subsets of X. (Received January 18, 1971.)

683-G3. MING-JUNG LEE, University of Rochester, Rochester, New York 14627. A generalized Mayer-Vietoris sequence. Preliminary report.

Let A be a monoid. Then a left (right) A-module can be considered as a covariant (contravariant) functor from A to the category of abelian groups. Watts (Proc. Conf. Categorical Algebra (La Jolla, Calif., 1965) Springer, New York, 1966, p. 331) generalized this idea by letting A be a small category. He showed that the simplicial homology (cohomology) groups are just the torsion (extension) groups of modules over the partially ordered set of the simplicial simplexes. In this paper, the analogous theorem for the singular theory is proved, but instead of partially ordered sets we use a small category with singular simplexes as objects. After setting up some spectral sequences based on the change of categories, a generalized Mayer-Vietoris sequence is obtained, involving the intersections and unions of infinitely many subsets of a space. As an application, a result of Milnor (Pacific J. Math. 12(1962), 337) follows as a corollary. (Received January 20, 1971.)

683-G4. RALPH JONES, University of Wisconsin, Madison, Wisconsin 53706. Riemann surfaces are unions of two open disks.

A Riemann surface R is a connected separable metric space such that every point of R has an open neighborhood homeomorphic to the open unit disk D lying in the plane. Theorem. There exist two open subsets of R, each homeomorphic to D, whose union is R. (Received January 22, 1971.)

683-G5. KAI WANG, University of Chicago, Chicago, Illinois 60637. Free smooth actions of S^1 and S^3 on homotopy spheres.

In the following, let $i = 1$ or 3 and $F = C$ or Q, resp. The word action means free smooth S^i-action. Let A denote the action on F^{n+1} by $g(u_0, \dots, u_n) = (gu_0, \dots, gu_n)$ where $g \in S^i$, the units of F and $u_j \in F$. Let $m = (i+1)(n+1) - 1$. We denote also by A the induced actions on $S^p \times S^q$, $S^p \times D^{q+1}$, $D^{p+1} \times S^q$ for p and q satisfied $x \equiv i \pmod{i+1}$ and $p + q + 1 = m$. Definition. Let (Σ^m, B) be a given action on homotopy sphere Σ^m which is decomposable if there is a suitable equivariant diffeomorphism f of $(S^p \times S^q, A)$ such that (Σ^m, B) is equivalent to $(S^p \times D^{q+1}, A) \cup_f (D^{p+1} \times S^q, A)$ with the naturally induced action, for some p, q with $p + q + 1 = m$. Theorem A. An action (Σ^m, B) is decomposable iff $p(\Sigma^m/B) \equiv (1+z^2)^{n+1} \pmod{z^{[n/4]+1}}$ for $i = 1$; $p(\Sigma^m/B) \equiv (1+z)^{2n+2}/(1+4z) \pmod{z^{[n/2]+1}}$ for $i = 3$ where p is the total Pontryagin class of the orbit space Σ^m/B and z is a generator of $H^{i+1}(\Sigma^m/B; Z)$. Theorem B. There are infinitely many distinct decomposable (resp. nondecomposable) actions on m-dim homotopy spheres for $m \geqq 13$. Theorem C. If Σ^m has a decomposable action, then there exist infinitely many distinct decomposable actions on it for $m \geqq 13$. Main theorem. There exist infinitely many distinct S^1-actions on the standard $(2n+1)$-dim sphere for $n \geqq 6$, and S^3-actions on the standard $(4n+3)$-dim sphere for $n \geqq 3$. (Received January 22, 1971.)

683-G6. ANDRZEJ GRANAS, University of Montreal, Montreal, Quebec, Canada. On some generalizations of the Leray-Schauder theory.

This will be a survey talk on some generalizations of the Leray-Schauder theory. (Received January 21, 1971.)

683-G7. PHIL HUNEKE, Ohio State University, Columbus, Ohio 43210. Commuting functions and their fixed points.

What conditions force functions (from a space to itself) which commute under composition to have common fixed points? A review of known theorems and recent examples will clarify the current status of this question and point out why local conditions appear to be insufficient to generate new results except under very particular global circumstances. (Received January 21, 1971.)

683-G8. RONALD J. KNILL, Tulane University, New Orleans, Louisiana 70118. On the homology of a fixed point set.

Let $g: X \times T \rightarrow X$ be a map with open domain in X and relatively compact image. The θ-homomorphism of Leray, a homotopy invariant of g, is discussed and used to extend Fuller's notion of index cycle to higher dimensions. (Received January 21, 1971.)

683-G9. KALYAN K. MUKHERJEA, University of California, Los Angeles, California 90024. New methods in coincidence theory.

Application of K-theory and cobordism to the study of coincidence of two maps between manifolds of different dimensions. Some extensions to coincidences of set-valued maps or correspondences and applications to algebraic geometry will also be included. (Received January 21, 1971.)

683-G10. ROGER D. NUSSBAUM, Rutgers University, New Brunswick, New Jersey 08903. An asymptotic fixed point theorem.

An asymptotic fixed point theorem is one in which the existence of fixed points is determined with the aid of assumptions on the iterates of the function. Employing new techniques, we obtain such a theorem which extends results of F. B. Browder, R. L. Frum-Ketkov and others. (Received January 21, 1971.)

683-G11. RICHARD B. THOMPSON, University of Arizona, Tucson, Arizona 85721. Fixed point theory via semicomplexes. Preliminary report.

The idea of defining a general class of spaces for the purpose of doing fixed point theory began with the quasi-complexes of S. Lefschetz in 1942 and was extended and localized, under the name of semicomplexes, by F. Browder and the author. The term semicomplex (SC) will be used as a generic name for all structures of this type. There are limitations on the applicability of SC's. For example, there exist compact, contractible, Hausdorff spaces, with the fixed point property, which support no SC-structure. Also, the class of tree-like continua which admit SC-structures has a geometric characterization which is known to exclude some plane tree-like continua. However, the various kinds of SC's are closed under several constructions, such as products and retractions, thus allowing a relatively simple proof that compact metric ANR's admit SC-structures. Applications of SC's to nonmetric situations such as HLC spaces--and hence to generalized manifolds and certain subsets of locally convex linear spaces--have been made. Finally, interesting questions arise in regard to the possibility of SC-structures in nonmetric settings and to topological applications of SC's outside the realm of fixed point theory. (Received January 21, 1971.)

683-G12. ROBIN BROOKS, Bowdoin College, Brunswick, Maine 04011. A lower bound for the number of solutions of $f(x) = a$.

The lower bound, $N(f, a)$, in question is the Δ_2-Nielsen number described in Brooks and Brown, "A lower bound for the Δ-Nielsen number," Trans. Amer. Math. Soc. 143 (1969), 555-564. Results regarding its computation were announced by the speaker in "The number of roots of $f(x) = a$," Bull. Amer. Math. Soc. 76 (1970), 1050-1052. For example: Theorem. If $f: X \to Y$ is a map of a path connected topological space X into a path connected topological manifold Y and $N(f,a) > 0$, then $N(f,a)$ is equal to the number of elements in the cokernel of the fundamental group homomorphism $f_{\#}: \pi(X) \to \pi(Y)$ induced by f. The proofs of some of these results will be sketched, as well as their relation to earlier results in fixed-point and coincidence theory. (Received January 21, 1971.)

683-G13. HENRY H. GLOVER and GUIDO MISLIN, Ohio State University, Columbus, Ohio 43210. Metastable annihilation in the homotopy groups of spheres. Preliminary report.

Let $\Sigma^{k+2}: \pi_{2k} S^{2n+1} \to \pi_{3k+2} S^{2n+k+3}$ denote the Freudenthal suspension map, and let $\Sigma^{\infty}: \pi_{2k} S^{2n+1} \to \pi_{2k}^{St} S^{2n+1}$ denote the canononical map into the stable stem. Theorem. kernel Σ^{k+2} = kernel Σ^{∞}. For $k \leq 4n + 1$ this result is contained in Freudenthal's theorem. Similarly, the p-primary parts of those kernels, $p > 2$, are well known to agree. The proof uses the embedding theorem of π-manifolds in the metastable range (H. Glover, to appear). We show that such embeddings can always be chosen with a fiber homotopically trivial normal bundle. We next consider a certain sphere fibration B over a sphere. By Browder's results we show that B has the homotopy type of a π-manifold B'. We then apply the results above to B' and study the Thom space of the normal bundle of the embedding of B' to obtain the theorem. The implication from embedding to suspension annihilation of homotopy groups is the converse of an implication known to M. Agoston (problems, Proc. Conf. Algebraic Topology, July, 1970). Similar results hold for spheres of even dimension and for odd dimensional homotopy groups. (Received January 25, 1971.)

683-G14. ORVILLE L. BIERMAN, University of Utah, Salt Lake City, Utah 84112. Manifolds with monotone union and monotone intersection properties. Preliminary report.

Let M be a compact manifold with boundary. M has the monotone intersection property provided that whenever $\{M_i\}$ is a sequence of manifolds such that: (a) $M_1 \overset{\circ}{\supset} M_2 \overset{\circ}{\supset} \ldots$, (b) for each i, M_i is homeomorphic to M, then $M_1 - \bigcap_{i=1}^{\infty} M_i \approx \dot{M}_1 \times [0, 1)$. M is trivially embedded in a manifold M_1 provided: (a) $M \overset{\circ}{\subset} M_1$,

(b) $M \approx M_1$, and (c) if X is a set in M such that $M - X \approx \dot{M} \times [0, 1)$, then $M_1 - X \approx \dot{M}_1 \times [0, 1)$. Theorem 1. M has the monotone intersection property if and only if whenever M is embedded in its interior it is trivially embedded in itself. Theorem 2. If M has the monotone intersection property then M also has the monotone union property. Theorem 3. If the boundary of M is a sphere and the dimension of M is not four, then M has the monotone intersection property. Theorem 4. If M^n has the monotone union property and M^n can be embedded in E^n, then the interior of M is an open n-cell. (Received January 25, 1971.)

683-G15. WILLIAM M. BOYCE, Bell Telephone Laboratories, Murray Hill, New Jersey 07974. Triods, commuting functions, and FPP-less plane continua. Preliminary report.

An unusual approach to the question of whether continua which do not separate the plane have the fixed-point property (FPP) raises the question of the existence of commuting continuous functions f and g on a triod (T or Y shaped space) which never agree. Under the assumption that the composition h = fg = gf has a finite fixed-point set H, the techniques of Glen Baxter are applied to study the possible behavior of f and g on H. (Received January 25, 1971.)

683-G16. JERREL K. YATES, University of Mississippi, University, Mississippi 38677. No nondegenerate space satisfying Axiom Ω is connected.

Let P denote the set of ordinals less than the first uncountable ordinal Ω. The space S satisfies Axiom Ω if there is a collection $\{G_x | x \text{ is in } P\}$ such that (1) if x is in P, then G_x is a collection of regions covering S, (2) if x and y are in P and x precedes y, then G_y is a subcollection of G_x and (3) if R is a region and each of A and B is a point of R, then there is an x in P such that if g is in G_x and A is in g then $\bar{g}$ is a subset of R and B is not in $\bar{g}$ unless B is A. Let S be a space that satisfies Axiom Ω and Axiom 0 of R. L. Moore's "Foundations of point set theory", Amer. Math. Soc. Colloq. Publ., Vol 13, Amer. Math. Soc., Providence, R. I., 1962. Theorem 1. If A is a point of S, then there is a collection H = $\{R_x | x \text{ is in } P\}$ of regions such that (1) if x and y are in P and x precedes y, then $\bar{R}_y$ is a subset of R_x, (2) if x is in P, then R_x is in G_x and (3) the common part of H is $\{A\}$. Theorem 2. If there is a point A in S such that for each region R containing A there is an x in P such that if each of g_1 and g_2 is in G_x, g_1 intersects g_2 and A is in g_1 then $\bar{g}_2$ is a subset of R, then S is not connected or S is degenerate. Theorem 3. S is not connected or S is degenerate. (Received January 25, 1971.)

683-G17. JOHN C. MARTIN, Rice University, Houston, Texas 77001. Substitution minimal flows.

Let θ be a substitution of length r on the b symbols $\{0,\ldots,b-1\}$ and $\mathfrak{X}_\theta = (X_\theta, T)$ the resulting symbolic flow, so that X_θ is infinite and $\mathfrak{X}_\theta$ minimal. θ is simple if $\theta(i)(n) \neq \theta(j)(n)$ $(i \neq j)$. $S = \{0,\ldots,b-1\}$ may be partitioned into $S_0,\ldots,S_{m(\theta)-1}$, so that if $i \in S_{n(i)}$ $(i \in S)$, then $(n(x(j)))_{j=0,1,\ldots}$ is periodic of period $m(\theta)$ $(x \in X_\theta)$. Let $A_{ijk} = \{\theta^j(p)(k)\,\theta^j(p)(k+1) : p \in S_i\}$, $P_\theta = \bigcup_{i,j,k} A_{ijk}$. Theorem 1. $\mathfrak{X}_\theta$ is a point-distal flow with a residual set of distal points. The structure transformation group of $\mathfrak{X}_\theta$ is $(Z_{m(\theta)} \times Z^r, T)$, where Z_m is the cyclic group of order m, and Z^r is the group of r-adic integers. Theorem 2. $\mathfrak{X}_\theta$ is almost automorphic if and only if there exist i, j, k so that for $p, q \in S_i$, $\theta^j(k)(p) = \theta^j(k)(q)$. Theorem 3. For

θ simple, $\mathfrak{X}_\theta$ is an AI extension of an equicontinuous flow if and only if the collection $\{A_{ijk}\}$ is a partition of P_θ. If b and r are prime, a third equivalent condition is that $\mathfrak{X}_\theta$ be an AI flow. In particular, therefore, the author constructs examples of point-distal flows with a residual set of distal points which are not AI flows. (See William A. Veech, "Point-distal flows," Amer. J. Math. 92(1970), 205-242.) (Received December 18, 1970.)

683-G18. RICHARD FREIMAN, University of Maryland, Baltimore, Maryland 21228. Proximally equicontinuous regular minimal sets over the circle.

This work is concerned solely with cascades, i.e., discrete flows. Definition 1. A minimal set (X, φ) is called regular if it is a universal minimal set for some admissible property, i.e., every minimal set satisfying the admissible property is a homomorphic image of the universal minimal set for that property. Definition 2. A minimal set (X, φ) is said to be proximally equicontinuous over the circle if (i) the proximal relation P is closed, and (ii) the quotient transformation group $(X/P, \varphi/P)$ is isomorphic to (S, M_g) where S is the circle represented as the group of complex numbers of modulus one, and $M_g: S \to S$ is defined by $M_g(z) = gz$. Theorem 1. Let (X, φ) be a proximally equicontinuous regular minimal set over the circle. Then the following are equivalent: (i) (X, φ) is isomorphic to (E, M_g') for some generator g of S, where (E, M_g') is the minimal set described in "A semigroup associated with a transformation group" (Trans. Amer. Math. Soc. 94(1960)), (ii) P' is contained in the diagonal of $X \times X$ and (X, φ) has no distal points, (iii) crd $(P(x)) = 2$ for some x in X, (iv) crd $(P(x)) = 2$ for all x in X, and (v) crd $(J) = 2$ where J is the set of idempotents of the unique minimal right ideal of the enveloping semigroup of (X, φ). (Received January 7, 1971.)

683-G19. NELSON G. MARKLEY, University of Maryland, College Park, Maryland 20740. F-minimal sets.

An F-minimal set is an almost automorphic minimal set with only one nontrivial proximal cell, denoted by F. Moreover, this proximal cell consists of uniformly asymptotic points. The author has obtained a method of constructing F-minimal sets from a special kind of almost automorphic functions. It can then be shown that F-minimal sets exist for a large class of groups and F's. Taking F to be the discrete two point space, one obtains a natural generalization of the Sturmian minimal sets, and the above construction yields results about their occurence; for example, if the phase group is the integers, than there are no generalized Sturmian minimal sets with an n-torus $n \geqq 2$ as their structure group. The author has also determined the structure of those minimal sets which are factors of the minimal right ideal of an F-minimal set. (Received January 7, 1971.)

683-G20. R. KANNAN, Purdue University, Lafayette, Indiana 47907. Some fixed point theorems in a reflexive Banach space.

Let X be a reflexive Banach space and let K be a nonempty bounded closed convex subset of X. The main theorem of the paper is as follows: If T be a mapping of K into itself such that (i) $\|Tx - Ty\| \leq \frac{1}{2}$ $\{\|x - Tx\| + \|y - Ty\|\}$, x, y $\in$ K, and (ii) for every nonempty closed convex subset F of K mapped into itself by T and containing more than one element there exists an $x \in F$ such that $\|x - Tx\| < \sup_{y \in F}\|y - Ty\|$. Then T has a unique fixed point in K. (Received January 25, 1971.)

683-G21. DENNIS R. DALUGE, University of Minnesota, Minneapolis, Minnesota 55455. The fundamental group of the space of conjugacy classes of a central analytic group.

Let G be a topological group and let $\bar{G}$ be the orbit space of the action of G on itself by inner automorphisms; that is, $\bar{G}$ is the space of conjugacy classes of G. For a locally compact central group G, that is, a locally compact group G with a compact adjoint group, $\bar{G}$ has been shown to be the natural domain for the family of characters of finite-dimensional representations of G (Grosser and Moskowitz, Bull. Amer. Math. Soc. 72(1966), 833). Let K(G) denote the closed commutator subgroup of G. Theorem. If G is a central analytic group, then the fundamental groups of the spaces $\bar{G}$ and G/K(G) are isomorphic. The proof depends upon the following special case. Theorem. If G is a compact semisimple analytic group, then $\bar{G}$ is simply connected. The covering space theory of Chevalley is used in the proof, along with the fact that each element of a compact semisimple analytic group is expressible as a commutator $xyx^{-1}y^{-1}$ of elements $x, y \in G$ (M. Goto, J. Math. Soc. Japan 1 (1949), 270). (Received January 26, 1971.)

683-G22. LAWRENCE M. FRANKLIN and N. P. BHATIA, University of Maryland, Baltimore, Maryland 21228. Separatrices and cross-sections in dynamical systems.

Markus has recently obtained some extensions to higher dimensions, of results presented in his paper "Global structure of ordinary differential equations in the plane" (Trans. Amer. Math. Soc. 76(1954)). This involves defining a separatrix for spaces of dimension higher than two. Using his definition he is able to prove that unstable flows in R^3 without separtrices are parallel. The chief drawback to his definition is that it requires the flow to be unstable. An alternative definition for a separatrix is presented here; one more along the lines of Markus' earlier paper. It involves the notion of the prolongation limit set J(x). Definition. Given a flow on a manifold, a trajectory C(x) is not a separatrix, if it lies in an invariant set U with the property that for each $y \in U$ we have $L(y) = J(y) = \emptyset$ or $L(y) = J(y) \neq \emptyset$. [L(x) denotes the limit set.] Theorem. Any C^2 flow in R^3 without separatrices admits a global cross-section to the orbits. The flow is equivalent to either (i) parallel lines, (ii) all points fixed, or (iii) rotation about a fixed axis. (Received January 5, 1971.)

683-G23. DOUGLAS W. CURTIS, Louisiana State University, Baton Rouge, Louisiana 70803. Pushing apart a locally finite collection of disjoint closed subsets in a normed linear space.

Theorem. Let $\{F_\alpha\}$ be a locally finite collection of disjoint closed subsets in a normed linear space X, and assume $\theta \notin \bigcup F_\alpha$. Then there exists a map $h: X \to [1, \infty)$ such that $H: X \to X$, defined by $H(x) = h(x) \cdot x$, is a homeomorphism, and $\rho(H(F_\alpha), H(F_\beta)) \geqq 1$ for $\alpha \neq \beta$. This problem (for two disjoint closed subsets in Hilbert space) was proposed by R. D. Anderson and F. Browder. In general, it may be impossible to push apart even two closed subsets with a bounded homeomorphism. In the Fréchet space $\underline{s}$, the countable infinite product of lines, with any of the usual invariant metrics, there exist disjoint closed subsets C, D such that for every homeomorphism $H: \underline{s} \to \underline{s}$, $\rho(H(C), H(D)) = 0$. (Received January 27, 1971.)

683-G24. HENRY H. GLOVER, Ohio State University, Columbus, Ohio 43210. Common fixed points of commuting periodic maps. Preliminary report.

Theorem. Let $Z_p \times Z_q$ act on the closed n-disk, D^n, or on real projective space of even dimension, RP(2n), as a group of homeomorphisms. Then there exists a fixed point for this action. This result generalizes the theorem of J. Joichi (Nieuw Arch. Wisk. 14(1966), 247-251) for the closed interval, and the theorem of Eustice, Glover, Mislin (to appear) for holomorphic maps of the n-polydisk. This last paper also gives counterexamples for the theorem above for projective spaces over the complex and quaternion fields. Question. Do nonperiodic commuting homeomorphisms of D^n or RP(2n) have a common fixed point? The proof of the theorem uses Smith theory to characterize the components of the fixed point set of dimension zero. This characterization together with the commutativity yields the result. (Received January 27, 1971.)

683-G25. FELIX E. BROWDER, University of Chicago, Chicago, Illinois 60637. Normal solvability and the solutions of nonlinear equations in Banach spaces.

A new geometrical method for determining the solvability of nonlinear equations in Banach spaces is presented, using the concept of normal solvability. Examples are given of mappings defined by analytic conditions which fall within the scope of this theory. (Received January 21, 1971.)

Miscellaneous Fields

683-H1. HSIN CHU, University of Maryland, College Park, Maryland 20742. A continuous flow acting on S^n.

The following result is established: "If a continuous flow acts on S^3 such that its action is almost periodic and effective, then there are exactly two closed orbits which are homeomorphic to a circle and the action must be orthogonal." In fact, we prove the following two theorems and the above result follows as a consequence. Theorem 1. Let R be an almost periodic, effective, continuous flow acting on S^n. Then there is an (n-1)-dim. orbit-closure under R in S^n if and only if $n = 3$. Theorem 2. Let G be a connected, locally compact group which acts on S^n effectively and almost periodically, where $n \neq 2k+3$, $k = 1, 2, \cdots, 14$. If there is an (n-1)-dim. orbit-closure under G in S^n, then G must be a Lie group and the action of G must be orthogonal. (Received December 8, 1970.)

683-H2. CHARLES C. PUGH, University of California, Berkeley, California 94720. On normally hyperbolic flows.

We discuss the problem of finding infinitesimal conditions for a flow to be normally hyperbolic at an invariant manifold V. A counterexample to a natural conjecture is explained. This is part of joint work with M. Hirsch and M. Shub. (Received January 7, 1971.)

683-H3. PATRICK EBERLEIN, University of California, Berkeley, California 94720. Geodesic flows in manifolds of negative curvature.

We study complete manifolds M with $K \leqq 0$ in terms of their simply connected Riemannian cover H and deckgroup D. We adjoin points at infinity, $H(\infty)$, for H, and $\bar{H} = H \cup H(\infty)$ is a topological n-cell with a natural topology. L(D), the limit set for D, is the set of cluster points in $H(\infty)$ of an orbit D(p), $p \in H$. L(D) is closely related to Ω, the set of nonwandering points of the geodesic flow in the unit tangent bundle of M, SM. We define duality for two points in L(D) and obtain a flow formulation. H satisfies Axiom 1 (Axiom 2) if there exists at least one (at most one) geodesic joining any two points of $H(\infty)$. If both axioms hold properties of D may be precisely stated in terms of geodesic flow. Theorem 1. Let H satisfy Axioms 1 and 2 and suppose that Ω does not consist of a single periodic orbit and its reverse. Then the goedesic flow is topologically transitive in Ω and periodic points are dense in Ω. Theorem 2. Let H satisfy Axiom 1 and let $\Omega = SM$, $M = H/D$. Then the geodesic flow is topologically transitive in SM. Theorem 2 is also true for certain manifolds without conjugate points. (Received January 8, 1971.)

683-H4. WILLIAM A. VEECH, Rice University, Houston, Texas 77001. Substitutions of nonconstant length.

A discussion of the properties of minimal sets arising from substitutions of nonconstant length, including the following sufficient condition for weak mixing. Let A, B be blocks of zeros and ones with the first letter of A being 0, and assume (A,B) does not have the form (0101...10, 1010...01). Let A, B have a, b and c, d zeros and ones, respectively, a, b, c, d $\geqq 1$, and let $\alpha = \binom{a\,b}{c\,d}$. Theorem. The minimal set arising from the substitution $0 \to A$, $1 \to B$ is weakly mixing if (i) $(a+b, c+d) = 1$, (ii) (trace α, det α) = 1, and (iii) $|\lambda_1|, |\lambda_2| \geqq 1$, where in (iii) λ_1 and λ_2 denote the eigenvalues of α. A converse to this result has recently been obtained by John Martin. (Received January 18, 1971.)

683-H5. LEONARD SHAPIRO, University of Minnesota, Minneapolis, Minnesota 55455. Solving ordinary differential equations on homogeneous spaces.

Let (P, π, M) be a locally trivial A-bundle, where A is an abelian Lie group, P is compact, and M is a homogeneous space of the nilpotent Lie group H (Lie algebra $\mathcal{H}$). Let X be an ergodic flow on M ($\mathcal{X}$ its associated vector field) which commutes with A, such that $\pi(\mathcal{X}) \in \mathcal{H}$. Assume that for $g \in C^{\infty}(M)$ there is $f \in C^{\infty}(M)$ and d constant such that (*) $\pi(\mathcal{X})f = g+d$. Then P is a homogeneous space of a Lie group G satisfying: (1) $A \in$ center of G, (2) G contains a lift of each element of H, (3) X arises from a one-parameter subgroup of G. In the case $M = \mathbb{R}^2/\mathbb{Z}^2$, $H = \mathbb{R}^2$, it is possible to determine precisely when (*) is solvable and also to prove a converse of the above statement, yielding a vector field $\mathcal{X}$ on a 3-dimensional nilmanifold P such that $\pi(\mathcal{X}) \in \mathcal{H}$ and there is no nilpotent Lie group satisfying (1) - (3). Partial generalizations to n dimensions and to nonnilpotent homogeneous spaces are also obtained. This is joint work with Professors J. Brezin, R. Ellis and L. Green. (Received January 21, 1971.)

683-H6. KENNETH R. BERG, University of Maryland, College Park, Maryland 20742. Quasi-disjointness, products, and inverse limits.

The notion of quasi-disjointness, introduced previously by the author, is further examined. If we denote by Q the class of ergodic processes which are quasi-disjoint from every ergodic process then we have that Q is closed under the formation of ergodic products and of countable inverse limits. (Received January 26, 1971.)

683-H7. PETER WALTERS, University of Maryland, College Park, Maryland 20742 and University of Warwick, Coventry, England. Some invariant σ-algebras for measure-preserving transformations.

For an invertible measure-preserving transformation T of a Lebesgue space $(X, \mathcal{B}, m)$ and a sequence $N = \{n_i\}_{i=1}^{\infty}$ of integers let $\mathcal{A}_N(T)$ denote the σ-algebra $\mathcal{A}_N(T) = \{A \in \mathcal{B} \mid m(T^{n_i} A \triangle A) \to 0\}$. We study the relationship of these σ-algebras and related σ-algebras to spectral theory, entropy theory, mixing properties and group extensions. It turns out that $\mathcal{A}_N(T)$ inherits some of the properties of the Pinster σ-algebra $\mathcal{P}(T)$ and some of the properties of the σ-algebra generated by the eigenfunctions. One such property is that if $\mathcal{C}$ is a sub σ-algebra of $\mathcal{B}$ with $T\mathcal{C} = \mathcal{C}$ and $\mathcal{A}_N(T_{\mathcal{C}}) = \mathcal{N}$ (the trivial σ-algebra) then $\mathcal{C}$ and $\mathcal{A}_N(T)$ are independent. One obtains interesting examples from Gaussian processes. (Received January 27, 1971.)

683-H8. CHARLES C. CONLEY, University of Wisconsin, Madison, Wisconsin 53706. The index of an isolated invariant set of a flow.

If an invariant set of a flow is the maximal invariant set in some neighborhood of itself it is called isolated. Such invariant sets admit a natural continuation to nearby flows. An index for isolated invariant sets, somewhat like the Morse index of an elementary critical point, can be defined; the index remains the same for continuations of the invariant set. Properties this index has and does not have are discussed along with an application to a (known) theorem arising in the study of shock waves. (Received January 27, 1971.)

ABSTRACTS PRESENTED TO THE SOCIETY

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Algebra & Theory of Numbers

71T-A14. AUGUSTO H. ORTIZ, University of Puerto Rico, Mayaguez, Puerto Rico 00708. A construction in general radical theory.

Given an arbitrary associative ring R let R[x] be the ring of polynomials over R in the commutative indeterminate x. For each radical property S define the function S^* which assigns to each ring R the ideal $S^*(R) = S(R[x]) \cap R$ of R. It is shown that the property S_A that a ring R be equal to $S^*(R)$ is a radical property. If S is semiprime, then S_A is semiprime also. If S is a special radical, then S_A is a special radical. S_A is properly contained in S. A necessary and sufficient condition that S and S_A coincide is given. The results are generalized to include extensions of R other than R[x]. One such extension is the semigroup ring R[A], where A is a semigroup with an identity adjoined. Hence one may consider polynomial rings in several indeterminates which need not commute with each other. (Received October 30, 1970.)

71T-A15. ROBERT L. HEMMINGER, Vanderbilt University, Nashville, Tennessee 37203. Whitney's line graph theorem for multigraphs. Preliminary report.

Let H be the class of all connected graphs other than triangles. Then it is well known that (1) two elements of H are isomorphic if and only if their line graphs are isomorphic, and (2) the line graph of a connected graph is isomorphic to the line graph of some element of H. Whitney ["Congruent graphs and the connectivity of graphs," Amer. J. Math. 54(1932), 150-168] proved the finite version and Jung ["Zu einem Isomorphiesatz von Whitney fur Graphen," Math. Ann. 164(1966), 270-271] proved the infinite version. Sabidussi ["Graph derivatives," Math. Z. 76(1961), 385-401] has dealt with the multigraph version by showing that the class M of connected multigraphs that have no terminal lines and no terminal multitriangles, has property (1). In this paper we show that the class N of connected multigraphs that have no multiple terminal lines and no terminal multitriangles, has property (2). The class N does not have property (1) but we describe a natural subclass of N that has both properties (1) and (2) and thus obtain the appropriate generalization of the Whitney-Jung Theorem. The same class works for the pseudograph (loops allowed) version since the line graph function fails to distinguish between loops and terminal lines. (Received November 2, 1970.)

71T-A16. D. JAMES SAMUELSON, University of Hawaii, Honolulu, Hawaii 96822. Some clusters of semiprimal algebras.

In this paper we enlarge the primal clusters of O'Keefe ["Independence in the small among universal algebras," Math. Ann. 154(1964)] to the more embracing semiprimal case. A subalgebra preserving mapping $f: A^n \to A$ of an algebra A is said to be surjective if for each $a \in A$ there exist elements $a_1, \ldots, a_n \in \mathrm{Span}(a)$ such that $f(a_1, \ldots, a_n) = a$; f is strongly surjective if it is surjective and if for each $a \in A$ not contained in a minimal subalgebra of A the $a_1, \ldots, a_n$ above can further be chosen to satisfy the "normalized" conditions $\mathrm{Span}(a_i) = \mathrm{Span}(a)$ for at most one $i = 1, \ldots, n$. A is strongly surjective if each operation of A is strongly surjective. Two algebras A and B of the same type have nonisomorphic structures if $A' \subseteq A$ and $B' \subseteq B$ implies $A' \not\cong B'$. Theorem. A family K of semiprimal algebras is a cluster if (a) each member of K is strongly surjective, (b) the members of K have pairwise nonisomorphic structures, and (c) for each $A \in K$, the intersection of all subalgebras of A is nonempty. (Received November 2, 1970.)

71T-A17. JAMES W. BREWER and EDGAR A. RUTTER, JR., University of Kansas, Lawrence, Kansas 66044. Descent for flatness. Preliminary report.

Let R and S be commutative rings with $R \subseteq S$. Following Ferrand in [C. R. Acad. Sci. Paris 269(1969), 946-949], we shall say that R has descent with respect to S provided the following condition is satisfied: If E is an R-module such that $E \otimes_R S$ is S-flat, then E is R-flat. We say that R has descent on finitely generated modules with respect to S provided the above condition holds for each finitely generated R-module E. Theorem 1. The following are equivalent: (1) R has descent on finitely generated modules with respect to S. (2) If E is a cyclic module such that $E \otimes_R S$ is S-flat, then E is R-flat. (3) If A is an ideal of R such that S/AS is S-flat, then R/A is R-flat. Corollary 1. If S is integral over R, then R has descent on finitely generated modules with respect to S. Theorem 2. If R has descent with respect to S, then each prime ideal of R is a contracted ideal of S. Theorem 3. If S is a pure R-module (see [P. Cohn, Math. Z. 71(1959), 380-398]), then R has descent with respect to S. (Received November 2, 1970.)

71T-A18. ANDRÉ JOYAL, University of Montreal, Montreal, Quebec, Canada. Spectral spaces and distributive lattices. Preliminary report.

A spectral space is a topological space X satisfying the following conditions: (1) The quasi-compact open subsets of X form a base closed under finite (possibly empty) intersections. (2) Every irreducible closed subset F possesses one and only one generic point x (i.e., $\overline{\{x\}} = F$). A morphism $X \xrightarrow{f} Y$ of spectral spaces is a quasi-compact continuous mapping (i.e., such that the inverse image of a quasi-compact open subset of Y is a quasi-compact open subset of X). The underlying space of a quasi-compact quasi-separated scheme is a spectral space. Given a spectral space X, the set $D(X)$ of all quasi-compact open subsets of X is a distributive lattice with 0 and 1. We obtain in this way a contravariant functor D from the category S_p of spectral spaces to the category of distributive lattices. Theorem. The functor D yields an equivalence of categories. Theorem. S_p is a complete category and the forgetting functor into the category of all topological spaces and continuous maps preserves left limits. Theorem. A topological space X is a spectral space if and only if X is a projective limit of finite T_0-spaces. (Received November 4, 1970.)

71T-A19. N. VANAJA, Madurai University, Madurai-2, Tamilnadu, India. On a conjecture of Azumaya. Preliminary report.

A left R-module Q is called ${}_RM$-projective if Hom(Q, -) is exact on every exact sequence $0 \to {}_RL \to {}_RM \to {}_RN \to 0$. Let $C^p(Q)$ denote the class of all modules ${}_RM$ such that Q is M-projective. G. Azumaya ["M-projectives and M-injectives," Trans. Amer. Math. Soc. (to appear)] conjectured that if $C^p(Q)$ is closed under formation of infinite direct products then Q has a projective cover. An example is constructed to show that this conjecture is not true in general. However, if the ring R is Jacobson semisimple, then the conjecture is shown to be true. We also show that if the ring R is noetherian then $C^p(Q)$ is closed under formation of infinite direct products for every R-module Q if and only if R is perfect. Further properties of $C^p(Q)$ are investigated. (Received November 6, 1970.) (Author introduced by Dr. M. Rajagopalan.)

71T-A20. K. M. RANGASWAMY, Madurai University, Madurai-2, Tamilnadu, India. Generalisation of a theorem of G. Nöebling.

An important recent result of Nöebling [Invent. Math. 6(1968), 41-55] states that the additive group of all bounded integer valued functions on any set X is a free abelian group. The following generalisation is obtained. Theorem. Let X be a nonempty set and R any ring with identity. Then the left module F of all finite valued functions from X to R is free as an R-module possessing a basis $\{\epsilon_i\}$ of characteristic functions. Following Nöebling, a submodule S of F is called Specker if, for each f in S, S contains all the ϵ_i which appear in any representation of f as a linear combination of characteristic functions. Corollary. Every specker R-module is free. (Received November 6, 1970.) (Author introduced by Dr. M. Rajagopalan.)

71T-A21. CHARLES C. LINDNER, Auburn University, Auburn, Alabama 36830. Finite embedding theorems for partial latin squares, quasigroups, and loops.

In this paper we prove that a finite partial commutative (idempotent commutative) latin square can be embedded in a finite commutative (idempotent commutative) latin square. These results are then used to show that the loop varieties defined by any nonempty subset of the identities $\{x(xy) = y, (yx)x = y\}$ and the quasigroup varieties defined by any nonempty subset of $\{x^2 = x, x(xy) = y, (yx)x = y\}$, except possibly $\{x(xy) = y, (yx)x = y\}$, have the strong finite embeddability property. It is then shown that the finitely presented algebras in these varieties are residually finite, hopfian, and have a solvable word problem. (Received November 10, 1970.)

71T-A22. MICHAEL RICH, Temple University, Philadelphia, Pennsylvania 19122. Associo-symmetric algebras of degree two.

An algebra A over a field F is called an associo-symmetric algebra if there is a map $g : A \times A \times A \to F$ such that $(xy)z = g(x, y, z)x(yz)$ for all x, y, z in A, and if $g(x, y, z) = g(x\pi, y\pi, z\pi)$ for all π in S_3. We have previously announced

(Abstract 70T-A191, these *Notices* 17(1970), 942) that a finite dimensional associo-symmetric algebra of degree > 2 or degree $= 1$ over a field of characteristics $\neq 2$ is associative. We are now able to extend this result to the degree 2 case also. Thus, the following arises: Theorem. If A is a finite dimensional semisimple associo-symmetric algebra over a field of characteristic $\neq 2$ then A is associative. (Received November 13, 1970.)

71T-A23. SELMER O. MOEN, University of Minnesota, Minneapolis, Minnesota 55455. Free derivation modules and a criterion for regularity. Preliminary report.

Let R be a projective complete intersection over an algebraically closed field k of characteristic zero; i.e., R is a quotient of a polynomial ring over k by an ideal generated by an R-sequence of forms. Let D^* be the module of k-derivations of R into itself. Theorem. Suppose D^* is free. Then R is regular. (Received November 16, 1970.)

71T-A24. E. FRIED and H. LAKSER, University of Manitoba, Winnipeg 19, Manitoba, Canada. Simple tournaments.

A tournament is a set T with a binary relation $\leq$ such that (1) $\leq$ is reflexive, (2) $\leq$ is antisymmetric, (3) given $a, b \in T$, either $a \leq b$ or $b \leq a$. If T, T' are tournaments, $\varphi: T \to T'$ is a homomorphism if $a \leq b$ implies $a\varphi \leq b\varphi$. A tournament T is simple if, for all tournaments T' and all homomorphisms $\varphi: T \to T'$ onto T', either φ is an isomorphism or $|T'| = 1$. A tournament T is strong if, given any $a, b \in T$ there are $a_1, \ldots, a_m \in T$ such that $a \leq a_1 \leq \ldots \leq a_m \leq b$. Theorem 1. Any tournament of cardinality n is a subtournament of a simple tournament of cardinality $2n+1$. Theorem 2. Any strong tournament of cardinality n is a subtournament of a simple tournament of cardinality 2n. Theorem 3. Any tournament that is not strong and has cardinality n is a subtournament of a simple tournament of cardinality $2n+2$. Corollary. There is a simple tournament of cardinality n iff $n \neq 4$. Theorem 4. If n is finite the number of nonisomorphic simple tournaments of cardinality 2n is at least the number of nonisomorphic strong tournaments of cardinality n. A similar statement holds for simple tournaments of finite odd cardinality. (Received November 13, 1970.) (Authors introduced by Professor George A. Grätzer.)

71T-A25. C. Y. TANG, University of Waterloo, Waterloo, Ontario, Canada. On the Frattini subgroups of generalized free products. Preliminary report.

Let $G = (\Pi_{i \in I}{}^* A_i)_H$, I an index set, be the generalized free products of the groups A_i amalgamating the subgroup H. Let $\Phi(G)$ be the Frattini subgroup of G. The following theorems are proved. Theorem 1. If $\Phi(G) \cap H = 1$ then $\Phi(G) = 1$. Theorem 2. Let A_i, $i \in I$, be finitely generated free groups. If H is finitely generated and N is the maximal G-normal subgroup contained in H then $\Phi(G) \subseteq N$. In particular if H is of infinite index in one A_i then $\Phi(G) = 1$. (Received November 6, 1970.)

71T-A26. SIN-MIN LEE, University of Manitoba, Winnipeg 19, Manitoba, Canada. On pseudo-rings which are disjoint unions of rings.

After O. Frink, we call a semiring $\langle R, +, \cdot \rangle$ a pseudo-ring if it satisfies the identity $a+b+c+d=a+c+b+d$. On this note we give an equational characterization for those pseudo-rings which can be expressed as the disjoint union of rings. We also determine the lattice of those equational subclasses of the above equational class which contains the equational class of associative rings. It is an 8 element Boolean lattice. (Received November 19, 1970.) (Author introduced by Professor George A. Grätzer.)

71T-A27. HIROSHI UEHARA, Oklahoma State University, Stillwater, Oklahoma 74074, FRANKLIN S. BRENNEMAN, Lehigh University, Bethlehem, Pennsylvania 18015 and LLOYD D. OLSON, North Dakota State University, Fargo, North Dakota 58102. A comparison theorem in fibred categories.

The following comparison theorem is a continuation of the study begun by the first two authors in "On a cotriple homology in a fibred category" (Publ. Res. Inst. Math. Sci. Ser. A (1968)). Let $(\mathfrak{X}, \mathfrak{B}, P)$ be a fibred category, where $P: \mathfrak{X} \to \mathfrak{B}$ is a functor from the category $\mathfrak{X}$ onto the category $\mathfrak{B}$. Let (G, ϵ, Δ) be a cotriple on $\mathfrak{X}$ and $(G_B, \epsilon_B, \Delta_B)$ the induced cotriple on the fibre $\mathfrak{X}_B$, for each B in $\mathfrak{B}$. We assume each $\mathfrak{X}_B$ is preadditive and G_B is a pointed functor. Let $\mathfrak{G}_{G_B}$ denote the projective class in $\mathfrak{X}_B$ determined by the cotriple $(G_B, \epsilon_B, \Delta_B)$. Theorem. If $(\mathfrak{Y}, \partial^{\mathfrak{Y}})$ and $(\mathfrak{Z}, \partial^{\mathfrak{Z}})$ are $\mathfrak{G}_{G_B}$ - and $\mathfrak{G}_{G_{B'}}$-projective resolutions of X and X' respectively, $P(X) = B$, $P(X') = B'$, and if $f: X \to X'$ is a morphism in $\mathfrak{X}$ such that $P(f) = \alpha: B \to B'$, then there exists an α-chain transformation $F: \mathfrak{Y} \to \mathfrak{Z}$ such that $\eta' \circ F_0 = f \circ \eta$ $(\eta: \mathfrak{Y}_0 \to X,\ \eta': \mathfrak{Z}_0 \to X')$ and any two such α-chain transformations are α-chain homotopic. An immediate consequence of this theorem is the existence of the cup-i-products, introduced by A. Zachairou, (preprints, Matematisk Institut, Aarhus Universitet, Denmark, 1969-1970), in the cobar construction $F(A^*)$ where A is a cocommutative Z_2-Hopf algebra. A second consequence is the existence of squaring operations in $\mathrm{Ext}_A^{**}(M, Z_2)$ where A is a cocommutative Z_2-Hopf algebra and M is a cocommutative A-module coalgebra. (Received November 19, 1970.)

71T-A28. ANDRZEJ EHRENFEUCHT, University of Southern California, Los Angeles, California 90007 and VANCE FABER, University of Colorado, Boulder, Colorado 80302. Do infinite nilpotent groups have equipotent abelian subgroups? Preliminary report.

Here, equipotent means of the same cardinality. We prove the following theorems concerning large abelian subgroups of nilpotent groups (see also V. Faber, "Large abelian subgroups of some infinite groups", Rocky Mount. J. Math., to appear, and Abstract 674-33, these *Notices* 17(1970), 530). Let $\mathfrak{m}$ be an infinite cardinal. Theorem 1. The following statements are equivalent: (A_1) Every periodic nilpotent group of cardinality $> \mathfrak{m}$ has an abelian subgroup of cardinality $> \mathfrak{m}$; (A_2) For every prime p, if V is an F_p vector space of dimension $> \mathfrak{m}$, if W is an F_p vector space of dimension $\leq \mathfrak{m}$, and if $\rho: V \to W$ is an alternating bilinear function, then ρ is degenerate on some subspace of dimension $> \mathfrak{m}$. Theorem 2. If one assumes the generalized continuum hypothesis, then for every finite field F and every cardinal $\aleph_\alpha$ there exists a vector space V over F of dimension $\aleph_{\alpha+1}$ and an alternating bilinear form ρ on V such that every ρ-degenerate subspace of V has dimension $\leq \aleph_\alpha$. In this case, (A_1) and (A_2) are both false for every $\mathfrak{m}$. (Received December 3, 1970.)

71T-A29. LAL M. CHAWLA, Kansas State University, Manhattan, Kansas 66502. An isomorphism theorem on universal algebras.

Let A be a nonempty set. Then any subset of $A^n = A \times A \times \ldots \times A$, n-times, is an n-ary relation on A. Let S be a nonempty set of n-ary relations. Any universal algebra (S, Ω) with carrier S and a system Ω of finitary operations in S is called here a universal algebra of relations. In this paper, we prove the following theorem concerning a class of universal algebras. This is a direct generalization of our previous result on groups, (L. M. Chawla, "An isomorphism theorem on groups", J. Natur. Sci. and Math. 1(1961), 110-112). Theorem. Any universal algebra (A, Ω) having at least one finitary algebraic operation $f_n(a_1, \ldots, a_n)$ which is a mapping of A^n onto A is isomorphic to a universal algebra of relations. (Received November 9, 1970.)

71T-A30. KHEE-MENG KOH, University of Manitoba, Winnipeg 19, Manitoba, Canada. Characterizations of distributive and modular lattices.

For notations and definitions of the representability of $\langle p_n \rangle$ sequence, see Abstract 69T-A54, these *Notices* 16(1969), 565. Let $\underset{\sim}{K}$ be the class of algebras of type $\langle 2,2 \rangle$; $\underset{\sim}{L}$ be the class of lattices. Then Lemma. If the sequence $\langle 0,0,2,m \rangle$ is representable in $\underset{\sim}{K}$ then $m \geq 9$. Theorem 1. Let $\mathfrak{A}$ be an algebra in $\underset{\sim}{K}$. Then $\mathfrak{A}$ represents $\langle 0,0,2,9 \rangle$ if and only if $\mathfrak{A}$ is a distributive lattice. Theorem 2. Let $\mathfrak{A} \in \underset{\sim}{L}$. Then $\mathfrak{A}$ represents $\langle 0,0,2,19 \rangle$ if and only if $\mathfrak{A}$ is modular. Following G. Grätzer, the sequence $\langle p_0, \ldots, p_n \rangle$ of cardinals is said to have the Minimal Extension Property (M.E.P.) with respect to the class $\underset{\sim}{C}$ if the following conditions hold: (1) There exists an algebra $\mathfrak{A}$ in $\underset{\sim}{C}$ such that $p_k(\mathfrak{A}) = p_k$ for $0 \leqq k \leqq n$. (2) If B is an algebra in $\underset{\sim}{C}$ satisfying $p_k(B) = p_k$ for $0 \leqq k \leqq n$ then $p_k(\mathfrak{A}) \leqq p_k(B)$ for each $k = 0, 1, 2, \ldots$. Corollary 1. The sequence $\langle 0,0,2,9 \rangle$ has the M.E.P. with respect to the class K. Corollary 2. The sequence $\langle 0,0,2,19 \rangle$ has the M.E.P. with respect to the class $\underset{\sim}{L}$. (Received November 5, 1970.) (Author introduced by Professor George A. Grätzer.)

71T-A31. JOHN McKAY, California Institute of Technology, Pasadena, California 91109. A new invariant for finite simple groups.

Examination of the number $m_2(G)$ of odd degree irreducible characters of various finite groups G reveals: Theorem. For the known simple groups of order $< 10^6$, for the known sporadic groups (possibly excepting F_{23}, F'_{24}), and for a simple group G from $L_2(q)$, ${}^2G_2(q)$, $Sz(q)$, and $Sp_4(q)$ (q odd), $m_2(G)$ is a power of two. This observation is not true in general (e.g. $m_2(L_3(11)) = 20$) but examination suggests Conjecture A. The number of irreducible characters of odd degree in a 2-block of full defect is a power of two; and Conjecture B. $m_2(G) = m_2(N_G(S_2))$ where S_2 is a Sylow 2-group of G. (Received November 19, 1970.) (Author introduced by Professor David B. Wales.)

71T-A32. ARTHUR M. HOBBS, 93-J Milford Street, Waterloo, Ontario, Canada and University of Waterloo, Waterloo, Ontario, Canada. Some Hamiltonian results in the square of a graph.

The square G^2 of graph G is the graph with vertex set V(G) in which 2 distinct vertices u, v are adjacent iff $d(u,v) \leqq 2$ in G. Conjecture 1. If G is a block, G^2 is Hamiltonian. Conjecture 2. If G is a block,

G^2 is Hamiltonian connected. Let $v_1, \ldots, v_k$, and u be distinct vertices. Form a rosette by joining each v_i to u by 2 or more suspended arcs. Theorem. If G is a rosette, then G^2 is Hamiltonian connected. Using this theorem and a catalog of all minimal blocks G with $|V(G)| \leq 10$, Conjecture 2 is verified for all of these blocks. Theorem. If G has minimum valency 2, G^2 contains a 2-factor. V_2 is the set of valency 2 vertices of G. Theorem. If G is Eulerian and $G - V_2$ is a forest, then G^2 is Hamiltonian. Not every Eulerian graph has a Hamiltonian square. Corollary. If G is a minimal block and Eulerian, then G^2 is Hamiltonian. A node is a vertex whose valency exceeds 2. Theorem. If G is connected and has no 2 adjacent nodes and at most 2 odd vertices, then G^2 is Hamiltonian. Theorem. If G is a block with at most 6 odd vertices and no 2 adjacent nodes, then G^2 is Hamiltonian. Let c(G) be the connectivity of G and suppose $|V(G)| = n$. Theorem. If k is a positive integer, then $c(G^k) \geq \min(n-1, kc(G))$. Corollary. If $c(G) \geq \frac{1}{4}n$, G^2 is Hamiltonian, and if $c(G) \geq \frac{1}{4}(n+1)$, G^2 is Hamiltonian connected. (Received November 23, 1970.) (Author introduced by Professor W. T. Tutte.)

71T-A33. V.P. CAMILLO and KENT R. FULLER, University of Iowa, Iowa City, Iowa 52240. Balanced and QF-1 algebras. Preliminary report.

A ring is QF-1 if every finitely generated faithful module has the double centralizer property. A ring is balanced if every homomorphic image is QF-1. Two theorems are proved. Let R be a finite dimensional algebra, then R is balanced if and only if every homomorphic image of R is quasi-Frobenius. If R is primary decomposable then R is QF-1 if and only if R is QF. (Received November 23, 1970.)

71T-A34. T. L. GOULDING, University of Florida, Gainesville, Florida 32601 and AUGUSTO H. ORTIZ, University of Puerto Rico, Mayaguez, Puerto Rico 00708. Structure of semiprime (p,q) radicals.

In this note, the structure of the semiprime (p, q) radicals is investigated. Let p(x) and q(x) be polynomials over the integers. An element a of an arbitrary associative ring R is called (p,q)-regular if $a \in p(a) \cdot R \cdot q(a)$. A ring R is (p, q)-regular if every element of R is (p, q)-regular. It is easy to prove that (p, q)-regularity is a radical property and also that it is a semiprime radical property (meaning that the radical of a ring is a semiprime ideal of the ring) if and only if the constant coefficients of p(x) and q(x) are ± 1. It is shown that every (p, q)-semisimple ring is isomorphic to a subdirect sum of rings which are either right primitive or left primitive. (Received November 23, 1970.)

71T-A35. JAMES R. WALL, University of Tennessee, Knoxville, Tennessee 37916. The semigroup of doubly stochastic matrices. Preliminary report.

Let D_n denote the semigroup of all $n \times n$ doubly stochastic matrices, and let (1/n) be the member of D_n with each entry equal to 1/n. Let $\mathcal{R}, \mathcal{L}, \mathcal{H}$, and $\mathcal{D}$ denote Green's relations on D_n. Theorem 1. Let $A, X \in D_n$. If $AX = A$ and A is irreducible, then either $A = (1/n)$ or else X is the $n \times n$ identity matrix. Theorem 2. For $A, B \in D_n$, $B \mathcal{R} A$ if and only if there exists an $n \times n$ permutation matrix P such that $B = AP$. The other Green's relations on D_n are characterized similarly. Known results concerning maximal subgroups of D_n, semigroup inverses, and the Moore-Penrose inverse follow as corollaries. (Received November 25, 1970.)

71T-A36. DRAGOMIR Ž. DJOKOVIĆ, University of Waterloo, Waterloo, Ontario, Canada. On the similarity between a linear transformation and its adjoint.

Let K be a field and $\alpha \to \bar{\alpha}$ an automorphism of K such that $\bar{\bar{\alpha}} = \alpha$ for all $\alpha \in K$. Let A be an n-square matrix over K and $A^* = \bar{A}'$ its conjugate transpose. Theorem. If A^* is similar to A then they are similar via a hermitian matrix, i.e., $A^* = H^{-1}AH$ where H is a nonsingular matrix such that $H^* = H$. This generalizes the result of J. W. Duke (Pacific J. Math. 31(1969), 321-323). (Received November 27, 1970.)

71T-A37. ROBERT WILLIS QUACKENBUSH and J. SICHLER, University of Manitoba, Winnipeg 19, Manitoba, Canada. Automorphism groups of commutative rings.

Theorem 1. For every group G there is an associative commutative ring R such that the group $\mathcal{A}(R)$ of all automorphisms of R is isomorphic to G. For a finite G a finitely generated R can be taken. Theorem 2. Let G be a group, card(G) = γ. Let ξ be an infinite cardinal number, $\xi \geqq \gamma$. Then there is an associative commutative ring R_ξ such that (i) card$(R_\xi) = \xi$, (ii) $\mathcal{A}(R_\xi) \cong G$. Corollary. For every group G there is a proper class of pairwise nonisomorphic rings R_α with $\mathcal{A}(R_\alpha) \cong G$. (Received November 27, 1970.)

71T-A38. C. J. EVERETT, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87544. Limit of nth power of the Wallis product.

From the well-known relation $n! = \sqrt{2\pi n}(n/e)^n \exp\theta_n/12n$, $0 < \theta_n < 1$, the formula $W_n = ((2n)!/n!^2 2^{2n})^2 \pi n$ for the Wallis product, and the inequality $1/\exp(1/4n) < W_n < 1/\exp(1/4n - 1/96n^3)$, it follows at once that $\lim (W_n)^n = \exp(-1/4)$. (Cf. "Inequalities for the Wallis product," Math. Mag. 43(1970), 30-33.) (Received December 7, 1970.)

71T-A39. ALBERT A. MULLIN, USARV HQ, USAICCV (LDSC), APO, San Francisco 96384. On an analogue of powerful numbers. Preliminary report.

By analogy with S. W. Golomb's notion of a powerful number (Amer. Math. Monthly 77(1970)) define an entropic number as follows: it is a natural number $\underline{n}$ such that if a prime appears in the mosaic of $\underline{n}$ then it appears at least twice in the mosaic of $\underline{n}$. Lemma 1. There exist infinitely many entropic numbers which are not powerful; e.g., 768 and 13,122. On the other hand, Lemma 2. Almost all entropic numbers are powerful. Let $\underline{E}(\underline{n})$ be the number of natural numbers $\leq \underline{n}$ which are entropic and let $\underline{P}(\underline{n})$ be the number of natural numbers $\leq \underline{n}$ which are powerful. Lemma 3. $\underline{E}(\underline{n}) \leq \underline{P}(\underline{n})$ for every natural number $\underline{n}$. Problems. Give useful estimates of $\underline{E}(\underline{n})$ and $\lim_{\underline{n}\to\infty} \underline{E}(\underline{n})/\underline{n}$, respectively. Questions. Do such estimates provide strong lower estimates of $\underline{P}(\underline{n})$ and $\lim_{\underline{n}\to\infty} \underline{P}(\underline{n})/\underline{n}$, respectively? (Received December 7, 1970.)

71T-A40. KENG TEH TAN, Queen's University, Kingston, Ontario, Canada. Some group-rings which are orders in artinian rings. Preliminary report.

Let Q(R) denote the left total quotient ring of a ring R. Theorem 1. If R is an order in an artinian ring and G is a free abelian group of finite rank, then the group ring RG is also an order in an artinian ring.

Corollary 2. If R is semiprime left Goldie and G as above, then RG is semiprime left Goldie. Theorem 3. If R is semiprime left Goldie and G an extension of a polycyclic group by a finite group, then Q(RG) exists and is artinian. This improves Theorem 3 of the author, see Abstract 70T-A171, these *Notices* 17(1970), 813. (Received December 7, 1970.) (Author introduced by Dr. Anthony Vito Geramita.)

71T-A41. CARL FAITH, Rutgers University, New Brunswick, New Jersey 08903. Big decompositions of modules. Preliminary report.

The category mod-R of right R-modules over a ring R is said to have a basis if there is a set $\{X_i\}_{i\in I}$ of objects such that every object M of mod-R is isomorphic to a direct sum of modules each of which is isomorphic to some X_j, with $j \in I$. A short time after the appearance of a 1967 J. Algebra paper with E. A. Walker, the author observed the Theorem. If mod-R has a basis, then R is right artinian. For, then R is right noetherian by loc. cit. Moreover, a theorem of S. Chase [Trans. Amer. Math. Soc. 97(1960), 457-473] (cf. also Chase, Pacific J. Math. 12(1962), 847-854) implies that mod-R has a basis only if R is right perfect. Then, by a theorem of Bass [Trans. Amer. Math. Soc. 95(1960), 466-488], R has nilpotent radical, and it follows from theorems of Levitzki (Compositio Math. 7(1939), 214-222, and Amer. J. Math. (1945), 437-442) and Hopkins [Ann. of Math. (2) 40(1939), 712-730] that R is right artinian. (Received December 10, 1970.)

71T-A42. BJARNI JONSSON, Vanderbilt University, Nashville, Tennessee 37203. The amalgamation property in varieties of modular lattices. Preliminary report.

A class K of algebras is said to have the amalgamation property provided, for any $A, B, C \in K$ and for any monomorphisms $f: A \to B$ and $g: A \to C$, there exist $D \in K$ and monomorphisms $f': B \to D$ and $g': C \to D$ such that $f'f = g'g$. Theorem. The class of all modular lattices does not have the amalgamation property. More generally, if a variety K of modular lattices contains the lattice of all subspaces of some projective plane in which Desargues' law fails, then K does not have the amalgamation property. (Received December 14, 1970.)

71T-A43. LEON J. OSTERWEIL, University of Maryland, College Park, Maryland 20742. An enumeration theorem and some applications to graph enumeration. Preliminary report.

Let $\{n_1, n_2, \ldots, n_t\}$ be a finite sequence of positive integers. Let p(x) be a counting polynomial for a set of figures. Form a collection of these figures by taking n_1 isomorphic copies of one figure in the set, together with n_2 isomorphic copies of a different figure in the set, together with n_3 isomorphic copies of a still different figure in the set, ... Denote the counting polynomial for all possible collections of this type by $N(n_1, n_2, \ldots, n_t)$. If $c_r = |\{i | n_i = r\}|$, then Theorem. $N(n_1, n_2, \ldots, n_t) = (c_1!c_2!\ldots c_t!)^{-1} \sum_P \prod_{B\in P} (-1)^{|B|-1}(|B|-1)!\, p(\prod_{i\in B} x^{n_i})$, where the summation is over all partitions, P, of $\{1, 2, \ldots, t\}$, and the product is over all blocks, B, of a given partition, P. This theorem is applied directly to the enumeration of unlabelled graphs having t equivalence classes of components, each class having n_i connected components which are isomorphic to each other. Definition. A p-gon tree is a connected graph each of whose lines is contained in exactly one p-gon, but in no other circuit. If p is an odd prime, this theorem can be used to enumerate p-gon trees having maximal condition on their point orders. (Such graphs have been enumerated by Norman: see R. Z. Norman, Ph.D. thesis, University of Michigan, 1954.) (Received December 14, 1970.)

71T-A44. WITHDRAWN.

71T-A45. SABAH FAKIR, Dalhousie University, Halifax, Nova Scotia, Canada. On pure-injective objects in Grothendieck categories. Preliminary report.

All terms are defined in (Stenström, "Purity in functor categories," J. Algebra 8(1968), 352-361). In a Grothendieck category, a monomorphism $u: A \to B$, is called: (1) a pure-essential extension, if it is pure and if for every morphism $v: B \to C$, $v \circ u$ is a pure monomorphism implies v is a monomorphism; (2) a maximal pure-essential extension, if it is pure-essential extension and if for every morphism $v: B \to C$, $v \circ u$ is pure-essential extension implies v is an isomorphism. Theorem. Let G and H be two locally finitely presented Grothendieck categories (in short l.f.p.g.). Let $T: G \to H$ be a full and faithful functor, preserving direct colimits, and having a left adjoint. Then T preserves and reflects pure monomorphisms, pure-essential extensions, and reflects maximal pure-essential extensions. Furthermore, if H has enough pure-injective objects, then the same holds for G. Proposition. Let G be a l.f.p.g. category. There exists an additive small category P, and a functor $T: G \to (P, Ab)$, satisfying the conditions of the theorem. Corollary. Any l.f.p.g. category has enough pure-injective objects, and has pure-injective envelopes. (Received December 17, 1970.) (Author introduced by Professor F. William Lawvere.)

71T-A46. PETER BURMEISTER, Mathematisches Institut der Universität Bonn, 53 Bonn, Federal Republic of Germany. On Problem 12 in G. Grätzer's book "Universal algebra".

Let $\underline{A} := (A, (f_i)_{i \in I})$ be a partial algebra of some finitary type $(k_i)_{i \in I}$. A congruence relation R of $\underline{A}$ is called closed or strong, if, for all $i \in I$, $(a_\varkappa \mid \varkappa \in k_i) \in \operatorname{def}(f_i)$ and $(a_\varkappa, b_\varkappa) \in R$ for all $\varkappa \in k_i$ always imply $(b_\varkappa \mid \varkappa \in k_i) \in \operatorname{def}(f_i)$. Theorem. For a given pair $(\underline{L}_1, \underline{L}_2)$ of lattices there exists a finitary partial algebra $\underline{A}$ such that $\underline{L}_1$ and $\underline{L}_2$ are isomorphic to the lattices $\Theta(\underline{A})$ of all congruence relations and $\Theta_s(\underline{A})$ of all strong congruence relations of $\underline{A}$ respectively, if and only if $\underline{L}_1$ is an algebraic lattice and $\underline{L}_2$ is isomorphic to a principal ideal of $\underline{L}_1$. This solves Problem 12 in G. Grätzer's book "Universal algebra" [van Nostrand, Princeton, N.J., 1968]. The proof uses the fact that every algebraic lattice is isomorphic to the congruence lattice of some full finitary algebra [cf. G. Grätzer and E. T. Schmidt, Acta Sci. Math. (Szeged) 24(1963)], and the following Lemma. If (A, f) is a full algebra, if $R \in \Theta((A, F))$ and $g := (\mathrm{id}_C)_{C \in A/R}$, then $\Theta((A, f \cup g)) = \Theta((A, f))$, and $\Theta_s((A, f \cup g))$ is the principal ideal of $\Theta((A, f))$ which is generated by R. (Received December 17, 1970.)

71T-A47. E. FRIED and GEORGE A. GRÄTZER, University of Manitoba, Winnipeg 19, Manitoba, Canada. A nonassociative extension of the class of distributive lattices. I.

Let us turn the three element tournament $\langle\{0, 1, 2\}; <\rangle$, $0<1<2<0$, into an algebra $\mathfrak{T}=\langle\{0, 1, 2\}; \wedge, \vee\rangle$ by $x \vee y = y$ and $x \wedge y = x$ for $x<y$, furthermore, $\wedge$ and $\vee$ are idempotent and commutative. Let $\underset{\sim}{T}$ denote the equational class generated by $\mathfrak{T}$. Note that $\underset{\sim}{T}$ is a proper extension of the class $\underset{\sim}{D}$ of distributive lattices, and neither of the two operations satisfy the associative identity. It is the purpose of our investigation to show that many of the most important properties of $\underset{\sim}{D}$ generalize to $\underset{\sim}{T}$. Theorem 1. For $\mathfrak{A} \in \underset{\sim}{T}$, $a, b, c \in A$, $a \vee b = a \vee c$ and $a \wedge b = a \wedge c$ imply $b = c$. Theorem 2. For $\mathfrak{A} \in \underset{\sim}{T}$, $a, b, c, d \in A$, $a \leqq b$ and $c \leqq d$ ($x \leqq y$ means that $x \wedge y = x$), $c \equiv d(\Theta(a, b))$ iff $a \wedge (c \wedge b) = a \wedge (d \wedge b)$ and $(a \vee c) \vee b = (a \vee d) \vee b$, which in turn is equivalent to $(((a \vee e_1) \vee e_2) \wedge e_3) \wedge e_4 = c$ and $(((b \vee e_1) \vee e_2) \wedge e_3) \wedge e_4 = d$, where $e_1 = a \wedge d$, $e_2 = a \vee c$, $e_3 = (b \vee d) \vee c$, $e_4 = d$. Using a result of A. Day, Theorem 2 implies Theorem 3. $\underset{\sim}{T}$ has the Congruence Extension Property (see G. Grätzer and H. Lakser, Abstract 70T-A66, these *Notices* 17(1970), 558). (Received December 21, 1970.)

71T-A48. ROBERT D. FRAY and ROBERT GILMER, Florida State University, Tallahassee, Florida 32306. On solvability by radicals of finite fields. Preliminary report.

Using the definition of solvability by radicals given by van der Waerden, Robert Gilmer has shown (see Abstract 70T-A212, these *Notices* 17(1970), 945) that the problem of determining when the field K is solvable by radicals over F, where K is a finite-dimensional, normal, separable, extension of the field F of characteristic $p \neq 0$, can be reduced to the case when K and F are finite fields. This paper investigates this problem when K and F are finite. In particular it is proved that there are infinitely many primes q for which $GF(p^{nq})$ is not solvable by radicals over $GF(p^n)$, and, if $p^n > 2$, there are infinitely many primes q for which $GF(p^{nq})$ is solvable by radicals over $GF(p^n)$. Also it is shown that for any field $GF(p^n)$ there exists a smallest subfield over which $GF(p^n)$ is solvable by radicals. A theorem which is fundamental to this investigation is the following: Let p be a prime and n and t positive integers with $(t,p)=1$. The field $GF(p^{nt})$ is solvable by radicals over $GF(p^n)$ if and only if there exists a decreasing sequence of positive integers $v_1, v_2, \dots, v_r = 1$ such that $(p, v_i) = 1$ for $i = 1, 2, \dots, r$, p^n belongs to the exponent $v_1 \pmod t$, and p^n belongs to the exponent $v_{i+1} \pmod{v_i}$ for $i = 1, 2, \dots, r-1$. (Received December 28, 1970.)

71T-A49. GRANT A. FRASER, University of California, Los Angeles, California 90024. Tensor products of distributive lattices.

Let A and B be distributive lattices. The tensor product of A and B in the category $\mathcal{D}$ of distributive lattices is denoted by $A \otimes_{\mathcal{D}} B$. Suppose first that A and B are chains. Then (1) the word problem for $A \otimes_{\mathcal{D}} B$ is solvable and (2) if A_1 is a sublattice of A and B_1 is a sublattice of B then $A_1 \otimes_{\mathcal{D}} B_1$ is canonically isomorphic to a sublattice of $A \otimes_{\mathcal{D}} B$. Examples are given to show that statements (1) and (2) are false when A and B are arbitrary distributive lattices. If A and B are distributive lattices then A and B may be regarded as join-semilattices. The tensor product of A and B in the category $\mathcal{J}$ of join-semilattices is denoted by $A \otimes_{\mathcal{J}} B$. Theorem. $A \otimes_{\mathcal{J}} B$ is a distributive lattice. The word problem for $A \otimes_{\mathcal{J}} B$ is solvable. If A_1 is a sublattice of

A and B_1 is a sublattice of B then $A_1 \otimes_{\mathscr{J}} B_1$ is canonically isomorphic to a sublattice of $A \otimes_{\mathscr{J}} B$. Let S(A) denote the Stone space of A and let $S'(A) = S(A) \cup \{A\}$. Theorem. $S'(A \otimes_{\mathscr{J}} B)$ is homeomorphic to $S'(A) \times S'(B)$. Let $\mathscr{D}_1$ be the category of distributive lattices with largest element 1 (and homomorphisms preserving 1). Theorem. If $A, B \in \mathscr{D}_1$ then $A \otimes_{\mathscr{J}} B$ is isomorphic to the free product of A and B in the category $\mathscr{D}_1$. (Received December 28, 1970.)

71T-A50. JOHN J. CURRANO, Roosevelt University, Chicago, Illinois 60605. Finite p-groups with isomorphic subgroups.

Let P be a finite p-group with subgroups Q and R of index p in P. Assume that there is an isomorphism, φ, of R onto Q which fixes no nonidentity subgroup of P. Generators and relations are given for P. It is shown that the proof can be modified to give generators and relations for a finite-dimensional nilpotent Lie algebra over the field of p elements which has isomorphic subalgebras of index p, where the isomorphism fixes no nonzero subalgebra. (Received December 29, 1970.)

71T-A51. N. GANESAN, No. 1, Umayal Lane, Chidambaram, Tamil Nadu, India and Annamalai University, Tamil Nadu, India. On finite abelian groups of order n and exponent k. Preliminary report.

As the order n and exponent k of any finite group have the same prime divisors and the exponent is a divisor of the order, let $n = \Pi_{i=1}^{r} p_i^{\alpha_i}$ and $k = \Pi_{i=1}^{r} p_i^{\beta_i}$ where p_i are distinct primes and α_i, β_i are positive integers with $\alpha_i \geqq \beta_i$ for all i. Using the direct sum decomposition of a finite abelian group G of order n and exponent k into its Sylow subgroups, the following results are obtained: Theorem 1. G has at least $\varphi(k)$ and at most $n \Pi_{i=1}^{r} (1 - 1/p_i^{\delta_i})$ distinct elements each of order k where $\varphi(k)$ is the totient function and δ_i is the greatest integer contained in α_i/β_i for all i. These bounds are sharp. Theorem 2. The number of nonisomorphic abelian groups of order n and exponent k is $\Pi_{i=1}^{r} P(\alpha_i - \beta_i, \beta_i)$ where P(a, b) denotes the number of different ways of expressing the positive integer a as a sum of positive integers each not exceeding the positive integer b and $P(0, b) = 1$. Corollary. There exists a unique abelian group of order n and exponent k if n/k or k is square-free. (Received December 29, 1970.)

71T-A52. SURJEET SINGH, Aligarh Muslim University, Aligarh, (UP), India and RAVINDER KUMAR, Ramjas College, Delhi - 7, India. (KE)-domains and their generalizations.

The concept of a (KE)-domain was introduced by S. Singh in ["Principal ideals and multiplication rings. II," Abstract 70T-A221, these *Notices* 17(1970), 950]. The present paper is in continuation to the above paper. Let R be a commutative ring with unity $1 \neq 0$. For any ideal A of R, A^* denotes the subring of R generated by $A \cup \{1\}$. A ring R is said to be an (ME)-ring if A^* is a multiplication ring for each proper ideal A of R. Following are the main theorems established: (I) A domain D is a (KE)-domain iff for each ideal A of D, A^* is a Dedekind domain or a Prüfer domain or a generalized Krull domain or an almost Krull domain. (II) Let R be a noetherian ring. Then R is an (ME)-ring iff one of the following holds: (i) R is a (KE)-domain, or (ii) $R = S \oplus T$, where S is a von Neumann regular ring of finite characteristic, and T is a (KE)-domain, or (iii) R is a von Neumann regular ring, or (iv) $R = S \oplus R_1 \oplus R_2 \oplus \ldots \oplus R_t$, where S is a von Neumann regular ring of finite characteristic and $R_1, R_2, \ldots, R_t$ are finite, local (ME)-rings of orders powers of distinct primes. (Received December 30, 1970.) (Authors introduced by Professor Klaus E. Eldridge.)

71T-A53. MARK BLONDEAU HEDRICK, University of Houston, Houston, Texas 77004. Nearly reducible and nearly decomposable: special classes of irreducible and fully indecomposable matrices.

The author studies the structural properties of nearly reducible and nearly decomposable matrices. A nearly reducible (decomposable) matrix is an irreducible (a fully indecomposable) matrix which becomes reducible (partly decomposable) when any positive entry is replaced by a zero. There are three main results. In Theorem 1, the author shows that the maximal number of positive entries (arcs) in an $n \times n$ nearly reducible matrix (a minimally connected graph with n vertices) is $2(n-1)$ and the matrix has a canonical form. In Theorem 2, he argues that the maximal number of positive entries in a nearly decomposable $n \times n$ matrix is $3(n-1)$ and is obtained uniquely at a canonical matrix. (The author has learned that H. Minc has independently obtained Theorem 2.) In Theorem 3, he obtains necessary and sufficient conditions for the permanent of an $n \times n$ nearly decomposable matrix A whose entries are 0 or 1 to equal $\sigma(A) - 2n + 2$ where $\sigma(A)$ is the number of positive entries in A. He then mentions some unsolved problems and indicates connections between each of these two types of matrices and graph theory. (Received December 31, 1970.) (Author introduced by Professor Richard D. Sinkhorn.)

71T-A54. RALPH P. STINEBRICKNER, State University of New York, Potsdam, New York 13676. On maximal Goldie subrings of semiprime Goldie rings which are not prime.

A maximal Goldie subring of a ring is defined to be a maximal element in the set of all right Goldie subrings properly contained in the ring. Let T be a semiprime right Goldie ring which is not prime, and let S be a maximal Goldie subring of T. Furthermore, let $A_1, A_2, \ldots, A_n$ be the maximal right annihilators of nonzero right ideals of T and $\ell(A_i)$ be the left annihilator of A_i, $i = 1, \ldots, n$. $S + A_i$ will denote the subring of T generated by S and A_i. The hypotheses of the following theorems provide a partition for a certain subset of the set of maximal Goldie subrings of T, and each conclusion mentions certain properties which members of a particular equivalence class have in common. Theorem. If $A_i \subseteq S$, $i = 1, \ldots, n$, then S is semiprime. Theorem. If $S + A_i = T$ and $\ell(A_i) \cap S \neq 0$, $i = 1, \ldots, n$, then S is semiprime and S and T have the same classical quotient ring. Theorem. If $S \subsetneq S + A_i \subsetneq T$ and $\ell(A_i) \cap S \neq 0$, $i = 1, \ldots, n$, then S is a subdirect sum of the rings $S/A_i \cap S$, $i = 1, \ldots, n$, and each $S/A_i \cap S$ has an infinite direct sum of nonzero right ideals. Theorem. If $\ell(A_i) \cap S = 0$ for some i, then T is the group theoretic direct sum of S and $\ell(A_i)$. Results of I. N. Herstein (Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8) 35 (1963), 23-26) are used frequently in the proofs. (Received January 11, 1971.)

71T-A55. MICHAEL SLATER, University of Bristol, Bristol BS8 1TW, England. Alternative extensions of the quaternions. Preliminary report.

Let F be a commut. assoc. ring with 1. Kleinfeld (Indian J. Math. 9 (1967), 443) asks for all center-preserving alternative extensions R of the 2×2 matrices F_2 over F. Let A be an assoc. F-algebra with unity e and center Fe. Let X be a unital left-A module, and let $[\cdot,\cdot]$ be an antisymmetric bilinear product on $X^2 \to Fe$. Suppose further $(ab - ba)x = 0$; $a[x, y] = [ax, y]$; $[y, z]x + [z, x]y + [x, y]z = 0$ for all a, b $\in$ A; x, y, z $\in$ X. Then construct $S = S(A, X)$ with F-module structure $(A_{11} + A_{10} + A_{01} + A_{00}) + (X_{10} + X_{01}) = A_2 + Y$, say, a direct sum of copies of A and X. Products are determined by $a_{ij} b_{pq} = \delta_{jp}(ab)_{iq}$; $a_{ii} x_{ij} = a_{ji} x_{ji} = x_{ij} a_{jj} = (ax)_{ij}$; $a_{ij} x_{ji} = x_{ji} a_{ij} = 0$; $x_{ij} y_{ij} = (-1)^j [x, y]_{ji}$; $x_{ij} y_{ji} = (-1)^i [x, y]_{ii}$. Theorem. Any S(A, X) is an R, and any R is isomorphic to some S(A, X). If F is a field, either $[\cdot,\cdot] = 0$ and $A_2 + Y$ is a split null extension of A_2, or dim X = 2, A = Fe, and S(A, X) is isomorphic to the split Cayley algebra over F. (Received January 11, 1971.)

71T-A56. J. VAN LEEUWEN, Mathematisch Instituut, Rijksuniversiteit, Utrecht, Netherlands. On compositions of semigroups. Preliminary report.

In L. A. M. Verbeek ["Semigroup extensions", thesis, 1968, Delft University of Technology, Netherlands; see also Abstract 67T-625, these *Notices* 14 (1967), 843-844 and "Union extensions of semigroups", Trans. Amer. Math. Soc. 150 (1970), 409-423] it was shown that there are $\leq$ 130 different compositions of semigroups. In this report it is proved that the numbers of possible compositions of semigroups is exactly equal to 130 and that for every k-composition α (i.e. a composition with k nonempty components) there exists a semigroup S with k + 1 elements that has composition α w.r.t. one of its idempotent elements. It is also proved that there are only 18 different compositions of semigroups possible for Rees-matrix semigroups. (Received January 12, 1971.) (Author introduced by Professor Leo A. M. Verbeek.)

71T-A57. KWANGL KOH, North Carolina State University, Raleigh, North Carolina 27607. On strongly harmonic rings.

Let R be a ring with 1. We say an ideal S in R is M-primary for some maximal ideal M in R provided that R/S is a local ring with a unique maximal ideal M/S and R/S' is not a local ring if S' is an ideal of R which is properly contained in S. We say R is strongly harmonic provided that for any pair of distinct maximal ideals M_1, M_2 there exist ideals A and B such that $A \nsubseteq M_1$, $B \nsubseteq M_2$ and $AB = 0$. Theorem 1. R is strongly harmonic if and only if O(M) is M-primary for every maximal ideal M where $O(M) = \{a \in R \mid aRx = 0$ for some $x \notin M\}$. Theorem 2. Let $\mathfrak{M}(R)$ be the maximal ideal space of R with the hull-kernel topology. If R is strongly harmonic then R is isomorphic to the global sections of a sheaf of rings $\mathcal{R} = \bigcup\{R/O(M) \mid M \in \mathfrak{M}(R)\}$ over $\mathfrak{M}(R)$. If A is a unitary (right) R-module then $\widetilde{A} = \bigcup\{A/A_M \mid M \in \mathfrak{M}(R)\}$ where $A_M = \{a \in A \mid aRx = 0$ for some $x \notin M\}$ is a sheaf of modules over $\mathfrak{M}(R)$ and $\xi_A : a \mapsto \hat{a}$ where $\hat{a}(M) = a + A_M$ is a semilinear isomorphism of A onto $\Gamma(\mathfrak{M}(R), \mathcal{R})$-module $\Gamma(\mathfrak{M}(R), \widetilde{A})$ in the sense that ξ_A is a group isomorphism satisfying $\xi_A(ar) = \hat{a}\hat{r}$ where $\hat{r}(M) = r + O(M)$ and, $\Gamma(\mathfrak{M}(R), \mathcal{R})$ = the global sections of $\mathcal{R}$ and $\Gamma(\mathfrak{M}(R), \widetilde{A})$ = the global sections of $\widetilde{A}$. (Received January 18, 1971.)

71T-A58. CHESTER E. TSAI and DAVID M. FOSTER, Michigan State University, East Lansing, Michigan 48823. Primary ideal theory for quadratic Jordan rings. Preliminary report.

An ideal P of a unital quadratic Jordan algebra (J, U, 1) over a commutative associative ring C with unity is prime if whenever A and B are ideals of J such that $AU_B \subseteq P$ then either $A \subseteq P$ or $B \subseteq P$ (see Tsai, "The prime radical in a Jordan ring," Proc. Amer. Math. Soc. 74(1968)). If A is an ideal of J, the radical r(A) of A is the intersection of all prime ideals of J which contain A. An ideal P of J is primary if whenever A and B are ideals of J with $AU_B \subseteq P$, then either $A \subseteq P$ or $B \subseteq r(P)$. For any ideal A of J, set $D^0(A) = A$ and inductively for $0 < k$, $D^k(A) = D^{k-1}(B)U_{D^{k-1}(A)}$. We say (J, U, 1) satisfies the Artin-Rees condition if for any two ideals A and B of J there exists a nonnegative integer h such that $A \cap D^h(B) \subseteq AU_B$. A Jordan algebra is noetherian if it satisfies the maximal condition on ideals. Theorem. Let (J, U, 1) be a noetherian quadratic Jordan algebra. Then every ideal in J can be represented as a finite intersection of primary ideals iff it satisfies the Artin-Rees condition. (Received January 18, 1971.)

Analysis

71T-B28. ANDRE de KORVIN and LAURENCE E. KUNES, Indiana State University, Terre Haute, Indiana 47809. Some nonweak integrals defined by linear functionals. II.

For notations the reader is referred to Abstract 71T-B26, these *Notices* 18(1971). Definition. Let f be a scalar function defined on S and let m_{y^*} be countably additive. $f \cdot x$ is called m-integrable if for every $y^* \in \sigma^*$ f is m_{y^*} integrable and for every $A \in \Sigma$ there exists $y \in F$ such that $y^*(y) = \langle \int_A f \, dm_{y^*}, x \rangle$. In this case $y = \int_A f \cdot x \, dm$. Lemma. If $v(A) = \int_A f \cdot x \, dm$ then v is a measure. Definition. f is called m-integrable if for every $x \in E$, $f \cdot x$ is m-integrable. The sequence $\{f_n\}$ will converge to f in measure if for every $\epsilon > 0$ and $\delta > 0$, there exists N such that for $n \geq N$, $\widetilde{m}\{s : |f(s) - f_n(s)| > \epsilon\} < \delta$. Theorem. Let $\{f_n\}$ be a sequence of m-integrable functions. Suppose $\widetilde{m}(S) < \infty$, that f_n converges to f in measure, that f is m_{y^*} integrable and that there exists an m_{y^*} integrable function g such that $|f_n - g|$ and $|f-g|$ are bounded by some constant B. Then f is m-integrable and $\int_A f_n \cdot x \, dm$ converges to $\int_A f \cdot x \, dm$ uniformly in $x \in \sigma$ (σ is the unit sphere in E) and $A \in \Sigma$. (Received October 28, 1970.)

71T-B29. KEITH A. EKBLAW, Boise State College, Boise, Idaho 83706. Polynomials of degree n and bounded index n. Preliminary report.

The following is shown. Theorem 1. Let $n \geq 1$ and let p be a polynomial of degree n and index n. If $a_1, a_2, \ldots, a_n$ are the zeros of p then there exists z_0 such that $|z_0 - a_k| < 2$ for $k = 1, 2, \ldots, n$. Theorem 2. Let $n \geq 1$ and let p be a polynomial of degree n. If $a_1, a_2, \ldots, a_n$ are the zeros of p and if there exists z_0 such that $|z_0 - a_k| < 1/n$ for $k = 1, 2, \ldots, n$ then p is of index n. Theorem 3. Let $p(z) = z^n + a_{n-1}z^{n-1} + \ldots + a_0$ be of index n. If $r = |a_{n-1}|/n + 4$ then $|a_k| < \binom{n}{k} r^{n-k}$ for $k = 0, 1, \ldots, n-2$. (Received October 30, 1970.)

71T-B30. MARIO C. MATOS, University of Rochester, Rochester, New York 14627. On holomorphic mappings defined on Baire topological vector spaces.

Let f be a mapping of the nonvoid open subset U of the topological vector space E into the Banach space F. Lemma. If f is G-holomorphic and B-continuous, then the mapping $h \in E \mapsto \hat{\delta}^n f(x;h) \in F$ is B-continuous for every $x \in U$ and $n \in N$. Theorem 1 (Zorn). If E is a Baire space, f is holomorphic if and only if it is G-holomorphic and B-continuous. Theorem 2 (Zorn). If E is a Baire space and f is G-holomorphic, the set of points of U where f is continuous, is both open and closed in U. Let now f be a mapping of the nonvoid open subset U of $E_1 \times E_2$ into F, where E_1 and E_2 are topological vector spaces. Theorem 3 (Hartogs). If E_1 and E_2 are Baire spaces, $E_1 \times E_2$ is a Baire space and E_1 is metrisable, then f is holomorphic if and only if it is separately holomorphic. The above theorems remain valid with the following changes: E, E_1 and E_2 are metrisable Baire spaces and F is a complete locally convex space. All the above spaces are complex and Hausdorff. (Received November 2, 1970.) (Author introduced by Professor Leopoldo Nachbin.)

71T-B31. ROBERT D. M. ACCOLA, Brown University, Providence, Rhode Island 02912. Vanishing properties of theta functions for abelian covers of Riemann surfaces. I.

Let W_1 be a closed Riemann surface of genus $p_1 \geq 2$ admitting an abelian group of automorphisms, G, of order n. Let $\underline{b}: W_1 \to W_1/G\ (= W_0)$ be the n-sheeted (possibly ramified) cover obtained by mapping W_1 onto the space of orbits, W_0 (of genus p_0). If $J(W_i)$, $i = 0, 1$, is the Jacobian of W_i, let $\underline{a}: J(W_0) \to J(W_1)$ be the homomorphism obtained by lifting divisors of degree zero from W_0 to W_1 via $\underline{b}$. A general theorem is proven from which can be derived many of the known vanishing properties for the theta function for $J(W_1)$ (most known cases concern $n = 2$). Also theta functions for W_1 evaluated on $\underline{a}J(W_0)$ $(\subset J(W_1))$ are expressed as products of theta functions for W_0 generalizing formulas of Riemann $(n = 2)$ and work of the author in the unramified case.
(Received November 3, 1970.)

71T-B32. DAVID B. SINGMASTER, Polytechnic of the South Bank, Borough Road, London SE 1, England. On the asymptotic behavior of $\sum_{n=0}^{\infty} (n+t)^a x^n$ as $x \to 1-$.

By extending a problem of Pólya and Szegö ("Aufgaben und Lehrsätze aus der Analysis," Band 1, 3te Auflage, Springer-Verlag, 1964, p. 15, no. 89) we obtain: Theorem. $\sum_{n=0}^{\infty} (n+t)^a x^n \sim \Gamma(a+1)\,(1-x)^{-a-1}$ as $x \to 1-$, for $a > -1$, $t > 0$. Further, we have that LHS $\leq$ RHS when $a \geq 1$, $0 < t \leq 1$ and $0 \leq x < 1$, while LHS $\geq$ RHS when $0 \leq a \leq 1$, $t \geq 1$ and $0 \leq x < 1$. Multiplying by x^r, $r \geq 0$, yields $\sum (n+t)^a x^{n+r} \sim \Gamma(a+1)(1-x)^{-a-1}$ and setting $r = t = 1$, we obtain $\sum n^a x^n \sim \Gamma(a+1)(1-x)^{-a-1}$ as $x \to 1-$, for $a > -1$, $t > 0$. (Received November 9, 1970.)

71T-B33. NADIM A. ASSAD and WILLIAM A. KIRK, University of Iowa, Iowa City, Iowa 52240. Fixed point theorems for set-valued transformations of contractive type. Preliminary report.

Let M be a complete, metrically convex, metric space, $\mathcal{T}(M)$ the family of nonempty bounded closed subsets of M, and let $K \subset M$. A mapping $\varphi: K \to \mathcal{T}(M)$ is said to be a contraction mapping if there exists a constant $k < 1$ such that $D(\varphi(x), \varphi(y)) \leq kd(x, y)$ where D denotes Hausdorff distance in $\mathcal{T}(M)$. Theorem 1. If

K is a nonempty closed subset of M and if $\varphi: K \to \mathcal{T}(M)$ is a contraction mapping for which $\varphi(x) \subset K$ for all x in the boundary of K, then there exists an $x_0 \in K$ such that $x_0 \in \varphi(x_0)$. This result may be applied to an approach of E. Lami Dozo: A Banach space is said to satisfy Opial's condition if $\liminf_{n\to\infty} \|x_n - x\| > \liminf_{n\to\infty} \|x_n - x_0\|$ when $x_n \to x_0$ weakly and $x \neq x_0$. Theorem 2. Let K be a nonempty, convex, weakly compact subset of H, where H is a closed convex subset of a Banach space which satisfies Opial's condition. Suppose T is a nonexpansive set-valued mapping ($D(Tx, Ty) \leq \|x-y\|$) defined on K whose values are nonempty compact subsets of H. If $Tx \subset K$ whenever x is in the relative boundary of K in H, then there exists an $x_0 \in K$ such that $x_0 \in Tx_0$. Theorem 2 improves Lami Dozo's generalization of a result of J. Markin. (Received November 9, 1970.)

71T-B34. LAWRENCE J. CRONE, DANIEL J. FLEMING and PETER G. JESSUP, Clarkson College of Technology, Potsdam, New York 13676. Fundamental biorthogonal sequences and K-norms on φ.

Let (x_i, f_i) be a fundamental biorthogonal sequence in a Banach space X. The span of the x_i's can be identified with φ the space of finitely nonzero sequences by the correspondence $x \Leftrightarrow s_x$ where $s_x = \sum_i f_i(x) e_i$ and $e_i = (\delta_{ij})_{j=1}^{\infty}$. The induced norm on φ is a K-norm (i.e. coordinate functionals are continuous on $(\varphi, \| \ \|)$). The multiplier algebra M(X) of such a biorthogonal sequence is defined to be the set of all continuous diagonal linear operators on φ with this norm. The form of M(X) is determined for any such biorthogonal sequence and a characterization is obtained to determine those sequence spaces which arise as multiplier algebras of fundamental or complete biorthogonal sequences of various types. For example, Theorem. A BK-algebra containing φ and e is the multiplier algebra of a K-norm on φ iff X is the δ-dual of a K-norm on φ. In the above theorem K-norm on φ can be replaced, in both places by any of: series summable (s.s.) K-norm on φ, strongly series summable (s.s.s.) K-norm on φ, s.s. M-basis, s.s.s. M-basis, Schauder basis, unconditional Schauder basis. Finally a method of constructing sequence spaces is given and used to construct a BK-space which contains φ densely but which has nonnorming coordinates. (Received November 10, 1970.)

71T-B35. DANIEL J. FLEMING and PETER G. JESSUP, Clarkson College of Technology, Potsdam, New York 13676. Perfect matrix methods.

Let $e_i = (\delta_{ij})_{j=1}^{\infty}$; $\Delta = (e_i)_{i=1}^{\infty}$ and let A be an infinite matrix which maps E into E where E is an FK-space with Schauder basis Δ. Let $E^{\delta} = \{t_f | f \in E^A\}$ where $t_f = (f(e_n))_{n=1}^{\infty}$. A matrix A is said to be of type E^{δ} if whenever $tA = 0$ for $t \in E^{\delta}$ then $t = 0$. Let $E_A = \{x | Ax \in E\}$ with the usual topology. A matrix A is perfect if $\overline{E} = E_A$. An element $t_f \in E^{\delta}$ is said to have property P if $(t_f A, x) = \sum_k \sum_n t_{fn} a_{nk} x_k$ converges for each $x \in E_A$. A matrix A is called associative if $Q = E^{\delta}$ and $f(Ax) = (t_f A, x)$ for all $f \in E^*$ and all $x \in E_A$ where $Q = \{t_f \in E^{\delta} | t_f$ has property P$\}$. Theorem. (i) If A is reversible then A is perfect iff A is of type E^{δ}. (ii) A is perfect iff $f(Ax) = (t_f A, x)$ for all $x \in E_A$ and each $f \in E^*$ such that $t_f \in Q$. (iii) A is associative iff E_A has basis Δ. An element $x \in E_A$ is perfect if $f(Ax) = (t_f A, x)$ for each $f \in E^*$ such that $t_f \in Q$. If in addition $Q = E^{\delta}$, x is associative. Theorem (i) $\sum_k x_k e_k$ converges weakly iff x is associative. (ii) x is in the closure of the finitely nonzero sequence in E_A iff x is perfect. Finally dropping the basis assumption on E it is shown that if weak and strong convergence coincide in E, then for $x \in E_A$ the series $\sum_k x_k e_k$ converges to x iff it converges to x weakly. (Received November 10, 1970.)

71T-B36. WILLIAM J. DAVIS, Ohio State University, Columbus, Ohio 43210. Remarks on finite rank projections.

Three problems stated in "Minimal projections," (E. W. Cheney and K. H. Price, Approximation Theory Conf., Lancaster, England, 1968) are solved. First, it is shown that there do not exist functionals $\{f_1, f_2, f_3\}$ in the ball of $(\ell_1^{(3)})^*$ such that, for every $x \in \ell_1^{(3)}$, $\|x\| \leq (f_1(x)^2 + f_2(x)^2 + f_3(x)^2)^{1/2}$. Next, the "principle of local reflexivity" of J. Lindenstrauss and H. Rosenthal ("The $\mathcal{L}p$ spaces," Israel J. Math. 7(1969), 325-349) is modified to give the Theorem. Let P be a finite rank projection on X^*, V a finite dimensional subspace of X^* and $\epsilon > 0$. There exists a finite rank projection R on X such that $\|R\| \leq \|P\| + \epsilon$, $\|(P - R^*)|V\| < \epsilon$ and $R^*(X^*) = P(X^*)$. As a corollary, it follows that if every n-dimensional subspace of X^* is complemented with norm $< K$, then every subspace of deficiency n in X is complemented with norm $< K$. Due to a result of Kadec (mentioned in the Cheney-Price paper) it follows that deficiency n subspaces are complemented with norm $< 1 + \sqrt{n} + \epsilon$ for every $\epsilon > 0$. Finally, it is shown that projection constants for finite dimensional spaces are determined by projections from finite dimensional superspaces. (Received November 23, 1970.)

71T-B37. SAMUEL ZAIDMAN, Université de Montréal, Montréal, Québec, Canada. An existence result for S^2-almost-periodic differential equations.

Let H be a hilbert space, and f(t), $-\infty < t < +\infty$, to H be a continuous function which is almost-periodic in the S^2-sense. Let B be a linear bounded symmetric operator which is positive definite in H. Then, there exists one and only one strong solution u(t) of the differential equation $u'(t) = Bu(t) + f(t)$, which is almost-periodic in the sense of Bochner. (Received November 16, 1970.)

71T-B38. JOHN T. HOFLER, University of South Carolina, Columbia, South Carolina 29208. Plasterable cones in locally convex spaces. Preliminary report.

Let (E, τ, K) be a locally convex space ordered by a cone K and with topological dual E'. B is a hyperbase for K if and only if there exists $f \in E'$ such that $B = f^{-1}(1) \cap K$. The following are generalizations of Banach space definitions from M. Krasnoselskii ["Positive solutions of operator equations," P. Nordhoff, Gröningen, 1964]. K allows plastering by the cone K_1 if and only if there exists a generating family P of seminorms for τ such that $K \setminus \{\theta\} \subseteq \operatorname{int} K_1$ and, furthermore, for each $p \in P$ there exists $a_p > 0$ such that $\theta \neq x \in K$ implies $\{x + h | p(h) \leq a_p p(x)\} \subseteq K_1$. Here a_p is independent of x. A linear functional f on (E, τ) is uniformly positive if for every τ-continuous seminorm, p, there exists $a_p > 0$ such that $f(x) \geq a_p p(x)$ $(x \in K)$. Theorem 1. The following are equivalent in (E, τ, K). (1) K has a bounded hyperbase. (2) There exists $f \in E'$ such that f is uniformly positive on K. (3) K allows plastering by a cone K_1. Theorem 2. K has a hyperbase if and only if there exists a generating family P of seminorms for τ such that for every $0 \neq x \in K$ and $p \in P$ there exists $\epsilon > 0$ such that $\{h \in E : p(h - x) \leq \epsilon\} \cap -K = \emptyset$. (Received November 2, 1970.)

71T-B39. JOAO B. PROLLA, University of Rochester, Rochester, New York 14627. The weighted Dieudonné theorem for density in tensor products.

Let X and Y be two completely regular Hausdorff spaces and V and W be two directed families of weights in the sense of L. Nachbin ("Elements of approximation theory," Van Nostrand, Princeton, N. J., 1967). Let $V \times W$ denote the set of all functions $(x, y) \mapsto v(x)w(y)$ on $X \times Y$. Let A be a locally convex topological algebra and let E and F be two locally convex topological modules over A. If $f \in CV_{\infty}(X, E)$ and $g \in CW_{\infty}(Y, F)$, the map $(x, y) \mapsto f(x) \otimes_A g(y)$ belongs to $C(V \times W)_{\infty}(X \times Y, E \otimes_A F)$ and it is denoted by $f \otimes_A g$. Theorem. The vector subspace of all finite sums of mappings of the form $f \otimes_A g$ is dense in $C(V \times W)_{\infty}(X \times Y, E \otimes_A F)$. (Received December 11, 1970.)

71T-B40. CASPER GOFFMAN and FON-CHE LIU, Division of Mathematical Sciences, Purdue University, Lafayette, Indiana 47907. On the localization property of square partial sums for multiple Fourier series.

It is shown that, for functions of n variables, localization holds for all functions in the Sobolev spaces W_p^1, $p \geqq n-1$. For each $p < n-1$ there is an $f \in W_p^1$ for which localization does not hold. For $n = 2$, there is an everywhere differentiable function for which localization does not hold. (Received November 18, 1970.)

71T-B41. JEFFREY B. RAUCH, Courant Institute, New York University, New York, New York 10012. Estimates and resolvent inequalities for hyperbolic initial-boundary value problems.

Consider the hyperbolic system with constant matrix coefficients $\partial_t u(t, x) = \sum_{j=1}^{n} A_j \partial_{x_j} u(t, x) + F(t, x)$ with initial condition $u(0, x) = 0$ and homogeneous boundary conditions on $x_1 = 0$ which satisfy the conditions of Hersh (J. Math. Mech. 12(1963)). Write the systems as $\partial_t u = Lu + F(t)$ with the boundary conditions hidden in $\mathcal{D}(L)$. Theorem. The $\mathcal{L}_2$ a priori estimate $\int_0^1 \int \|u(t, x)\|^2 dx dt \leq c \int_0^1 \int \|F(t, x)\|^2 dx dt$ with c independent of F holds $\Leftrightarrow (\exists c')(\forall \tau, \operatorname{Re} \tau > 0)(\forall \varphi \in \mathcal{D}(L)) \|(\tau - L)\varphi\| \geqq \operatorname{Re} \tau \|\varphi\|$. Similar results are obtained for related problems. (Received November 9, 1970.)

71T-B42. MICHAEL A. GOLBERG, University of Nevada, Las Vegas, Nevada 89109. Invariant imbedding and Ricatti transformations. Preliminary report.

In this paper we develop several initial value procedures for solving a class of boundary value problems connected with first order differential and difference equations in a Banach space. The methods are based on functional relations which the author has recently proved in connection with the theory of invariant imbedding (M. Golberg, "Some invariance principles for two-point boundary value problems," to appear in J. Math. Anal. Appl.). These relations give rise to a generalization of the Ricatti transformations given in (P. Nelson, M. Scott, "Internal values in particle transport by the method of invariant imbedding," to appear in J. Math. Anal. Appl.). The main feature of these techniques is that it enables us to solve for the internal values of a family of problems with a single numerical integration. We include applications to two-point and multi-point problems for differential equations, integral boundary conditions, integro-differential equations and two-point problems for difference equations. (Received November 20, 1970.)

71T-B43. ROBERT C. JAMES, Claremont Graduate School, Claremont, California 91711. Quasicomplements.

Subspaces X and Y of a normed linear space T are quasicomplements iff $X+Y$ is dense in T and $\overline{X} \cap \overline{Y} = \{0\}$. Results of Murray and Mackey were extended by Guarii and Kadec, who showed that if X and Y are infinite-dimensional subspaces of a separable Banach space B, $X \subset Y$, and B/Y is infinite-dimensional, then X has a quasicomplement $\widetilde{Y}$ that is nearly isometric to Y and Y has a quasicomplement $\widetilde{X}$ that is nearly isometric to X. This is extended by showing that if X and Y are subspaces of a separable normed linear space and both $X \cap Y$ and $T/\mathrm{cl}(\overline{X+Y})$ are infinite-dimensional, then there is a subspace $\widetilde{Y}$ which is nearly isometric to Y and is a quasicomplement for all Z such that $X \cap Y \subset Z \subset \mathrm{cl}(\overline{X+Y})$. Thus if a separable Banach space B contains a subspace H_1 isomorphic to Hilbert space and $H_1 \subset X \subset B$ with $B/\overline{X}$ infinite-dimensional, then there is a subspace H_2 isomorphic to Hilbert space for which H_2 is a quasicomplement for every Z with $H_1 \subset Z \subset \overline{X}$. If X and Y are quasicomplements in a reflexive separable Banach space B, but their closures are not complements, then there are subspaces Y_1 and Y_2 such that Y/Y_1 and Y_2/Y are infinite-dimensional and X and Z are quasicomplements if $Y_1 \subset Z \subset Y_2$. Reflexivity is not needed to prove the existence of Y_1. (Received November 16, 1970.)

71T-B44. WITHDRAWN.

71T-B45. JOSEPH M. LAMBERT, Penn State University, York, Pennsylvania 17403. Property SAIN and ℓ_1.

Property SAIN was introduced by Deutsch and Morris [J. Approximation Theory 2 (1969), 355-373]. A geometric approach to property SAIN was taken by Holmes and Lambert, Abstract 677-41-4, these *Notices* 17(1970), 780. In that report it was shown that (ℓ_1, M, L) is SAIN whenever M is the subspace of ℓ_1, consisting of the elements having only finitely many nonzero coordinates and L is any finite dimensional subspace of C_0. It was conjectured that (ℓ_1, M, L) would be SAIN where M is as above and L is any finite dimensional subspace of ℓ_∞. This conjecture is shown to be true. To this end a special version of Yamabe's theorem [Osaka Math. J. 2(1950), 15-17] is proven. Let F be a set, M a subset of F, then F is said to have M-core at f in F, if given m in M, there exists $\delta > 0$ such that $(1+\delta)f - \delta m$ is in F. Theorem. Let X be a Banach space, F a convex set contained in X. Let y_i, $i=1,\ldots,n$, be linear functionals on F. Let M be a norm dense convex subset of F. Let f be in F, F having M-core at f, then given $\epsilon > 0$, there exists m in M such that $y_i(f) = y_i(m)$, $i=1,\ldots,n$, and $\|f-m\| < \epsilon$. (Received November 27, 1970.)

71T-B46. ANTHONY LEUNG, University of Wisconsin, Madison, Wisconsin 53706. Connection formulas for asymptotic solutions of second order turning points in unbounded domains. Preliminary report.

Asympotic expansions, as $\epsilon \to 0^+$ or $x \to \infty$, for fundamental systems of solutions for $\epsilon^2 u''(x) - p(x)u(x) = 0$ are obtainable by Evgrafov's and Fedoryuk's method on unbounded canonical domains with neighborhoods deleted around turning points. When p(x) is a polynomial, they also found a "lateral connection" formula for two fundamental systems of solutions with known asymptotic expansions valid in the interior of two different unbounded overlapping canonical regions with a common first order turning point at their boundaries. However, their connecting methods are not applicable to second order turning points. This paper employs techniques of Wasow and of R. Lee to find central connection formulas with a solution having known asymptotic expansion in a bounded full neighborhood of a second order turning point. With the help of this result, lateral connection formulas are also established. (Received November 30, 1970.) (Author introduced by Professor Wolfgang R. Wasow.)

71T-B47. HARI M. SRIVASTAVA and J. P. SINGHAL, University of Victoria, Victoria, British Columbia, Canada. Some extensions of the Mehler formula. Preliminary report.

In the literature on the classical Hermite polynomials $\{H_n(z) \mid n=0,1,2,\dots\}$, the bilinear generating function $(*)$ $\sum_{n=0}^{\infty} H_n(x)\, H_n(y)(t^n/n!) = (1-4t^2)^{-\frac{1}{2}} \exp\{(1-4t^2)^{-1} \cdot [4xyt-4(x^2+y^2)t^2]\}$ is well known as Mehler's formula. Recently, L. Carlitz ["Some extensions of the Mehler formula," to appear; see also Boll. Un. Mat. Ital. 3(1970), 43-46] gave a number of interesting extensions of the formula $(*)$. By using certain operational techniques, the authors prove here an elegant unification of these extensions. It is also shown how rapidly several of Carlitz's formulas would follow from these considerations. (Received November 30, 1970.)

71T-B48. DAVID S. BROWDER, University of Oregon, Eugene, Oregon 97403. Derived algebras in L_1 of a compact group. Preliminary report.

Let G be a compact group and let A be a subalgebra of $L_1(G)$. Suppose A is a Banach algebra under its own norm and an essential left Banach L_1-module. Let D_A denote the derived algebra of A, analogous to Helgason's derived algebra (Ann. of Math. (2) 64(1956), 240-254.) Let A^z denote the center of A and let $\mathcal{D}_A$ denote its derived algebra. Also, let S_A denote the ideal of functions in A with unconditionally convergent Fourier series. Then the following statements hold: (1) $S_A^z \subset \mathcal{D}_A$. (2) $D_{C(G)} = D_{K(G)} = K(G)$, where $K(G)$ is the algebra of functions with absolutely convergent Fourier series. (3) $D_{L_p} = L_2$ for $1 \leqq p \leqq 2$. (4) $D_{L_p} \subset S_{L_p}$ for $1 \leqq p < \infty$. (5) $\mathcal{D}_{L_p} = S_{L_p}^z$ for $1 < p < \infty$, generalizing a result of G. F. Bachelis and J. E. Gilbert (to appear). Each of the inclusions: $D_{L_p}^z \subset \mathcal{D}_{L_p}$, $D_{L_p} \subset S_{L_p}$, $K(G) \subset S_{C(G)}$, and $L_2 \subset S_{L_1}$ is proper for the group S_3^{∞}. It is known that these inclusions are all equalities if G is abelian. (Received December 4, 1970.)(Author introduced by Professor Kenneth A. Ross.)

71T-B49. BENJAMIN VOLK, 13-15 Dickens Street, Far Rockaway, New York 11691. A coefficient region.

The exact coefficient region of the cubic polynomial, with nonnegative coefficients, that does not vanish in the interior of the unit disc which is centered at the origin, was found. The interest is in the proof. (Received December 4, 1970.)

71T-B50. JOHN K. PERRYMAN and STEPHEN WEST, University of Texas, Arlington, Texas 76013. A set of integral operators for functions which are bounded by Me^{bt^3} for large t.

A set of integral transformations is developed which is useful in solving differential equations that contain functions which grow like e^{t^3}. A complex inversion formula is developed for these transformations and a short table of transform pairs is computed for two of the transformations. (Received December 4, 1970.)

71T-B51. WILLIAM ERB DIETRICH, University of Texas, Austin, Texas 78712. On the ideal structure of Banach algebras.

For Banach algebras in a class which includes all group and function algebras, we show that the family of ideals of A with the same hull is typically quite large, containing ascending and descending chains of arbitrary length through any ideal in the family, and that typically a closed ideal of A whose hull meets the Silov boundary of A cannot be finitely generated algebraically. For example, if G is an LCA group, there is an ideal of $L^1(G)$ strictly between any two ideals $I \subsetneq J$ which have the same hull whenever either I or J is closed. Further, a closed ideal of $L^1(G)$ can be finitely generated only if its hull is open-closed. (Received December 4, 1970.)

71T-B52. PETER HESS, University of Chicago, Chicago, Illinois 60637. Existence theorems of Fredholm alternative type for nonlinear Hammerstein integral equations.

In operator-theoretic terms, a nonlinear Hammerstein integral equation $u(x) + \int_G K(x,y)a(y,u(y))dy = f(x)$ can be written as a functional equation $u + TAu = f$, defined in a Banach space X of functions on G, and with the linear and nonlinear mappings T, A being given by $(Tu)(x) = \int_G K(x,y)u(y)dy$; $(Au)(x) = a(x,u(x))$. A solvability criterion of Fredholm alternative type has been obtained by J. Nečas (see a forthcoming paper) for T compact. Here two results are stated for bounded T. Theorem 1. Let X be a real Hilbert space, T a bounded monotone selfadjoint operator in X, $B: X \to X$ pseudomonotone, odd, homogeneous and continuous, and $N: X \to X$ bounded, continuous with $\|Nu\| / \|u\| \to 0$ ($\|u\| \to \infty$), and such that $B + N$ is pseudomonotone. Then the range of $I + T(B+N)$ is X, provided $u + TBu = 0$ only for $u = 0$. Theorem 2. Let X be a real reflexive Banach space, X^* its conjugate, L a bounded linear monotone operator from X^* to X, and $C: X^* \to X$ a compact linear mapping. Let further B and N be bounded continuous mappings of X to X^*, B homogeneous and odd and N such that $\|Nu\| / \|u\| \to 0$ as $\|u\| \to \infty$. Suppose that B and $B + N$ are strictly monotone. If $u + (L+C)Bu = 0$ implies that $u = 0$, then the equation $u + (L+C)(B+N)u = f$ is solvable for each $f \in X$. The proofs are based on homotopy arguments. (Received December 7, 1970.)

71T-B53. ANDRE de KORVIN and RICHARD J. EASTON, Indiana State University, Terre Haute, Indiana 47809. Vector valued absolutely continuous functions on idempotent semigroups. I.

For the definitions and notations in the scalar setting see de Korvin, Abstracts 70T-B135 and 70T-B136, these *Notices* 17(1970), 661. Definition. Let u and v be finitely additive set functions defined on Σ, where u is scalar valued and v is X-valued. Then $v << u$ if v is the limit in the variation norm of functions of the form $\Sigma v_i \cdot x_i$, where $x_i \in X$, v_i is finitely additive, scalar valued which is $\epsilon - \delta$, $v_i << u$. An analogous definition is given for X-valued functions G defined on S which are absolutely continuous with respect to a positive definite function F. Characterizations for $v << u$ are given and the following theorem is proved. The map $m \to \hat{m}$ is defined by $\hat{m}(f) = m(A_f)$, $f \in S$. Theorem. The map $m \to \hat{m}$ is a linear isometry of $BV(\Sigma, X)$ onto $BV(S, X)$ and moreover $m << u$ if and only if $\hat{m} << \hat{u}$. (Received December 7, 1970.)

71T-B54. RONALD A. KNIGHT, Oklahoma State University, Stillwater, Oklahoma 74074. On a class of planar dynamical systems.

For definitions and notations refer to those given by Ahmad (Pacific J. Math. 32(1970), 561-574). The following theorem characterizes planar flows of characteristic 0^+. Theorem. A flow (R^2, π) is of characteristic 0^+ if and only if either case (i) or (ii) is satisfied. (i) The set S of rest points is compact and one of the following holds: (R^2, π) is parallelizable; $S = \{s_0\}$ where s_0 is either a global Poincaré center or a local Poincaré center such that the set consisting of s_0 and all periodic points is a connected global attractor; or $\{s\}$ is a stable for each $s \in \partial S$ and S is a global attractor. (ii) S is not compact and each of the following holds: $\{s\}$ is stable for each $s \in \partial S$; each $x \in \partial A^+(S)$ is dispersive; and the flow restricted to $R^2 - \overline{A^+(S)}$ is parallelizable. These conditions are sharp. There exists a planar flow having a noncompact set of rest points and satisfying all of the properties exhibited by Ahmad in the reference above. (Received December 7, 1970.)

71T-B55. BLAISE MONTANDON, University of Iowa, Iowa City, Iowa 52240. The method of averaging and integral manifolds of coupled harmonic oscillators with weak nonlinear damping-perturbation terms.

The system of differential equations describing n forced or unforced coupled harmonic oscillators with "small" nonlinearities is transformed by a linear decoupling transformation followed by a change to a mixture of rectangular, polar, and rotating rectangular coordinates. The nature of integral manifolds for the decoupled system of differential equations is discussed. Theorems of Hale and Diliberto which are applicable to the transformed system are cited and used to unify the search for (almost) periodic solutions with that for (almost) periodic surfaces. The spirit here is one of replacing the search for stable (almost) periodic solutions with a search for stable one dimensional manifolds. Thus the search for stable solutions becomes a special case in the search for stable manifolds! An example (an autonomous, nonconservative system with two degrees of freedom) is considered which demonstrates the problem of unboundedness in the RHS of the DE for the angular variables when some of the radial variables are near zero. Tables of means of some trigonometric polynomials are presented along with their derivation. The defining equations of the decoupling transformation are derived. (Received December 8, 1970.) (Author introduced by Professor Anthony J. Schaeffer.)

71T-B56. N. K. SHARMA, Indiana University, Bloomington, Indiana 47401. Norlund methods associated with analytic functions. Preliminary report.

Let Δ denote the Banach algebra of conservative triangular matrices. $B[c]$ the Banach algebra of bounded operators on c where c is the space of convergent sequences. Let D denote the closed unit disk $\{z = |z| \leqq 1\}$ and let A denote the triangular matrix $\{a_{nk}\}$ where $a_{n,n-1} = 1$ $\forall n$ & $a_{nk} = 0$ otherwise. Let f be analytic in D and let $f(z) = \sum_n \alpha_n z^n$ be its power series expansion in D. Then we define $f(A) = \sum_n \alpha_n A^n$. Let $\mathfrak{A} = \{f(A) = f$ analytic in D$\}$. Theorem 1. $\mathfrak{A}$ = commutant of A in Δ = commutant of A in $B[c]$. Theorem 2. f(A) is invertible iff f(z) has no zero in D. Several well-known results about Norlund methods associated with polynomials can now be deduced. (Received December 8, 1970.)(Author introduced by Professor Billy E. Rhoades.)

71T-B57. ALVIN F. BARR, 243 Avent Street, Oxford, Mississippi 38655 and University of Mississippi, University, Mississippi 38677. On the starlikeness of certain classes of analytic functions.

Let $P(z) = \Pi_{k=1}^{n}(z-z_k)$ be a polynomial of degree $n>0$, $|z_k| \geqq 1$, $f(z) \in S^*$, the class of normalized starlike univalent functions and S_λ be the radius of starlikeness of order λ, $0 \leqq \lambda < 1$. Theorem 1. If $F(z) = z[P(z)]^{\beta/n}$, $\beta = \rho e^{i\alpha}$, α real, then $S_\lambda \geqq (1-\lambda)/\rho$ for $\rho\cos\alpha = \lambda - 1$, $S_\lambda \geqq \frac{1}{2}(\rho - \sqrt{(\rho^2 + 4(\lambda-1)(\lambda - 1 - \rho\cos\alpha))})/(\lambda - 1 - \rho\cos\alpha)$, otherwise. Conditions for equality are obtained. Theorem 1 generalizes a result by T. Basgoze, J. London Math. Soc. (2) 1(1969), 140-144. Theorem 2. Let $F(z) = f(z)[P(z)]^{\beta/n}$, $\beta = \rho e^{i\alpha}$, $f(z) \in S^*$. Then $S_\lambda \geqq (1-\lambda)/(2+\rho)$ for $\rho\cos\alpha = 1+\lambda$, $S_\lambda \geqq \frac{1}{2}(2+\rho - \sqrt{((2+\rho)^2 - 4(1-\lambda)(1+\lambda-\rho\cos\alpha))})/(1+\lambda-\rho\cos\alpha)$, otherwise. Equality iff $f(z)$ is the Koebe function and all the zeros are concentrated at one point on the unit circle. Theorem 3. Let $F(z) = z[f(z)/z]^{\beta}[P(z)]^{\beta/n}$, $\beta = \rho e^{i\alpha}$, $f(z) \in S^*$. Then $S_\lambda \geqq (1-\lambda)/3\rho$, $\rho\cos\alpha = 1-\lambda$, $S_\lambda \geqq \frac{1}{2}(3\rho - \sqrt{(9\rho^2 - 4(1-\lambda)(\rho\cos\alpha - 1 + \lambda))})/(\rho\cos\alpha - 1 + \lambda)$. Equality is attained as in Theorem 2. Other results have been obtained. (Received December 14, 1970.)

71T-B58. JAMES K. BROOKS, University of Florida, Gainesville, Florida 32601. On the existence of a control measure for strongly bounded vector measures.

Let X denote a Banach space, Σ an algebra of subsets of a set S. μ is strongly bounded (s-bounded) if $\lim \mu(A_i) = 0$ whenever $\{A_i\}$ is a disjoint sequence of sets from Σ. Theorem. Let $\mu : \Sigma \to X$ be finitely additive and s-bounded. Then there exists a positive bounded finitely additive set function ν defined on Σ such that: (i) $\lim_{\nu(E)\to 0} \mu(E) = 0$; (ii) $\nu(E) \leqq \sup\{\|\mu(F)\| : F \subseteq E,\ F \in \Sigma\}$, $E \in \Sigma$. Corollary. Let $\mu : \Sigma \to X$ be finitely additive. μ is s-bounded if and only if the range of μ is a conditionally weakly compact subset of X. The above theorem extends the Bartle-Dunford-Schwartz theorem for countably additive vector measures. (Received December 14, 1970.)

71T-B59. MARTIN SCHECHTER, Belfer Graduate School, Yeshiva University, New York, New York 10033. Compact Sobolev embeddings.

Let Ω be a domain in Euclidean space E^n, and for each $x \in E^n$ let s_x be the ball of radius 1 and center x. Let s, p, q be real numbers such that $1 > 1/p \geqq 1/q > 1/p - s/n$. Theorem. The embedding of $H^{s,p}(\Omega)$ [resp. $W^{s,p}(\Omega)$] into $L^q(\Omega)$ is compact if and only if the volume of $\Omega \cap S_x \to 0$ as $|x| \to \infty$. This is also a sufficient condition for the embeddings of $H_0^{s,p}$ or $W_0^{s,p}$ in $L^q(\Omega)$ to be compact. (Received December 16, 1970.)

71T-B60. RICHARD F. BASENER, Brown University, Providence, Rhode Island 02912. An example concerning peak points. Preliminary report.

In his thesis "One-point parts and the peak point conjecture" (Yale, 1968), Cole constructed a function algebra A on a compact metric space X satisfying: (i) the maximal ideal space of A is X; (ii) $A \neq C(X)$; (iii) every point of X is a peak point for A. Cole later extended this result to give an example of the same general type with the algebra doubly generated (unpublished). In this note, a concrete example of such a finitely generated algebra is constructed. For a compact set $X \subseteq \mathbb{C}^n$, $R(X)$ denotes the uniform closure on X of the algebra of rational functions holomorphic in a neighborhood of X. For a compact subset S of the open unit disk, let $X_S = \{(z,w) \in \mathbb{C}^2 \mid |z|^2 + |w|^2 = 1,\ z \in S\}$. If S is chosen so that R(S) is a regular algebra, then $R(X_S)$ is shown to satisfy properties (i), (ii), (iii) with respect to X_S. (Received December 21, 1970.)

71T-B61. PAUL A. FUHRMANN, Tel Aviv University, Tel Aviv, Israel. On reachability of linear systems.

Let H, H_1 be Hilbert spaces, A the infinitesimal generator of a strongly continuous one parameter semigroup of contractions in H, T its infinitesimal cogenerator, i.e. the Cayley transform of A. Let B be a bounded operator from H_1 to H. We consider the continuous system $\dot{x}(t) = Ax(t) + Bu(t)$ and the discrete system $x_{n+1} = Tx_n + Bu_n$. Theorem. The continuous system is reachable if and only if the discrete system is reachable. Here reachability means the zero state can be controlled arbitrarily close to any given state. (Received December 21, 1970.)

71T-B62. EDWARD NEWBERGER, State University College of New York, Buffalo, New York 14222. Asymptotic Gevrey classes and the Cauchy problem.

This paper nontrivially extends the results of J. Leray and Y. Ohya ["Systèmes linéaires, hyperboliques non stricts," Deuxième Colloq. l'Anal. Fonct., Centre Belge Recherches Math., Librarie Universitaire, Louvain, 1964, pp. 105-144] to non-quasi-analytic classes other than Gevrey classes. The notion of asymptotic Gevrey class is introduced. These classes provide finer asymptotic estimates for functions than Gevrey classes, the "asymptoticness" being with respect to increasing order of spatial differentiation. Three main ideas run through this paper: (I) To find necessary and sufficient conditions for two sequences to determine different classes (classes à la Mandelbrojt's classes of infinitely differentiable functions). (II) To find analytic conditions on the sequence so that we can solve a Cauchy problem. (III) To make these analytic conditions independent of the choice of the determining sequence of the class. The proof of our existence theorem has points in common with the proof of the Cauchy-Kovalewsky theorem. One difference, however, is that in the proof of the Cauchy-Kovalewsky theorem there is one majorizing Cauchy problem whereas in the proof of our existence theorem there is a majorizing Cauchy problem and then another Cauchy problem which majorizes it, that is, a majorizing majorizing Cauchy problem. (Received December 21, 1970.)

71T-B63. SANDY GRABINER, Claremont Graduate School, Claremont, California 91711. Ranges of operator iterates. Preliminary report.

Suppose that T is a quasi-nilpotent, but not nilpotent, operator on a Banach space E. If k is a positive integer, then $T^{k+1}(E)$ has infinite deficiency in $T^k(E)$. The proof is by the following induction: if all such $T^{k+1}(E)$ have deficiency greater than m, then they all have deficiency greater than m+1. (The case $m = 0$ is treated in the author's paper: "Ranges of quasi-nilpotent operators," Illinois J. Math., to appear). Similar arguments can be used to prove that $T^{k+1}(E)$ has infinite deficiency for other classes of operators including, in particular, compact nonnilpotent operators. (Received December 23, 1970.)

71T-B64. MUSTAFA A. AKCOGLU and LOUIS SUCHESTON, Ohio State University, Columbus, Ohio 43210. On operator convergence in Hilbert space and in Lebesgue space.

We are here concerned with relations between weak convergence, and strong convergence of averages of subsequences. Theorem 1. Let T be a linear operator on a Hilbert space H and assume either (A) $\|T\| \leq 1$, or (B) there is a constant $\delta > 0$ such that $\limsup \|T^n h\| > \delta$ for each h in H with $\|h\| = 1$. Then the following conditions (i) and (ii) are equivalent: (i) T^n converges weakly; (ii) $n^{-1}(T^{k_1} + \ldots + T^{k_n})$ converges strongly for every strictly increasing sequence of positive integers k_i. The main result is the following: Theorem 2. Let T be a linear operator on L_1 of a measure space, with $\|T\| \leq 1$. Then again the conditions (i) and (ii) are equivalent. The proof uses the Hilbert space result, and also a discussion of the existence of positive fixed points for the modulus operator of T. One obtains as Corollary. Let T be a positive linear contraction operator on L_1. The following are equivalent: (iii) $T^n f$ converges weakly to zero for every f in L_1 with zero integral; (iv) $n^{-1}(T^{k_1} f + \ldots + T^{k_n} f)$ converges strongly to zero for every f in L_1 with zero integral and every strictly increasing sequence of positive integers k_i. (Paper to appear in the volume of Periodica Mathematica Hungarica devoted to the memory of Alfred Renyi.) (Received December 29, 1970.)

71T-B65. TSOY-WO MA, University of Western Australia, Nedlands, Western Australia, 6009. Homotopy extension theorem for set-valued compact fields in locally convex spaces.

The Homotopy Extension Theorem III 2.2 (see Granas, Rozprawy Mat. 30(1962), 1-89) is extended from single-valued to set-valued maps. Influenced by Leray-Schauder, a different notion of singularity for set-valued compact fields is introduced. As applications, sweeping theorem, Borsuk-Ulam's Theorem, invariance of domains and fixed point theorems for set-valued maps are derived. The crucial point is to generalise Dugundji's Extension Theorem (Pacific J. Math. 1(1951), 353-367) from single-valued to set-valued maps. (Received January 4, 1971.)

71T-B66. GARY M. SAMPSON, California Institute of Technology, Pasadena, California 91109. Sharp estimates for convolutions in terms of decreasing functions.

By the convolution product of two Lebesgue measurable functions f and g we mean $(f*g)(x) = \int_{-\infty}^{+\infty} f(t)g(x-t)dt$. We obtain new sharp inequalities for $\int_0^u g_1 \cdots g_n$ where each $g_i = f_{i1} * \ldots * f_{in}$ with applications to Fourier transforms. (Received November 23, 1970.)

71T-B67. HEINRICH W. GUGGENHEIMER, Polytechnic Institute of Brooklyn, Brooklyn, New York 11201. Geometric theory of differential equations. V: Two point boundary value problems.

We consider nth order homogeneous linear equations defined on the real axis and boundary value problems $x^{(j)}(v) = \lambda x^{(j)}(u)$, $j = 0, 1, \ldots, n-1$. Let x_j be the solution of the DE given by $x_j^{(k)}(u) = \delta_j^k$ and W^u the Wronskian of the x_j's. (1) Nontrivial solutions exist only for λ an eigenvalue of $W^u(v)$. (2) For $n = 2$, $x'' + px = 0$, existence of a solution for λ implies the same for λ^{-1}. (3) Let $\varphi(u)$ be the conjugate point of u and $\chi(u) = \inf t$, $t > \varphi(u)$, $x_1'(t) > 0$ or $x_0(t) > 0$. For $p > 0$, $\lambda > 0$, no nontrivial solution can exist for $u < v \leq \chi(u)$. (4) If the boundary value problem for $x''+px=0$, $p>0$, has a nontrivial solution for $\lambda_0>1$ on $[u,v]$, then for any λ, $1\leq\lambda\leq\lambda_0$, there exists a v^* in $[u,v]$ so that the problem for λ has a nontrivial solution on $[u,v^*]$. (5) For $p>0$, $\lambda=\pm 1$, fixed u, there are nontrivial solutions only for a discrete set of v's. For $\lambda\neq 1$, almost always there are solutions only for a nowhere dense set of v's. (6) For $n>2$ and all u, v, there are nontrivial solutions satisfying n-1 of the n-boundary conditions for $\lambda=1$. (Received January 11, 1971.)

71T-B68. R. P. SINGAL, Thapar Institute of Engineering and Technology, Patiala, India. Integral expressions for Kampe' de Feriet hypergeometric function of 3rd order.

Making use of the fractional derivative and fractional integration by parts for functions of one and two variables defined by Erdelyi [Quart. J. Math. 10(1939), 176-189] and Koschmieder [Acta Math. 79(1947), 241-254] respectively, integral expressions for $F^{0,3}_{1,1}(x,y)$ are obtained, where $F^{0,3}_{1,1}(x,y)$ [P. Appell et J. Kampe' de Feriet, "Fonction hypergeometriques et hyperspheriques," Gauthier-Villars, Paris, 1926] is a Kampe' de Feriet hypergeometric function of order 3. In the end these integral expressions are obtained by another method, independent of fractional integration by parts, also. (Received January 7, 1971.) (Author introduced by Dr. Brij M. Nayar.)

71T-B69. CHARLES CHENEY, Indiana State University, Terre Haute, Indiana 47809. Mappings on B*-algebras. Preliminary report.

Let M and N be B*-algebras, with identity, with $N\subseteq M$. For notation, terminology, and related results, see Abstract 69T-B24, these *Notices* 16(1969), 410 and Abstract 664-4, these *Notices* 16(1969), 501. Theorem. Suppose that $\{\sigma_\alpha\}$ and $\{\rho_\alpha\}$ are two complete collections of positive linear functionals on M, such that for every α there is a constant $M_\alpha>0$ such that $\sigma_\alpha(x^*x)\leq M_\alpha\rho_\alpha(x^*x)$ for all x in M. Then under additional hypotheses on the two collections of functionals, there exist complete normed algebras N_α and expectation-like mappings $\varphi_\alpha : M\to N_\alpha$ such that N is dense in each N_α and $\sigma_\alpha(ax)=\rho_\alpha(a\varphi_\alpha(x))$ for all $a\in N$ and $x\in M$. This can be considered as a Radon-Nikodym theorem for positive linear functionals. Corollary. Let M and N be B*-algebras with identity and ρ and σ positive linear functionals on M and N, respectively. Let $T:M\to N$ be a continuous positive hermitian linear map and suppose that there is a constant $K>0$ such that $\sigma(T(x^*x))\leq K\rho(x^*x)$ for all $x\in M$. Then if ρ and σ satisfy certain conditions, then there exists an algebra L and an expectation-like map $\varphi : M\to L$ such that $\sigma(T(ax))=\rho(a\varphi(x))$ for all $a\in L$ and $x\in M$. (Received January 11, 1971.)

71T-B70. SHAWKY E. SHAMMA, University of West Florida, Pensacola, Florida 32504. On the asymptotic solutions of a generalized eigenvalue problem.

Previously [see Abstract 672-654, these *Notices* 17(1970), 271, and SIAM J. Appl. Math. 20(1971)] we considered solutions $u_n(x,y)$ of Laplace's equation, and of bounded gradient, in the interior and exterior of a smooth closed plane curve c, and coupled acron c by the jump-conditions $[u]=0$, $[\partial u/\partial\nu]=\lambda_n gu$, where g is a sufficiently smooth, positive, periodic and prescribed function of arc-length, and u_n,λ_n are eigenfunctions and eigenvalues to be determined. It is shown that $\lambda_n=O(n)$ and u_n is trigonometric. In this note we show that the asymptotic behavior of the positive eigenvalues and the corresponding normalized eigenfunctions is specified by the following two sequences: $\lambda_{2n} = 4n\pi/J + \epsilon_n$, $u_{2n}(s) = \sqrt{2/J}\cos(2n\pi\xi + \varkappa_n) + r_n(s)$, $\lambda_{2n+1} = 4n\pi/J + \epsilon'_n$, $u_{2n+1}(s) = \sqrt{2/J}\sin(2n\pi\xi + \varkappa_n) + r'_n(s)$, $n = N, N+1, \ldots$, where N is a chosen sufficiently large positive integer, $\varkappa_n$ are constants independent of s, ϵ_n and $\epsilon'_n \to 0$ as $n \to \infty$, $r_n(s)$ and $r'_n(s) \to 0$ uniformly as $n \to \infty$ and $\xi = [\int_0^s g(t)dt/\int g(t)dt]$. (Received January 12, 1971.)

71T-B71. EDMUND E. GRANIRER, University of British Columbia, Vancouver 8, British Columbia, Canada. Exposed points for the set LIM for locally compact groups and semigroups. Preliminary report.

Theorem. Let G be a separable amenable locally compact noncompact group. Let T LIM denote the set of topological LIM's on $L_\infty(G)$. Let $f_n \in L_\infty(G)$, α_n reals. Then the set $M = \{\Psi \in \text{T LIM};\ \Psi f_n = \alpha_n,\ n = 1, 2, \ldots\}$ is empty or nonnorm separable (in particular M has no exposed points). Theorem. Let S be a countable left amenable semigroup $K \subset \beta S$, $M_K = \{\Psi \in \text{LIM};\ \text{support } \Psi \subset K\}$. Then M_K has an exposed point for some compact Baire set K iff S contains a finite left ideal. M_K has an exposed point for some compact S-invariant $K \subset \beta S$ iff $S/(r)$ is a finite group (S need not contain a finite left ideal) ($a, b \in S$, $a(r)b$ iff $as = bs$ for some $s \in S$), which generalizes a result of Ching Chou. The results are in terms of representations of S on locally convex spaces and have wider applications. (Received January 14, 1971.)

71T-B72. GÜNTHER W. GOES, Illinois Institute of Technology, Chicago, Illinois 60616. Representation of null sequences by C_α-summable sequences. Preliminary report.

The following theorem is proved: Let $\alpha > -1$ and let $x = (x_k)$ be a complex sequence. Then $k^{-\alpha}x_k \to 0$ $(k \to \infty)$ if and only if there exist complex sequences $a = (a_k)$ and $b = (b_k)$ such that $x = a+b$ and the Cesàro-sums of order α, $(C, \alpha) - \sum_{k=1}^{\infty} a_k$ and $(C, \alpha) - \sum_{k=1}^{\infty} (-1)^k b_k$ exist. This statement supersedes the theorem in "Some representations of null sequences" (Abstract 71T-B24, these *Notices* 18(1971), 254). The proof is based mainly on the fact that a bounded sequence $x = (x_k)$ together with the sequence $((-1)^k x_k)$ are sequences of bounded variation of order $(\alpha + 1)$ if and only if $\sum_{k=1}^{\infty} k^{\alpha}|x_k| < \infty$. (Received January 18, 1971.)

71T-B73. LEONARD Y. H. YAP, University of Singapore, Singapore 10, Singapore. On Fréchet subalgebras of the group algebras. Preliminary report.

Let G be a nondiscrete locally compact unimodular group. Various classes of subalgebras of $L_1(G)$ are studied. We give some sample results here. Let $\{1\} \subsetneq P \subset [1, \infty)$ and for $p \in P$ let $A_p(G) = L_1(G) \cap L_p(G)$ with the norm $\|f\| = \|f\|_1 + \|f\|_p$. Let $A(P, G) = \bigcap \{A_p(G) : p \in P\}$ with the projective topology. Theorem 1. $A(P, G)$ is a Fréchet algebra under convolution; $A(P, G)$ is a Segal algebra if and only if the convex hull of P is closed and bounded. Theorem 2. $A(P, G)$ fails to have the factorization property. Corollary. $A(P, G)$ contains a maximal ideal which is neither prime, regular, nor closed. The last two facts generalize some results of the author (Studia Math. 35(1970), 165-175). (Received January 18, 1971.)

71T-B74. HARI M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada. On the product of two Laguerre polynomials. Preliminary report.

Making use of some of his earlier results [H. M. Srivastava, "An extension of the Hille-Hardy formula", Math. Comp. 23(1969), 305-311; see also Abstract 68T-449, these *Notices* 15(1968), 634-635], which generalize the well-known Hille-Hardy formula for the Laguerre polynomials, the author derives here elegant expressions for the product of two Laguerre polynomials as finite series involving these polynomials. It is also shown that one of the results obtained in this paper can be applied to deduce a generalization of a convolution type theorem for the inverse Weierstrass-Laguerre transform proved recently by the author [H. M. Srivastava, "The Weierstrass-Laguerre transform," Proc. Nat. Acad. Sci. U.S.A. 68(1971), to appear]. (Received January 18, 1971.)

71T-B75. MAXWELL O. READE, University of Michigan, Ann Arbor, Michigan 48104 and TOSHIO UMEZAWA, Saitama University, Urawa, Japan. Univalent analytic functions convex in a direction. Preliminary report.

Let $f(z) = z + a_2 z^2 + \dots$ be analytic in the unit disc Δ, with $[f(z)f'(z)/z \neq 0]$ there, and let α denote a real number. In this note, the influence of conditions of the form (I) $\mathrm{Re}\,(1 + z f''(z)/f'(z)) \geqq \alpha$, (II) $\mathrm{Re}\,(1 + z f''(z)/f'(z)) \leqq \alpha$, (III) $\mathrm{Re}\,(z f'(z)/f(z)) \geqq \alpha$, and (IV) $\mathrm{Re}\,(z f'(z)/f(z)) \leqq \alpha$ on the univalence of $f(z)$ in Δ are studied. A typical result is the following one. Theorem. Each function $f(z)$ that satisfies (I) in Δ is univalent there if and only if $-1/2 \leqq \alpha \leqq 3/2$. This result is sharp. Moreover, each $f(z)$ satisfying (I) in Δ, with $-1/2 \leqq \alpha \leqq 3/2$, is univalent and convex in one direction (at least). The results contained in this note complement earlier ones due to Ogawa [J. Nara Gakugei Univ. Nat. Sci. 10(1961), 7-12] and Umezawa [J. Math. Soc. Japan 4(1952), 194-202]. (Received January 20, 1971.)

71T-B76. G. D. LAKHANI and R. S. L. SRIVASTAVA, Indian Institute of Technology, Kanpur, India. Factorization and invariant subspaces. Preliminary report.

Sz.-Nagy and Foias proved ["Sur les contractions de l'espace de Hilbert, IX," Acta Sci. Math. (Szeged) 25(1964), 283-312] that if T is a linear transformation (bounded by 1) defined in a Hilbert space of dimension greater than 1 such that the sequences $\{T^n\}$ and $\{T^{*n}\}$ converge strongly to zero as $n \to \infty$, then T has a nontrivial invariant subspace. Using a different technique, it is shown in this paper that the same conclusion can be obtained if either of the sequences converge to zero strongly. (Received January 20, 1971.) (Author introduced by Professor S. A. Naimpally.)

Applied Mathematics

71T-C3. JOHN E. OSBORN, University of Maryland, College Park, Maryland 20742. On the condition numbers of a sequence of nonselfadjoint eigenvalue problems. Preliminary report.

Let L be the ordinary differential operator on $[a,b]$ defined by $Lx=(-1)^{m/2}x^{(m)}$, where m is even, and a set of selfadjoint Sturm type boundary conditions. The domain of L is the appropriate linear manifold in $L_2[a,b]$; let $(\cdot\,,\cdot)$ denote the L_2 inner product. Assume L is positive definite. L has eigenvalues $\mu_1, \mu_2, \cdots$ and orthonormal eigenvectors $x_1, x_2, \cdots$. Let A be a (not necessarily selfadjoint) differential operator of order j where $j \leq m-2$. There will be a constant c such that the eigenvalues of $L+A$ are contained in the circles $\{\lambda \mid |\lambda - \mu_k| \leq c\mu_k^{j/m}\}$; assume these circles are mutually disjoint. For $n=1,2,\ldots$ define $(L+A)_n = P_n(L+A)|_{v_n}$ where $v_n = \operatorname{span}(x_1, \cdots, x_n)$ and P_n is the projection onto v_n. Let $z_1(n), \cdots, z_n(n)$ be the unit eigenvectors of $(L+A)_n$ and let Λ_n and λ_n be the greatest and least eigenvalue respectively of the Gram matrix $(z_i(n), z_j(n))$. Theorem. $(\Lambda_n/\lambda_n)^{1/2}$ is bounded in n. $(\Lambda_n/\lambda_n)^{1/2}$ is the condition number of the eigenvalue problem $(L+A)_n x = \lambda x$. (Received November 3, 1970.)

71T-C4. GORDON H. BRADLEY, Administrative Sciences Department, Yale University, New Haven, Connecticut 06520. Transformation of integer programs to single constraint integer programs.

It is shown that any bounded integer linear programming problem can be transformed to an equivalent integer linear programming problem with the same number of variables. Let A be an $(m) \times (n)$ integer matrix and b, d integer vectors. Theorem. There exists an integer vector h such that the set of solutions to the system of constraints $Ax=b$, $0 \leq x \leq d$ and x integer, is identical to the set of solutions to $h^TAx=h^Tb$, $0 \leq x \leq d$ and x integer. Theorem. There exist an integer vector f and integers π_1 and π_2 such that the set of solutions to minimize c^Tx subject to $Ax=b$, $0 \leq x \leq d$ and x integer, is identical to the set of solutions to minimize f^Tx subject to $\pi_1 \leq f^Tx \leq \pi_2$, $0 \leq x \leq d$ and x integer; further, for any feasible solution $\bar{x}$, $c^T\bar{x}=f^T\bar{x}$. The results are extended to problems with nonlinear constraints. (Received November 20, 1970.)

Geometry

71T-D4. CHARLES FEFFERMAN and MAX A. JODEIT, JR., University of Chicago, Chicago, Illinois 60637. A measure inequality for convex sets. Preliminary report.

Let C be a closed convex and symmetric $(C=-C)$ set in R^n, and let T be a linear transformation with Euclidean norm no greater than one. The sets C and TC (the image of C under T) meet the unit sphere in sets A, B respectively. The innocent-looking inequality of the title, namely that the surface area of A is at least that of B, is proved using Fourier transform methods and Anderson's theorem (Proc. Amer. Math. Soc. 6(1955), 170). (Received October 30, 1970.)

71T-D5. JOHN DeCICCO, Illinois Institute of Technology, Chicago, Illinois 60616 and ROBERT V. ANDERSON, Université du Québec à Montréal, Montréal 110, Québec, Canada. Affine and projective cartograms of a surface.

A cartogram T between two surfaces Σ and $\bar{\Sigma}$ of a Euclidean space E_3 is said to be projective if and only if it converts every geodesic of Σ into a geodesic of $\bar{\Sigma}$. Necessary and sufficient conditions are derived for a cartogram to be projective, in terms of the affine connections of Σ. If C_1 and C_2 are two curves tangent at a point P of Σ such that their order of contact is one, then the ratio of their scalar geodesic curvatures is invariant under a projective cartogram T. This is an extension of the theorem of Memhke-Segré. An affine cartogram T is one such that whenever an arbitrary vector λ undergoes a parallel displacement along a curve C of Σ, then, under T, the image vector $\bar{\lambda}$ undergoes a parallel displacement along the image $\bar{C}$ of C. An Appell transformation between two surfaces is depicted by an affine cartogram T if and only if the rate of change of the time t is constant. A cartogram T between two surfaces is both projective and conformal if and only if it is a homothetic cartogram T. If Σ is a developable surface and is related to $\bar{\Sigma}$ by an affine cartogram T, then $\bar{\Sigma}$ is also developable. (Received January 12, 1971.)

71T-D6. RICHARD H. ESCOBALES, JR., University of Notre Dame, Notre Dame, Indiana 46556. Hypersurfaces and Riemannian submersions. Preliminary report.

Let M be a Riemannian manifold of dimension n+p and B a Riemannian manifold of dimension n. Let f be a Riemannian submersion from M onto B. Assume the fibers are minimal submanifolds of M. Under these conditions the following proposition obtains: A hypersurface P of B has constant mean curvature if and only if the hypersurface $f^{-1}(P)$ of M has constant mean curvature. (Received January 18, 1971.)

Logic and Foundations

71T-E9. CARL G. JOCKUSCH, JR., University of Illinois, Urbana, Illinois 61801 and ROBERT I. SOARE, University of Illinois at Chicago Circle, Chicago, Illinois 60680. Degrees of members of Π_1^0 classes.

This paper deals with the degrees of members of Π_1^0 classes of sets or functions which lack recursive members. For instance, it is shown that if $\underset{\sim}{a}$ is a degree and $\underset{\sim}{0} < \underset{\sim}{a} \leqq \underset{\sim}{0}'$, then there exists a Π_1^0 class of sets which has a member of degree $\underset{\sim}{a}$ but none of degree $\underset{\sim}{0}$. By way of contrast there is no Π_1^0 class of functions which has members of all nonzero r.e. degrees but no recursive members. Furthermore, each nonempty Π_1^0 class of sets has a member of r.e. degree but not necessarily of r.e. degree less than $\underset{\sim}{0}'$. As corollaries results are derived about degrees of theories and degrees of models such as: (1) There is an axiomatizable, essentially undecidable theory with a complete extension of minimal degree; (2) (Scott-Tennenbaum): There is a complete extension of Peano arithmetic of degree $\underset{\sim}{0}'$ but none of r.e. degree $\underset{\sim}{<} \underset{\sim}{0}'$; (3) There is no nonstandard model of Peano arithmetic of r.e. degree $\underset{\sim}{<} \underset{\sim}{0}'$. Finally the recursion theorem is applied to yield new information about standard constructions such as Yates' simple nonhypersimple set S recursive in a given nonrecursive r.e. set A. The recursion theorem shows that A is automatically recursive in S, so that one need not code A into S as Yates does. (Received November 5, 1970.)

71T-E10. CHRISTOPHER J. ASH, Monash University, Clayton, Victoria 3168, Australia. Undecidable $\aleph_0$-categorical theories.

Let M be any set of positive natural numbers. Let L be the language with the single binary predicate letter P. Theorem. There is an $\aleph_0$-categorical theory T in the language L with the same degree of undecidability as M. (The problem of finding such theories was posed by Grzegorczyk, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 16(1968), 687.) A model for such a theory may be obtained as follows. Consider the class, K, of all structures $\langle A, R, S_n\rangle_{n\in M}$, where R, S_n are binary, n-ary relations on A respectively, for which R is irreflexive and symmetric and, for each n, S_n is symmetric and only holds for n-tuples which constitute n-element subsets of A which are maximal subsets with respect to the property that every two distinct elements are related by R. The class K contains a denumerable universal homogeneous structure $\langle A', R', S'_n\rangle_{n\in M}$ whose theory is recursive in M and is $\aleph_0$-categorical. The same remarks apply to the theory of the reduct $\langle A', R'\rangle$. $n \in M$ iff $\langle A', R'\rangle$ has a subset of cardinal n which is maximal w.r.t. the property that every two distinct elements are related by R'. This theory thus has the required properties. A similar construction yields a p.o. set with an $\aleph_0$-categorical theory of the same degree as M. (Received November 16, 1970.)

71T-E11. CHARLES H. APPLEBAUM, Bowling Green State University, Bowling Green, Ohio 43403. A result for pi-groups.

Let $\epsilon = \{0,1,2,\dots\}$. For $\alpha,\beta \subset \epsilon$, we say α is recursively equivalent to β $[\alpha \simeq \beta]$ if there exists a function f mapping α one-to-one onto β and f has a one-to-one partial recursive extension. For $\alpha \subset \epsilon$, $P(\alpha)$ is the ω-group of Gödel numbers of finite permutations of α, as introduced by Hassett ["Recursive equivalence types and groups," J. Symbolic Logic 34(1969), 13-20]. For $\alpha,\beta\subset\epsilon$, $P(\alpha) \cong_{\omega} P(\beta)$ if there exists a group isomorphism ψ between $P(\alpha)$ and $P(\beta)$ such that ψ has a one-to-one partial recursive extension. Hassett proved in the same paper that for $\alpha,\beta\subset\epsilon$, $\alpha\simeq\beta \Rightarrow P(\alpha) \cong_{\omega} P(\beta)$ and this implication can be reserved if α and β are nonempty isolated sets. It can now be stated that the converse is true in the general case. Theorem. Let α and β be nonempty subsets of ϵ. Then $\alpha\simeq\beta \Leftrightarrow P(\alpha) \cong_{\omega} P(\beta)$. (Received November 17, 1970.)

71T-E12. BRUNO J. SCARPELLINI, Mathematics Institute, University of Basel, Rheinsprung 21, 4000 Basel, Switzerland. A system of classical analysis.

We use the language of second order arithmetic. (x,y) represents the pairing function $\frac{1}{2}((x+y)^2+3x+y)$. $\alpha(x,y)$ is short for $\alpha((x,y))$. Define prim. rec. functors Δ_1, Δ_2 as follows: (1) $\Delta_1(\alpha,x)(s,t)$ is $\alpha(s,t)$ if $s<x$, is 0 if $x \leqq s$. (2) $\Delta_2(\alpha,x,\beta)(s,t)$ is $\alpha(s,t)$ if $s<x$, is $\beta(t)$ if $s=x$, is 0 if $x<s$. $\bar{\alpha}(x)$ and $\bar{\alpha}(x) * \beta$ are short for $\Delta_1(\alpha,x)$ and $\Delta_2(\alpha,x,\beta)$ resp. Let $M(\alpha)$, $R(\alpha,x)$, $A(\alpha,x)$ be three formulas with x not free in M; they may contain parameters. $\bar{M}(\alpha,x)$ is short for $(s)_{\leqq x} M(\lambda y\, \alpha(s,y))$. $I_1, I_2, I_3, B_1, B_2, B_3$ are in that order:

$\bar{M}(\alpha, x) \to R(\bar{\alpha}(x), x) \vee \neg R(\bar{\alpha}(x), x)$, $\bar{M}(\alpha, x+1) \wedge R(\bar{\alpha}(x), x) \to R(\bar{\alpha}(x+1), x+1)$, $(x)\bar{M}(\alpha, x) \to (Ex)R(\bar{\alpha}(x), x)$, $\bar{M}(\alpha, x) \wedge R(\bar{\alpha}(x), x) \to A(\bar{\alpha}(x), x)$, $\bar{M}(\alpha, x) \wedge (\beta)\ (M(\beta) \to A(\bar{\alpha}(x) * \beta, x+1)) \to A(\bar{\alpha}(x), x)$, $(\alpha, x)(\bar{M}(\alpha, x) \to A(\bar{\alpha}(x), x))$. Let Z_0 be intuitionistic number theory provided with the axiom of choice. The system EBI has the same axioms as Z_0, the rules of Z_0 plus an additional rule EBIR^*: for any M, R, A, if I_1, I_2, I_3, B_1, B_2 have already been proved then we can infer B_3. Theorem. EBI is as strong as classical analysis. The proof is via a constructive model for barrecursion of higher types whose construction is performed within EBI. (Received November 17, 1970.)

71T-E13. BARUCH GERSHUNI, Bloch Street 38, Tel Aviv, Israel. The action of the $\bigcup$-operator upon plural classes.

We consider only plural classes P, the elements (elements of the first degree), the elements of the elements (elements of the second degree), shortly the generalized elements of which are plural classes or individua. The element of an individuum is itself. We form, according to the usual (conventional) definition of the operator $\bigcup$, the sequence of plural classes $P_{01} P_1 = \bigcup P_{01} P_2 = \bigcup P_1 = \bigcup^2 P_0$ .... The class P_1 is called the reduced of the first order of P_0; P_2 is called the reduced of the 2nd order of P_0; a. s. o. Axiom. For any class P_0 there is a natural number n such that the reduced of the nth order $P_n = \bigcup^n P_0$ has mere individuals as elements separated from each other by simple commas. There is a minimal such number m. Then P_{m-1} has at least two elements separated from each other by a semicolon; a. s. o. It is valid: $\ell = \ell' = m - k + 1$ where ℓ is the highest number of points (incl. the comma) of an ETT comma junctive appearing in P_k and ℓ' is the highest order of elementhood in P_k. ℓ is the degree of this junctive. The formal definition of the operator $\bigcup$ is: the operator lowering ℓ by one unity and leaving the other comma junctives of P unaltered. The operator $\bigcup$ is called the reduction operator. The old name of $\bigcup$, the "union operator", seems not to be appropriate. (Received November 16, 1970.)

71T-E14. STEPHEN H. HECHLER, Case Western Reserve University, Cleveland, Ohio 44106. On classifying almost-disjoint families.

A family of infinite subsets of N (the set of natural numbers) will be called almost disjoint iff any two of its members have a finite intersection. Such a family $\mathfrak{F}$ will be said to (strongly) separate over a finite decomposition $\mathcal{D}$ of N iff the set $\{F \in \mathfrak{F} : \exists D \in \mathcal{D}\ F \subseteq D\}$ is (infinite) nonempty. A family will be called (strongly) n-separable iff it (strongly) separates over every n element decomposition of N. Theorem. If a family is n-separable, it is strongly n-1-separable. Thus given any family $\mathfrak{F}$ we may distinguish three cases: (1) For every $n \in N$, $\mathfrak{F}$ is strongly n-separable. (2) There exists an $n \in N$ such that $\mathfrak{F}$ is strongly m-separable for all $m \leqq n$, but is non n+1-separable. (3) There exists an $n \in N$ such that $\mathfrak{F}$ is n+1-separable but not strongly n+1-separable and is therefore not n+2-separable but is strongly m-separable for all $m \leqq n$. Theorem. If every infinite maximal almost-disjoint family has cardinality $2^{\aleph_0}$, then there exist maximal almost disjoint families of each of the above types (for all $n \in N$). We also look at various generalizations and consequences of these notions e.g., Theorem. If $\mathfrak{F}$ is strongly 2-separable then every maximal pairwise-disjoint family $\mathcal{J} \subseteq \mathfrak{F}$ is infinite. Finally, to obtain an example of a "very nonseparable" family, we prove that if Martin's axiom holds, then there exists a maximal almost-disjoint family $\mathfrak{F}$ and a bijection φ from N onto Q (the set of rationals) such that for each $F \in \mathfrak{F}$ the set $\varphi[F]$ is everywhere dense. (Received November 27, 1970.)

71T-E15. SAHARON SHELAH, University of California, Los Angeles, California 90024. Two-cardinal and power like models: compactness and large group of automorphisms.

Let Q_i, $i < \alpha < \omega_1$, Q^i, $i < \beta < \omega_1$, be one-place designated predicates and $<$ a two-place designated predicate, and λ_i, $i < \alpha$, λ^i, $i < \beta$, be infinite cardinals. Let K be the class of models M whose language include the designated predicates, $|Q_i^M| = \lambda_i$, and $<^M$ order $(Q^i)^M$ in a λ^i-like order. Definition. K is μ-compact if: if $|T| \leq \mu$, T is a set of sentences, and every finite subset of T has a model in K, then T has a model in K. Theorem 1 (Fuhrken). If for every i, $(\lambda_i)^{\aleph_0} = \lambda_i$, $\mu_n < \lambda^i \Rightarrow \Pi_{n<\omega}\mu_n < \lambda^i$ then K is $\aleph_0$-compact. Theorem 2. If for every i, $\mu \leq \lambda_i$, $\mu < \lambda^i$ and K is $\aleph_0$-compact, then K is μ-compact (will essentially appear in Israel J. Math). Theorem 3. If K is μ-compact, $|T| \leq \mu$, T has a model in K, $|A| \leq \mu$, $<^A$ order A, then T has a model M in K, $A \subset |M|$, and every automorphism of $(A, <^A)$ can be extended to an automorphism of M. This generalizes Ebbinghaus, Abstract 70T-E68, these *Notices* 17(1970),837 which generalizes the results of Ehrenfeuch and Mostowski). Moreover in the model M at most $|T| + 2^{\aleph_0}$ types are realized by finite sequences of elements. (This generalizes a result of Ehrenfeucht.) (Received December 7, 1970.) (Author introduced by Professor Chen Chung Chang.)

71T-E16. JOHN MYHILL, State University of New York at Buffalo, Amherst, New York 14226. Reducibility and R.E.T.'s.

Notation is as in Dekker and the author's monograph "Recursive equivalence types", Univ. California Publ. Math. 3(1960). α and β are infinite recursively enumerable sets. In the monograph (p.121) it is proved that Req $\alpha' \leq$ Req $\beta' \rightarrow \alpha R_1 \beta$. We prove (1) the converse proposition is false; (2) Req $\alpha' \leq$ Req $\beta' \Leftrightarrow \alpha R_1 \beta$ by a function with recursive range; (3) $\alpha R_m \beta \Leftrightarrow$ R Req $\alpha' \leq$ R Req β'. Proofs of (2) and (3) are elementary; (1) follows from (2) by a result of R. W. Robinson (see Rogers "Theory of recursive functions", McGraw-Hill, New York, 1967, p. 101). (Received December 14, 1970.)

71T-E17. KENNETH KUNEN, University of Wisconsin, Madison, Wisconsin 53706. A partition theorem. Preliminary report.

Let κ be a real-valued measurable cardinal and μ a normal measure on κ. Let $A \subseteq [\kappa]^2$. Then either (i) there is a subset, $X \subseteq \kappa$, such that $\mu(X) > 0$ and $[X]^2 \subseteq A$, or (ii) for all countable ordinals α, there is an $X \subseteq \kappa$ such that X has order type α and $[X]^2 \cap A = 0$. The proof uses a generalization of the zero-one law. (Received December 14, 1970.)

71T-E18. KENNETH A. BOWEN, Syracuse University, Syracuse, New York 13210. Cut elimination in transfinite type theory.

Using ZF as metalanguage, for any ordinal $\mathcal{O} \geq 1$, a system $TT^{\mathcal{O}}$ of monadic cumulative, simple transfinite type theory is formulated in Gentzen's sequentzen style. Theorem 1. $TT^{\mathcal{O}}$ is consistent. A nonextensional semantics is provided for each $TT^{\mathcal{O}}$. Theorem 2. $TT^{\mathcal{O}}$ is complete relative to the given semantics. A semivaluation is a mapping from a subset of the formulae of $TT^{\mathcal{O}}$ to $\{t, f\}$ which reflects the usual definition of truth and is defined on all subformulae of formulae in its domain; it is total if it is defined on all formulae.

Theorem 3. If every semivaluation of $TT^{\mathcal{O}}$ is extendable to a total valuation, the cut rule is redundant in $TT^{\mathcal{O}}$.

Theorem 4. Cut elimination holds in $TT^{\mathcal{O}}$. (Received December 15, 1970.)

71T-E19. PETER H. KRAUSS, State University of New York, New Paltz, New York 12561. Universally complete theories of algebras. I. Preliminary report.

For unexplained notation see Abstract 70T-E48, these *Notices* 17(1970), 831. Consider algebras $\mathfrak{A}$ with finitely many operations, and the corresponding first order language L. A set Σ of sentences (of L) is called universally complete if (i) Σ is significantly consistent and (ii) for every universal sentence σ, either $\Sigma \models \sigma$ or $\Sigma \cup \{\sigma\}$ is not significantly consistent. Theorem 1. Let Σ be a significantly consistent set of sentences. Then the following are equivalent: (a) Σ is universally complete. (b) For all $\mathfrak{A}, \mathfrak{B} \in \mathrm{Mod}\,\Sigma$, if $\bar{\bar{\mathfrak{A}}}, \bar{\bar{\mathfrak{B}}} \geq \omega$ then $\mathfrak{A} \equiv_{\forall} \mathfrak{B}$, and for any $\mathfrak{A} \in \mathrm{Mod}\,\Sigma$, where $\bar{\bar{\mathfrak{A}}} < \omega$, there exists $\mathfrak{B} \in \mathrm{Mod}\,\Sigma$, where $\bar{\bar{\mathfrak{B}}} \geq \omega$, such that $\mathfrak{A} \in \mathrm{IS}\mathfrak{B}$. (c) For all $\mathfrak{A}, \mathfrak{B} \in \mathrm{Mod}\,\Sigma$, if $\bar{\bar{\mathfrak{A}}}, \bar{\bar{\mathfrak{B}}} \geq \omega$ then $\mathfrak{A} \equiv_{\forall} \mathfrak{B}$, and $\mathrm{Mod}\,\Sigma$ has the embedding property. Corollary 2. Let Σ be a significantly consistent set of universal sentences. Then Σ is universally complete iff for all $\mathfrak{A}, \mathfrak{B} \in \mathrm{Mod}\,\Sigma$, if $\bar{\bar{\mathfrak{A}}}, \bar{\bar{\mathfrak{B}}} \geq \omega$ then $\mathfrak{A} \equiv_{\forall} \mathfrak{B}$, and there exists $\mathfrak{A}$ such that $\bar{\bar{\mathfrak{A}}} \geq \omega$ and $\Sigma = \mathrm{Th}_{\forall}\mathfrak{A}$. Remark. The results of this note can be generalized to algebraic systems with both operations and relations. (Received December 18, 1970.)

71T-E20. GIORGIO M. GERMANO and ANDREA MAGGIOLO SCHETTINI, Laboratorio di Cibernetica del C.N.R., Naples, Italy. Eliminability of concluding formulas in Markov's normal algorithms.

Consider the functions which map natural numbers on natural numbers. Definition. The function $f(x_1, \ldots, x_k)$ is computable by the normal algorithm F, with exit i iff $F(*^1D^1_{n_1} \ldots *^1D^1_{n_k}) \simeq *^iD_{f(n_1, \ldots, n_k)}$, where D^i_n is the digit $I^i \ldots I^i O^i$ representing the number n in the alphabet $\{I^i, O^i\}$. Definition. The function $f(x_1, \ldots, x_k)$ is computable by the normal algorithm F iff there is an i such that f is computable by F with exit i. Theorem. For a function f the following conditions are equivalent: (a) f is partial recursive; (b) f is computable by some normal algorithm with exit 1; (c) f is computable by some normal algorithm; (d) f is computable by some normal algorithm without concluding formulas. The main part of the proof consists of stating that every partial recursive function is computable by some normal algorithm without concluding formulas. This is proved by defining a kind of programming language consisting of a branching algorithm, a connection algorithm, a loop algorithm and a juxtaposition algorithm, without concluding formulas. In this language it is easy to write program schemes which correspond to the defining schemes of recursive number theory. (Received December 21, 1970.)

71T-E21. LAWRENCE H. LANDWEBER and FREDERICK A. HOSCH, Department of Computer Sciences, University of Wisconsin, Madison, Wisconsin 53706. Finite shift solutions for sequential conditions. Preliminary report.

A decision procedure is given for determining whether or not a condition C(X,Y), stated in sequential calculus, admits an h-shift, but no (h+1)-shift solution for Y, for some finite h (Buchi and Landweber, "Solving sequential conditions," Trans. Amer. Math. Soc. 138(1969), 295-311). Here a subset of ω is interpreted as an

ω-sequence of members of $\{T, F\}$. An h-shift solution of C for Y then is an operator $Y = A(X)$ that can be presented in the form $Yt = F(\overline{X}(t-h))$, where Yt is the t-th element of the sequence Y, $\overline{X}(n) = \Lambda$ if $n < 0$, $\overline{X}(n) = X0, \ldots, Xn$ if $n \geqq 0$. The decision procedure involves showing that the question of whether or not the condition C has an h-shift solution for Y is equivalent to the question of whether or not a finite automata graph G is solvable with delay h. The procedure for deciding whether or not a finite automata graph is solvable with finite delay, as given by Even and Meyer in "Sequential boolean equations" (IEEE Trans. Computers C-18(1969), 230-240), can then be used. (Received December 21, 1970.)

71T-E22. DAVID ELLERMAN, Boston University, Boston, Massachusetts 02215. Ultraproducts of intuitionistic structures.

Given a set $\{\langle G_i, R_i, \models_i, P_i\rangle\}_{i \in I}$ of first order intuitionistic structures (also called Kripke structures; for notation see Fitting, "Intuitionistic logic model theory and forcing," North-Holland, 1969) and an ultrafilter F on I, the ultraproduct of the structures w.r.t. F is the structure $\langle G, R, \models, P\rangle$ where: (1) $\langle G; R\rangle = \Pi\langle G_i, R_i\rangle/F$ (classical ultraproduct), (2) for $\Gamma \in G$, $P(\Gamma) = \Pi P_i(\Gamma(i))/F$ (classical), and (3) for any formula $\varphi(x_1, \ldots, x_n)$ with all free variables displayed, $\Gamma \models \varphi((f_1), \ldots, (f_n))$ iff $\varphi((f_1), \ldots, (f_n)) \in \hat{p}(\Gamma)$ and $\{i : \Gamma(i) \models_i \varphi(f_1(i), \ldots, f_n(i))\} \in F$. For $\Gamma R \Gamma'$ the canonical embedding $P(\Gamma) \to P(\Gamma')$ is taken as the inclusion. The theorem that $\langle G, R, \models, P\rangle$ is a first order intuitionistic structure is the 'fundamental theorem' for intuitionistic ultraproducts. Now one may, for example, use these ultraproducts to prove the intuitionistic compactness theorem by generalizing the classical ultraproduct proof. (Received December 28, 1970.) (Author introduced by Professor Rohit J. Parikh.)

71T-E23. WILFRID A. HODGES, Bedford College, Regent's Park, London NW 1, England. Possible orderings of a set of indiscernibles.

Let A be an L-structure, $\varphi(v_0, \ldots, v_{n-1})$ a formula of L, X an infinite subset of A's domain, and $<$, $<'$ two linear orderings of X. Suppose that for every permutation $\pi \in S_n$, both $(X, <)$ and $(X, <')$ are sets of indiscernibles for the formula φ_π, i.e. $\varphi(v_{\pi(0)}, \ldots, v_{\pi(n-1)})$. Suppose also that increasing n-tuples in $(X, <)$ satisfy some but not all of the formulae φ_π, $\pi \in S_n$. Then there is a subset Y of X, with $|X - Y| \leqq \max(n + 3, 13)$, such that $(Y, <')$ comes from $(Y, <)$ by at most (i) reversing the ordering, and (ii) moving an initial segment to the end. (Received January 4, 1971.)

71T-E24. JEROME I. MALITZ and WILLIAM N. REINHARDT, University of Colorado, Boulder, Colorado 80302. A complete $L^Q_{\omega_1}$-theory having maximal models in every power admitting a maximal structure.

$L^Q_{\omega_1}$ is the extension of first order predicate calculus obtained by adding the quantifier "there exist at least ω_1 x's such that ...." We call $\mathfrak{A}$ $L^Q_{\omega_1}$-maximal iff $\mathfrak{A}$ has no proper extensions with the same true $L^Q_{\omega_1}$ sentences. Let M be the class of cardinals $\varkappa$ for which there is a maximal structure $\mathfrak{A}$ with countable type and power $\varkappa$. Let Sp(T) be the class of cardinals $\varkappa$ for which there is an $L^Q_{\omega_1}$-maximal model of T of power $\varkappa$. Theorem 1. There is a complete $L^Q_{\omega_1}$-theory T with countable type such that Sp(T) = M. Theorem 2 (G.C.H.). Let T be as in Theorem 1. Then T has an $L^Q_{\omega_1}$-maximal model in every uncountable power $\varkappa$ which is less than the first uncountable measurable cardinal and is not weakly compact. (Received January 4, 1971.)

71T-E25. WITHDRAWN.

71T-E26. KENNETH J. DANHOF, Southern Illinois University, Carbondale, Illinois 62901. Concepts in normal theories. Preliminary report.

For an elementary theory T, formulas of T define elementary concepts on the models of T. For A a model of T and c and d such concepts, we write $c \leq d(A)$ if each automorphism of dA is an automorphism of cA ($KdA \subseteq KcA$). $\leq$ induces a partial order on $C_d(A) = \{c;\ d \leq c(A)\}$. T is normal relative d if for models A, A' of T, dA = dA' and $A \cong A'$ imply KA = KA'. Employing results of Buchi (for T normal relative d and A a model of T, KA is a normal subgroup of KdA and $G_d(A) = KdA/KA$ is finite), we have: Theorem. (i) For T normal relative d and A a model of T, there is a lattice anti-isomorphism from $C_d(A)$ onto the subgroups of $G_d(A)$. (ii) If T is also complete, there is a model A of T such that $C_d(A)$ characterizes elementary definability (in the sense of Beth) in T down through d. This theorem has applications to the Galois theory of fields. (Received January 11, 1971.)

71T-E27. JOHN M. MacINTYRE, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. The nonaxiomatizability of the theory of the metadegrees.

For terminology see G. E. Sacks, "Post's problem, admissible ordinals, and regularity," Trans. Amer. Math. Soc. 124 (1966), 1-23. Definition. A set of recursive ordinals is generic iff it is generic with respect to Kreisel's forcing construction of a nonregular subgeneric set (see Sacks, p. 14). By generalizing and relativizing an argument due to Hugill and Lachlan we obtain: Theorem 1. If $\underline{a}$ is the metadegree of a generic set and D is a countable distributive lattice, then there is a metadegree $\underline{b}$ such that $\{\underline{c} \mid \underline{a} <_M \underline{c} <_M \underline{b}\}$ is order isomorphic to D. By the theory of the metadegrees we mean the set of first order sentences in the language with one predicate symbol $\leq$ (of two variables) which are true in the model whose domain is the set of all metadegrees where the predicate symbol $\leq$ is interpreted by $\leq_M$. Theorem 1 then implies: Theorem 2. The theory of the metadegrees is nonaxiomatizable. Furthermore, Theorem 3. For every countable admissible α, the theory of the α-degrees is nonaxiomatizable. (Received January 11, 1971.)

71T-E28. JAMES E. BAUMGARTNER, Dartmouth College, Hanover, New Hampshire 03755. A possible extension of Cantor's theorem on the rationals. Preliminary report.

A set A of real numbers is $\aleph_1$-dense if A has power $\aleph_1$ and between every two members of A there are exactly $\aleph_1$ members of A. Let (*) denote the assertion that any two $\aleph_1$-dense sets of reals are order-isomorphic. It follows from a result of Sierpinski that (*) fails very badly if $2^{\aleph_0} = \aleph_1$. The same method can be used to show that it is relatively consistent with ZFC that $2^{\aleph_0} > \aleph_1$ and (*) still fails very badly. Theorem 1. Assume $2^{\aleph_0} = \aleph_1$. If A and B are $\aleph_1$-dense sets of reals, then there exists a partial order P satisfying the countable chain condition such that the sentence "A and B are order-isomorphic" is forced (with P as the set of conditions). Theorem 2. It is relatively consistent with ZFC that $2^{\aleph_0} = \aleph_2$ and (*) holds. Theorem 2 is deduced from Theorem 1 using the same techniques Solovay and Tennenbaum ["Iterated Cohen extensions and Souslin's problem," to appear] used to show the consistency of Martin's axiom. It is not known whether Martin's axiom implies (*). (Received January 18, 1971.)

Statistics and Probability

71T-F2. A. K. GUPTA, University of Arizona, Tucson, Arizona 85721. On the test for reality of a covariance matrix in a certain complex Gaussian distribution. Preliminary report.

Goodman (Ann. Math. Statist. 34(1963), 152-176) has derived the complex Gaussian distribution of a complex matrix variate $\underset{\sim}{Z} = \underset{\sim}{X}_1 + i\,\underset{\sim}{X}_2$, $(i = (-1)^{1/2})$ with covariance matrix $\underset{\sim}{\Sigma} = \underset{\sim}{\Sigma}_1 + i\,\underset{\sim}{\Sigma}_2$ which is hermitian positive definite. The problem of testing the independence of $\underset{\sim}{X}_1$ and $\underset{\sim}{X}_2$ sets of variates (i.e. testing $H_0 : \underset{\sim}{\Sigma}_2 = \underset{\sim}{0}$) has been considered by Khatri (Ann. Math. Statist. 36(1965), 115-119). In this paper, we derive the exact distribution of the likelihood ratio criterion and tabulate the lower percentage points for $p = 4, 5$, where p is the number of variables, and for selected values of $N - q$. (Received November 9, 1970.) (Author introduced by Professor John L. Denny, Jr.)

71T-F3. JOHN S. KALME, U. S. Naval Academy, Annapolis, Maryland 21402. Approximation of signal quadrature components by a semigroup of operators.

Let a signal $x(u)$ be represented by quadrature components as $x_c(u)\cos(\omega_c u) + x_s(u)\sin(\omega_c u)$. $x_c(u)$ is the SSB representation of the signal $x(u)$. Let $\hat{x}(u) = \pi^{-1}\,\mathrm{PV}\int_{-\infty}^{\infty} x(w)(w-u)^{-1}dw$, $[V(t)f](u) = \pi^{-1}t\int_{-\infty}^{\infty} f(u-w)(t^2+w^2)^{-1}dw$. Then $V(t)$ defines a semigroup of operators on L^p. (See Abstract 70T-F10, these *Notices* 17 (1970), 676.) Let $z(u) = x(u) - i\hat{x}(u)$ be the analytic signal of $x(u)$. Then $x_c(u) = \mathrm{Re}[z(u)\exp(i\omega_c u)]$, $x_s(u) = \mathrm{Im}[z(u)\exp(i\omega_c u)]$. Theorem. Let $\{x(u)\}$ be a real stationary gaussian random process. Let $R(h) = E(X(u+h)X(u))$. Suppose $R(h)$ satisfies the conditions in the above abstract. Then $\exists$ a process $\{Y_c(u)\}$ equivalent to $\{X_c(u)\} \ni$ almost all trajectories of $\{Y_c(u)\}$ are continuous and a.s. if $Y_c(u) \in L^p$, then $\int_0^{\infty} t^{-1-\sigma p}(\int_{-\infty}^{\infty}|[V(t)Y_c](u) - Y_c(u)|^p du)dt < \infty$. Similar statements hold for $\{X(u)\}$ and $\{X_s(u)\}$. The above theorem shows how close signals can be approximated by a semigroup of operators, and generalizes some classical theorems in approximation due to D. Jackson and S. Bernstein. (Received January 11, 1971.)

Topology

71T-G18. ROBERT M. DIEFFENBACH, University of Iowa, Iowa City, Iowa 52240. Regular neighborhoods of 3-manifolds.

Let M be an orientable closed 3-manifold piecewise-linearly embedded in a (q+3)-manifold Q. If N is an orientable regular neighborhood of M in Q, then N is an SO(q) disc bundle over M. Specifically it is shown that there exists a solid torus $T \subset M$, an SO(q) bundle map $\gamma : \dot{T} \times B^q \to \dot{\overline{M-T}} \times B^q$, and a PL homeomorphism $F : N \to T \times B^q \cup_\gamma \overline{M-T} \times B^q$ such that $F(x) = (x, 0)$ for all $x \in M$. The map γ is defined in the following manner. Suppose $T \subset M$ is a solid torus and that $a \times b : S^1 \times S^1 \to \dot{T}$ is a homeomorphism such that $a : S^1 \to \dot{T}$ represents a meridian on $\dot{T}$. Let $g : \dot{T} \to SO(q)$ be a map (unique up to homotopy in SO(q)) satisfying $g \cdot a \not\simeq 0$, $g \cdot b \simeq 0$. Then $\gamma : \dot{T} \times B^q \to \dot{\overline{M-T}} \times B^q$ is the map $\gamma(x, y) = (x, g_x(y))$ where $g_x = g(x) \in SO(q)$. Using the bundle representation, regular neighborhoods are classified. Theorem. If J is a centerline of T and if $T' \subset M$ is another solid torus with centerline J', then there exists a 0-preserving homeomorphism $G : T \times B^q \cup_\gamma \overline{M-T} \times B^q \to T' \times B^q \cup_\gamma \overline{M-T'} \times B^q$ if and only if $J \sim J' \bmod Z_2$. In particular there exists a 0-preserving homeomorphism $G : T \times B^q \cup_\gamma \overline{M-T} \times B^q \to M \times B^q$ if and only if $J \sim 0 \bmod Z_2$. It follows that whenever $H_1(M; Z_2) \approx 0$, then in codimension $\geqq 3$, M has only one orientable regular neighborhood, namely a product. (Received November 2, 1970.) (Author introduced by Professor Thomas M. Price.)

71T-G19. S. A. NAIMPALLY and P. L. SHARMA, Indian Institute of Technology, Kanpur-16, U.P., India. Construction of Lodato proximities.

P(X) denotes the power-set of a set X. In an R_0 topological space $(X, \mathcal{T})$, the set $\{\overline{x}\}$ for $x \in X$ is an R_0-class. Theorem. In an R_0 topological space $(X, \mathcal{T})$, a binary relation δ_m on P(X) defined by: $(A, B) \in \delta_m$ iff $\overline{A} \cap \overline{B} \neq \emptyset$ or each of $\overline{A}$ and $\overline{B}$ is a union of m or more R_0-classes, is a compatible Lodato proximity on X for each infinite cardinal m. Theorem. In an R_0 topological space $(X, \mathcal{T})$, a binary relation δ on P(X) defined by: $(A, B) \in \delta$ iff $\overline{A} \cap \overline{B} \neq \emptyset$ or each of $\overline{A}$ and $\overline{B}$ is a union of infinitely many R_0-classes, is the coarsest compatible Lodato proximity on X. Corollary. An R_0 topological space $(X, \mathcal{T})$ has a unique compatible Lodato proximity iff, of any two disjoint closed subsets, at least one is a union of only finitely many R_0-classes. Corollary. A T_1 topological space $(X, \mathcal{T})$ has a unique compatible Lodato proximity iff any two infinite closed subsets of X intersect. Theorem. Any T_4 topological space which is not countably compact has at least c distinct T_2-compactifications (where c is the cardinality of the real line). Several methods of constructing compatible proximities and compatible Lodato proximities are given. (Received November 2, 1970.)

71T-G20. P. L. SHARMA, Indian Institute of Technology, Kanpur-16, U.P., India. Proximity bases.

P(X) denotes the power-set of a set X. Definition. A binary relation $\mathcal{B}$ on P(X) is a proximity-base iff (i) $A \cap B \neq \emptyset$ implies $(A, B) \in \mathcal{B}$ and (ii) $(A, B) \notin \mathcal{B}$ implies there exists $E \subseteq X$ such that $(A, E) \notin \mathcal{B}$ and $(X - E, B) \notin \mathcal{B}$. Theorem. If $\mathcal{B}$ is a proximity-base on X then there exists a coarsest proximity $\delta(\mathcal{B})$ on X finer than $\mathcal{B}$. Theorem. The collection of all proximities on X forms a complete lattice. Theorem. Let $(X, \mathcal{T})$ be a Tychonoff space. The collection of all T_2 compactifications of X forms a complete sup-semilattice. The collection forms a complete lattice iff $\mathcal{T}$ is locally compact. Theorem. Let f be a map from a proximity space (X, δ_1) into a proximity space (Y, δ_2). Let $\mathcal{B}$ be a proximity-base for δ_2. Then f is p-continuous iff $(A, B) \notin \mathcal{B}$ implies $(f^{-1}(A), f^{-1}(B)) \notin \delta_1$. Theorem. Let $\{(X_a, \delta_a) : a \in I\}$ be a collection of proximity spaces. Let $Z = \Pi\{X_a : a \in I\}$. Then $\mathcal{B} = \{(A, B) : (P_a(A), P_a(B)) \in \delta_a$ for each $a \in I\}$ is a proximity-base for the product proximity. Theorem. Every proximity on a set X is a sup of pseudometrizable proximities on X. Several problems on proximities are solved. (Received November 2, 1970.) (Author introduced by Professor S. A. Naimpally.)

71T-G21. DANIEL A. MORAN, Department of Pure Mathematics, University of Cambridge, Cambridge, England. A residual ANR which is not strongly residual.

An appeal to recent work of V. Nicholson (Trans. Amer. Math. Soc. 143(1969), 259-268) shows that a simple arc embedded as a residual set in S^3 as in Example (1.2) of Artin and Fox (Ann. of Math. (2) 49(1948), 979-990) fails to be strongly residual there. This gives an embarrassingly easy answer to the question raised in (Proc. Amer. Math. Soc. 25(1970), 752-754), and also demonstrates the dependence of strong residuality upon local properties of the embedding of a residual set. (Received November 6, 1970.)

71T-G22. OFELIA T. ALAS, Universidade de São Paulo, Caixa Postal 11072, São Paulo, Brazil. Topological groups and uniform continuity.

Let G be a nondiscrete topological group, e be the neutral element of G and m be the least cardinal number such that there is a family (of cardinality m) of open subsets of G whose intersection is not an open set. Suppose that m is an infinite nonstrongly inaccessible cardinal. Definition. G has the property K if any continuous real-valued function on G is right uniformly continuous. Lemma. If G has the property K, then any locally finite open covering of G has cardinality less than m. Theorem 1. If G is paracompact and any locally finite open covering of G has cardinality less than m, then G has the property K. Theorem 2. If G has the property K and m is the pseudoweight at the point e, then G is paracompact. Remark. Other similar results for universal uniformities for paracompact spaces are proved. (Received November 6, 1970.)

71T-G23. ROBERT A. HERRMANN, U.S. Naval Academy, Annapolis, Maryland 21402. Some results using the nonstandard theory of remoteness. I.

Let U be our universe of discourse and $u(p)$ the monad of the standard point $p \in X \in U$. Definitions. A *point q is said to be remotely near to A iff $q^* \in A \subset X \in U$ and there exists a $p \in X - A$ such that $q \in u(p)$. A *point q is said to be very remote to A iff there does not exist a $p \in X$ such that $q \in u(p)$. Using the nonstandard methods and definitions of M. Machover [Lecture Notes in Math., Vol. 94, Springer-Verlag, 1969] with some modifications, we have the following: Theorem 1. Let f be a continuous mapping from the space $X \in U$ into the space $Y \in U$. If $q^* \in A$ is remotely near to A, then $g(q)$ is remotely near to $f[A]$, where g is the unique extension of f such that $g: \hat{X} \to \hat{Y}$. Theorem 2. Let X be T_2 and $X \in U$. A nontrivial subset A in X ($A \neq \emptyset, X$) is not closed iff there exists a *point q remote and remotely near to A. Theorem 3. Let X be T_2 and $X \in U$. X is connected iff for each nontrivial open $G \subset X$ there exists a *point q remote and remotely near to G. Theorem 4. Let F be a nontrivial closed subset in $X \in U$. F is not compact in X iff there exists a *point q very remote to F. (Received November 9, 1970.)

71T-G24. PETER S. LANDWEBER, Rutgers University, New Brunswick, New Jersey 08903 and CONNOR LAZAROV, Herbert Lehman College, City University of New York, Bronx, New York 10468. The cobordism of G-manifolds. Preliminary report.

Let G be a finite group which is either (1) cyclic, or (2) metacyclic with generators a and b satisfying $b^n = 1$, $a^m = 1$, $bab^{-1} = a^r$, and $(n, m) = 1$. We assume further, for simplicity, that $r^s - 1$ is a unit in Z_m for

$s=1,2,\ldots,n-1$. Let $\mathfrak{F}$ be a family of subgroups of G satisfying the condition that if K is in $\mathfrak{F}$ and K' is a subgroup of K, then K' is in $\mathfrak{F}$. We consider $\Omega_*^U(G,\mathfrak{F})$, the bordism group of stably complex G-manifolds M, such that every isotropy group of M is conjugate to an element of $\mathfrak{F}$. Theorem. For any $\mathfrak{F}$, $\Omega_+^U(G,\mathfrak{F})$ is a free Ω_*^U-module, and $\Omega_-^U(G,\mathfrak{F})$ has projective dimension one over Ω_*^U. Further, if $\mathfrak{F}$ is the family of all subgroups of G, then $\Omega_-^U(G,\mathfrak{F})=0$. (Received November 10, 1970.)

71T-G25. SURENDRA-NATH PATNAIK, Indian Institute of Technology, Hauz Khas, New Delhi 29, India. A Lefschetz fixed point theorem for spaces with finite group action.

Let X be a compact polyhedron on which a finite group G acts as a group of homeomorphisms. The orbit space X/G carries an induced simplicial decomposition. A fixed point x_0 of an arbitrary (single-valued, continuous) map $f: X \to X/G$ is given by $x_0 \in Gx_0$ where Gx_0 denotes the orbit determined by $x_0 \in X$, i.e., Gx_0 is the image under the identification map $p: X \to X/G$ of x_0. There is the diagram $H_i(X) \xrightarrow{f_{*i}} H_i(X/G) \xrightarrow{\lambda_{*i}} H_i(X)$ where λ_{*i} is the transfer homomorphism determined by p_{*i}. (Simplicial homology with coefficients in a field is used.) Definition. The Lefschetz number L(f) associated with the map $f: X \to X/G$ is given by $L(f) = \sum_i (-1)^i \operatorname{trace}(\lambda_{*i} f_{*i})$. Theorem. If $L(f) \neq 0$, there exists a fixed point under a map homotopic to the map $f: X \to X/G$. Some applications of this theorem are given. (Received November 12, 1970.)

71T-G26. NORBERT H. SCHLOMIUK, Université de Montréal, Montréal, Québec, Canada. On a conjecture in algebraic homotopy. Preliminary report.

In the paper "Principal cofibrations in the category of simplicial groups", Trans. Amer. Math. Soc. 146(1969), 151-165, we introduced a notion of μ-homotopy of simplicial homomorphisms and we conjectured the equivalence of μ-homotopy and loop-homotopy. We have proved this conjecture. (Received November 12, 1970.)

71T-G27. PHILIP T. CHURCH, Syracuse University, Syracuse, New York 13210 and KLAUS LAMOTKE, Universität Kolns, 5 Koln 41, Weyertal 86-90, Germany. Almost free G-manifolds.

Let G be a compact, connected Lie group, and let M^n be a closed, oriented, connected G-manifold. We consider actions in both the topological and smooth categories. Assume that the action is free, except for isolated fixed points, and that the orbit space is a topological manifold. We deduce that the number of fixed points is even, and $G=S^1=\mathcal{U}(1)$ with $\dim M/G=3$ or $G=S^3=Sp(1)$ with $\dim M/G=5$. Theorem. For every even natural number 2k and closed, connected, oriented (smooth) 3-manifold N^3, there is, up to equivariant homeomorphism (diffeomorphism), one and only one S^1-manifold M with 2k fixed points and orbit space homeomorphic (diffeomorphic) to N^3. In the topological S^3 case the analog is true, but in the smooth S^3 case we found for each 2k and N^5 two actions, and we are unable to decide whether or not these are distinct. (Received November 13, 1970.)

71T-G28. ERIC MENDELSOHN, University of Toronto, Toronto, Ontario, Canada. Any group is the collineation group of a projective plane.

Theorem 1. If G is a group and α a cardinal $\alpha \geq \aleph_0$ then there exists a projective plane $\mathcal{P} = (P, L, I)$ such that the colineation group of $\mathcal{P}$, $A(\mathcal{P}) \cong G$, and $|P| = \alpha|G|$. Theorem 2. If G_i, $i \in I$, is a well ordered sequence of groups then there exist projective planes $\mathcal{P}_i$, such that $\mathcal{P}_i$ is a subplane of $\mathcal{P}_j$ whenever $i < j$ and $A(\mathcal{P}_i) \cong G_i$. Theorem 3. Let G_1 and G_2 be given. Then there exist projective planes $\mathcal{P}_1 = (P_1, L_1, I_1)$ and $\mathcal{P}_2 = (P_2, L_2, I_2)$ and a pair of onto maps $\varphi : P_1 \to P_2$ and $\psi : L_1 \to L_2$ such that $pI_1\ell \Rightarrow \varphi(p)I_\alpha\psi(\ell)$, with $A(\mathcal{P}_1) \cong G_1$ and $A(\mathcal{P}_2) \cong G_2$. (Received November 4, 1970.)

71T-G29. M. K. SINGAL, Meerut University, Meerut, India and ASHA MATHUR, University of Delhi, Delhi, India. Some weaker forms of countable compactness.

Two types of topological spaces called countably C-compact and functionally countably compact spaces have been considered. A space is said to be functionally countably compact if whenever $\mathcal{U}$ is a countable open filterbase on X such that the intersection A of the elements of $\mathcal{U}$ is equal to the intersection of closures of the elements of $\mathcal{U}$, then $\mathcal{U}$ is a base for neighborhoods of A. $(X, \mathcal{T})$ is said to be countably C-compact if every countable $\mathcal{T}$-open cover of every closed subset of X has a finite subfamily the closures of whose members cover the set. Countably compact $\Rightarrow$ countably C-compact $\Rightarrow$ functionally countably compact $\Rightarrow$ minimal-E_1. Functionally countably compact E_1-spaces are characterised by the property that every continuous function defined on such a space into an E_1-space is closed. Countably C-compact spaces also have this property. Several results related to minimal-E_1, countably C-compact and functionally countably compact spaces have been obtained. (Received November 19, 1970.) (Authors introduced by Miss Shashi Prabha Arya.)

71T-G30. M. K. SINGAL, Meerut University, Meerut, India and SHASHI PRABHA ARYA, University of Delhi, Delhi, India. Another sum theorem for topological spaces.

Let $\mathcal{P}$ be a weakly hereditary topological property (that is, a property which when possessed by the space is also possessed by every closed subspace of it) satisfying the following: ' If $\{F_\lambda : \lambda \in \Lambda\}$ be a locally finite closed covering of X such that each F_λ has property $\mathcal{P}$, then X has property $\mathcal{P}$'. Theorem. If X be regular and $\mathcal{V}$ be a locally finite open covering of X such that each $V \in \mathcal{V}$ possesses the property $\mathcal{P}$ and Frontier V is Lindelöf for each $V \in \mathcal{V}$, then X possesses $\mathcal{P}$. The theorem is applicable to several important classes of topological spaces. (Received November 19, 1970.)

71T-G31. M. K. SINGAL, Meerut University, Meerut, India and PUSHPA JAIN, University of Delhi, Delhi, India. A note on subparacompact spaces.

A space X is said to be subparacompact (cf. D. K. Burke, Proc. Amer. Math. Soc. 23(1969), 655-663) if every open covering of X has a σ-locally finite closed refinement. Subparacompactness is equivalent to σ-paracompactness of Arhangel'skii (Russian Math. Surveys 21(1966), 115-162) and it is also equivalent to F_σ-screenability of McAuley (Proc. Amer. Math. Soc. 9(1958), 796-799). Theorem 1. If X is the union of a locally finite family of closed subparacompact sets, then X is subparacompact. Theorem 2. If $\mathcal{V}$ be an order locally finite open covering of X such that $\overline{V}$ is subparacompact for each $V \in \mathcal{V}$, then X is subparacompact. Theorem 3. If X is regular and $\mathcal{V}$ is an order locally finite open covering of X such that each $V \in \mathcal{V}$ is subparacompact and Frontier V is compact for each $V \in \mathcal{V}$, then X is subparacompact. Theorem 4. If X is regular and $\mathcal{V}$ is a locally finite open covering of X such that each $V \in \mathcal{V}$ is subparacompact and Frontier V is Lindelöf for each $V \in \mathcal{V}$, then X is subparacompact. Theorem 5. The disjoint topological sum of subparacompact spaces is subparacompact. (Received November 19, 1970.) (Authors introduced by Miss Shashi Prabha Arya.)

71T-G32. ASHA RANI, Meerut University, Meerut, India and SHASHI PRABHA ARYA, University of Delhi, Delhi, India. On pairwise almost regular spaces.

The concept of almost regularity (cf. M. K. Singal and Shashi Prabha Arya, Glasnik Mat. Ser. III 4(1969), 89-99) has been studied in bitopological spaces. A bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be pairwise almost regular if for every (i, j)-regularly closed set F (that is $F = \mathcal{T}_i\text{-cl}\,\mathcal{T}_j\text{-int } F$) and a point $x \notin F$ there exists a $\mathcal{T}_j$-open set V and a disjoint $\mathcal{T}_i$-open set U such that $x \in U$, $F \subseteq V$, $i, j = 1, 2$, $i \neq j$. $(X, \mathcal{T}_1, \mathcal{T}_2)$ is pairwise regular if and only if it is pairwise almost regular and pairwise semiregular. Every pairwise almost regular, pairwise Hausdorff space is pairwise Urysohn. Every pairwise dense or bi-open subspace of a pairwise almost regular space is pairwise almost regular. With a given bitopological space, there are associated three bitopological spaces in a natural way all of which coincide with each other if the given space be pairwise almost regular. (Received November 19, 1970.)

71T-G33. BEN FITZPATRICK, JR., Auburn University, Auburn, Alabama 36830. On countable dense homogeneity.

Ralph Bennett [Abstract 679-G8, these *Notices* 17(1970), 1035] calls a topological space X countable dense homogeneous provided that X is separable and for each two countable dense subsets A and B of X there is a homeomorphism from X onto X that takes A onto B. Theorem. If X is a connected, locally compact metric space that is countable dense homogeneous, then X is locally connected. (Received November 27, 1970.)

71T-G34. WARREN H. WHITE, Instituto de Matematica Pura e Aplicada, Rio de Janeiro ZC-58, GB. Brasil. A slippery wild torus. Preliminary report.

Let $T_0 = \{(z, w) \in C \times C : |z| = |w| = 1\}$ be the standard torus in $S^3 = \{x \in C \times C : \|x\| = \sqrt{2}\}$, and let $g_{a,b} : T_0 \to T_0$ be the homeomorphism sending $(e^{i\pi t_1}, e^{i\pi t_2})$ into $(e^{i\pi(t_1+a)}, e^{i\pi(t_2+b)})$. Let $f : T_0 \to T \subset S^3$ be a topological embedding such that each homeomorphism $f \circ g_{a,b} \circ f^{-1} : T \to T$ can be extended to a homeomorphism $G_{a,b} : S^3 \to S^3$. We say that T is tame if there is a homeomorphism $h : S^3 \to S^3$ such that $h(T) = T_0$. Theorem 1. If each $G_{a,b}$ is smooth, then T is tame. Theorem 2. If $G_{a,b}$ is continuous in a and b, then T is tame. Theorem 3. In general, T need not be tame. (Received November 27, 1970.)

71T-G35. JOHN L. BRYANT and R. CHRISTOPHER LACHER, Florida State University, Tallahassee, Florida 32306. Embeddings with mapping cylinder neighborhoods.

Suppose that M is an m-manifold topologically embedded in an n-manifold N $(m < n)$. Then M has a mapping cylinder neighborhood in N at the point $x \in M$ if there exist a neighborhood V of x in M, an open $(n-1)$-manifold U, a proper map φ of U onto V, and a homeomorphism h of the mapping cylinder Z_φ of φ onto a neighborhood of x in N such that $h|V = \text{identity}$. Theorem. Suppose that K is a 1-complex topologically embedded in E^4 such that K has a mapping cylinder neighborhood at each point of $K - K^0$ ($K^0 = 0$-skeleton of K). Then K is tame. (Received November 30, 1970.)

71T-G36. T. G. RAGHAVAN, Thiagarajar College of Engineering, Madurai - 15, India. Metrization of quasi-metric spaces.

The following results have been established using bitopological methods: (1) If (X, τ) is a quasi-metric space such that each conjugate open (= open in the conjugate quasi-metric) cover has a conjugate open refinement which is τ-locally finite, then τ is metrizable. (2) If (X, τ) is a quasi-metric space such that each countable conjugate open cover has a conjugate open refinement that is τ-locally finite, then τ is metrizable. (3) If (X, τ) is a quasi-metric space such that the topology of the conjugate quasi-metric is countably compact or equivalently sequentially compact, then τ is metrizable. (Received December 2, 1970.) (Author introduced by Professor M. Rajagopalan.)

71T-G37. RUSSELL GRANT WOODS, University of Manitoba, Winnipeg 19, Manitoba, Canada. Irreducible images of remainders of Stone-Cech compactifications.

Let $E(X)$ denote the absolute ("projective cover") of the completely regular Hausdorff space X. Let $R(X)$ denote the Boolean algebra of regular closed subsets of X. A map from X to Y is irreducible if it maps proper closed subsets of X to proper closed subsets of Y. Theorem. Let f be the perfect irreducible map from $E(\beta X)$ onto βX. The restriction of f to $E(\beta X) - E(X)$ is a perfect irreducible map onto $\beta X - X$ if and only if the map $A \to \mathrm{cl}_{\beta X} A - X$ is a Boolean algebra homomorphism from $R(X)$ into $R(\beta X - X)$. Corollary 1. If X is metric, or realcompact, or nowhere locally compact, there is a perfect irreducible map from $\beta E(X) - E(X)$ onto $\beta X - X$. Corollary 2. [CH] Let Q denote the rationals, I the irrationals, and N the countable discrete space. Then $E(\beta Q - Q)$ can be partitioned into two subspaces, one homeomorphic to $E(I)$ and the other homeomorphic to $\beta N - N$. Corollary 3. [CH] If X is locally compact, σ-compact and noncompact, and if $|R(X)| = 2^{\aleph_0}$, then $\beta N - N$ can be mapped irreducibly onto $\beta X - X$. Corollary 4. [CH] The absolute of a compact metric space without isolated points is homeomorphic to βT, where T is the set of remote points of βQ. (Received December 7, 1970.)

71T-G38. T. THIRIVIKRAMAN, Madurai University, Madurai-2, India. Net theoretic approach to topological problems. II. Preliminary report.

For any infinite cardinal m, a net $S: D \to X$ is called m-directed if any subcollection of elements of D of cardinality $\leq m$ has an upper bound. In this paper, convergence of various types of m-directed nets are studied and therefrom various known classes of spaces (like Lindelof spaces, real-compact spaces, P-spaces, absolutely closed spaces) are characterised. Some equivalent forms of Ulam's conjecture are pointed out. As a typical one, a cardinal m is nonmeasurable if and only if in every space of power m, every $\aleph_0$-directed universal net is convergent. New classes of spaces like 'weakly Lindelof spaces', 'α'-spaces', 'α''-spaces' are introduced and properties studied. 'Largest extensions' with respect to these and above properties are characterised as spaces of certain equivalence classes of nets. (Received December 7, 1970.) (Author introduced by Professor M. Rajagopalan.)

71T-G39. ALAN D. WEINSTEIN, University of California, Berkeley, California 94720. Global generating functions and closed orbits for periodic hamiltonian systems. Preliminary report.

Global generating functions for near-identity symplectic diffeomorphisms (Bull. Amer. Math. Soc. 75(1969), 1040-1041; Advances in Math, to appear) are used to prove fixed-point and periodic-point theorems for symplectic diffeormorphisms and hamiltonian flows. Theorem 1. Let (M, Ω) be a symplectic manifold with $H^1(M, R) = 0$. For each symplectic diffeomorphism of M sufficiently C^1-close to the identity, there is a real-valued function on M whose critical point set equals the fixed point set of the diffeomorphism. Theorem 2. Let (M, Ω) be a symplectic manifold and let the function $E: M \to R$ have c as a regular value with $H^1(E^{-1}(c), R) = 0$. If the vector field X_E defined by $X_E \lrcorner\, \Omega = dE$ has all its orbits on $E^{-1}(c)$ nearly closed with period T, then there is a real valued function on $E^{-1}(c)$ whose critical set equals the set of closed orbits of X_E on $E^{-1}(c)$ with period near T. Theorems 1 and 2 permit the application of critical point theory to existence problems for fixed points and closed orbits. The hypotheses of Theorem 2 (imprecisely stated above) are satisfied for slightly perturbed periodic hamiltonian systems. (Received December 14, 1970.)

71T-G40. LUDVIK JANOS, University of Florida, Gainesville, Florida 32601. On rigidity of metrics.

A metric space (x, ρ) is said to be rigid if it has no isometry except the identity. It is said to be strongly rigid if $\rho(x,y) = \rho(a,b)$ and $x \neq y$ imply $\{x,y\} = \{a,b\}$. If X is a metrizable topological space, the fact that X admits a rigid or strongly rigid metric is a topological property of X. It is obvious that a discrete space admits a strongly rigid metric iff its cardinality is $\leq c$. Theorem 1. The real line R, $[0,1]$ and S_1 admit a rigid but not a strongly rigid metric. Theorem 2. If a metrizable space X of cardinality $\leq c$ can be represented as a countable product of discrete spaces, then it admits a strongly rigid metric. In particular the Cantor set C admits such a metric. (This result is due to N. Passell.) Theorem 3. If $(X_\alpha \mid \alpha \in \mathfrak{A})$ is a family of connected and mutually non homeomorphic spaces X_α admitting a rigid metric, then its free topological union $\bigvee_\alpha X_\alpha$ admits also a rigid metric. (Received December 14, 1970.)

71T-G41. KOK-KEONG TAN, Dalhousie University, Halifax, Nova Scotia, Canada. Remarks on mappings with diminishing orbital diameters. Preliminary report.

The notions of nonexpansive mappings, asymptotically regular mappings, mappings with diminishing orbital diameters and nonisometric mappings have been generalized to a Hausdorff topological space X whose topology is generated by a family $\{d_\lambda\}_{\lambda \in \Gamma}$ of pseudometrics on X. Main results are: (1) If X is compact and $f: X \to X$ is nonexpansive w.r.t. $\{d_\lambda\}_{\lambda \in \Gamma}$, then the following are equivalent: (i) f is asymptotically regular w.r.t. $\{d_\lambda\}_{\lambda \in \Gamma}$; (ii) f has diminishing orbital diameters w.r.t. $\{d_\lambda\}_{\lambda \in \Gamma}$; and (iii) f is nonisometric w.r.t. $\{d_\lambda\}_{\lambda \in \Gamma}$. (2) If X is compact and F is an abelian semigroup of nonexpansive mappings on X with diminishing orbital diameters w.r.t. $\{d_\lambda\}_{\lambda \in \Gamma}$, then F has a common fixed point. (Received December 14, 1970.)

71T-G42. YIM-MING WONG, University of Hong Kong, Hong Kong. Uniform continuity on metrizable spaces.

Let X, Y be two metric spaces and C(X,Y) (resp. UC(X,Y)) be the set of all continuous (resp. uniformly continuous) functions from X into Y. A sequence $\{x_i\}$ is said to be nonisolated if there is a sequence $\{y_i\}$ equivalent to $\{x_i\}$ such that $\{i : x_i \neq y_i\}$ is infinite. Theorem. $C(X,Y) = UC(X,Y)$ for any metric space Y iff one of the following conditions is valid: (a) $C(X,R) = UC(X,R)$ where R is the real line. (b) Every two nonempty disjoint closed sets in X have a positive distance. (c) Every nonisolated sequence in X has a convergent subsequence. (d) X has the Lebesgue covering property. Let $(X, \mathcal{T})$ be a metrizable space. Denote X_d the metric space with a metric d on X which induces $\mathcal{T}$. Theorem. For some d such that $C(X_d, Y) = UC(X_d, Y)$ for any metric space Y iff one of the following conditions is valid: (a) Every closed subset in X has a countable open neighbourhood base. (b) X is the union of a compact set and isolated points. Theorem. For every d which induces $\mathcal{T}$, $C(X_d, Y) = UC(X_d, Y)$ for any metric space Y iff X is compact. (Received December 15, 1970.) (Author introduced by Dr. K. F. Ng.)

71T-G43. TOGO NISHIURA and CHOON-JAI RHEE, Wayne State University, Detroit, Michigan 48202. The hyperspace of a pseudoarc is a Cantor manifold.

It is known that the dimension of the hyperspace of subcontinua of a pseudoarc is two [C. Eberhart and S. B. Nadler, Jr., "The dimension of certain hyperspaces," to appear]. The following improvement is established by the authors: Theorem. The hyperspace of subcontinua of a pseudoarc is a two-dimensional Cantor manifold. (Received December 21, 1970.)

71T-G44. PAUL L. STRONG, University of Illinois, Urbana, Illinois 61801. Images of separable metric spaces. Preliminary report.

A k-network in a space is a collection of subsets γ such that for $K \subset U$, K compact and U open, there exists an $A \in \gamma$ such that $K \subset A \subset U$. The following is a result of E. A. Michael ["$\aleph_0$-spaces," J. Math. Mech. 15(1966), 983-1002]. Theorem. A T_3-space X is the image of a separable metric space under a quotient mapping if and only if X is a k-space with a countable k-network. This theorem is also true for Hausdorff spaces; in fact, we have the following: Theorem. A Hausdorff space X is the image of a separable metric space under a pseudo-open (resp. quotient) mapping if and only if X is A Fréchet (resp. sequential) space with a countable k-network. (For terminology, see Franklin, "Spaces in which sequences suffice," Fund. Math. 57(1965), 107-115.) (Received December 21, 1970.)

71T-G45. RAYMOND E. SMITHSON, University of Wyoming, Laramie, Wyoming 82070. The lattice invariance of the fixed point property. Preliminary report.

It is shown that for T_1-spaces the fixed point property is lattice invariant. Further, an example is given which shows that the theorem may fail if we do not assume that the spaces are T_1. (Received December 21, 1970.)

71T-G46. PAUL F. DUVALL, PETER FLETCHER and ROBERT A. McCOY, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Isotopy Galois spaces. Preliminary report.

Definition. Let X be a topological space and for each closed set C of X let C' be the group of autohomeomorphisms of X which are supported on X - C. For each $x \in X$, let $C'(x) = \{g(x) : g \in C'\}$. Then $(X, \mathcal{T})$ is a Galois space (an orbit attraction space) provided that for each closed set C and each $x \in X - C$, $C'(x) \neq \{x\}$ $(\overline{C'(x)} \cap C \neq \emptyset)$. If for each closed set C and each $x \in X - C$ there is $h \in C'$ such that $h(x) \neq x$ and such that h is isotopic to the identity, then X is an isotopy Galois space. Results. If X is an isotopy Galois space and Y is a Tychonoff space, then $X \times Y$ is an isotopy Galois space. There is a metrizable Galois space X such that $X \times X$ is not a Galois space. Each locally compact metric space which admits a fixed point free flow and each locally compact topological group which is not 0-dimensional is an isotopy Galois space. Each nondegenerate minimal set in a flow on a locally compact metric space is an orbit attraction space. In particular the solenoid is an orbit attraction space and an isotopy Galois space. (Received December 21, 1970.)

71T-G47. CHARLES E. AULL, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. The oldest separation axiom.

The fourth axiom of F. Riesz introduced by F. Riesz in 1906 is as follows: for $x, y \in A'$ (derived set of A) there exists a subset B of A, such that $x \in B'$ and $y \notin B'$. A topological space will be said to be T_R if it is T_1 and satisfies the fourth axiom of Riesz. The following are proved. A T_R space X is Hausdorff iff any finer topological space is T_R. A minimal T_R space has the property that every open ultrafilter converges. A T_R space that is a dense subset of a compact T_R space has a T_R one point compactification iff it is a k' space of Arhangel'skii. Corollary. A completely regular Hausdorff space X such that for some $x \in X$, $X \sim [x]$ is a k' space (includes Fréchet of S_6 spaces) has a coarser topology which is (a) compact, (b) every compact subset is closed (based on a result of Wilansky), (c) satisfies T_R and (d) except for one point, pairs of distinct points are separated by disjoint open sets. (Received December 21, 1970.)

71T-G48. PETER J. NYIKOS, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. The N-compactification of Prabir Roy's space Δ. Preliminary report.

An N-compact space is one which can be embedded as a closed subspace in a product of countable discrete spaces. Associated with each space X is a unique N-compact space νX (called the N-compactification of X) which plays the same role that βX does for compact Hausdorff spaces: νX is the reflection of X in the category of N-compact spaces. In particular, if $\operatorname{ind} X = 0$, the map from X into νX is an embedding. A conjecture of long standing is that every N-compact X satisfies $\dim \beta X = 0$. Prabir Roy has described a complete metric space Δ [Trans. Amer. Math. Soc. 134(1968), 117-132] such that $\operatorname{ind} \Delta = 0$, $\dim \Delta = \dim \beta\Delta = 1$. The author has shown [Abstract 70T-G147, these *Notices* 17(1970)] that Δ is not N-compact, thus eliminating Δ as a counterexample to the conjecture. But $\nu\Delta$ remained a promising candidate for the honor. We now give an explicit construction of $\nu\Delta$ and prove that $\dim \nu\Delta = \operatorname{ind} \nu\Delta = 0$, so that $\dim \beta(\nu\Delta) = 0$. (Received December 21, 1970.)

71T-G49. ANITA H. SOMERS, University of North Carolina, Chapel Hill, North Carolina 27514. An example concerning subparacompact spaces. Preliminary report.

Definition. A space X is called subparacompact if every open cover of X has a σ-locally finite closed refinement. Definition. A collection $\{F_\lambda : \lambda \in \Lambda\}$ of subsets of a space X, with Λ well-ordered, is called linearly locally finite if for every $\lambda_0 \in \Lambda$, $\{F_\lambda : \lambda < \lambda_0\}$ is locally finite. The following example answers a question of D. K. Burke ["Subparacompact spaces," Proc. Washington State Univ. Conf. on General Topology, 1970, pp. 39-49]. Example. There exists a completely regular T_1 space which is not subparacompact and in which every open cover has a linearly locally finite closed refinement. (Received December 21, 1970.)

71T-G50. JERROLD TUNNELL, Harvey Mudd College, Claremont, California 91711. Countable compactness properties and closed refinements.

Certain closed refinement properties are shown to imply countable paracompactness and countable metacompactness as follows: Theorem 1. If every countable open cover has a locally finite closed refinement, the space is countably paracompact. Theorem 2. If every countable open cover has a σ-closure preserving closed refinement, the space is countably metacompact. Hodel's result (Abstract 69T-G138, these *Notices* 16(1969), 988) that F_σ-screenable spaces are countably metacompact is a corollary of Theorem 2. A space is paralindelöf if every open cover has a locally finite open refinement. Theorem 3. Every countably paracompact paralindelöf Hausdorff space is regular. An example of a compact space which has an open cover with no σ-closure preserving closed refinement is given. (Received December 24, 1970.) (Author introduced by Professor John Greever.)

71T-G51. CHARLES D. BASS, University of Tennessee, Knoxville, Tennessee 37916. Squeezing disks in E^3. Preliminary report.

Let Δ_2 denote the unit disk in the plane and let $L_t = \{(x,y) \in \Delta_2 | x = t\}$. Definition. An embedding g of Δ_2 into E^3 supports a squeezing map if there exists a map f of E^3 onto itself such that (i) $f|E^3 - g(\Delta_2)$ is a homeomorphism; (ii) $fg(L_t)$ is a singleton for each t; (iii) $t' \neq t$ implies $fg(L_t) \neq fg(L_{t'})$. Eaton and Daverman have shown that each disk in E^3 is the image of an embedding of Δ_2 which supports a squeezing map. In this paper conditions are given which are sufficient to imply that an embedding $g: \Delta_2 \to E^3$ supports a squeezing map. In particular, the following results are obtained as corollaries. Every homeomorphism of Δ_2 onto a subdisk of Gillman's 2-sphere supports a squeezing map and every homeomorphism of Δ_2 onto a subdisk of Alford's 2-sphere supports a squeezing map. (Received December 28, 1970.)

71T-G52. JEONG SHENG YANG, University of South Carolina, Columbia, South Carolina 29208. Topological groups without equal uniformities. Preliminary report.

A topological group is said to be a group without equal uniformities if its left and right uniformities are not equal. Theorem. A topological group G is a group without equal uniformities if and only if there are nets $\{x_a\}$ and $\{y_a\}$ in G such that $\{x_a y_a\}$ converges to the identity element e but e is not a cluster point of the net $\{y_a x_a\}$. Corollary. A topological group has equal uniformities if and only if its associated Hausdorff topological group has equal uniformities. An example of a topological semidirect product of two abelian groups is given to show how the theorem works. (Received December 29, 1970.)

71T-G53. PETER FLETCHER, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061 and WILLIAM F. LINDGREN, Southern Illinois University, Carbondale, Illinois 62901. Transitive quasi-uniformities. Preliminary report.

A quasi-uniformity is said to be transitive iff it has a base consisting of transitive entourages. A quasi-uniform space $(X,\mathcal{U})$ has the Lebesgue property iff for each $\mathcal{T}_{\mathcal{U}}$-open cover $\mathcal{C}$ of X there exists $U \in \mathcal{U}$ such that $\{U(x): x \in X\}$ refines $\mathcal{C}$. Theorem 1. If $(X,\mathcal{U})$ is a quasi-uniform space with the Lebesgue property, then it is complete. A topological space $(X,\mathcal{T})$ is orthocompact iff for each open cover $\mathcal{C}$ of X there is an open refinement $\mathcal{P}$ of $\mathcal{C}$ such that if $x \in X$ then $\bigcap \{U \in \mathcal{P}: x \in U\} \in \mathcal{T}$. Theorem 2. A topological space $(X,\mathcal{T})$ has a compatible transitive quasi-uniformity with the Lebesgue property iff it is orthocompact. Let $(X,\mathcal{T})$ be a topological space and $\mathcal{C}$ be a collection of open sets such that if $x \in X$, then $\bigcap \{C \in \mathcal{C}: x \in C\} \in \mathcal{T}$. Then $\mathcal{C}$ is a Q-collection. If $\mathcal{B}$ is a base for $\mathcal{T}$, and there exists a sequence $\{\mathcal{B}_i\}_{i=1}^{\infty}$ of Q-collections such that $\mathcal{B} = \bigcup_{i=1}^{\infty} \mathcal{B}_i$, then $\mathcal{B}$ is a σ-Q-base for $\mathcal{T}$. Theorem 3. Let $(X,\mathcal{T})$ be a T_1 topological space. Then the following are equivalent: (i) There exists a δ-Q-base for $\mathcal{T}$. (ii) $(X,\mathcal{T})$ has a compatible transitive quasi-uniformity with a countable base. (iii) $(X,\mathcal{T})$ is generated by a non-Archimedean quasi-metric. (Received January 18, 1971.)

71T-G54. WILLIAM L. VOXMAN, University of Idaho, Moscow, Idaho 83843. A fixed point theorem for decomposition spaces. Preliminary report.

In this note the relationship between the fixed point property and UV decompositions of compact metric spaces is studied. Theorem. Suppose K is an n-dimensional finite simplicial complex with the fixed point property, and let G be a UV^{n-1} decomposition of K. Then K/G has the fixed point property. An example is given to show that the following "converse" is false: Suppose G is a UV^{ω} decomposition of a finite simplicial complex K such that K/G has the fixed point property. Then K has the fixed point property. (Received January 4, 1971.)

71T-G55. MELVIN C. THORNTON, University of Nebraska, Lincoln, Nebraska 68508. Torsion topological groups with minimal open sets.

Torsion topological groups with the additional property that the intersection of open sets is open are considered and their topological structure is determined: (1) The topology is uniquely determined by a normal subgroup. (2) Each group is uniquely an extension of an indiscrete group by a discrete group. (3) The topology may be changed within limits without changing the dual group. (Received January 5, 1971.)

71T-G56. KENNETH C. MILLETT, University of California, Santa Barbara, California 93106. Homotopy sequences of fibrations.

The homotopy sequence of a fibration is generalized to include pairs, triads and squares of fibrations. In accomplishing this the (three dimensional) homotopy lattice of a cube is described and is used to define an associated lattice for a fibration. The standard exact sequences are briefly described. Finally, a potpourri of examples is presented, including some calculations concerning the effect of Lundell's nonstable Bott map on the nonstable homotopy of U(n), with the intention of indicating the breadth of relevance and the usefulness of this method. (Received January 11, 1971.)

71T-G57. RALPH JONES, University of Wisconsin, Madison, Wisconsin 53706. Open n-manifolds are unions of n open n-cells.

Theorem. Every open connected triangulable n-manifold is a union of n open n-cells. Also for n = 1, 2, 3, this is the best possible result. This extends the result announced in Abstract 70T-G179, these *Notices* 17(1970), 976, and answers the question raised there. (Received January 18, 1971.)

71T-G58. PHILIP T. CHURCH, Syracuse University, Syracuse, New York 13210. Differentiable monotone maps on manifolds. III.

Let N^n be connected, let $f: M^n \to N^n$ be C^r ($r \geqq n$) and proper, and let $G = Z$ or Z_2. A map f is locally monotone (resp., locally G-acyclic) at $x \in M^n$ if and only if there are open neighborhoods $\mathcal{U}$ of x and $\mathcal{V}$ of f(x) such that $\mathcal{U} \xrightarrow{f} \mathcal{V}$ is proper monotone (onto) (resp., G-acyclic). Let P_f (resp., $Q_f(G)$) = $\{x \in M^n$: f at x not locally monotone (resp., locally G-acyclic)$\}$, and let $\mathcal{W}_f$ (resp., $X_f(G)$, Y_f) = $f^{-1}(\{y \in N^n: f^{-1}(y)$ is not connected (resp., G-acyclic, homotopy cellular)}). Theorems. (1) Either $P_f = \emptyset$ (and so $f = hg$, where g is C^r monotone and h is a C^r k-to-1 diffeocovering map) or $\dim P_f \geqq n-2$. (2) If $n \geqq 4$, either $Q_f(G) = \emptyset$ (and so f has the factorization with g G-acyclic) or $\dim Q_f(G) \geqq n/2$. (3) If f is onto, either $\mathscr{V}_f = \emptyset$ (i.e. f is monotone) or $\dim \mathscr{V}_f = n$. (4) Either $X_f(G) = \emptyset$ (i.e. f is G-acyclic) or $\dim X_f(G) \geqq n/2$. (5) If f is Z-acyclic, either $Y_f = \emptyset$ (i.e. f is homotopy cellular) or $\dim Y_f \geqq n-1$. All the dimension statements are best possible. (Received January 13, 1971.)

71T-G59. ROBERT M. STEPHENSON, JR., University of North Carolina, Chapel Hill, North Carolina 27514. A correction concerning almost realcompact spaces.

According to Z. Frolík's Theorem 6 in [Czech. Math. J. 13 (88) (1963), 127-137], if $\mathcal{a}$ is a collection of subsets of a Hausdorff space and if each member of $\mathcal{a}$ is almost realcompact, then $\bigcap \mathcal{a}$ is almost realcompact. In this note it is shown that Theorem 6 is valid only for regular spaces. (Received January 15, 1971.)

71T-G60. STANLEY P. FRANKLIN, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. and DAVID J. LUTZER and B. V. S. THOMAS, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. On subcategories of TOP. Preliminary report.

A categorical characterization of a category S as a subcategory of TOP (the category of topological spaces and continuous functions) is one which enables the identification of S in TOP without requiring the reconstruction of the topological structures of its objects. Categorical characterizations are given for each of the (full, replete) subcategories of TOP (or perhaps T2 or $T3\frac{1}{2}$) whose objects are one of the following classes of spaces: metrizable, separable metrizable, first countable T1, compact Hausdorff, locally compact Hausdorff, paracompact, pseudocompact, Lindelöf, regular, perfectly normal, collectwise normal. Sample results. The metrizable spaces are the objects of the smallest, nontrivial, hereditary, co-productive, countably productive, right-fitting (i.e. preserved under perfect maps) subcategory of T2; the paracompact spaces are the objects of the largest left-fitting subcategory of T2 which is preserved by closed epis in TOP. Of course categorical descriptions are given for such concepts as closed map, perfect map, subspace, etc. (Received January 15, 1971.)

71T-G61. LEWIS E. WARD, JR., University of Oregon, Eugene, Oregon 97403. Fixed points of monotone surjections.

Theorem 1. If M is a hereditarily locally connected continuum, e is an endpoint of M and $f(M) = M$ is a monotone mapping such that $f(e) = e$, then there exists $x \in M - \{e\}$ such that $f(x) = x$. Theorem 2. If M is a dendroid, e is an endpoint of M and f and g are commuting monotone surjections on M such that $e = f(e) = g(e)$, then there exists $x \in M - \{e\}$ such that $x = f(x) = g(x)$. (Received January 18, 1971.)

71T-G62. JACK D. WILSON, University of Mississippi, University, Mississippi 38677. A notion of convexity in certain topologically flat spaces.

A space is said to cyclicly descriptive if there is a point Ω and a collection G of simple closed curves such that (1) each element of G contains Ω, (2) no two elements of G have three points in common, and (3) if P and Q are two points distinct from Ω, then some element of G contains them (Abstract 682-54-16, these *Notices* 18 (1971), 209). Let Σ denote a cyclicly descriptive space that satisfies Axioms 0-4 of R. L. Moore's "Foundations of point set theory," Amer. Math. Soc. Colloq. Publ., Vol. 13, Amer. Math. Soc., Providence, R. I., 1962. G-convexity, a notion of convexity, is defined relative to the collection G. Let Σ' denote a space whose points are the points of Σ and whose regions are the simple domains of Σ. A simple closed curve in Σ is a simple closed curve in Σ'. For every space Σ, Σ' is topologically equivalent to the sphere. A point set is G-convex in Σ if and only if it is G-convex in Σ'. Thus G-convexity in Σ (which may be nonmetrizable) is equivalent to G-convexity in the sphere. (Received January 18, 1971.) (Author introduced by Professor David Edwin Cook.)

71T-G63. CARLOS R. BORGES, University of California, Davis, California 95616. Connectivity of function spaces.

We prove, among others, the following results: Theorem 1. If X is any space and Y is a metrizable AE (metrizable) space then Y^X with either the compact-open topology or the pointwise convergence topology is an AE (metrizable). Theorem 2. If X is any space and Y is equiconnected then Y^X with either of the above topologies is equiconnected. We also obtain the corresponding local analogues. (Received January 18, 1971.)

71T-G64. THOMAS R. KRAMER, Duke University, Durham, North Carolina 27706. On countably subparacompact spaces. Preliminary report.

D. Burke studied subparacompact spaces in [Proc. Amer. Math. Soc. 23 (1969), 655-663]. A space is countably subparacompact (c.s.) iff every countable open cover of it has a σ-discrete closed refinement. R. Hodel introduced c. s. spaces in [Proc. Amer. Math. Soc. 25 (1970), 842-845]. A space is subnormal iff every finite open cover of it has a countable closed refinement. Theorem. The following are equivalent: Every countable open cover of a space has a ((a) σ-discrete, (b) σ-locally finite, (c) σ-closure preserving, (d) countable) closed refinement. Theorem. A space is c.s. iff it is countably σ-paracompact, also iff it is countably metacompact and subnormal. Example. Let Ω be the first uncountable ordinal, $W^* = [1, \Omega]$ and $W = [1, \Omega)$. Put the order topology on both W and W*. Then $W \times W^*$ is not subnormal (and hence not c.s.), although W* is compact Hausdorff and W is countably compact and normal. Two sets of sufficient conditions on X and Y are found under which $X \times Y$ is c.s. Some elementary theorems concerning subsets, covers and images of c.s. spaces are presented. A peculiar inverse mapping theorem is established. An improved proof of Burke's Theorem is offered. (Received January 20, 1971.)

71T-G65. FRANKLIN D. TALL, University of Toronto, Toronto 181, Canada. Normal nonmetrizable Moore spaces with dense metrizable subspaces. Preliminary report.

Theorem. If there is a normal nonmetrizable Moore space, then there is one with a dense metrizable subspace. (Received January 20, 1971.)

ERRATA

Volume 17

DIANA L. DUBROVSKY. Computability and models of p-adically closed fields, Abstract 70T-E72, Page 965.

Line 13 should read: "p-adically closed field $H \subset N^*$ such that $(\mathbb{Q}_H, N) \equiv (H, N^*)$."

K. R. KELLUM. Almost continuous functions of Baire class 1. Preliminary report, Abstract 70T-B212, Page 955.

In line 6, after the words "any open set containing f" insert "which is the complement of a closed vertical ray."

HARI M. SRIVASTAVA. A pair of dual series equations involving generalized Bateman k-functions, Abstract 70T-B216, Page 956.

Line 3: Equations (*) and (**) and the parametric conditions that follow them should be replaced by: "(*) $\sum_{n=0}^{\infty} \{A_n/\Gamma(2\beta+\sigma+n+1)\} k_{2(n+\alpha)}^{2(\sigma+\sigma)}(x) = f(x)$, $0 \leqq x < y$, (**) $\sum_{n=0}^{\infty} \{A_n/\Gamma(2\nu+\sigma+n+1)\} k_{2(n+\beta)}^{2(\beta+\sigma)}(x) = g(x)$, $y < x < \infty$, where $\alpha + \sigma + 1 > 0$, $\beta > \nu > \alpha - \frac{1}{2}m$, $2\nu + \sigma + 1 > 0$, $\sigma + 1 \leqq 0$, m is a nonnegative integer."

Volume 18

WITOLD M. BOGDANOWICZ. Measurability and linear lattices of functions closed under convergence everywhere, Abstract 682-26-2, Page 139.

Replace the period at the end of the abstract by a comma and add the following: "moreover for any two functions $f, g \in L$ the function $(f:g)$ is measurable with respect to the sigma ring V."

CHUNG-CHUN YANG. On meromorphic functions with values distributed almost on a line, Abstract 682-30-16, Page 148.

The conclusion of the theorem: "then the order of f is no greater than one and in fact, $T(r,f) = O(r \log r)$ as $r \to \infty$" should be replaced by "then $T(r,f) = O(r)^{\alpha}$ (if $1 < \alpha < 2$) or $T(r,f) = O(r \log r)$ (if $\alpha \leqq 1$)."

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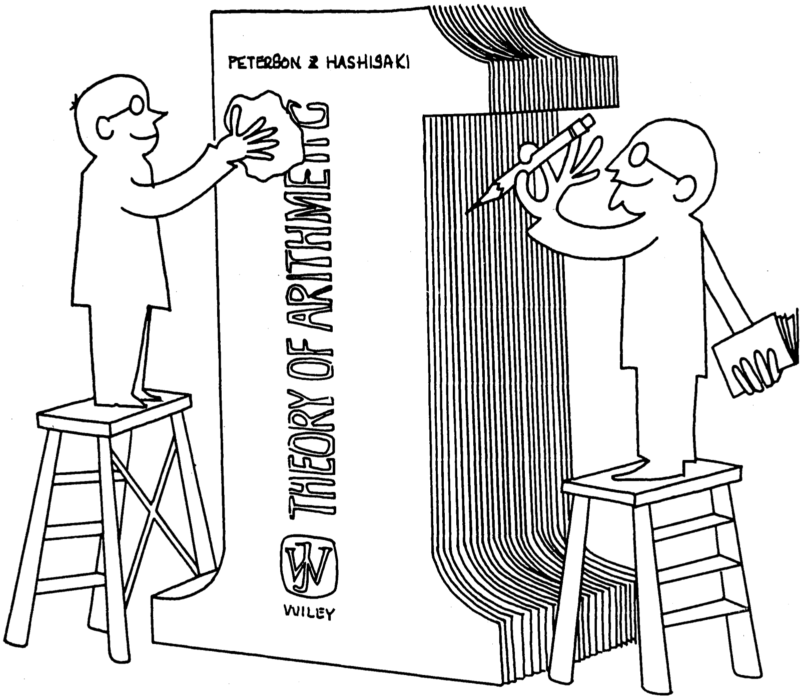

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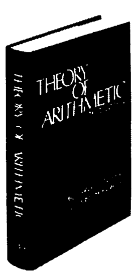

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