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April 2007

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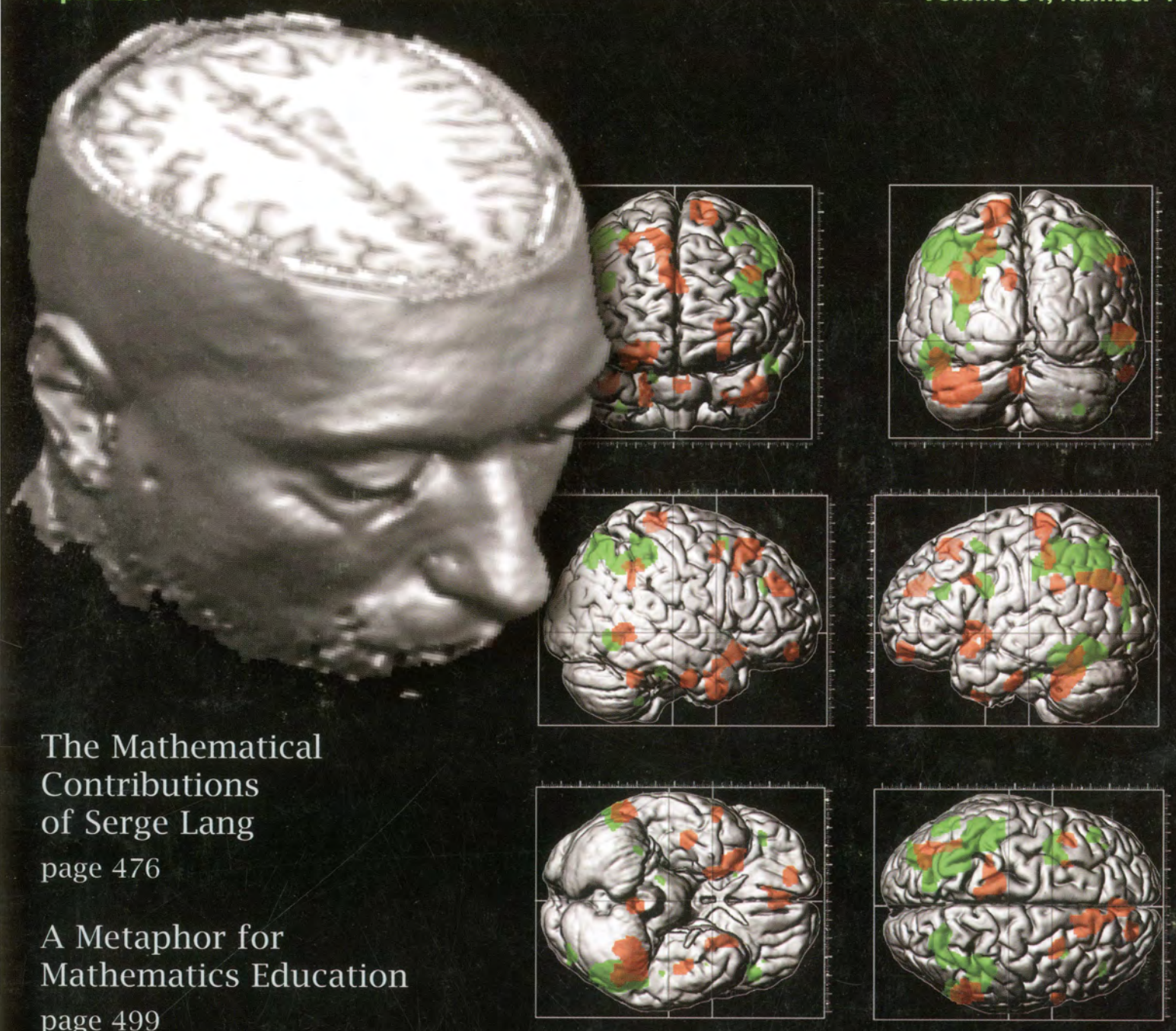

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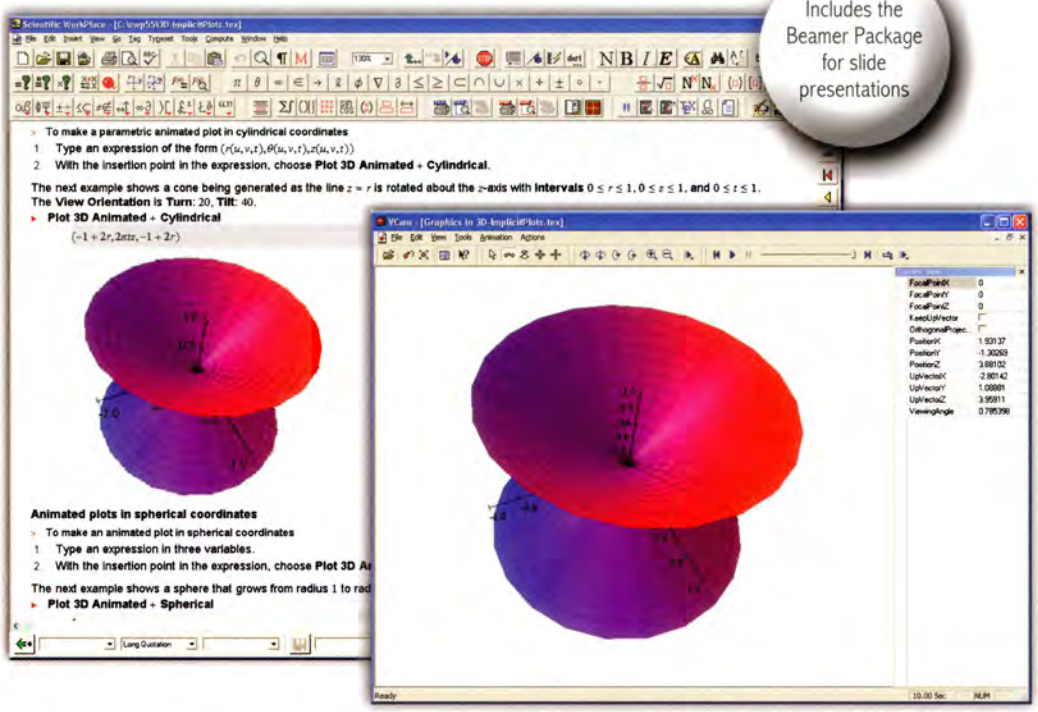

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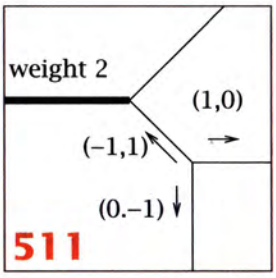

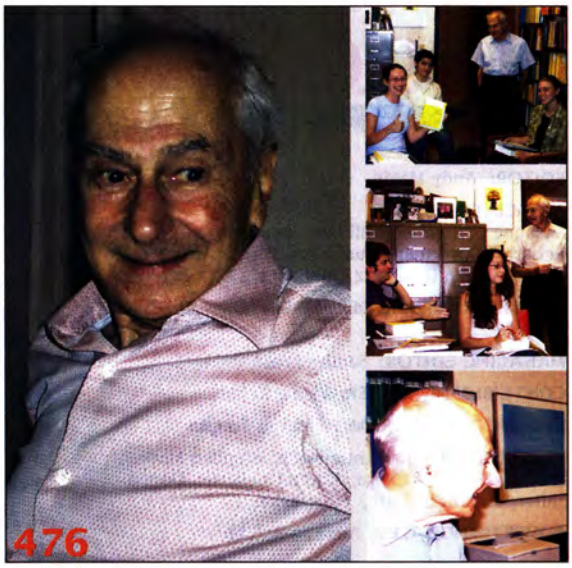

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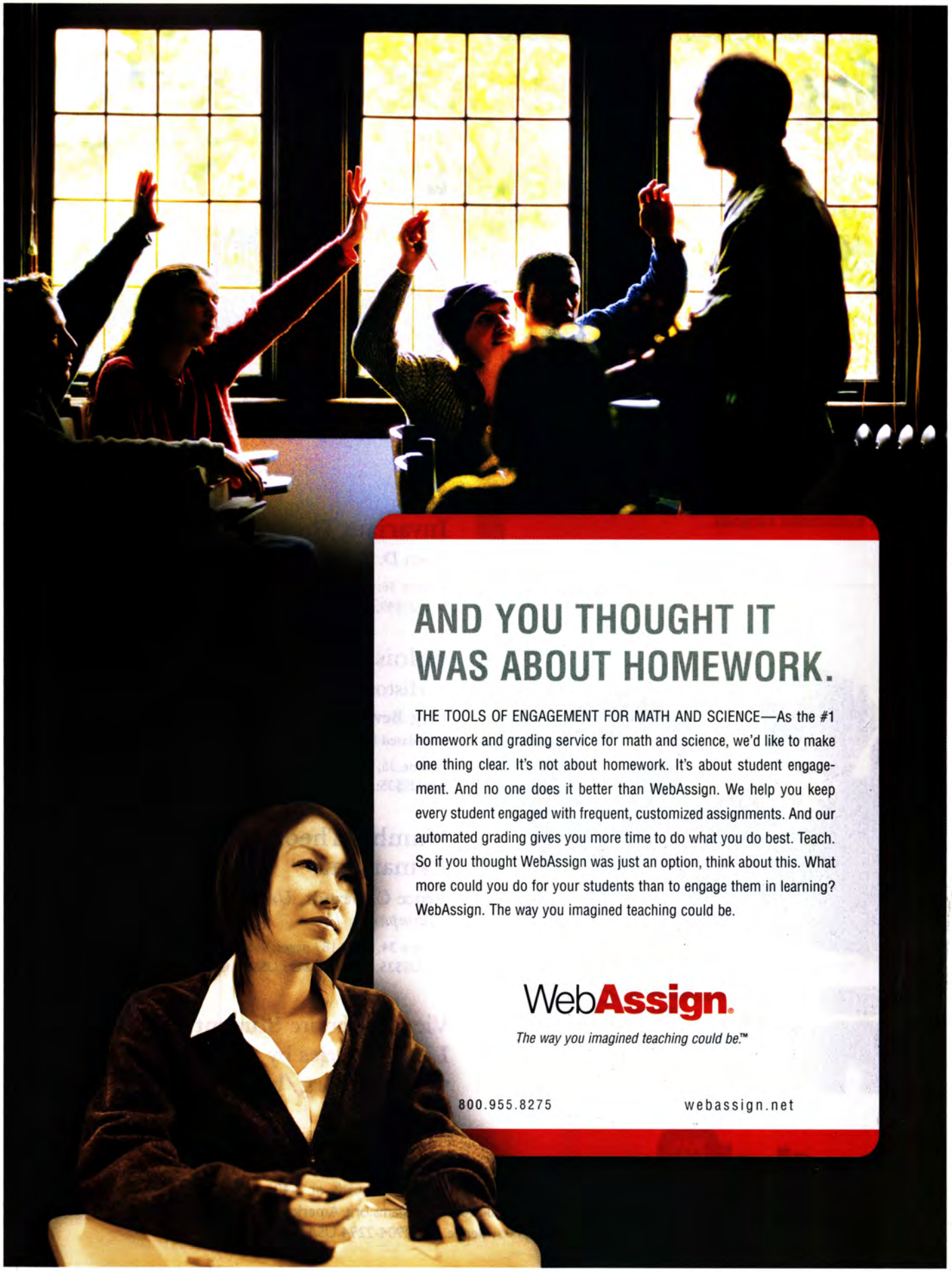

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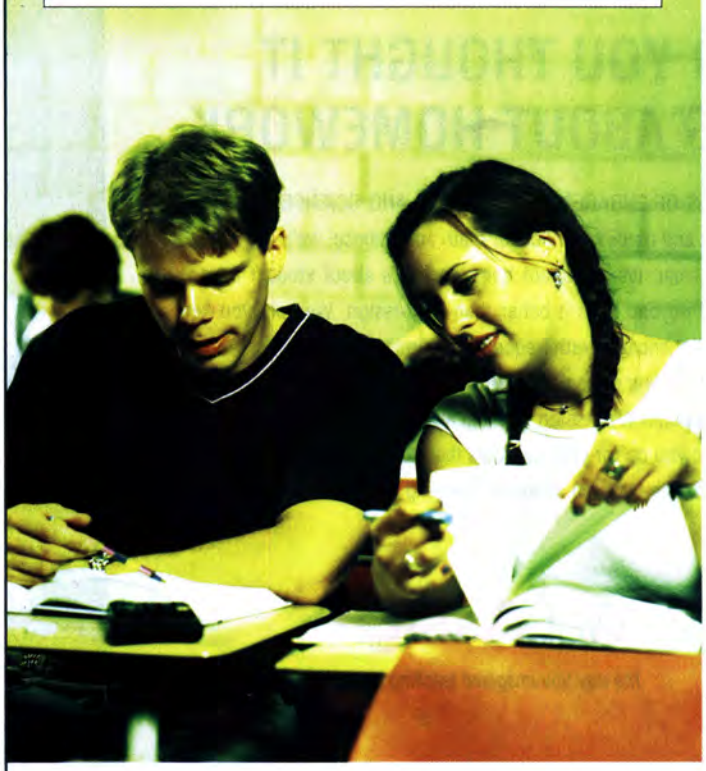

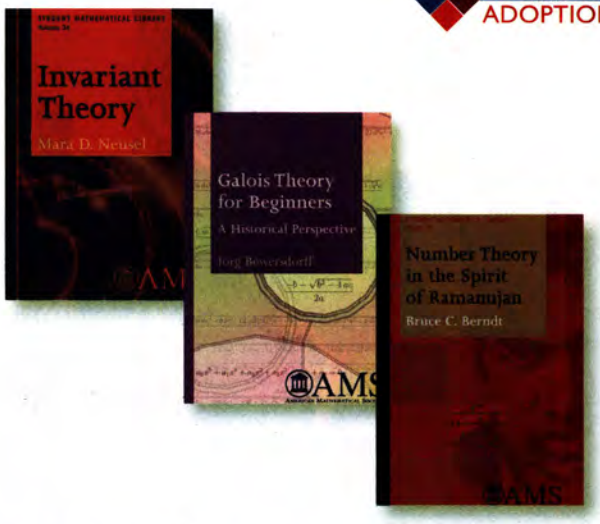

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Volume 36; 2007; 314 pages; Softcover; ISBN: 978-0-8218-4132-7; List US$49; AMS members US$39; Order code STML/36

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Pure and Applied Mathematics

Toward the end of the recent International Congress of Mathematicians in Madrid, there was a panel discussion about whether pure and applied mathematics are drifting apart. The panelists were Lennart Carleson, Ronald Coifman, Yuri Manin, Helmut Neunzert, and Peter Sarnak. John Ball, the president of the International Mathematical Union, moderated. The following reflections were inspired by that session.

I studied at the Courant Institute under Harold Grad, a distinguished applied mathematician. However, I think of myself as having one foot in pure mathematics and the other in applied.

The majority of the audience as well as the panelists were pure mathematicians. So perhaps it would be helpful to ask, What is applied mathematics? A very good answer was provided by the late Kurt Friedrichs, who distinguished himself in both pure and applied mathematics. He used to say, "Applied mathematics consists in solving exact problems approximately and approximate problems exactly." Initial and boundary value problems associated with the Navier-Stokes equations are an example of problems that are extremely difficult to solve exactly and where we look for approximate solutions. Hence computing is an important part of applied mathematics. The Bhatnagar-Gross-Krook equation in kinetic theory and plasma physics is an example of a solvable problem that approximates an intractable one.

Apropos of definitions, Peter Lax remarked once, quoting Joe Keller, that pure mathematics is a branch of applied mathematics; this was echoed by one of the ICM panelists. Some of the greatest mathematicians of the past—Newton, Euler, Lagrange, Gauss, and Riemann—and more recently Hilbert, Weyl, Wiener, von Neumann, and Kolmogorov did both pure and applied mathematics. We are of this world, and nothing comes entirely from within ourselves without reference to the external world.

Sarnak said that proofs are essential in pure math, but they are essential in applied math too, except that the path one takes is rather different. As Carleson said, applied math relies heavily on the inductive method, as opposed to the deductive method preferred by pure mathematicians. In pure mathematics the emphasis is on rigor. However, ideas are far more important. Ideas come from intuition, of course, which in turn comes from living and breathing the subject. Sarnak and his colleague Weinan E are quite right in insisting that even applied mathematicians need basic training in mathematics—roughly what is taught in good North American graduate schools in the first two years of study—but that is not enough. As Grad used to say, one must also immerse oneself completely in the subject to which one wants to apply mathematics. For instance, since he worked in magneto-fluid dynamics, to stay in touch with the experimentalists he regularly visited Oak Ridge and Los Alamos. It is by gaining a thorough understanding of the problems arising in the subject proper that one develops a feeling for it, and with it, intuition. So, as Coifman said, in subjects like biology and informatics we are in a pre-Newtonian age.

In applied mathematics the emphasis on rigor and proof must come at the appropriate stage. Let us consider an example. Feynman had great intuition but didn't care much for rigor or proofs. He says in one of his autobiographical writings that while he was at Cornell he used to talk to William Feller and Mark Kac, the famous probabilists. It is a happy circumstance, for science in general and mathematics in particular, that Feller and Kac didn't dismiss Feynman as a sloppy crackpot but instead patiently listened to him. Thus was born the great Feynman-Kac formula. The moral, I think, is that pure mathematicians, while insisting correctly on rigor and proofs, must be patient and show some respect toward intuition born out of a deep knowledge of a subject. Attitudes like that of the late Paul Halmos— "Applied mathematics is bad mathematics" —are shortsighted. For their part, applied mathematicians, while using intuition as their guide, must recognize the need for and the importance of proofs. On the other hand, it is rare that a single individual embodies all the requisite qualities to a high degree. So often what is needed is a joining of hands of people with disparate abilities, strengths, and points of view rather than a separation or drifting apart.

It is interesting to note that one doesn't always know whether one's work is useful or not. For example, G. H. Hardy is considered the quintessential pure mathematician. However, Joseph and Maria Mayer (she won the Nobel Prize in physics) use, in their classic book on statistical mechanics, a result of Hardy and Ramanujan in the evaluation of the partition function of an imperfect gas. Some say that Hardy's distaste for applications and the pride he took in the uselessness of his work had roots in his pacifism. The use of poison gas in the First World War, which he regarded as an application of chemistry, appalled him and may have had the unfortunate effect of turning him against applications altogether. One cannot but admire such loftiness, however.

Acknowledgment. It is a pleasure to thank Peter Lax for helpful comments.

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Letters to the Editor

Retarded Differential Equations and Quantum Mechanics

G. W. Johnson and I wish to draw attention to the work of C. K. Raju that is related to some of the ideas discussed by Sir Michael Atiyah in his talk "The Nature of Space", which we reported on in the June/July 2006 issue of the *Notices*. Ideas suggesting a link between retarded differential equations and quantum mechanics were put forward some years ago by Raju, and we, along with Atiyah, believe they deserve attention. Interested readers are encouraged to read, in particular, the following papers written by Raju:

1. *Time: Towards a Consistent Theory*, Kluwer Academic, Dordrecht, 1994 (Fundamental Theories of Physics, vol. 65), ch. 5b "Electromagnetic time" (pp. 116–122), and ch. 6b "Quantum mechanical time" (pp. 161–189).
2. *The Eleven Pictures of Time*, Sage, 2003, pp. 298–302.
3. "The electrodynamic 2-body problem and the origin of quantum mechanics", *Foundations of Physics*, **34**, (June 2004), 937–962.

—*Mark E. Walker*
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(Received December 29, 2006)

The Proof of the Poincaré Conjecture

In recent months there has been considerable attention devoted to the proof of the Poincaré conjecture that was started by Hamilton and completed by Perelman, both among mathematicians and in the media. Although most of the reports are largely true to fact, some incorporate false statements or innuendo which I believe are irresponsible. I am writing this letter to set the record straight.

Let me say first that the Hamilton-Perelman proof of the Poincaré conjecture is a great triumph for mathematics in general and for geometric analysis in particular. I am privileged to have participated in the nurturing of geometric analysis from its infancy to its adulthood. There were the good old days when ideas were shared and new frontiers were explored. It was during this period that the Ricci flow was introduced and investigated by Hamilton. Thirty years later, geometric analysis has reached maturity, and the proof of the Poincaré conjecture is perhaps its most spectacular success to date. I expect many more successes to come.

The achievements of Hamilton and Perelman in solving the conjecture, especially their major breakthroughs on singularities of nonlinear parabolic systems and the structure theorem for 3-dimensional manifolds, are unparalleled. They far exceed the established standards for Fields Medals. I fully support, and have always said so, the award of the Fields Medal to Perelman. (In my view, Hamilton clearly deserves the Fields Medal also, but he is not eligible at this time because of the age restriction.) For anyone to suggest in words or a cartoon that my position has ever been anything but that is both offensive and completely untrue.

Proving the Poincaré conjecture is an intricate and daunting process. In a work of this scale, it is understandable that when Perelman released his manuscripts on arxiv.org, several key steps were merely sketched or outlined. These manuscripts posed a tremendous challenge for the math community to digest. For two years many top experts in the field of geometric analysis worked hard and made steady progress in understanding and clarifying Perelman's papers. At the end of 2005, Cao and Zhu completed a three-hundred-plus-pages manuscript that provided a complete account of the Hamilton-Perelman proof of the Poincaré conjecture. This paper provides the proof in a form that finally can be understood by researchers in the field.

This past summer while I was in China, I held a press conference and also gave a public lecture on the Poincaré conjecture. My press conference addressed a group of Chinese reporters. Its intention was to encourage young Chinese mathematicians and scientists to be more ambitious and seek the frontiers of research being done worldwide and not just in China. Young mathematicians in China need not just encouragement but a better perspective of what the most exciting and promising directions of research are. My public lecture in Beijing on June 20 [2006] was addressed to the mathematics community and a large group of string theorists. In that talk I focused on the achievements of Hamilton and Perelman. Since Cao and Zhu managed to put together in writing the details of the deep ideas of Hamilton and Perelman, I praised them as well, hoping this would encourage their fellow mathematicians in China.

Over the years I have inherited from my teacher S. S. Chern the strong belief that it is the duty of any mature mathematician to train the next generation. Since he and I both come from China and there are many talented young Chinese mathematicians who are not exposed to modern mathematics, we have spent a lot of time helping mathematicians and students in China. We devoted a lot of time discussing the challenges and working together to address them. Thanks to his leadership, there are now many outstanding Chinese-trained mathematicians in Western universities. Over the last twenty years, Chern and I have also been trying to develop mathematics within Chinese universities. Because of the Cultural Revolution, the recovery has been slow. But thanks also to the help of many friends from the West, the situation is improving.

There have been uninformed reports on how the Cao-Zhu paper was handled by the *Asian Journal of Mathematics*, as well as on the joint work by Lian, Liu, and me on the mirror symmetry conjecture. Regarding the former, rumor has it that the normal peer review process had been tossed out the window. On the contrary, it took the journal several months to go through the established process until the paper was accepted for publication. After receiving the submission in December 2005, I asked, without success, several leading experts on geometric flows, including Perelman, if they would referee the paper. Under the circumstances, I myself took on the referee's task. After attending more than sixty hours of

lectures by Zhu and numerous hours of studying the paper on my own, I convinced myself that the paper was correct. This was in April 2006, and only then did I recommend the publication of the paper to the whole editorial board. The paper was then accepted according to the standard editorial procedure of the journal, by which acceptance was automatic unless an objection was voiced within a few days of the chief editor's recommendation.

I must add that this procedure of the *Asian Journal of Mathematics* of requiring consent from the whole editorial board is more stringent than several leading mathematical journals, where the chief editor would consult only a few members closest to the subject of the submitted paper. It is also a common practice for editors to expedite the reviewing process for important solicited papers.

Regarding my joint work with Lian and Liu, some reports have been particularly incomplete, biased, and unfair. This is not the place for me to respond to these false allegations, but I urge anyone else interested to look up the responses of Lian and Liu (B. Lian and K. Liu, On the Mirror Conjecture, `http://www.doctoryau.com`), which contain an account of the mathematics and history surrounding this conjecture.

In an age of instantaneous communication, the solution of a great problem like the Poincaré conjecture would inevitably draw attention from the media, with some reports more meritorious than others. Regardless of what has been said, however, what we in the mathematics community can cherish is the good fortune of having borne witness to this historic achievement of Hamilton and Perelman.

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(Received December 29, 2006)

Letters of Recommendation Terminology

From the chair of a hiring committee to everyone who writes letters for job candidates: Recommending someone "in the strongest possible terms" has become a cliché. I urge you in the strongest possible terms to find other ways of expressing your enthusiasm.

—Gerald Folland
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(Received January 2007)

Submitting Letters to the Editor

The *Notices* invites readers to submit letters and opinion pieces on topics related to mathematics. Electronic submissions are preferred (`notices-letters@ams.org`); see the masthead for postal mail addresses. Opinion pieces are usually one printed page in length (about 800 words). Letters are normally less than one page long, and shorter letters are preferred.

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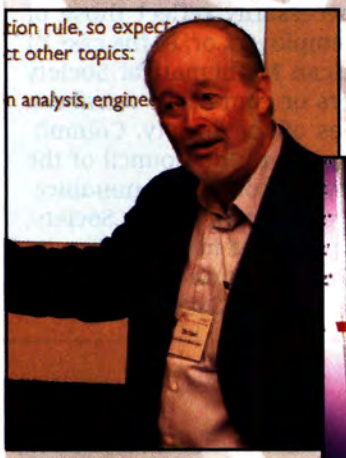
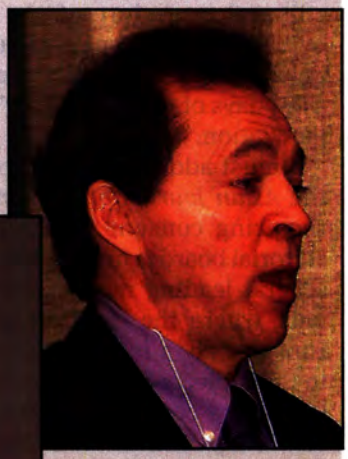
2007 Joint Mathematics Meetings in New Orleans





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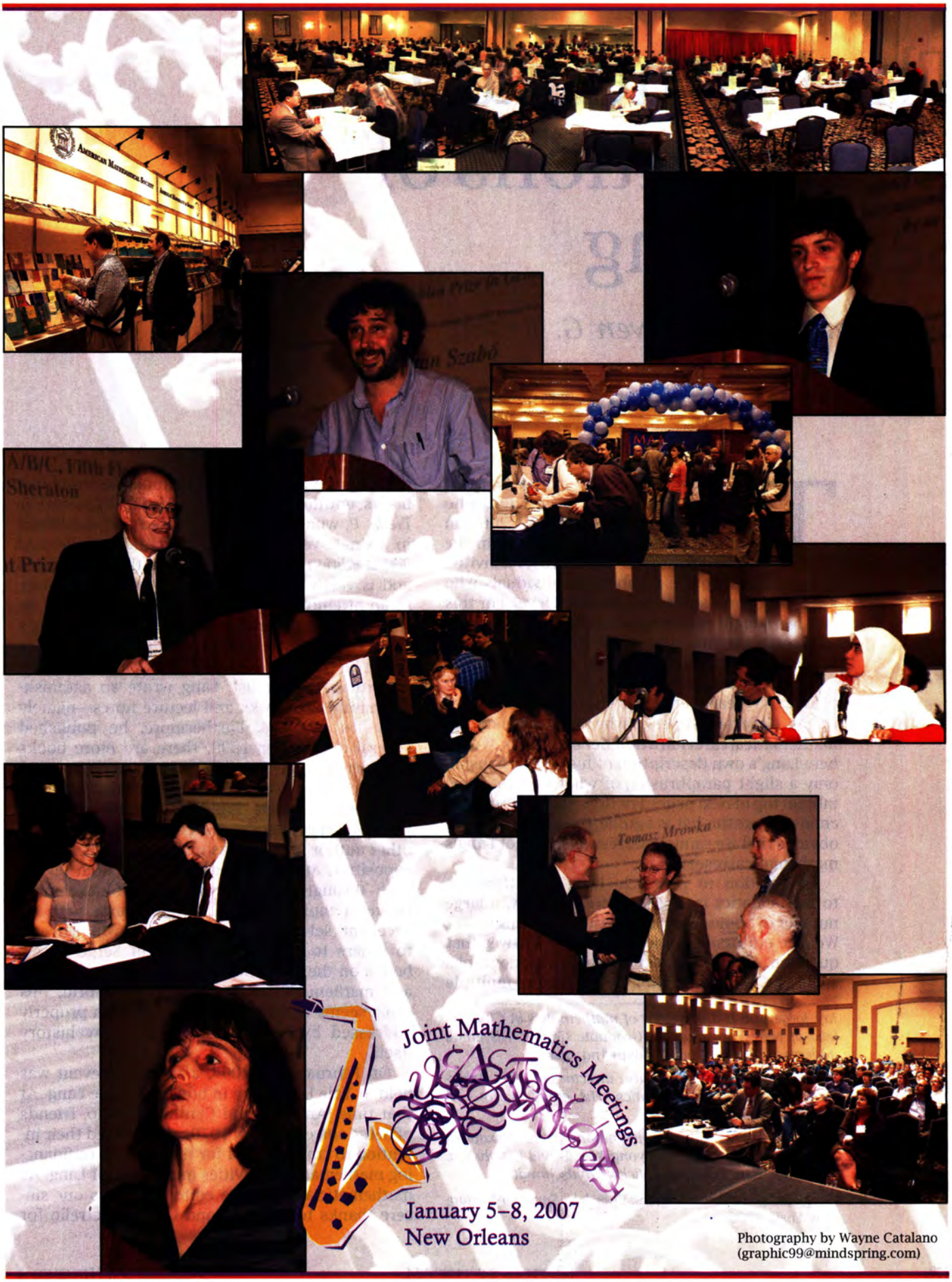

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The Mathematical Contributions of Serge Lang

Jay Jorgenson and Steven G. Krantz

This article is the second in a two-part series in memory of Serge Lang, who passed away on September 12, 2005. In the first article, which appeared in the May 2006 issue of the *Notices*, we invited contributions from a number of individuals who knew Serge on a somewhat personal level. For this part, we sought expositions which would describe, with certain technical details as necessary, aspects of Serge's contribution to mathematical research.

To begin understanding the breadth and depth of Serge's research endeavors, we refer to Volume I of his *Collected Papers*,[1] where he outlined his mathematical career in a number of periods. We list here Lang's own description of his research, using only a slight paraphrasing of what he wrote (see table at top of next page). For this article, the editors choose to use this list as a guide, though it should be obvious that we cannot address all facets of Lang's mathematical research.

In addition to research, Lang's contribution to mathematics includes, as we all know, a large number of books. How many books did Lang write? We (the editors) are not sure how to answer that question. Should we count political monographs such as *Challenges*? How do we count multiple editions and revisions? For example, he wrote two books, entitled *Cyclotomic Fields I* and *Cyclotomic Fields II*, which were later revised and published in a single volume, yet his text *Algebra: Revised Third Edition* has grown to more than 900 pages and is vastly different from the original version. In an attempt to determine how many books he wrote, we consulted the bibliography from Lang's *Collected Papers*, where he highlighted the entries which he considered to be a book or lecture note. According to that list, Lang wrote an astonishing number of books and lecture notes—namely sixty—as of 1999. Furthermore, he published several items after 1999, there are more books in production at this time, and a few unfinished manuscripts exist. In 1999, when Lang received the Leroy P. Steele Prize for Mathematical Exposition, the citation stated that "perhaps no other author has done as much for mathematical exposition at the graduate and research levels, both through timely expositions of developing research topics ... and through texts with an excellent selection of topics." We will leave it to others to assess the impact of Serge Lang's books on the education of mathematics students and mathematicians throughout the world; this topic seems to be a point of discussion properly addressed by historians as well as by history itself.

On February 17, 2006, a memorial event was held at Yale University in honor of Serge Lang. At that time, Anthony and Cynthia Petrello, friends of Serge since the early 1970s, announced their intention to create a fund for the purpose of financing mathematical activities in memory of Lang. As mathematicians, we the editors express our sincere thanks to Anthony and Cynthia Petrello for

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Based on mathematical contributions from: Alexandru Buium, Jay Jorgenson, Minhyong Kim, David E. Rohrlich, John Tate, Paul Vojta, and Michel Waldschmidt.

[1]Serge Lang: Collected Papers. Vols. I–V, *Springer-Verlag, New York, 2000.*

1.	1951–1954	Thesis on quasi-algebraic closure and related matters
2.	1954–1962	Algebraic geometry and abelian (or group) varieties; geometric class field theory
3.	1963–1975	Transcendental numbers and Diophantine approximation on algebraic groups
4.	1970	First paper on analytic number theory
5.	1975	$SL_2(\mathbf{R})$
6.	1972–1977	Frobenius distributions
7.	1973–1981	Modular curves, modular units
8.	1974, 1982–1991	Diophantine geometry, complex hyperbolic spaces, and Nevanlinna theory
9.	1985, 1988	Riemann-Roch and Arakelov theory
10.	1992–2000+	Analytic number theory and connections with spectral analysis, heat kernel, differential geometry, Lie groups, and symmetric spaces

Chronological description of Serge Lang's mathematics career.

their generous support of mathematical research. As teachers, we see the story of Serge, Anthony, and Cynthia as a wonderful example of the type of lifelong friendship that can develop between instructors and students. As editors of the two articles on Lang, we are in awe at being shown yet another way in which Serge has influenced the people he encountered. We look forward to seeing the results from the development of the "Lang Fund" and its impact on the mathematical community.

Serge Lang's Early Years

John Tate

These remarks are taken from my talk at the Lang Memorial at Yale on Lang and his work in the early years, roughly 1950–1960. Lang's papers from this period fill less than one half of the first of the five volumes of his collected works. His productivity was remarkably constant for more than fifty years, but my interaction with him was mostly early on. We were together at Princeton as graduate students and postdocs from 1947 to 1953 and in Paris during 1957–58.

In the forward of his *Collected Papers*, Lang takes the opportunity to "express once more" his appreciation for having been Emil Artin's student, saying "I could not have had a better start in my mathematical life." His Ph.D. thesis was on quasi-algebraic closure and its generalizations. He called a field k a C_l field if for every integer $d > 0$, every homogeneous polynomial of degree d in more than d^l variables with coefficients in k has a nontrivial zero in k. A field is C_0 if and only if it is algebraically closed. Artin's realization that Tsen's proof (1933) that the Brauer group of a function field in one variable over an algebraically closed constant field is trivial was achieved by showing that such a field is C_1, and he called that property quasi-algebraic closure. In analogy with Tsen's theorem, he conjectured that the field of all roots of unity is C_1. This is still an open question. In his thesis Lang proved various properties of C_l fields and showed that a function field in j variables over a C_l field is C_{l+j}. He also proved that the maximal unramified extension of a local field with perfect residue field is C_1. Artin had also conjectured that a local field with finite residue field is C_2. Lang could prove this for power series fields, but not for p-adic fields. In 1966 it became clear why he failed. G. Terjanian produced an example of a quartic form in 18 variables over the field $\mathbf{Q}_2$ of 2-adic numbers with no zero. Terjanian found his example a few months after giving a talk in the Bourbaki Seminar (November 1965) on a remarkable theorem of Ax and Kochen. Call a field $C_l(d)$ if the defining property of C_i above holds for forms of degree d. Using ultrafilters to relate $\prod_p \mathbf{Q}_p$ to $\prod_p \mathbf{F}_p((t))$, they showed that for each prime p, $\mathbf{Q}_p$ has property $C_2(d)$ for all but a finite set of d. In that sense, Artin was almost right.

Lang got his Ph.D. in 1951. After that he was a postdoc at Princeton for a year and then spent a year at the Institute for Advanced Study before going to Chicago, where he was mentored by Weil—if anyone could mentor Serge after his Ph.D. He and Weil wrote a joint paper generalizing Weil's theorem on the number of points on a curve over a finite field. They show that the number N of rational points on a projective variety in $\mathbb{P}^n$ of dimension r and degree d defined over a finite field satisfies

$$|N - q^r| \le (d-1)(d-2)q^{r-\frac{1}{2}} + Aq^{r-1},$$

where q is the number of elements in the finite field and A is a constant depending only on n, d, and r. (Here and in the following, "variety defined over k" means essentially the same thing as a geometrically irreducible k-variety.) From this result

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they derive corollaries for arbitrary abstract varieties. They show, for example, that a variety over a finite field has a rational zero-cycle of degree 1. The paper is a very small step towards Weil's conjectures on the number of points of varieties over finite fields which were proved by Deligne.

Lang proved for abelian varieties in 1955 and soon after for arbitrary group varieties that over a finite field k a homogeneous space for such a variety has a k-rational point. A consequence is that for a variety V over k, a "canonical map" $\alpha : V \to \mathrm{Alb}(V)$ of V into its Albanese variety can be defined over k and is then unique up to translation by a k-rational point on $\mathrm{Alb}(V)$.

In 1955 Lang dedicated to Artin a paper in which he generalized Artin's reciprocity law to most unramified abelian extensions (those "of Albanese type") of a function field K of arbitrary dimension over a finite constant field k. Let V/k be a projective normal model for K/k. For each finite separable extension L/K, let V_L be the normalization of V in L; and let Z_L denote the group of 0-cycles on V_L rational over the constant field k_L of L, Z_L^0 those of degree 0, and Z_L^a the kernel of the canonical map of Z_L^0 into $A_L(k_L)$, the group of k_L-rational points on the Albanese variety A_L of V_L. Let $C_L := Z_L/Z_L^a$ denote the group of classes of 0-cycles on V_L defined over k_L. Lang calls L/K "of Albanese type" if its "geometric part" $L\bar{k}/K\bar{k}$ is obtained by pullback, via a canonical map $\alpha : V = V_K \to A_K$, from a separable isogeny $B \to A_K$ defined over the algebraic closure $\bar{k}$ of k. Such an extension is abelian if the isogeny and α are defined over k and the kernel of the isogeny consists of k-rational points. If the Neron-Severi group of V is torsion free, then every finite abelian extension of degree prime to the characteristic is of Albanese type. Lang shows that the map which associates to an extension L/K its trace group $S_K^L C_L$ gives a one-one correspondence between the set of abelian extensions L/K of Albanese type and the set of subgroups of finite index in C_K. He also shows, in exact analogy with Artin's reciprocity law, that the homomorphism $Z_K \to \mathrm{Gal}(L/K)$ which takes each prime rational 0-cycle P to its associated Frobenius automorphism $(P, L/K)$ vanishes on the Albanese kernel Z_K^a and induces an isomorphism $C_K/S_K^L C_L \approx \mathrm{Gal}(L/K)$. Moreover, from Lang's geometric point of view, this reciprocity law becomes transparent and quite easy to prove.

A year later, in his first paper written in French, Lang defined the analogue of Artin's nonabelian L-functions for Galois coverings $f : W \to V$ and proved with them the analogue of Tchebotarov's density theorem. He also generalized his reciprocity law for abelian coverings $W \to V$ of Albanese type described above to any covering obtained by pullback of a separable isogeny $B \to A$ of commutative group varieties, via a map $\alpha : V \to A$ defined outside a divisor. These coverings, which he calls "de type (α, A)" can be highly ramified, and Lang notes that in the case where V is a curve, taking Rosenlicht's generalized Jacobians for A and throwing in constant field extensions, one gets all abelian coverings, so that his theory recovers the classical class field theory over global function fields.

These papers were the beginning of higher-dimensional class field theory and earned Lang a Cole Prize in 1959. In his acceptance remarks, he acknowledges his indebtedness to others. In his paper on unramified class field theory, he expresses his great and sincere appreciation to Chow, Matsusaka, and Weil for discussions on the algebraic aspects of the Picard and Albanese varieties and for proving the theorems he needed for the work. (In the L-series paper he also thanks Serre, writing "Je ne voudrais pas terminer cette introduction sans exprimer ma reconnaissance à J.-P. Serre, qui a bien voulu se charger de la corection des phautes d'orthografe." I too thank Serre here for pointing out an error in my first description of these results of Lang.)

During the next couple of years Lang collaborated with many people. In a paper "Sur les revêtements non ramifiés" he and Serre proved that coverings in characteristic p behave more or less as in characteristic 0, provided the variety is projective, and applied this to abelian varieties in order to show that every covering is given by an isogeny. After a paper with Chow on the birational invariance of good reduction, Lang collaborated with me on a study of the Galois cohomology of abelian varieties. We were able to show, for each positive integer m, the existence of a curve of genus 1 over a suitable algebraic number field K, the degrees of whose divisors defined over K are exactly the multiples of m. Over a p-adic field F we essentially proved the prime-to-p part of the duality theorem

$$\mathrm{Hom}(H^1(\mathrm{Gal}(\bar{F}/F), A(\bar{F})), \mathbf{Q}/\mathbf{Z}) = \hat{A}(F)$$

for dual abelian varieties A and $\hat{A}$ without stating it that way.

Then, after a paper with Kolchin applying the theory of torsors for algebraic groups to the Galois theory of differential fields, Lang published with Néron a definitive account of the "theorem of the base": the finite generation of the Néron-Severi group of divisors modulo algebraic equivalence on a variety. Néron had proved this earlier, but here the proof is made more transparent. Using a criterion of Weil, they show that the theorem follows from the Mordell-Weil theorem for abelian varieties over function fields, which they prove in the usual way.

I view this as the end of Lang's first period of research, in which he applied Weil's algebraic

geometry to class field theory and to questions of rational points on varieties with great success. At this time (around 1960) he began to consider questions of integral points on curves and varieties and function field analogues of the Thue-Siegel-Roth theorem on Diophantine approximations, a new direction in which his ideas of generalization and unification of classical results had great influence.

In addition to doing outstanding research, Lang has had a tremendous influence as a communicator, teacher, and writer, as everyone knows. In closing I would like to mention a few examples of these things from the period I am writing about. Wonderful as it was, our graduate training with Artin was almost totally one-dimensional and nongeometric: number fields and function fields in one variable. In the following years Serge helped me become comfortable with higher-dimensional things such as the Jacobian, Picard and Albanese varieties over arbitrary ground fields and also with "reduction mod $\mathfrak{p}$", which was not such a simple matter working with Weil's *Foundations* in those days before schemes. Serre tells me it was Lang who made him appreciate the importance of the Frobenius automorphism. In general, Serge, who travelled regularly to Paris, Bonn, Moscow, and Berkeley, was an excellent source of information about what was happening in the world of mathematics. He was an energetic communicator who seemed driven to publish. The Artin-Tate book on class field theory is a good example. Lang took the original notes of the seminar, typed them, continued for years to urge their publication (over my perfectionist and unrealistic objections), and finally arranged for their publication by Addison-Wesley. His name should be on the cover.

Serge Lang's Early Work on Diophantine and Algebraic Geometry

Alexandru Buium

In this article we review some of Serge Lang's early work on Diophantine equations and algebraic geometry. We are mainly concerned here with papers written before the 1970s (roughly the first volume of his *Collected Papers* [44]).

Diophantine equations are polynomial equations $f(x_1, \ldots, x_n) = 0$ with rational coefficients or, more generally, with coefficients in fields K that have an "arithmetic flavor" (e.g., number fields, function fields, local fields, finite fields, etc.). The main problem is to determine whether such an equation has solutions with coordinates in K and, more generally, to "count" or "construct" all such solutions. Morally, one expects "many" solutions if the degree d of f is "small" with respect to the number n of variables, and one expects "few" solutions if d is "big" with respect to n. In the language of algebraic geometry, systems of polynomial equations correspond to varieties (or schemes) V over K, and solutions correspond to K-points $P \in V(K)$ of our varieties. According to conjectures made by Lang in the 1980s, the above conditions on d and n, controlling the size of the set of solutions to $f = 0$, should be replaced by precise algebro-geometric and complex analytic properties of the varieties in question.

Most of Lang's early work stems from his interest in Diophantine equations. In his thesis [22] Lang obtained remarkable new results on polynomial equations of low degree over local fields and function fields. Diophantine equations naturally led Lang [23] to the study of algebraic groups and their homogeneous spaces. Closely related to these are Lang's class field theory of function fields of characteristic p [24] and his work with Néron [29] on the Mordell-Weil theorem over function fields of any characteristic. From the latter Lang passed to his investigation, in the line of Mordell and Siegel, of finiteness of integral points [30] and division points [31] on curves. In [30] [31], he formulated his celebrated conjecture on *subvarieties of semiabelian varieties*. (Later [41], [42], [43] Lang came back to this circle of ideas by formulating his Diophantine conjectures for *arbitrary varieties*; we will not review this aspect of Lang's later work here.) Problems on integral points are intertwined with problems in the theory of Diophantine approximation, while the latter shares its spirit and methods with transcendence theory; in both these theories Lang made important contributions [32], [33], [34], [38], [39], [40], [5].

These are but a few of the themes that Lang pursued in his early work. Lang's impact on these themes was substantial. Not only did he contribute fundamental results, but, at the same time, he reorganized and systematized each of these subjects by attempting to clearly define their scope, formulate their basic problems, and make sweeping conjectures. In what follows we will review these themes in some detail.

Equations of Small Degree over Local Fields

E. Artin defined quasi-algebraically closed fields (in Lang's terminology, C_1 fields [22]) as fields K such that any form of degree d in n variables with $n > d$ and coefficients in K has a nontrivial

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zero in K. He noted that a method of Tsen [60] implies that function fields of one variable over an algebraically closed field are C_1 and conjectured that finite fields are C_1; this was proved by Chevalley [9]. Artin also conjectured that certain "local fields", such as $\mathbf{Q}_p^{ur}$ (the maximum unramified extension of the p-adic field), are C_1. This conjecture was proved by Lang in his thesis [22]; here is his strategy. Lang first uses Witt coordinates to transform the equation $f = 0$ with coefficients in $K = \mathbf{Q}_p^{ur}$ into an infinite system of equations $f_0 = f_1 = f_2 = \cdots = 0$ in infinitely many variables with coefficients in the algebraic closure k of the prime field $\mathbf{F}_p$. He is able to solve this infinite system in an algebraically closed extension k_1 of k by purely algebro-geometric considerations; here the hypothesis on the degree d is used to control the dimensions of the algebraic sets defined by various truncations of the infinite system. From the solution of the system in k_1 he gets a solution of $f = 0$ in K_1, a complete field with residue field k_1. Next he specializes this solution to a solution in $\hat{K}$, the completion of K. Finally, from a solution in $\hat{K}$ he gets solutions in K itself via a beautiful argument involving a variant of "Newton approximation". The proof outlined above contained a number of fruitful new ideas that provided starting points for further developments by other mathematicians. The step involving "Newton approximation" is one of the origins of M. Artin's work on approximating formal solutions to algebraic equations [4], which was crucial in his work on moduli. The viewpoint whereby equations over $\hat{K}$ can be transformed into infinite systems of equations over k was put into a general context by M. Greenberg [19] and has been intensely used ever since; in particular, it plays a role in Raynaud's work [54], [55] on Lang's conjecture [31] on division points; cf. the discussion below.

Points on Homogeneous Spaces over Finite Fields

In [23] Lang proves his celebrated theorem that if $\mathbf{F}_q$ is a finite field, then any homogeneous space $H/\mathbf{F}_q$ for an algebraic group $G/\mathbf{F}_q$ has an $\mathbf{F}_q$-point. The proof is beautifully simple and runs as follows: Let $G(k) \to G(k)$, $x \mapsto x^{(q)}$ be the q-power Frobenius map on k-points where k is the algebraic closure of $\mathbf{F}_q$. Then Lang proves that the map $G(k) \to G(k)$, $x \mapsto x^{-1}x^{(q)}$ is surjective. He then takes any point $y_0 \in H(k)$. By transitivity of the G-action there is a point $x_0 \in G(k)$ such that $x_0 y_0^{(q)} = y_0$. By surjectivity of $x \mapsto x^{-1}x^{(q)}$ there is a point $x_1 \in G(k)$ such that $x_0 = x_1^{-1}x_1^{(q)}$. Then $x_1^{-1}x_1^{(q)}y_0^{(q)} = y_0$; hence $(x_1 y_0)^{(q)} = x_1 y_0$, so one finds the desired point $x_1 y_0 \in H(\mathbf{F}_q)$. Lang's theorem generalizes a result of F. K. Schmidt about elliptic curves and also generalizes a result of Châtelet, who had proved that if a variety over $\mathbf{F}_q$ becomes isomorphic over the algebraic closure of $\mathbf{F}_q$ to a projective space $\mathbf{P}^n$, then the variety is already isomorphic over $\mathbf{F}_q$ to $\mathbf{P}^n$.

In [24], [25], [23] Lang uses the map $x \mapsto x^{-1}x^{(q)}$ as a key ingredient in his class field theory for function fields over finite fields (equivalently for coverings of varieties over finite fields). He shows that abelian coverings are essentially induced by appropriate isogenies of commutative algebraic groups (of which $x \mapsto x^{-1}x^{(q)}$ is a basic example), and he introduces his reciprocity mapping, which turns out to have the expected properties. It is interesting to note that this function field theory was developed (long) after E. Artin's work in the 1920s [3] on its number field prototype; usually, in the history of number theory, the number field theorems are being proved *after* their function field analogues. (Cf. the discussion of the Mordell conjecture below.) In the case at hand, the function field analogue had to wait until the necessary algebro-geometric tools (especially the algebraic theory of abelian varieties) became available.

Finite Generation of Points of Abelian Varieties over Function Fields

Lang was actively involved [26], [27], [28] in establishing the foundations of the algebraic theory of abelian varieties, following the pioneering work of Weil, Chow, and Matsusaka. With this theory at hand, Lang and Néron [29] were able to provide elegant proofs for some basic finiteness theorems in algebraic geometry. Néron had proved [50] the "Néron-Severi" theorem of the base stating that the group $D(V)$ of divisors on a variety V (over an algebraically closed field k) modulo the group $D_a(V)$ of divisors algebraically equivalent to 0 is finitely generated. In [29] Lang and Néron show how to reduce the proof of the finite generation of $D(V)/D_a(V)$ to proving the finite generation of a group of the form $A(K)/\tau B(k)$, where A is an abelian variety over a function field K over k and (B, τ) is the K/k-trace of A. Then they prove the finite generation of $A(K)/\tau B(k)$, which is the "Mordell-Weil" theorem in the function field context. Recall that the number field version of the Mordell-Weil theorem asserts that for any abelian variety A over a number field K, the group $A(K)$ is finitely generated. The latter was conjectured by Poincaré and proved by Mordell [48] in case $\dim A = 1$, $K = \mathbf{Q}$, and by Weil [66] in general. Here again, the number field theorem preceded the function field theorem.

Integral Points, Rational Points, and Division Points

Lang became interested in questions related to the Mordell conjecture around 1960; this conjecture and the impact of Lang's insights into it have

a long history. We briefly sketch the evolution of this circle of ideas below; our discussion is inherently incomplete and is meant to give only a hint as to the role of some of Lang's early ideas on the subject. Mordell [48] had conjectured that a nonsingular projective curve V of genus $g \geq 2$ over a number field K has only finitely many points in K. In particular, an equation $f = 0$ where f is a nonsingular homogeneous polynomial of degree ≥ 4 in 3 variables with $\mathbf{Q}$-coefficients should have, up to scaling, only finitely many solutions in $\mathbf{Q}$. Siegel [58] proved, using Diophantine approximations, that if V is an affine curve over a number field K, of genus $g \geq 1$, then V has only finitely many integral points (i.e. points with coordinates in the ring of integers of K). Mahler [45] conjectured that the same holds for S-integral points (S a finite set of places), and he proved this for $g = 1$ and $K = \mathbf{Q}$. In [30] Lang proves Mahler's conjecture by revisiting the arguments of Siegel and Mahler in the light of the new developments in Diophantine approximations (Roth's Theorem) and abelian varieties (especially the Lang-Néron paper [29]). Lang also makes, in [30], a conjecture which later, in [31], he strengthens to what came to be known as the *Lang conjecture* (on subvarieties of semiabelian varieties):

(*) *Let G be a semiabelian variety over an algebraically closed field F of characteristic 0, let $V \subset G$ be a subvariety, and let $\Gamma \subset G$ be a finite rank subgroup. Then V contains finitely many translates X_i of algebraic subgroups of G such that $V(F) \cap \Gamma \subset \cup_i X_i(F)$.*

Here, by Γ of finite rank, one understands that $\dim_{\mathbf{Q}} \Gamma \otimes \mathbf{Q} < \infty$. The weaker version of Conjecture (*) stated in [30] only assumes Γ is finitely generated; this is a reformulation and generalization of a conjecture of Chabauty [8]. A proof in case V is a curve, G is a linear torus, and Γ is finitely generated is given by Lang in [36] using Diophantine approximations. Lang remarks in [30] that Conjecture (*) (for Γ finitely generated) implies the Mordell conjecture; indeed if V is a curve of genus ≥ 2 defined over a number field K, then one lets F be the algebraic closure of K, embeds V into its Jacobian A, and notes that $V(K) = A(K) \cap V(F)$. But $A(K)$ is finitely generated by the Mordell-Weil Theorem, so $V(K)$ is finite and the Mordell conjecture follows. In a similar vein, Lang conjectures in [30] that if V is a nonsingular projective curve over a function field K of characteristic zero such that $V(K)$ is infinite, then V can be defined over the constants of K; this is the *Mordell conjecture over function fields of characteristic zero*. Lang proves in [30] that a curve as in the latter conjecture cannot have infinitely many points of bounded height; this implies an analogue of Siegel's Theorem [58] for curves over function fields of characteristic zero. In the same paper [30] Lang conjectures that

(**) *If V is an affine open set of an abelian variety over a number field K and S is a finite set of places of K, then the set of S-integral places of V is finite.*

The Mordell conjecture for curves over function fields of characteristic zero was proved by Manin [46]. Subsequently other proofs and new insights were provided [18], [56], [51], [2], [59], [12], etc. In Manin's work the question arose whether a curve, embedded into its Jacobian, contains only finitely many torsion points. This was independently asked by Mumford and came to be known as the *Manin-Mumford conjecture*. In [31] Lang stated Conjecture (*) so as to make the Mordell conjecture and the Manin-Mumford conjecture special cases of one and the same conjecture. The Manin-Mumford conjecture was proved by Raynaud [54], and in the same year Faltings [13] proved the original Mordell conjecture. Faltings's proof did not involve Diophantine approximations; later Vojta [61] provided a completely different proof of the Mordell conjecture involving Diophantine approximations. This led to another breakthrough by Faltings [14] in the higher-dimensional case, followed by more work of Vojta [62]. The full Lang conjecture (*) was subsequently proved by McQuillan [47], based partly on ideas of Hindry [20] and Raynaud about how to reduce the case "Γ of finite rank" to the case "Γ finitely generated". For curves over function fields in characteristic p, a remarkably short proof of a variant of the Lang conjecture (*) was given by Voloch [63]; cf. also [1]. Conjecture (**) was proved by Faltings [14]; function field analogues of Conjecture (**) were proved in [52], [7], [64].

We would like to close this discussion by noting that the Lang conjecture (*) is a purely algebro-geometric statement, i.e., a statement about varieties over an algebraically closed field F, and one could expect a purely algebro-geometric proof of this conjecture (in which the arithmetic of a global field of definition contained in F doesn't play any role). Implicit in Lang's viewpoint was that the Mordell conjecture could be attacked via the Lang conjecture (*); the whole arithmetic in the proof could then be concentrated into the Mordell-Weil Theorem. The decisive breakthroughs in the subject (by Manin and Faltings) went the other way around: the Mordell conjecture was proved directly (and then the Lang conjecture (*) followed by work of Raynaud, Hindry, McQuillan). A proof along Lang's original plan of attack was given by the author in [6], where the Lang conjecture (*), in the case A has no quotient defined over a number field and V is nonsingular, was proved "without global field arithmetic"; no such proof is known for A defined over a number field. The proof in [6] had a key complex analytic ingredient; Hrushovski [21] saw

how to replace this ingredient by an argument from mathematical logic (model theory) which also applied to function fields in characteristic p. In characteristic zero, Hrushovski's model theoretic argument can be rephrased, in its turn, as an entirely algebro-geometric argument [53].

Diophantine Approximations and Transcendence

A surprising discovery of Lang's [32] was that if β is a quadratic real irrational number and $c \geq 1$, then the number of integers q with $|q| \leq B$ such that $0 < q\beta - p < c|q|^{-1}$ for some integer p is a multiple of $\log B + O(1)$. This was in sharp contrast with the known fact that for c sufficiently small the inequality $|q\beta - p| < c/q$ has only finitely many solutions. Lang further explored asymptotic approximations in [33], [35], [34]. As already mentioned, Diophantine approximations are closely related to Lang's conjecture on intersections of subvarieties of algebraic groups with finitely generated subgroups. Partially motivated by this circle of ideas, Lang conjectured [37] strong inequalities for heights of points in algebraic groups; these conjectures roughly replaced heights in inequalities following from work of Mahler, Siegel, and Roth by logarithmic heights. The subject of Diophantine approximations evolved spectacularly after the 1970s, mainly due to breakthroughs by Baker, Bertrand, Bombieri, Brownawell, Faltings, Feldman, Lang, Masser, Nesterenko, Philippon, Roy, Schmidt, Vojta, Waldschmidt, Wüstholz, and many others. In particular, the following conjecture of Lang was recently proven by David and Hirata-Kohno [11], [10] (following work of Ably and followed by a generalization by Gaudron [16]): if E is an elliptic curve over a number field K and ϕ is a rational function on E/K, then there is a constant $c > 0$ such that for any point $P \in E(K)$ which is not a pole of ϕ, one has

$$|\phi(P)| \geq (\hat{h}(P) + 2)^{-c},$$

where $\hat{h}$ is the Néron-Tate height. Lang's original form of this conjecture [37] actually makes the constant c more explicit in terms of E/K and ϕ; in this more precise form the conjecture is still open. We refer to [11] for a history of work on this conjecture of Lang. For an in-depth presentation of Diophantine approximations on linear groups up to the year 2000, we refer to [65]. For more on Diophantine approximations on abelian varieties, we refer to [4], [43], [49].

In [38] Lang proved a conjecture of Cartier stating that if G is an algebraic group over a number field K and $\alpha \in (\mathrm{Lie}\, G)(K)$ is such that $t \mapsto \exp_G(t\alpha)$ is not an algebraic function, then $\exp(\alpha)$ is transcendental over K. For G a linear group this reduces to the classical result about the exponential function. The novelty comes from the nonlinear case; in case G is an abelian variety, Lang's result is a transcendence result for values of theta functions. Lang derived the above theorem from his transcendence criterion generalizing the method of Gelfond [17] and Schneider [57]. His criterion says the following: Let K be a number field and let $g_1, \ldots, g_n$ be meromorphic functions on $\mathbf{C}$ of finite order ρ such that the field $K(g_1, \ldots, g_n)$ has transcendence degree ≥ 2 over K. Assume d/dt sends $K[g_1, \ldots, g_n]$ into itself. Let $w_1, \ldots, w_m \in \mathbf{C}$ be distinct complex numbers such that $g_i(w_j) \in K$. Then $m \leq 10\rho[K : \mathbf{Q}]$. Using ideas of Schneider, Lang extended his transcendence criterion to meromorphic functions of several variables in [39], [40]. In particular, in [40] he derives the celebrated "Theorem on the 6 exponentials": if $\beta_1, \beta_2 \in \mathbf{C}$ are $\mathbf{Q}$-linearly independent and $z_1, z_2, z_3 \in \mathbf{C}$ are $\mathbf{Q}$-linearly independent, then not all 6 numbers $e^{\beta_i z_j}$ are algebraic. (Apparently this had been known to Siegel; Lang rediscovered the result, and his proof was the first published proof.) In the same vein, Lang proves that if A is an abelian variety over a number field K and $\Gamma \subset A(K)$ is a subgroup of rank ≥ 7 contained in a 1-parameter subgroup of A, then this 1-parameter subgroup is algebraic, i.e., an elliptic curve. Later, using deep analytic arguments, Bombieri and Lang [5] extended this theory to s-parameter subgroups. For a comprehensive survey of transcendence up to the year 1997, we refer to [15].

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Serge Lang's Contributions to the Theory of Transcendental Numbers[2]

Michel Waldschmidt

When Serge Lang started to work on transcendental number theory in the early 1960s, the subject was not fashionable. It became fashionable only a few years later, thanks to the work of S. Lang certainly, but also to the contributions of A. Baker. At that time the subject was considered as very technical, not part of the main stream, and only a few specialists were dealing with it. The proofs were somewhat mysterious: why was it possible to prove some results while other conjectures resisted?

With his outstanding insight and his remarkable pedagogical gifts, Lang comes into the picture and contributes to the subject in at least two very different ways: on the one hand, he simplifies the arguments (sometimes excessively) and produces the first very clear proofs which can be taught easily; on the other hand, he introduces new tools, like group varieties, which put the topic closer to the interests of many a mathematician.

His proof of the Six Exponentials Theorem is a good illustration of the simplicity he introduced in the subject. His arguments are clear; one understands for instance why the construction of an auxiliary function is such a useful tool. Probably nobody knows so far why the arguments do not lead to a proof of the four exponentials conjecture, but this is something which will be clarified only later. Several mathematicians knew the Six Exponentials Theorem; Lang was the first to publish its proof (a few years later, K. Ramachandra rediscovered it).

Another nice example is the so-called Schneider-Lang criterion. Schneider had produced a general statement on the algebraic values of meromorphic functions in 1949. This statement of Schneider is powerful; it includes a number of transcendence results, and it was the first result containing at the same time the Hermite–Lindemann Theorem on the transcendence of $\log \alpha$, the Gel'fond–Schneider solution of Hilbert's seventh problem on the transcendence of α^β, and the Six Exponentials Theorem. However, Schneider's criterion was quite complicated; the statement itself included a number of technical hypotheses. Later, in 1957 (in his book on transcendental number theory), Schneider produced a simplified version dealing with functions satisfying differential equations (at the cost of losing the Six Exponentials Theorem from the corollaries, but Schneider did not state this theorem explicitly anyway). S. Lang found nice hypotheses which enabled him to produce a simple and elegant result.

Lang also extended this Schneider-Lang criterion to several variables, again using ideas of Schneider (which he introduced in 1941 for proving the transcendence of the values $B(a,b)$ of Euler's Beta function at rational points). Lang's extension to several variables involves Cartesian products. M. Nagata suggested a stronger statement involving algebraic hypersurfaces. This conjecture was settled by E. Bombieri in 1970 using a generalization in several variables of Schwarz's Lemma, which was obtained by Bombieri and Lang using also some deep L^2 estimates from Hörmander. It is ironic that Bombieri's Theorem is not required but that the statement with Cartesian product suffices for the very surprising proof of Baker's Theorem (and its extension to elliptic curves) found by D. Bertrand and D. W. Masser in 1980.

The introduction by S. Lang of group varieties in transcendental number theory followed a conjecture of Cartier, who asked him whether it would be possible to extend the Hermite-Lindemann Theorem from the multiplicative group to a commutative algebraic group over the field of algebraic numbers. This is the result that Lang proved in 1962. At that time there were a few transcendence results (by Siegel and Schneider) on elliptic functions and even Abelian functions. But Lang's introduction of algebraic groups in this context was the start of a number of important developments in the subject.

Among the contributions of Lang to transcendental number theory (also to Diophantine approximation and Diophantine geometry), the least are not his many conjectures which shed a new light on the subject. On the contrary, he had a way of considering what the situation should be, which was impressive. Indeed, he succeeded in getting rid of the limits from the existing results and methods. He made very few errors in his predictions, especially if we compare them with the large number of conjectures he proposed. His description of the subject will be a guideline for a very long time.

[2] *Editor's note: An expanded version of Waldschmidt's contribution is published in: "Les contributions de Serge Lang à la théorie des nombres transcendants", Gaz.* Math. **108** (2006), 35–46.

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Lang's Work on Modular Units and on Frobenius Distributions[3]

David E. Rohrlich

In 1972 Lang joined the Department of Mathematics of Yale University, where he remained a faculty member until his retirement. The move to Yale coincided with a change of direction in Lang's research, a change which reflected a broader trend in number theory as a whole: Whereas the theory of automorphic forms had previously been the exclusive domain of specialists, by the early seventies modular forms and the Langlands program were playing a central role in the thinking of number theorists of a variety of stripes. In Lang's case these influences were particularly apparent in the work with Kubert on modular units and in the work with Trotter on Frobenius distributions.

Modular Units

Two brief notes on automorphisms of the modular function field (articles [1971c] and [1973] in the *Collected Papers*) signaled Lang's developing interest in modular functions, but his primary contribution in this domain was the joint work with Kubert on modular units, expounded in a long series of papers from 1975 to 1979 and subsequently compiled in their book *Modular Units*, published in 1981. The work has two distinct components: the function theory of modular units on the one hand and the application to elliptic units on the other.

The Function-Theoretic Component

In principle the problem considered by Kubert and Lang can be formulated for any compact Riemann surface X and any finite nonempty set S of points on X. Let C_S be the subgroup of the divisor class group of X consisting of the classes of divisors of degree 0 which are supported on S. If one prefers, one can think of C_S as the subgroup of the Jacobian of X generated by the image of S under an Albanese embedding. In any case, the problem is to determine if C_S is finite, and when it is finite to compute its order.

In practice this problem is rarely of interest: if the genus of X is ≥ 1, then for most choices of S we can expect that C_S will be the free abelian group of rank $|S| - 1$, and there is nothing further to say. However, in the work of Kubert and Lang X is a modular curve and S its set of cusps. In this case Manin [18] and Drinfeld [9] had already proved the finiteness of C_S, but their proof rested on a clever use of the Hecke operators and gave no information about the order of C_S. Kubert and Lang found an altogether different proof of the Manin-Drinfeld theorem in which the whole point was to exhibit a large family of functions on X with divisorial support on S. (These functions, by the way, are the "modular units". If R_S is the subring of the function field of X consisting of functions holomorphic outside S, then the modular units are indeed the elements of the unit group $R_S^\times$ of R_S.) In optimal cases, in particular when X is the modular curve usually denoted $X(N)$ and N is a power of a prime $p \geq 5$, Kubert and Lang were able to deduce an explicit formula for $|C_S|$ in terms of certain "Bernoulli-Cartan numbers" closely related to the generalized Bernoulli numbers $b_{2,\chi}$ which appear in formulas for the value of a Dirichlet L-function $L(s,\chi)$ at $s = -1$.

This work found immediate application in the proof by Mazur and Wiles [19] of the main conjecture of classical Iwasawa theory, and since then it has found many other applications as well. But quite apart from its usefulness, the work can be appreciated as a counterpoint to the "Manin-Mumford conjecture", enunciated by Lang in an earlier phase of his career (see [1965b]) in response to questions posed by the eponymous authors. The conjecture asserts that the image of a curve X of genus ≥ 2 under an Albanese embedding intersects the torsion subgroup of the Jacobian of X in only finitely many points. A strong form of the conjecture was proved by Raynaud in 1983 [23], and the subject was subsequently enriched by Coleman's theory of "torsion packets" [5]: a torsion packet on X is an equivalence class for the equivalence relation

$$P \equiv Q \Leftrightarrow n(P - Q) \text{ principal for some } n \geq 1$$

on the points of X. Of particular relevance here is the proof by Baker [1] of a conjecture of Coleman, Kaskel, and Ribet, from which it follows that for most values of N (including in particular $N = p^n$ with p outside a small finite set) the cuspidal torsion packet on $X(N)$ consists precisely of the cusps. Thus the results of Kubert and Lang provide one of the relatively rare examples of a curve for which the order of the subgroup of the Jacobian generated by the image under an Albanese embedding of a nontrivial torsion packet on the curve has been calculated explicitly.

Elliptic Units

Let us now view modular functions f as functions on the complex upper half-plane $\mathfrak{H}$ rather than on the modular curves. Given an imaginary quadratic field K, we can then embed K in $\mathbf{C}$ and evaluate f at points $\tau \in K \cap \mathfrak{H}$. It has been known since the time of Kronecker and Weber that for

[3] *Editor's note: In addition to contributions to both* Notices *articles about Lang, David Rohrlich has also written the following piece: "Serge Lang", Gaz. Math.* **108** *(2006), 33–34.*

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appropriate choices of f and τ the values $f(\tau)$ generate ray class fields L of K, and if f is in addition a modular unit, then $f(\tau)$ is a unit of L. Roughly speaking, the group of *elliptic units* of L is the group of units obtained in this way, and a major theme of the theory is that the index of the group of elliptic units in the group of all units of L should be closely related to the class number of L. Achieving an optimal statement of this sort has proved to be an incremental process. Kubert and Lang built on the work of Siegel [28], Ramachandra [22], and especially Robert [24], and they also drew inspiration from Sinnott [29], who had solved the analogous problem which arises when the base field K is replaced by $\mathbf{Q}$ (the role of the elliptic units is then played by the cyclotomic units). In the end it was Lang's doctoral student Kersey who obtained some of the definitive results of the theory, for example the determination of a group of roots of elliptic units in the Hilbert class field H of K such that the index of this group in the group of all units of H is precisely the class number of H. Kersey was in effect a third author of the part of *Modular Units* having to do with class number formulas.

In recent years the theory of elliptic units has to some extent been subsumed in and overshadowed by broader developments in Iwasawa theory, notably Rubin's proof [25] of the one-variable and two-variable main conjectures in the Iwasawa theory of imaginary quadratic fields. But the need for explicit formulas at finite level never really ends. The very recent work of Á. Lozano-Robledo [17] and A. L. Folsom [12] attests to the ongoing vitality of the problems considered more than a quarter of a century ago by Robert and Kubert-Lang.

Frobenius Distributions

The circle of ideas known as the "Lang-Trotter conjectures" comprises two distinct themes that are developed respectively in the book *Frobenius Distributions in* GL_2*-Extensions* (reproduced in its entirety as article [1976d] of the *Collected Papers*) and the paper "Primitive points on elliptic curves" [1977b]. What these two works have in common, besides their joint authorship with Trotter, is that both are concerned with Frobenius distributions arising from elliptic curves. Here the term "Frobenius distribution" is used broadly to include any function $p \mapsto a(p)$ from prime numbers to integers which arises naturally in algebraic number theory or Diophantine geometry. Henceforth E denotes an elliptic curve over $\mathbf{Q}$ and Δ its minimal discriminant.

Frobenius Distributions in GL_2-Extensions

In this subsection we assume that E does not have complex multiplication. For $p \nmid \Delta$ put

$$a(p) = 1 + p - |\tilde{E}(\mathbf{F}_p)|,$$

where $\tilde{E}$ is the reduction of E modulo p. Given an integer t and an imaginary quadratic field K, Lang and Trotter consider the counting functions $N_t(x)$ and $N_K(x)$ corresponding to what they call the "fixed trace" and "imaginary quadratic" distributions of the map $p \mapsto a(p)$. By definition, $N_t(x)$ is the number of primes $p \le x$ ($p \nmid \Delta$) such that $a(p) = t$, and $N_K(x)$ is the number of primes $p \le x$ ($p \nmid \Delta$) such that the polynomial $X^2 - a(p)X + p$ factors into linear factors in K. Of course this polynomial is just the characteristic polynomial of a Frobenius element $\sigma_p \in \mathrm{Gal}(\bar{\mathbf{Q}}/\mathbf{Q})$ acting on the ℓ-adic Tate modules of E ($\ell \neq p$), and the "trace" in "fixed trace distribution" is an allusion to this interpretation of $a(p)$. In fact Lang and Trotter define $N_t(x)$ and $N_K(x)$ for any strictly compatible family of ℓ-adic representations $\rho_\ell : \mathrm{Gal}(\bar{\mathbf{Q}}/\mathbf{Q}) \to \mathrm{GL}(2, \mathbf{Z}_\ell)$ such that the image of the product representation into $\mathrm{GL}(2, \hat{\mathbf{Z}})$ is an open subgroup of $\mathrm{GL}(2, \hat{\mathbf{Z}})$ and such that the characteristic polynomial of $\rho_\ell(\sigma_p)$ has the form $X^2 - a(p)X + p$ with $a(p) \in \mathbf{Z}$ and $|a(p)| \le 2\sqrt{p}$. The authors ask by the way whether any such families exist besides the ones coming from elliptic curves, and to my knowledge their question has not been explicitly addressed in the literature. In any case, given this framework Lang and Trotter define certain constants $c_t \ge 0$ and $c_K > 0$ (depending on the compatible family $\{\rho_\ell\}$ as well as on t and K) and conjecture that

$$N_t(x) \sim c_t \sqrt{x}/\log x$$

and

$$N_K(x) \sim c_K \sqrt{x}/\log x$$

for $x \to \infty$. Here we are following the convention of Lang-Trotter that if $c_t = 0$, then the relation $N_t(x) \sim c_t \sqrt{x}/\log x$ means that $N_t(x)$ is constant for large x (in other words the underlying set of primes is finite).

Let us refer to this conjecture as the *first Lang-Trotter conjecture*; the conjecture on primitive points discussed below is then the *second Lang-Trotter conjecture*. An important aspect of the first conjecture is the precise definition of the constants c_t and c_K, which is based on a probabilistic model. This feature distinguishes the conjecture from, say, Tuskina's earlier attempt [30] to study the asymptotics of supersingular primes (the case $t = 0$) without a probabilistic model and without any prediction about the value of c_0. A similar comment pertains to V. K. Murty's paper [21], which is in other respects a vast generalization of the Lang-Trotter conjecture. On the other hand, in their study of the asymptotics of supersingular primes for modular

abelian varieties, Bayer and González [2] do consider a probabilistic model generalizing that of Lang and Trotter.

What is immediately striking about the first Lang-Trotter conjecture is its apparent utter inaccessibility. As Lang once remarked, the conjecture is contained "in the error term of the Riemann hypothesis." Nonetheless, there are a few results (referring for the most part to an elliptic curve E rather than to an abstract compatible family $\{\rho_\ell\}$ satisfying the Lang-Trotter axioms) that have some bearing on the conjecture. To begin with, the theorem of Elkies [10] that an elliptic curve over $\mathbf{Q}$ has infinitely many supersingular primes is at least consistent with the conjecture, because Lang and Trotter show that $c_0 > 0$ in this case. Also consistent are a number of "little oh" results about $N_t(x)$ and $N_K(x)$, starting with Serre's observation that these functions are $o(x/\log x)$ (i.e., the underlying sets of primes have density 0) and even $o(x/(\log x)^\gamma)$ with $\gamma > 1$ (see [26], [27]). Further improvements in the bound for $N_t(x)$ were made by Wan [31] and V. K. Murty [20], and in the case of supersingular primes the bound $N_0(x) = O(x^{3/4})$ was obtained by Elkies, Kaneko, and M. R. Murty [11]. There are also results giving the conjectured growth rate "on average" for $N_0(x)$ (Fouvry-Murty [13]) and more generally for $N_t(x)$ (David-Pappalardi [8]), the average being taken over a natural two-parameter family of elliptic curves. As for $N_K(x)$, the best upper bound to date is in the recent paper of Cocojaru, Fouvry, and M. R. Murty [4], who also give estimates under the generalized Riemann hypothesis. Finally, and in an altogether different direction, analogous problems for Drinfeld modules have been investigated by Brown [3] and David [6], [7].

Primitive Points on Elliptic Curves

The first Lang-Trotter conjecture can be viewed as an analogue of the Chebotarev density theorem in which finite Galois extensions of $\mathbf{Q}$ are replaced by the infinite Galois extensions generated by division points on elliptic curves. The second Lang-Trotter conjecture also has a classical antecedent, but it is Artin's primitive root conjecture, which predicts the density of the set of primes p such that a given nonzero integer a is a primitive root modulo p. In particular, if $a \neq -1$ and a is not a square, then this density is conjectured to be positive. The analogue proposed by Lang and Trotter involves an elliptic curve E over $\mathbf{Q}$ of positive rank and a given point $P \in E(\mathbf{Q})$ of infinite order. There is no longer any need to assume that E is without complex multiplication. Consider the set of primes $p \nmid \Delta$ such that $\tilde{E}(\mathbf{F}_p)$ is generated by the reduction of P modulo p (and is therefore in particular cyclic). Lang and Trotter conjecture that this set has a density, and they explain how to compute the conjectured density using reasoning analogous to Artin's. More generally, Lang and Trotter consider an arbitrary free abelian subgroup Γ of $E(\mathbf{Q})$. If we let $N_\Gamma(x)$ denote the number of primes $p \leq x$ ($p \nmid \Delta$) such that $\tilde{E}(\mathbf{F}_p)$ coincides with the reduction of Γ modulo p, then the general form of the conjecture is that

$$N_\Gamma(x) \sim c_\Gamma x/\log x$$

for some constant c_Γ.

Just as Hooley [16] was able to prove Artin's primitive root conjecture by assuming the generalized Riemann hypothesis, R. Gupta and M. R. Murty [14] were able to prove a conditional result in the Lang-Trotter setting: Under the generalized Riemann hypothesis we have $N_\Gamma(x) \sim c_\Gamma x/\log x$ whenever the rank of Γ is ≥ 18. In fact in the case of elliptic curves with complex multiplication, Gupta and Murty obtain an asymptotic relation of this sort even when the rank of Γ is one, but for a slightly different counting function, say $N'_\Gamma(x)$, which differs from $N_\Gamma(x)$ in that we count only primes which split in the field of complex multiplication. Unconditionally, Gupta and Murty prove that if the rank of Γ is ≥ 6, then $N'_\Gamma(x) \gg x/(\log x)^2$.

An analogue of the second Lang-Trotter conjecture can also be formulated for an elliptic curve over a global field of positive characteristic, and in very recent work Hall and Voloch [15] have proved the analogue whenever the rank of Γ is ≥ 6.

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Serge Lang's Work in Diophantine Geometry

Paul Vojta

I first knew of Serge Lang through his books: *Algebra, Algebraic Number Theory*, and *Elliptic Functions*. Later, as I was finishing my degree and getting ready to join him at Yale, he was finishing his book *Fundamentals of Diophantine Geometry*, a substantial rewrite of his earlier *Diophantine Geometry*. In Serge's world view, the way you choose to look at a theorem is often more important than the theorem itself. As the title suggests, Serge's outlook on number theory was decidedly geometric. While others at the time shared this viewpoint (e.g., Weil, Tate, Serre), it is easy to forget that others did not, as Mordell's review of the earlier *Diophantine Geometry* attests.

A few years later Serge wrote *Introduction to Arakelov Theory*, which, together with Cornell-Silverman and Soulé-Abramovich-Burnol-Kramer, forms the short list of key introductory books in this area.

Beyond books, Serge's influence on number theory derives more from his conjectures than from his theorems, although he had quite a few of those, too. His earliest major conjecture in this area was that a projective variety over a number field embedded in $\mathbf{C}$ is *Mordellic* (i.e., had only finitely many points rational over any given number field containing the field of definition of the variety) if and only if the corresponding complex projective variety is Kobayashi hyperbolic. Recall that the *Kobayashi semidistance* on a complex space X is the largest semidistance satisfying the property that all holomorphic maps from $\mathbf{D}$ to X are distance nonincreasing, where $\mathbf{D}$ is the unit disk in $\mathbf{C}$ with the Poincaré metric. A complex space is then *Kobayashi hyperbolic* if its Kobayashi semidistance is actually a distance. For example, a compact Riemann surface of genus g is Kobayashi hyperbolic if and only if $g \geq 2$, exactly the condition of Mordell's conjecture. Later Serge extended this conjecture to include subfields of $\mathbf{C}$ finitely generated over $\mathbf{Q}$.

In 1978 Brody showed that a compact complex space X is Kobayashi hyperbolic if and only if there are no nonconstant holomorphic maps from $\mathbf{C}$ to X, thus simplifying the above conjecture to the assertion that X is Mordellic if and only if there are no nonconstant holomorphic maps $\mathbf{C} \to X$. In general, this conjecture is still open, although it has been proved for curves and more generally for closed subvarieties of abelian

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varieties. It is also closely related to conjectures and theorems in Nevanlinna theory.

The above special case came as a consequence of a proof of another of Lang's conjectures. Let A be a semiabelian variety over $\mathbf{C}$, let X be a closed subvariety of A, and let Γ be a subgroup of $A(\mathbf{C})$ such that $\dim_{\mathbf{Q}} \Gamma \otimes_{\mathbf{Z}} \mathbf{Q}$ is finite. Then $\Gamma \cap X(\mathbf{C})$ is contained in a finite union of translated semiabelian subvarieties of A contained in X. This was proved by Faltings, Vojta, and McQuillan (who stated it over the algebraic closure $\mathbf{Q}^a$ of $\mathbf{Q}$, but the general case follows by the function field variant of the same theorem). This gives finiteness statements for $X(k)$ in the abelian case by letting $\Gamma = A(k)$ and correctly anticipated the fact that working with rational points (or integral points in the semiabelian case) really boils down to finite rank of Γ. The conjecture was also proved for function fields of characteristic $p > 0$ by Hrushovsky (suitably restated). Returning to the case over $\mathbf{Q}^a$, this conjecture of Lang has been combined with Bogomolov's conjecture on points in $X(\mathbf{Q}^a)$ of small height to give an elegant result conjectured by Poonen and proved by Rémond.

Another conjecture of Lang that has received a lot of attention is his conjecture (originally posed as a question by Bombieri) that if X is a pseudo-canonical projective variety defined over a subfield of $\mathbf{C}$ finitely generated over $\mathbf{Q}$, then X is pseudo-Mordellic. Here a variety is *pseudo-canonical* if it is of general type. This follows Griffiths's definition that a variety is *canonical* if its canonical bundle is ample; in Lang's terminology, *pseudo* means outside of a proper Zariski-closed subset, so pseudo-ample means big and therefore pseudo-canonical means general type.[4] Likewise pseudo-Mordellic means that the rational points over any given subfield of $\mathbf{C}$ finitely generated over $\mathbf{Q}$ and containing the field of definition of the variety are not Zariski dense. This is sometimes called the "weak Lang conjecture"; the strong version asserts that there is a proper Zariski-closed subset Z of X such that $X(k) \setminus Z(k)$ is finite for all fields k as above.

A number of consequences of these conjectures have been proved over the years. For example, Caporaso, Harris, and Mazur showed that if the weak Lang conjecture holds, then for all integers $g \geq 2$ and for all number fields k there is a bound $B(g,k) \in \mathbf{Z}$ such that $\#C(k) \leq B(g,k)$ for all curves C of genus g over k. If the strong Lang conjecture is true, they showed in addition that for all $g \geq 2$ there is a bound $N(g) \in \mathbf{Z}$ such that for all k there are only finitely many smooth projective curves of genus g over k (up to isomorphism) with $\#C(k) > N(g)$. It should be noted that some people (e.g., Bogomolov) believe this conjecture to be false.

Serge never let controversy stop him, though, and he formulated additional conjectures regarding the proper Zariski-closed subset in the strong form of his conjecture. Let X be a projective variety over a field $k \subseteq \mathbf{C}$. We define the *algebraic special set* to be the Zariski closure of the union of the images of all nonconstant rational maps of group varieties over k^a into $X \times_k k^a$. He then conjectured that X is pseudo-canonical if and only if its special set is not all of X. He further conjectured that if X is pseudo-canonical, then the proper Zariski-closed subset in his strong conjecture can be taken to be the special set and also (assuming $k = \mathbf{C}$) that all nonconstant holomorphic maps $\mathbf{C} \to X$ lie in the special set (pseudo-Brody hyperbolic). This last conjecture thus generalizes his conjecture on Kobayashi hyperbolicity. A recurring theme in these conjectures is that one has algebraic criteria for analytic conditions of hyperbolicity.

In recent years Serge set aside these conjectures in favor of working on the heat kernel, but he returned to an aspect of them during his last few years. Recall that Roth's theorem (in its simplest form) states that if α is an algebraic number and $\epsilon > 0$, then there are only finitely many $p/q \in \mathbf{Q}$ (with p and q relatively prime integers and $q > 0$) such that $-\log|\alpha - p/q| > (2+\epsilon)\log q$. In the 1960s Lang conjectured that this could be improved to $2\log q + c \log\log q$ for some constant c, possibly $1 + \epsilon$, and subsequent computations with Trotter backed this up. Furthermore, based on a theorem of Khinchin, he conjectured that the error term $\epsilon \log q$ in Roth could be improved to $\log \psi(q)$ for any given increasing function ψ for which the sum $\sum (q \log \psi(q))^{-1}$ converges. His philosophy was that Khinchin's theorem applied to almost all real numbers, in a measure theoretic sense, and that algebraic numbers should behave likewise.

When the conjecture with Nevanlinna theory came on the scene, he posed the corresponding question in that context. It was proved by P.-M. Wong and Lang, and subsequent refinements have been obtained by Hinkkanen, Ye, and others. The original question on Diophantine approximation still remains open though.

In the last two years, Lang and van Frankenhuijsen started work on the question of what the best error term should be for the abc conjecture. For some time it has been well known that a Khinchin-type error term is too strong. Instead, they suggest $O(\sqrt{h/\log h})$, where h is the logarithmic height of the point $[a : b : c]$. Sadly, van Frankenhuijsen will have to continue work on this on his own.

[4] *As Serge would say, "The notation is functorial with respect to the ideas."*

Diophantine Geometry As Galois Theory in the Mathematics of Serge Lang

Minhyong Kim

Lang's conception of Diophantine geometry is rather compactly represented by the following celebrated conjecture [4]:

> Let V be a subvariety of a semi-abelian variety A, $G \subset A$ a finitely generated subgroup, and $\mathrm{Div}(G)$ the subgroup of A consisting of the division points of G. Then $V \cap \mathrm{Div}(G)$ is contained in a finite union of subvarieties of V of the form $B_i + x_i$, where each B_i is a semi-abelian subvariety of A and $x_i \in A$.

There is a wealth of literature at this point surveying the various ideas and techniques employed in its resolution, making it unnecessary to review them here in any detail [10], [39]. However, it is still worth taking note of the valuable *generality* of the formulation, evidently arising from a profound instinct for the plausible structures of mathematics. To this end, we remark merely that it was exactly this generality that made possible the astounding interaction with geometric model theory in the1990s [3]. That is to say, analogies to model-theoretic conjectures and structure theorems would have been far harder to detect if attention were restricted, for example, to situations where the intersection is expected to be finite. Nevertheless, in view of the sparse subset of the complex net of ideas surrounding this conjecture that we wish to highlight in the present article, our intention is to focus exactly on the case where A is compact and V does *not* contain any translate of a connected nontrivial subgroup. The motivating example, of course, is a compact hyperbolic curve embedded in its Jacobian. Compare then the two simple cases of the conjecture that are amalgamated into the general formulation:

(1) $V \cap A[\infty]$, the intersection between V and the torsion points of A, is finite.
(2) $V \cap G$ is finite.

Lang expected conjecture (1) to be resolved using Galois theory alone. This insight was based upon work of Ihara, Serre, and Tate ([28], VIII.6) dealing with the analogous problem for a torus and comes down to the conjecture, still unresolved, that the image of the Galois representation in $\mathrm{Aut}(A[\infty]) \simeq \mathrm{GL}_{2g}(\hat{\mathbf{Z}})$ contains an open subgroup of the homotheties $\hat{\mathbf{Z}}^*$. Even while assertion (1) is already a theorem of Raynaud [44], significant progress along the lines originally envisioned by Lang was made in [2] by replacing $A[\infty]$ with $A[p^\infty]$, the points of p-power torsion, and making crucial use of p-adic Hodge theory.

It is perhaps useful to reflect briefly on the overall context of Galois-theoretic methods in Diophantine geometry, of course without attempting to do justice to the full range of interactions and implications. Initially, that Galois theory is relevant to the study of Diophantine problems should surprise no one. After all, if we are interested in $X(F)$, the set of rational points of a variety X over a number field F, what is more natural than to observe that $X(F)$ is merely the fixed point set of $\Gamma := \mathrm{Gal}(\bar{F}/F)$ acting on $X(\bar{F})$? Since the latter is an object of classical geometry, such an expression might be expected to nicely circumscribe the subset $X(F)$ of interest. This view is of course very naive, and the action of Γ on $X(\bar{F})$ is notoriously difficult to use in any direct fashion. The action on *torsion points* of commutative group varieties on the other hand, while still difficult, is considerably more tractable, partly because a finite abelian subgroup behaves relatively well under specialization. Such an arithmetic variation exerts tight control on the fields generated by the torsion points, shaping Galois theory into a powerful tool for investigations surrounding (1).

On the other hand, for conjecture (2), where the points to be studied are not torsion, it is not at all clear that Galois theory can be as useful. In fact, my impression is that Lang expected *analytic geometry* of some sort to be the main input to conjectures of type (2). This is indicated, for example, by the absence of any reference to arithmetic in the formulation. We might even say that implicit in the conjecture is an important idea that we will refer to as the *analytic strategy*:

> (a) Replace the difficult Diophantine set $V(F)$ by the geometric intersection $V \cap G$.
> (b) Try to prove this intersection finite by analytic means.

In this form, the strategy appears to have been extraordinarily efficient over function fields, as in the work of Buium [4]. Even Hrushovski's [15] proof can be interpreted in a similar light where passage to the completion of suitable theories is analogous to the move from algebra to analysis (since the theory of fields is not good enough). These examples should already suffice to convince us that it is best left open as to what kind of analytic means are most appropriate in a given situation. The proof over number fields by Faltings [9], as well as the curve case by Vojta [47], utilizes rather heavy Archimedean analytic geometry. Naturally, the work of Vojta and Faltings draws us away from the realm of traditional Galois theory. However, in Chabauty's theorem [5], where V is a curve and the rank of G is strictly

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less than the dimension of A, it is elementary non-Archimedean analysis, more specifically p-adic abelian integrals, that completes the proof. Lang makes clear in several different places ([28], [28], notes to Chapter 8; [36], I.6) that Chabauty's theorem was a definite factor in the formulation of his conjecture. This then invites a return to our main theme, as we remind ourselves that non-Archimedean analysis has come to be viewed profitably over the last several decades as a projection of *analysis on Galois groups*, a perspective of which Lang was well aware ([33], Chapter 4). As such, it has something quite substantial to say about nontorsion points, at least on elliptic curves ([18], for example). Hodge theory is again a key ingredient, this time as the medium in which to realize such a projection [43].

It is by now known even to the general public that a careful study of Galois actions underlies the theorem of Wiles [49] and roughly one-half of the difficulties in the theorem of Faltings [8]. There, Galois representations must be studied in conjunction with an array of intricate auxiliary constructions. However, the most basic step in the Galois-theoretic description of nontorsion points, remarkable in its simplicity, goes through the Kummer exact sequence

$$0 \to A[m](F) \to A(F) \to A(F) \xrightarrow{\delta} H^1(\Gamma, A[m]) \to H^1(\Gamma, A) \to \dots.$$

In this case, an easy study of specialization allows us to locate the image of δ inside a subgroup $H^1(\Gamma_S, A[m])$ of cohomology classes with restricted ramification which then form a finite group. We deduce thereby the finiteness of $A(F)/mA(F)$, the weak Mordell-Weil theorem. Apparently, a streamlined presentation of this proof, systematically emphasizing the role of Galois cohomology, first appears in Lang's paper with Tate [38]. There they also emphasize the interpretation of Galois cohomology groups as classifying spaces for *torsors*, in this case, for A and $A[m]$. (We recall that a torsor for a group U in some category is an object corresponding to a set with simply transitive U-action, where the extra structure of the category, such as Galois actions, prevents them from being trivial. See, for example, [40], III.4.) This construction has been generalized in one direction to study nontorsion algebraic cycles by associating to them extensions of motives [7]. More pertinent to the present discussion, however, is a version of the Kummer map that avoids any attempt to abelianize, taking values in fact in *nonabelian* cohomology classes.

In the course of preparing this article, I looked into Lang's *magnum opus* [28] for the first time in many years and was a bit surprised to find a section entitled "Non-abelian Kummer theory". What is nonabelian there is the Galois group that needs to be considered if one does not assume a priori that the torsion points of the group variety are rational over the ground field. The field of m-division points of the rational points will then have a Galois group H of the form

$$0 \to A[m] \to H \to M \to 0$$

where $M \subset \mathrm{GL}_{2g}(\mathbf{Z}/m)$. Thus, "non-abelian" in this context is used in the same sense as in the reference to nonabelian Iwasawa theory. But what is necessary for hyperbolic curves is yet another layer of noncommutativity, this time in the coefficients of the action. Given a variety X with a rational point b, we can certainly consider the étale fundamental group $\hat{\pi}_1(\bar{X}, b)$ classifying finite étale covers of $\bar{X}$. But the same category associates to any other point $x \in X(F)$ the set of étale paths

$$\hat{\pi}_1(\bar{X}; b, x)$$

from b to x which is naturally a torsor for $\hat{\pi}_1(\bar{X}, b)$. All these live inside the category of pro-finite sets with Galois action. There is then a nonabelian continuous cohomology set $H^1(\Gamma, \hat{\pi}_1(\bar{X}, b))$ that classifies torsors and a nonabelian Kummer map

$$\delta^{na} : X(F) \to H^1(\Gamma, \hat{\pi}_1(\bar{X}, b))$$

sending a point x to the class of the torsor $\hat{\pi}_1(\bar{X}; b, x)$. This is obviously a basic construction whose importance, however, has begun to emerge only in the last twenty or so years. It relies very much on the flexible use of varying base points in Grothendieck's theory of the fundamental group, and it appears to have taken some time after the inception of the arithmetic π_1 theory [45] for the importance of such a variation to be properly appreciated [12], [6], [16]. In fact, the impetus for taking it seriously came also for the most part from Hodge theory [13], [14]. As far as Diophantine problems are concerned, in a letter to Faltings [12] written shortly after the proof of the Mordell conjecture, Grothendieck proposed the remarkable conjecture that δ^{na} should be a bijection for compact hyperbolic curves. He expected such a statement to be directly relevant to the Mordell problem and probably its variants like conjecture (2). This expectation appears still to be rather reasonable. For one thing, it is evident that the conjecture is a hyperbolic analogue of the finiteness conjecture for Tate-Shafarevich groups. And then profound progress is represented by the work of Nakamura, Tamagawa, and Mochizuki [42], [46], [41], where a statement of this sort is proved when points in the number field are replaced by dominant maps from other varieties. Some marginal insight might also be gleaned from [22] and [23], where a unipotent analogue of the Kummer map is related to Diophantine finiteness theorems. There, the ambient space inside which the analysis takes place is a classifying variety

$H^1_f(\Gamma_v, U_n^{et})$ of torsors for the local unipotent étale fundamental group (rather than the Jacobian), while the finitely generated group G is replaced by the image of a map

$$H^1_f(\Gamma_S, U_n^{et}) \to H^1_f(\Gamma_v, U_n^{et})$$

coming from a space of global torsors. Thereby, one obtains a new manifestation of the analytic strategy proving $X \cap \mathrm{Im}[H^1_f(\Gamma_S, U_n^{et})]$ to be finite in some very special circumstances and in general for a hyperbolic curve over $\mathbf{Q}$ if one admits standard conjectures from the theory of mixed motives (for example, the Fontaine-Mazur conjecture on geometric Galois representations). Fortunately, Chabauty's original method fits naturally into this setting as the technical foundation of the analytic part now becomes nonabelian p-adic Hodge theory and iterated integrals. Incidentally, some sense of the Diophantine content of these ideas can already be gained by deriving the *injectivity* of δ^{na} from the Mordell-Weil theorem.

It should be clear at this point that the Galois theory of the title refers in general to the theory of the fundamental group. Serge Lang was profoundly concerned with the fundamental group for a good part of his mathematical life. A rather haphazard list of evidence might be comprised of:

—his foundational work on unramified class field theory for varieties over finite fields, where he proves the surjectivity of the reciprocity map among many other things [25], [26];

—his study of the ubiquitous "Lang torsor" [27];

—his work with Serre on fundamental groups of proper varieties in arbitrary characteristic [37];

—his extensive study with Kubert of the modular function field [24];

—his work with Katz [21] on finiteness theorems for relative π_1's that made possible the subsequent proof by Bloch [1] and then Kato and Saito [19], [20] of the finiteness of CH_0 for arithmetic schemes.

Besides these influential papers, the reader is referred to his beautiful AMS colloquium lectures [31] for a global perspective on the role of covering spaces in arithmetic.

Even towards the end of his life when his published work went in an increasingly analytic direction, he had a keen interest both in fundamental groups and in the analogy between hyperbolic manifolds and number fields wherein fundamental groups play a central role. In my last year of graduate school, he strongly urged me to study the work of Kato and Saito (and apply it to Arakelov theory!) even though it had been years since he had himself been involved with such questions. From the spring of 2004 I recall a characteristically animated exchange in the course of which he explained to me a theorem of Geyer [11] stating that abelian subgroups of $\mathrm{Gal}(\bar{\mathbf{Q}}/\mathbf{Q})$ are pro-cyclic. It was clear that he perceived this fact to fit nicely into his vivid ideas about the heat kernel [17], but in a manner that I failed (and still fail) to comprehend properly. (He was unfortunately secretive with his deeper reflections on the arithmetic significance of his later work, allowing only informal glimpses here and there. It is tempting but probably premature to speculate about a Galois theory that encompasses even Archimedean analysis.) The preoccupation with hyperbolic geometry that was evident even from the 1970s ([30], [34], [35], and [36], Chapters 8 and 9) could rather generally be construed as reflecting a persistent intuition about the relevance of fundamental groups to Diophantine problems. (An intuition that was shared by Grothendieck [12] and even Weil [48].)

As for the direct application of nonabelian fundamental groups to Diophantine geometry that we have outlined here, one can convincingly place it into the general framework of Lang's inquiries. He is discussing the theorem of Siegel in the following paragraph from the notes to Chapter 8 of [28]:

> The general version used here was presented in [28] following Siegel's (and Mahler's) method. The Jacobian replaces the theta function, as usual, and the mechanism of the covering already used by Siegel appears here in its full formal clarity. It is striking to observe that in [25], I used the Jacobian in a formally analogous way to deal with the class field theory in function fields. In that case, Artin's reciprocity law was reduced to a formal computation in the isogeny $u \mapsto u^{(q)} - u$ of the Jacobian. In the present case, the heart of the proof is reduced to a formal computation of heights in the isogeny $u \mapsto mu + a$.

We have emphasized above the importance of the Kummer map

$$x \mapsto [\hat{\pi}_1(\bar{X}; b, x)] \in H^1(\Gamma, \hat{\pi}_1(\bar{X}; b)).$$

When X is defined over a finite field $\mathbf{F}_q$ and we replace $\hat{\pi}_1(\bar{X}, b)$ by its abelian quotient $H_1(\bar{X}, \hat{\mathbf{Z}})$, the map takes values in

$$H^1(\mathrm{Gal}(\bar{\mathbf{F}}_q/\mathbf{F}_q), H_1(\bar{X}, \hat{\mathbf{Z}})) = H_1(\bar{X}, \hat{\mathbf{Z}})/[(\mathrm{Fr} - 1)H_1(\bar{X}, \hat{\mathbf{Z}})],$$

$\mathrm{Fr} \in \mathrm{Gal}(\bar{\mathbf{F}}_q/\mathbf{F}_q)$ being the Frobenius element. But this last group is nothing but the kernel

$$\hat{\pi}_1^{ab}(X)^0$$

of the structure map

$$\hat{\pi}_1^{ab}(X) \to \hat{\pi}_1(\mathrm{Spec}(\mathbf{F}_q)).$$

Thus the abelian quotient of the Kummer map becomes identified with the *reciprocity* map [19]

$$\mathrm{CH}_0(X)^0 \to \hat{\pi}_1^{ab}(X)^0$$

of unramified class field theory evaluated on the cycle $(x) - (b)$. In other words, the reciprocity map is merely an "abelianized" Kummer map in this situation. There is no choice but to interprete the reciprocity law [19], [20] as an "abelianized Grothendieck conjecture" over finite fields.

Of course it is hard to imagine exactly what Lang himself found striking in the analogy when he wrote the lines quoted above. What is not hard to imagine is that he would have been very much at home with the ideas surrounding Grothendieck's conjecture and the nonabelian Kummer map.

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Serge Lang and the Heat Kernel

Jay Jorgenson

Beginning in the early 1990s, Serge Lang viewed the heat kernel and heat kernel techniques as a potentially unlimited source of mathematics which would touch many fields of study. In [19] we presented an argument supporting the term "the ubiquitous heat kernel" by citing numerous results where the heat kernel played a prominent role. In Lang's own writing, one can see the incorporation of the heat kernel in several places, including: The Weierstrass approximation theorem and Poisson summation formula in [26], the explicit formulas for number theory in [25], the Gamma function in [27], and the background for the entire development in [28]. Lang's fascination with the heat kernel was so thorough that, according to Peter Jones, Serge began referring to himself as "an analyst" when asked to describe his research interests.

From the early 1990s until his death in 2005, Serge's own research activities can be described as addressing two points: Analytic aspects of regularized products and harmonic series, and geometric constructions of zeta functions. Both endeavors included heat kernels and heat kernel analysis, and the projects together focused on the long-term goal of developing a theory of "ladders" of zeta functions. I had the unique honor of working with Serge on these and other projects for nearly fifteen years, and I will now describe a portion of the mathematics Serge and I had in mind.

Regularized Products and Harmonic Series

Children learn how to multiply numbers, developing the ability to compute the product of a finite set of numbers. The astute student will realize they actually can evaluate the product of a countably infinite set of numbers provided that all but a finite number of terms in the product are equal to one. In undergraduate analysis courses, students study a slight perturbation of the elementary setting, namely when the terms in the countably infinite sequence approach one sufficiently fast. The convergence result established also demonstrates that when evaluating the infinite product, one can simply multiply a sufficiently large number of terms in the sequence and obtain an answer quite close to the theoretical result which exists for the infinite product. This connection with the elementary situation is intuitively consistent with that which is learned in childhood.

In another direction, one can seek other mathematical means by which one can determine the product of a finite sequence and then study the situations when the definition extends. For example, let $A = \{a_k\}$ with $k = 1, \ldots, n$ be a finite sequence of real, positive numbers, and let

$$\zeta_A(s) = \sum_{k=1}^{n} a_k^{-s}, \tag{1}$$

which we consider as a function of a complex variable s. Elementary calculus applies to show that

$$\prod_{k=1}^{n} a_k = \exp(-\zeta_A'(0)). \tag{2}$$

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In words, a special value of the zeta function (1) can be used to realize a product of the elements of the finite set of numbers A.

To generalize (2), we seek to describe the countably infinite sets of numbers for which (2) makes sense. Perhaps the first example to consider is $A = \mathbf{Z}_{>0}$, the set of natural integers, so then $\zeta_A(s)$ is the Riemann zeta function. Classically, it is known that the Riemann zeta function admits a meromorphic continuation to all $s \in \mathbf{C}$ and is holomorphic at $s = 0$. Furthermore, it can be easily shown (using the functional equation of the Riemann zeta function) that $-\zeta'_{\mathbf{Z}_{>0}}(0) = \log(\sqrt{2\pi})$, which leads to the remark "infinity factorial is equal to $\sqrt{2\pi}$". Of course, such a comment needs to be understood in the sense of (2) and meromorphic continuation.

More generally, we define a countably infinite sequence $A = \{a_k\}$ to have a *zeta regularized product,* or *regularized product,* if the zeta function

$$\zeta_A(s) = \sum_{k=1}^{\infty} a_k^{-s}$$

converges for $s \in \mathbf{C}$ with $\mathrm{Re}(s)$ sufficiently large, admits a meromorphic continuation at $s = 0$, and is holomorphic at $s = 0$. With these conditions, the regularized product of the elements of A is defined by the special value of the zeta function as in (2). The problem which naturally arises is to determine the conditions on the sequence A for which a regularized product exists. For this, we rewrite the zeta function ζ_A as

$$\zeta_A(s) = \frac{1}{\Gamma(s)} \int_0^{\infty} \theta_A(t) t^s \frac{dt}{t} \tag{3}$$

where

$$\theta_A(t) = \sum_{k=1}^{\infty} e^{-a_k t} \tag{4}$$

and $\Gamma(s)$ is the Gamma function. As further background, we refer to the articles [1] and [29], which discuss some of the important roles played by the zeta regularized products.

The first part of my work with Lang appeared in [14] and [15]. The paper [14] establishes general conditions for theta functions which will lead to regularized products as well as to regularized harmonic series. As an example of the type of general computations in [14], a connection is made relating zeta regularized products with Weierstrass products from complex analysis, which we call the Lerch formula, thus establishing a relation with the elementary notion of infinite products of numbers which approach one sufficiently fast.

Further analysis in [15] and [17] leaned toward a type of formal analytic number theory associated with regularized products. Functional equations of zeta functions were shown to be equivalent to inversion formulas for the associated theta functions, so then one is drawn to study theta functions rather than zeta functions in this context. In the development of the explicit formulas, the Weil functional comes from evaluating a complex integral involving the multiplicative factors in the functional equations of zeta functions. If the multiplicative factors are assumed to be expressed in terms of regularized products, then in [15] it is shown that the Weil functional can be evaluated in terms of a Parseval-type formula, so again one is seeking theta functions with certain properties, this time for the factors in the functional equations.

Quite naturally, the problem which arises is to characterize sequences from which one can obtain regularized products. From (3), this problem amounts to determining sequences whose associated theta function admits certain asymptotic behaviors as t approaches zero and infinity. From Riemannian geometry, theta functions naturally appear as the trace of heat kernels associated to certain differential and pseuo-differential operators. The asymptotic conditions defined in [14] were established with the heat kernel in mind.

From analytic number theory, where one does not know if an appropriate operator exists, an obvious sequence to study is given by the nontrivial zeros of a zeta function. In [2] Cramér studied the theta function

$$\theta_{\mathbf{Q}}(z) = \sum_{\rho} e^{\rho z}, \tag{5}$$

where the sum is over the zeros of the Riemann zeta function with positive imaginary part, which we denote by $A^+(\mathbf{Q})$, and with $z \in \mathbf{C}$, again having positive imaginary part. Cramér's results in [2] consisted of two main parts: The theta function (5) admits a branched meromorphic continuation to all $z \in \mathbf{C}$, and the singularities of the continuation of (5) are at points of the form $\pm i \log p^n$ for a positive integer n and a prime number p; and if we set $z = it$ for $t \in \mathbf{R}_{>0}$, then (5) satisfies the asymptotic conditions required of a theta function (4) in order to form a regularized product from the sequence $A^+(\mathbf{Q})$. Cramér's theorem forms a key component in Deninger's program [3], which has the goal of developing a cohomological approach to analytic number theory (see also [29] and [31]).

In [16] and [18] Lang and I extended Cramér's theorem to a wide range of zeta functions which satisfy very general conditions. In addition, in [20] we extended Guinand's results from [7], [8], and [9] which proved, among other theorems, several functional relations for Cramér's function. The point of view taken in [16], [18] and [20] is to "load an induction", as Lang would say, by placing various hypotheses on the factors in the functional equation; specifically, it was assumed that the factors in the functional equations could

be expressed as regularized products. With this hypothesis and other general assumptions such as the existence of an Euler product, or Bessel sum, expansion, it was proved that the original zeta function could be expressed as regularized products. As an example, since the Gamma function can be expressed as a regularized product, which comes from the first example mentioned above, we concluded that the Riemann zeta function is also expressible as regularized products. A similar argument applies to virtually all zeta functions from number theory.

Turning to geometry, one can define a (Selberg) zeta function associated to any finite volume hyperbolic Riemann surface (see [10] and references therein for a complete development of the Selberg zeta function in this context). In the case when the surface is compact, the known functional equation and Euler product expansions allow one to apply the results from [16] to conclude that the Selberg zeta function can be expressed as regularized products, re-proving a known result from [4] and [30]. For noncompact yet finite volume surfaces, attempts were made to express the Selberg zeta function as a regularized product, but the successful results required the surface to be arithmetic. Using the results from [16], we were able to conclude that the Selberg zeta function in general is a regularized product through the induction we defined. The most interesting example for us was the Selberg zeta function associated to the discrete group $\mathrm{PSL}(2,\mathbf{Z})$, since the functional equation involves the Riemann zeta function, thus producing the following finite "ladder" of functions: the Gamma function, the Riemann zeta function, and the Selberg zeta function for $\mathrm{PSL}(2,\mathbf{Z})$. As stated, a direct calculation shows that the Gamma function is a regularized product, and the induction hypothesis from [16] implies that the Riemann zeta function and then the Selberg zeta function for $\mathrm{PSL}(2,\mathbf{Z})$ are expressible as regularized products.

Constructions from Geometry

What is the next step in the "ladder" of zeta functions? Lang and I believed that one can construct zeta functions using heat kernel analysis on symmetric spaces, resulting in an infinite "ladder" of zeta functions with functional equations involving zeta functions on the lower levels. Consider a general setting involving a symmetric space X and discrete group Γ with finite volume and noncompact quotient $\Gamma\backslash X$. As described in [23], the procedure we propose is the following: Start with a heat kernel on X, periodize with respect to Γ, evaluate a regularized trace of the heat kernel, and then compute a certain integral transform (the Gauss transform, which is the Laplace transform with a quadratic change of variables) of the regularized trace of the heat kernel. The resulting object is our proposed zeta function associated to $\Gamma\backslash X$. In the work we were undertaking, Lang and I were focusing our attention on the symmetric spaces associated to $\mathrm{SL}(n,\mathbf{C})$, though one certainly could consider the spaces associated to $\mathrm{SL}(n,\mathbf{R})$. By taking $\Gamma = \mathrm{SL}(n,\mathbf{Z})$ and $G = \mathrm{SL}(n,\mathbf{R})$, we felt that one could obtain an infinite "ladder" of zeta-type functions for each $n \geq 2$, with the case $n = 2$ yielding the Selberg zeta function. Furthermore, we believe that the functional equation for the zeta function at level n will involve all zeta functions from lower levels, as demonstrated by the case $n = 2$.

In [21], [22], and [23], Lang and I initiated our program of study, which I plan to continue. In particular, in [22] we defined Eisenstein series obtained by "twisting with heat kernels rather than automorphic forms," as Lang would say, and we proposed what can be viewed as a repackaging of spectral decompositions in terms of heat Eisenstein series. At this stage, we felt that one could already see new relations involving zeta functions, namely that the constant term of Fourier expansions of heat Eisenstein series should involve the Selberg zeta function of lower levels. One implication of such a result would be an identity involving L-functions and zeta functions which would follow by comparing the Fourier coefficients in the heat Eisenstein series with the sum of Fourier coefficients of Eisenstein series attached to automorphic forms, which are known to be expressible in terms of L-functions (see [6] and references therein).

Recent Developments

Throughout our time together Lang remained optimistic that our proposed "ladder" of zeta functions would provide new ideas in analytic number theory. Admittedly, many of our concepts have yet to be fully tested; only future endeavors will determine if Lang's belief in our program of study was well founded.

Beyond our own work, there have been many developments in mathematics which Lang would have pointed to as providing further support for his faith in the heat kernel. Certainly, the successful completion of the Poincaré conjecture is one instance where heat kernel ideas have played a role. In [5] the author states the need for a "second independent proof of the Moonshine conjectures" and states that the heat kernel could provide one possibility. Lang would have been thrilled by this statement. Serge was very taken by the article [24], where the authors consider spectral theory and spectral expansions on the spaces $n\mathbf{Z}\backslash\mathbf{Z}$, showing that even in the case when $n = 1$ the results are nontrivial. In my own work with Kramer, we have used the heat kernel associated to the hyperbolic metric to obtain new expressions

for the analytic invariants of the Arakelov theory of algebraic curves; see [11] and [12]. In recent developments, we have shown that by taking the Rankin-Selberg integral of an identity from [11], one obtains theta-function type expressions involving certain L-functions attached to nonholomorphic Maass forms, ultimately obtaining an identity in terms of the certain L-function attached to an orthonormal basis of holomorphic weight two forms; the full development of this identity is presented in [13]. In some ways, the work in [13] relates to one of the first steps envisioned by Lang and me, namely the uncovering of new relations, possibly regularized in some sense, involving known and new zeta functions.

Soon after Lang and I completed the article [19], we discussed at length the ideas and hopes we had for the results one could obtain from "ladders". At one point he said to me, "I wish I were thirty years old again so I could concentrate on the heat kernel." Given the way Serge devoted his life to mathematics, let us take that statement as summarizing his sincere and profound belief in the strength of the heat kernel and heat kernel analysis.

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A Metaphor for Mathematics Education

Greg McColm

Most mathematicians don't worry about the philosophy of mathematics. If we have not resolved the crises of the early twentieth century, at least we've learned to live with them. And it's not clear what contemporary philosophers have to say to mathematicians: while philosophy and logic still walk together—perhaps more in parallel than in tandem—and while cognitive scientists collaborate with epistemologists, philosophy seems to offer mathematics in general less than it used to.

Yet philosophy *can* offer self-awareness.

This seems a rather odd commodity for philosophy to offer to mathematics, for presumably we mathematicians are aware of what we're doing. We compose definitions and conjectures, prove theorems and develop algorithms, write papers and teach students, and even serve on committees. Of course we know what we're doing, don't we?

Besides, contemporary philosophy of mathematics doesn't seem to offer self-awareness; its primary concern is what mathematical objects (indeed, set theoretic objects) *are*, if anything. But that's because the philosophers are trying to understand the mathematics we have safely filed away in journals.

But we may need self-awareness. Consider the following sad tale.

In 1844 Hermann Grassman presented his "calculus of extension" in his *Ausdehnungslelire*, which began with an alarmingly philosophical introduction and continued into an algebra which his contemporaries found rather strange. Famous people receiving copies reacted almost characteristically: Gauss said he had done it already, Möbius confessed to being philosophy-phobic and passed it to a friend who apparently never responded, Cauchy seems to have misplaced his copy or gotten confused or something, and Hamilton was favorably but ineffectually impressed. It appears that Grassman's contemporaries understood at best a gist of his algebra; certainly they didn't see what we see in hindsight: an embryonic vector calculus. In 1862 Grassman published a revision of the *Ausdehnungslelire*, even longer, in a more Euclidean format, sans philosophy (but still, according to reviewers, opaque), which also fell flat. Grassman spent a quarter century producing variations of a single difficult and largely unread book, as well as other articles that mathematicians found hard to read, all the while writing patient letters to famous people, most of which seem to have gone unanswered.

Michael Crowe[1] suggests that Grassman's problem may have been his lack of students and credentials (he studied philology and theology in Berlin and taught at technical schools, but never got a university post) and the novelty of his approach. And there was the opacity of his exposition. Put another way, he did not persuade his contemporaries to make the effort to follow up on his work, and he did not develop a navigable path for his contemporaries to follow to his discovery: his problems were essentially *pedagogical*.

Here is where philosophy might help: a "navigable path"? Through what? And to what?

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[1] *This account from M. J. Crowe,* A History of the Vector Calculus: The Evolution of the Idea of a Vectorial System, *Dover, rev. ed., 1994. Crowe's suggestion is on pp. 94–95.*

Mathematicians tend to act as if we discover things more than inventing them. We behave as if there really is, out there, somewhere, that celebrated sponsor of many episodes of *Sesame Street*, The Number Seven. This *Platonic* view is that mathematics is somehow generated by abstract ideas or "forms", like The Number Seven. On the other hand, the *Aristotelian* view is that forms do not generate objects, but rather the other way around: (our knowledge of) mathematical objects somehow arise from observing phenomena. These views are two poles of a vast, wide spectrum of philosophies, but Plato and Aristotle still dominate the landscape, with philosophers tending to lean one way or the other.

After two millennia, there is still no consensus on whether linear operators on Hilbert spaces are as real as lions in the Serengeti. But there is a compromise: while what is "real" may be unknown or even unknowable, our perceptions and our thoughts about our perceptions are knowable, and that is enough for science[2]—at least, for a science that concerns itself with explanation rather than "truth".

Thus Hilbert says that while we might not be able to visualize the "cardinality of the continuum", we can engineer a nomenclature that allows us to manipulate it.[3] We see this as engineering because of its quirks, e.g., when a notion doesn't point at the object we thought it pointed at, when several quite different notions point at the same object, or simply when our nomenclature trips over itself.[4] The mathematics in our literature is our own creation, distinct from the "real" mathematics "out there" that our literature is "about".

We propose a metaphor capturing this distinction.

Imagine a plain on which a vast, invisible edifice supposedly rises up to the sky. We know it's there because of its effect on the plain and the climate around it. People plant near what seems to be the base of the edifice the seeds of a vine that grows up the invisible walls, feeling its way along the nooks, crannies, and statuary, slowly producing an outline of... something there. This vine does not grow at will, for it needs constant care. It needs water and fertilizer and even directing, hence gardeners. These caretakers constantly prune and poke at it, from the ground or while standing on totteringly tall ladders. The result is that the shape of the vine and the outline it appears to make reflect not only the edifice within but also the interests and agendas of the gardeners.

The vine forms a history of a dialogue with the universe. X explores a line of thought, Y conducts an experiment, and Z writes a book. When X comes up with some new results, what her colleagues see is not what X glimpsed through a glass darkly, but what she wrote on paper. When Y applies those results in his laboratory, he is using something whose origin is part edifice, part vine. And then Z, squatting among stacks of papers she is trying to resolve into a coherent work, deals regularly with the paper trails of darkly glimpsed phenomena and so conducts a dialogue of her own. All three produce the substance of the vine.

There are three ways that the vine grows. The vine grows extensively: tendrils explore the way, pushing forward, upward, and inward. This is the frontier growth that we read about in newspapers and history books. The vine grows expansively: the branches may form an outline of a turret, but tendrils still poke around, finding more patterns and shapes on the surface (and occasional passageways), filling in the spaces between the major initial discoveries in a field. This is the "mature science" of Thomas Kuhn.[5] And the vine grows intensively, reorganizing its structure by merging branches to form great boughs (the abstractions made of many smaller preceding theories), producing the mathematics of synthesizers, expositors, and teachers.

We are the caretakers. If we see ourselves as the creators and maintainers of this ancient cultural creation I call a vine, which we use indirectly to study a reality we may not directly apprehend, then we can see that by understanding the philosophical (and sociological!) problem of how the vine behaves, we are in a better position to do our jobs. The philosophers may have something critical to say to us after all.

Let's turn to pedagogy.

Grassman's predicament is a familiar one to many teachers. It's all so clear, but the students can't or won't follow. In fact, introducing something new to one's students is a bit like introducing something new to one's colleagues. Imagine that you are one of several caretakers, tending a vine as it sends tendrils creeping up an invisible wall. This is the sort of research that many of us do: accompanied by five to fifty colleagues worldwide, we explore a section of the wall. We report our

[2] *The philosopher most strongly associated with this sort of view is Immanuel Kant: see his* Prolegomena to Any Future Metaphysics, *which is slightly more accessible and a lot shorter than his* Critique of Pure Reason. *For a more clear, plausible, and moderate distillation, see A. F. Chalmers,* What Is This Thing Called Science?, *Univ. of Queensland Press, 2nd ed., 1976.*

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[4] *See I. Lakatos,* Proofs and Refutations: The Logic of Mathematical Discovery, *(J. Worrall and E. Zahar, eds.), Cambridge Univ. Press, 1976, for examples.*

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results to journals that specialize in the upper north-northwestern face, although the journals on the westernmost of the northern turrets have been known to take our papers. A vertical mass of vine on the north-northwestern section of the edifice appears to be growing up a wall, and the clump to the west seems to be growing up some kind of cylinder. After many such assessments, the American Mathematical Society has divided up the sections of the vine as if that division reflects the architecture of the edifice itself, and the National Science Foundation divides its funding according to fashion and probable economic return associated with various sections of the vine.

But suppose your tendril encounters something unexpected, like an open space amidst some projections. A barred passageway? Exhilarated but anxious, you explore this strange opening. You write an article about it, and then... the first journal thinks you dropped a minus sign, the second thinks you've lost your marbles, and the third journal thinks your results aren't "interesting". As your champagne goes flat, you may wonder what's wrong with everybody.

Here's what the referees think. Perhaps your tendrils lost hold of the edifice and are now growing into a pointless tangle. Or perhaps there is something there, but they cannot navigate the snarl. (And unless there is some kind of authority behind it, mathematicians tend to be wary of muddle: like our students, we want reassurance—perhaps credentials printed on parchment—of the competence of our guide before we invest hours of our time.) In order to lead one's colleagues to something new, they have to be led via familiar-looking paths. There is a theory of "processing fluency"[6] that suggests that both the difficulty of exposition and the apparent unfamiliarity of the topic may undermine the (perceived) credibility of the thesis.

Grassman found the turret that everybody wanted (how to do calculus in Euclidean space), but the approach was difficult, and Grassman never figured out how to lead anyone there. Later, when the vine had grown up closer to the turret and there were more places to approach this turret (from Gibbs's mechanics to Heaviside's electricity), that place was discovered by two people who more effectively described the path and sold the idea of visiting the place—especially Willard Gibbs, who bombarded his colleagues with fusillades of preprints. So thanks more to Gibbs and Heaviside than to Grassman, vector calculus is now entrenched in the curriculum.

Since we can only see the vine, our colleagues can only follow the green we have grown for them, down the paths we have made. The problem is pedagogical, and if there is an ignored elephant in Western philosophy's living room, it is pedagogy.[7] Much mathematical "research" consists of teaching mathematics to our colleagues. Part of Grassman's problem was that while he did find a route to the turret, he did not develop his bit of vine into a stem that led cleanly up it.

Discovery only generates unorganized reams of reports of trailblazing here and there. Science is a social activity, so there is more to science than discovery. There is also organization. There are plenary talks, expository articles, advanced topics courses, multiyear projects culminating in retrospective research monographs, followed by graduate texts. Visitors from other fields pick up nuggets to be baked in academic kilns and presented, in spruced up or dumbed down form, for strange applications the original discoverers never imagined. And then it's on to the massive apparatus of undergraduate education.

The undergraduate curriculum is not at all like the confusing tangle of the canopy. Students start at ground level, where the underbrush has been cleared away and where the base of the vine is an orderly array of well-tended trunks, with helpful and well-tended limbs ("examples" and "exercises") to assist climbing novices. Our students slowly work their way up from the park we maintain for beginners, and slowly the problems get more complicated, less well defined, with more stray branches (or are they stray branches?) and so on, until finally they are just like us.

Consider one of the conundrums of first year calculus: what do we do about limits? We can skip past the rigors of ϵ and δ, but this seems to deprive or even frustrate our more curious and demanding students.[8] So traditionally we had our students stretch towards the ϵ-δ branch in Calculus I, and then towards an ϵ-N branch in Calculus II, and then back towards ϵ-δ in Calculus III, so that after all this yoga they can actually grasp the branch in Advanced Calculus.

But many students flail and give up, so most calculus classes have dropped the epsilons. Some teachers see an alternative route using Abraham Robinson's nonstandard analysis, with some reports of success.[9] Whatever one's position on the debate over epsilons versus infinitesimals, we do

[6] *See R. Reber, N. Schwarz and P. Winkielman, "Processing fluency and aesthetic pleasure: Is beauty in the perceiver's processing experience?"*, Personality and Social Psychology Review ***8***:*4 (2004), 364-382.*

[7] *Eastern philosophy, on the other hand, has devoted a great deal of attention to pedagogy.*

[8] *J. E. Szydlik, "Mathematical beliefs and conceptual understanding of the limit of a function"*, J. Res. Math. Ed. ***13***:*3 (2000), 258-276.*

[9] *K. Sullivan, "The teaching of elementary calculus using the nonstandard analysis approach"*, Amer. Math. Monthly ***85***:*5 (1976), 370-375.*

see here the arboreal aspect of mathematics.[10] Out of historical accident[11] two vine branches embrace the same invisible rampart, and now we debate which vine is the more intelligible, the more powerful, the more practical, etc. (Some combatants even debate which is more like the invisible rampart, as if that was a resoluble issue.) Both growths are human creations, the results of our endeavor to navigate and make out this difficult but apparently critical invisible structure, and ultimately the debate is over the utility of each part of vine in accomplishing this navigation and perception.

Like our students, we prefer to stand on an orderly array of well-tended boughs; even the Zen masters of complexity display their talents by cleaning up the messes they encounter; even the great and messy problem solvers need a safe place to stand. And if each researcher sees her or his own array of branches, an outsider often just sees—a tangle. If we are to persuade outsiders to join us, we have to remember our audience and design our section of vine to entice visitors. Mathematical truth—whatever that is—is not what we deal with daily; it is our mathematical gardening that we share. And like all human productions, mathematical gardening require users' manuals, packaging, and salesmanship.

Acknowledgement. Many people helped me with the topics and format of this article. I would like to thank Edwin Clark, Brian Curtin, Sherman Dorn, Mourad Ismail, Mile Krajcevski, John Jungerman, Penelope Maddy, Roy Weatherford, Morris Zelditch (who pointed out to me the importance of expansive growth), and Fred Zerla for their invaluable advice, comments, and counsel.

[10] *Hilbert, remember, would call this an engineering aspect.*

[11] *See, e.g., I. Kleiner, "History of the infinitely small and the infinitely large in calculus",* Ed. Stud. Math. ***48*** *(2001), 137–174.*

About the Cover

Mathematical exercises

April is Mathematics Awareness Month, and this year's theme is the connection between mathematics and the brain. The webpage is at `http://www.mathaware.org/mam/` and there ought to be some interesting short essays posted there. What those who chose this theme presumably had in mind was the sophisticated mathematics that has gone into the technology that analyzes brain functions. But also of interest is the possibility that analyis of brain activity can tell us something about how humans do mathematics. In his book on mathematical invention, the eminent French mathematician Jacques Hadamard asked, "Will it ever happen that mathematicians know enough of that subject of the physiology of the brain and that neurophysiologists know enough of mathematical discovery for efficient cooperation to be possible?" The answer seems now "Very likely".

Mauro Pesenti and colleagues have studied thoroughly the brain activity of a calculating prodigy named Rüdiger Gamm, and it is his brain displayed on the cover of this month's issue. The sorts of things he does remarkably well in his head include multi-digit multiplication, computation of sines, and calendrical calculations. The areas of the brain displayed in green are those used by both Gamm and nonexpert control subjects while doing mental arithmetic, and those in red are those used only by Gamm. What is interesting is that the areas used only by Gamm are generally those associated to episodic long-term memory,

—continued on page 539

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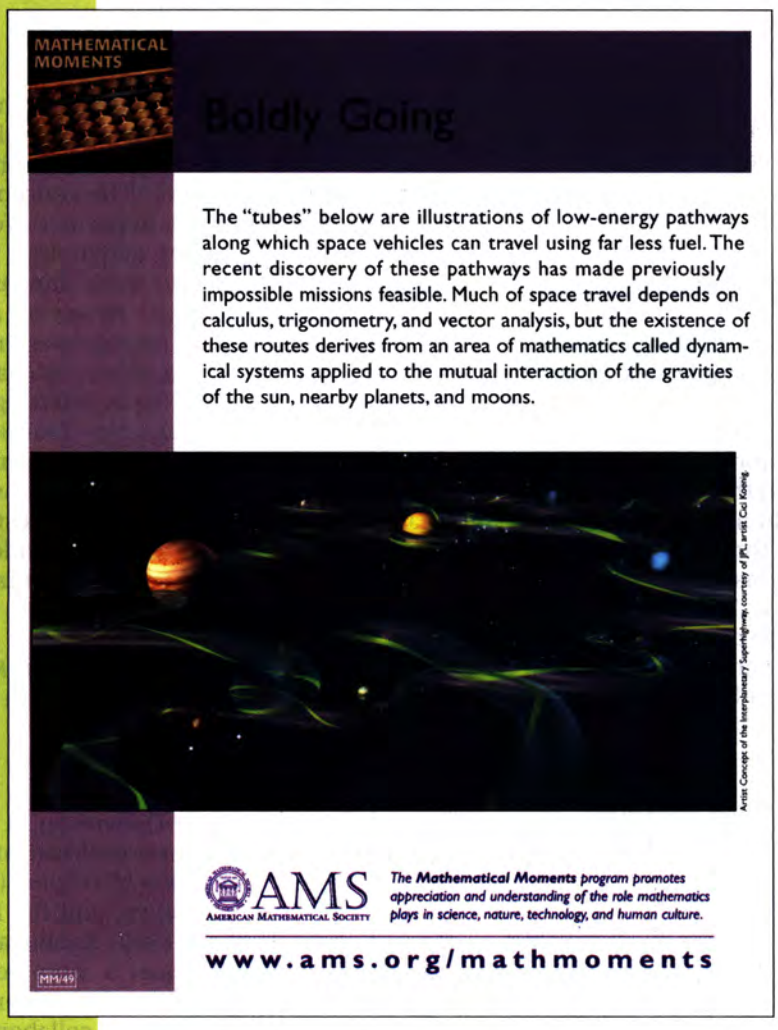

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Book Review

Shadows of Reality:
The Fourth Dimension in Relativity, Cubism, and Modern Thought

Reviewed by Tony Phillips

Shadows of Reality: The Fourth Dimension in Relativity, Cubism, and Modern Thought
Tony Robbin
Yale University Press, 2006
$40.00
160 pages, ISBN 978-0300110395

The fourth dimension? As La Rochefoucauld observed of true love, many people talk about it, but few have seen it. H. S. M. Coxeter, whose *Regular Polytopes* is the primary reference on four-dimensional constructions, says [Cox, ix]: "we can never fully comprehend them by direct observation." But he goes on: "In attempting to do so, however, we seem to peek through a chink in the wall of our physical limitations, into a new world of dazzling beauty." This is how the fourth dimension appeals to a geometer; for the popular imagination, fired by late nineteenth and early twentieth century science-fiction, theosophy and charlatanism, it became a fad with transcendental overtones. Just as UFOs are interpreted today as manifestations of a superior, alien civilization, so the fourth dimension gave room, lots of room, for the spirit world.

Tony Robbin tells this story in *Shadows of Reality: The Fourth Dimension in Relativity, Cubism, and Modern Thought*, along with an analysis of the impact of four-dimensional thought on modern art, and its importance in modern science.

An additional and more idiosyncratic theme is sounded by the invocation, in the book's title, of Plato's myth of the cave. The prisoners in the cave can see only shadows projected on the wall before them; and as Socrates explains to Glaucon: "to them the truth would be literally nothing but the shadows of the images." Thus our own truth may be only the projection of a more meaningful four-dimensional reality. But there is more. The concept itself of *projection* has enormous importance for Robbin; he systematically opposes, and prefers, projection (from four dimensions to three dimensions, and then usually to two to get things on a page) to the dual perspective on four-dimensional reality, three-dimensional *slicing*, which he characterizes as "the Flatland model". He is referring to Edwin A. Abbott's book *Flatland*, where four dimensions are explained by analogy with the Flatlanders' conception of the three-dimensional space we humans inhabit. "Consider this book a modest proposal to rid our thinking of the slicing model of four-dimensional figures and spacetime in favor of the projection model." I will come back to this point later.

The jacket blurb for *Shadows of Reality* describes the work as "a revisionist math history as well as a revisionist art history."

"Revisionist Art History"

Chapter 3, "The Fourth Dimension in Painting", contains an analysis of three well-known paintings by Picasso: *Les Demoiselles d'Avignon* (1907), the *Portrait of Ambroise Vollard*, and the *Portrait of Henry Kahnweiler* (both 1910). Robbin argues that "at one propitious moment a more serious and sophisticated engagement with the fourth dimension pushed [Picasso] and his collaborators into the discovery of cubism," and that that moment occurred between the painting of *Les Demoiselles* and that of the two portraits. The source of the geometry is pinned down: the *Traité élémentaire de géométrie à quatre dimensions* by Esprit Jouffret

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(1903) [Jouf], and the bearer of this knowledge to Picasso is identified as Maurice Princet, a person with some mathematical training who was a member of the Picasso group.

This is indeed revisionist, because it claims a hitherto unrecognized mathematical influence on the most famous artist of the twentieth century.

The interaction between geometry and art at the beginning of the twentieth century has already been studied in encyclopedic detail by Linda Dalrymple Henderson [Hen]. When it comes to the fourth dimension and Picasso, Henderson is much more cautious: "Although Picasso has denied ever discussing mathematics or the fourth dimension with Princet, Princet was a member of the group around Picasso by at least the middle of 1907, and probably earlier. It thus seems highly unlikely that during the several years which followed, Picasso did not hear some talk of the fourth dimension..." But she presents eye-witness and contemporary testimony from Jean Metzinger, an artist member of the group, which states that the influence ran the other way: "Cézanne showed us forms living in the reality of light, Picasso brings us a material account of their real life in the mind—he lays out a free, mobile perspective, from which that ingenious mathematician Maurice Princet has deduced a whole geometry." [Metz]

Henderson gives solid documentation of the interest in mathematical ideas among the artists of "Picasso's circle", especially after they drifted away from him in 1911. And if images like Jouffret's figures were part of the visual ambience in the 1907–1910 period, they may well have caught Picasso's omnivorous eye. But for a specific mathematical influence on Picasso there is no evidence, and Robbin presents no reasonable argument beyond his own gut feeling: "Indeed, as a painter looking at the visual evidence I find that Picasso... clearly adopted Jouffret's methods in 1910." So we have to take his statement in the preface—"Pablo Picasso not only looked at the projections of four-dimensional cubes in a mathematics book when he invented cubism, he also read the text, embracing not just the images but the ideas"—as pure, unadvertised invention. Let us leave the last word on this topic to the artist himself: "Mathematics, trigonometry, chemistry, psychoanalysis, music and whatnot, have been related to cubism to give it an easier interpretation. All this has been pure literature, not to say nonsense, which brought bad results, blinding people with theories." [Pic]

"Revisionist Math History"

The agenda is stated in the preface. "Contrary to popular exposition, it is the projection model that revolutionized thought at the beginning of the twentieth century. The ideas developed as part of this projection metaphor continue to be the basis for the most advanced contemporary thought in mathematics and physics." The rest of the paragraph amplifies this assertion, naming projective geometry, Picasso (as cited above), Minkowski ("had the projection model in the back of his head when he used four-dimensional geometry to codify special relativity"), de Bruijn (his "projection algorithms for generating quasicrystals revolutionized the way mathematicians think about patterns and lattices"), Penrose ("showed that a light ray is more like a projected line than a regular line in space," with this insight leading to "the most provocative and profound restructuring of physics since the discoveries of Albert Einstein"), quantum information theory, and quantum foam. "Such new projection models present us with an understanding that cannot be reduced to a Flatland model without introducing hopeless paradox."

These items are fleshed out in the main text of the book, roughly one per chapter. I will look in detail at Chapter 4 ("The Truth"), which covers the geometry of relativity, and at Chapter 6, "Patterns, Crystals and Projections".

Relativity, and Time as the Fourth Dimension

For mathematicians, dimension is often just one of the parameters of the object under investigation. Their work may be set in arbitrary dimension ("in dimension n") or, say, in dimension 24, as in the work of Henry Cohn and Abhinav Kumar, who caused a stir in 2004 [Cohn] when they nailed down the closest regular packing of 24-dimensional balls in 24-dimensional space. So we really should be talking about "a fourth dimension", and not "the fourth dimension". For the general public, on the other hand, four dimensions are esoteric enough. They are usually thought of as the familiar three plus one more; hence "the fourth dimension". So where is this fourth dimension? Nineteenth-century work on 4-dimensional polytopes (reviewed in [Cox]) clearly posited an extra spatial dimension. But at the same time a tradition, going back to Laplace, considered *time* as "the fourth dimension". In fact partial differential equations relating space derivatives to time derivatives leave us no choice: their natural domain of definition is four-dimensional space-time.

Here there is some point to Robbin's anti-slicing strictures. Special relativity implies that space-time cannot be described as a stack of three-dimensional constant-time slices. More precisely, the most general Lorentz transformation, relating two overlapping (x, y, z, t) coordinate systems of which one may be traveling at constant speed with respect to the other, mixes the (x, y, z)s and the ts in such a way that the $\{t = \text{constant}\}$ slices are not preserved. But projections do not help. Here is Minkowski as quoted by Robbin: "We are compelled to admit that

it is only in four dimensions that the relations here taken under consideration reveal their inner being in full simplicity, and that on a three-dimensional space forced on us *a priori* they cast only a very complicated projection." Minkowski is saying that projections from four dimensions to three are *not* very useful in understanding relativistic reality. Robbin gets the word "projection" back into play by describing Lorentz transformations as being "revealed when one geometric description of space is projected onto another." But the projections here are isomorphisms from one four-dimensional coordinate system to another; they are not the kind of projections, onto lower-dimensional spaces, that Robbin is contrasting to slicing.

Patterns, Crystals and Projections

Chapter 6 begins with a description of nonperiodic tilings and how they may be generated by "matching rules" and by repeated dissection and inflation. It then goes on to discuss the fascinating and illuminating relationship, discovered by Nicolaas de Bruijn [de B], between nonperiodic tilings in dimension n and projections of approximations of irrational slices of periodic tilings: cubical lattices in dimension $2n$ or $2n + 1$. The simplest example involves dimensions 1 and 2: take the (x, y)-plane with its usual tiling by unit squares; the vertices are at points (n, m) with integer coordinates. Draw the straight line $y = \varphi x$ with irrational slope $\varphi = 1.6180339\ldots$ (the "golden mean" number). Starting at $(0, 0)$, cover the line with the smallest possible subset of the tiling. The squares are shown here shaded.

Because the slope is irrational, the line does not pass through any lattice vertex other than $(0, 0)$, so the choice of covering tile is always unambiguous. Now project the center of each covering tile perpendicularly onto the irrational line. The projected points will define a tiling of the line by intervals: a long interval (L) if the two consecutive tiles were adjacent vertically, and a short one (S) if they were adjacent horizontally. A trigonometric calculation shows that the length ratio of S to L is φ. Our tiling by intervals is non-periodic because φ is irrational, but it has other amazing properties because φ is such a special number. For example, if we represent the short tile by S, and the long one by L, the tiling of the positive x-axis corresponds to a sequence of Ls and Ss. It turns out (the argument requires some linear algebra; see [web]) that the sequence may also be generated as follows: start with S, and copy the string over and over, each time rewriting every S as L, and every L as LS:

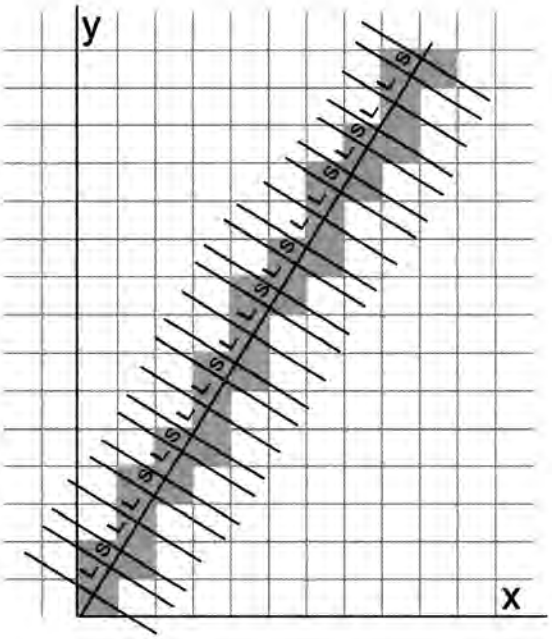

Figure 1.

S
L
LS
LSL
LSLLS
LSLLSLSL
LSLLSLSLLSLLS
LSLLSLSLLSLLSLSLLSLSL
LSLLSLSLLSLLSLSLLSLLSLSLLSLLSLSLLSLLS
...

Figure 2.

The sequence may also be generated like the Fibonacci numbers, starting with S and L and making each succeeding line the concatenation of its two predecessors. And there is more.

Instead of drawing a line with golden slope through the 2-dimensional lattice of unit squares, let us slice a plane (a golden plane—see below) through the analogous 5-dimensional lattice, cover the plane with the smallest possible subset of the 5-cubes of that lattice, and project the center points of those cubes orthogonally onto the plane. The projected points turn out to be the vertices of Penrose's non-periodic tiling [de B]. In this tiling, the plane is covered by copies of two rhombs (equilateral parallelograms), a fat one, with lesser angle $2\pi/5$, and a skinny one $\pi/5$, whose areas, just like the lengths of L and S, are in the golden ratio.

(We use the plane in 5-space spanned by the vectors

$$(1, -\cos(\pi/5), \cos(2\pi/5), \\ -\cos(3\pi/5), \cos(4\pi/5))$$

and

$$(0, -\sin(\pi/5), \sin(2\pi/5), \\ -\sin(3\pi/5), \sin(4\pi/5))$$

[web2]; equivalently, in terms of the golden mean, we may scale those vectors to

$$(2, -\varphi, \varphi - 1, \varphi - 1, -\varphi)$$

and

$$(0, -1, \varphi, -\varphi, 1).$$

A straightforward linear algebra computation shows that the 10 different unit squares in the 5-dimensional lattice project to the 10 possible orientations (5 each) of the 2 rhombs in the Penrose tiling.)

De Bruijn's construction of the Penrose tiling is intellectually satisfying. Instead of the arbitrary-sounding matching rules, which add one tile at a time, or the stepwise generation of tilings of finite portions of the plane by dissection and inflation, we have a single, intelligible definition tiling the entire plane. Furthermore the magical-seeming recurrence properties of the tiling are anchored back to the well-understood behavior of the fractional part of consecutive integral multiples of an irrational number, which has been studied since Kronecker. As an additional bonus, the construction also yields a new insight into the psychological phenomenon noted by Martin Kemp [Kem] in describing how a rhombic Penrose tiling affects the eye: "We can, for instance, play Necker cube-type games with apparent octagons, and facet the surface into a kind of cubist medley of receding and advancing planes." The surface in 5-space, which projects to the tiling, is made up of square facets lined up with the coordinate planes. Three by three these facets determine a cube, which projects correctly; but these cubes live in 5 dimensions, and they fit together in a manner inconceivable in 3-space.

Robbin puts the impact of de Bruijn's discoveries eloquently but inaccurately: "Once one accepts the counterintuitive notion that quasicrystals are projections of regular, periodic, cubic lattices from higher-dimensional space, all the other counterintuitive properties soon become clear in a wave of lucid understanding." The inaccuracy, which is fundamental, stems from Robbin's failure to recognize that these tilings are projections of *sections* of cubic lattices. At the end of the chapter the language becomes even more vivid, but the thesis is still fundamentally wrong-headed: "Quasicrystals show us that objects and systems described by projective geometry cannot be made Euclidean without introducing the most mystical and anthropomorphic properties into the system. Making what is generated or accurately modeled by projective geometry into a traditional Euclidean static model takes us away from science and moves us towards fetishism."

There are smaller-scale problems also, in Robbin's portrayal of Penrose's tilings and de Bruijn's construction. The main ideas are there, with correct references, but a naïve and attentive reader can only be bewildered by the presentation. We read things that are obviously just plain wrong ("The mechanics of the loom enforce a periodic repeat"), things that turn out to be wrong ("Originally, the Amman bars were equidistant"), and things that are misleading ("It was long thought to be impossible to have a tiling made up of pentagons") or unnecessarily obscure ("The dual net is a mathematical device that is often used to analyze pattern"—no further description). The figures only add to the confusion. Figure 6.7 is mislabelled, and here is Figure 6.8:

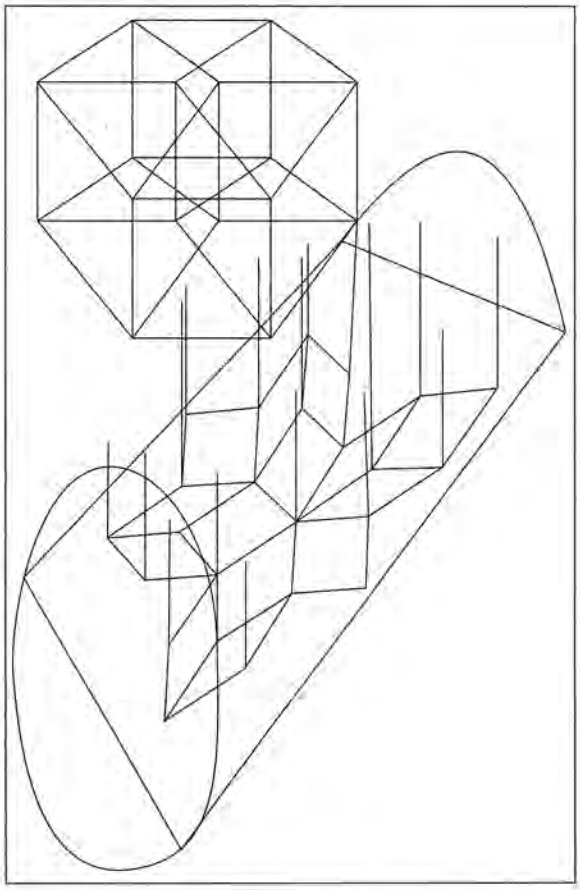

Figure 3. Robbin's Figure 6.8.

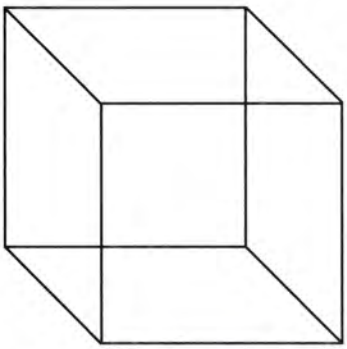

Figure 4. "Necker" cube: the eye imposes one of two three-dimensional interpretations of this figure as a cube, and oscillates between them.

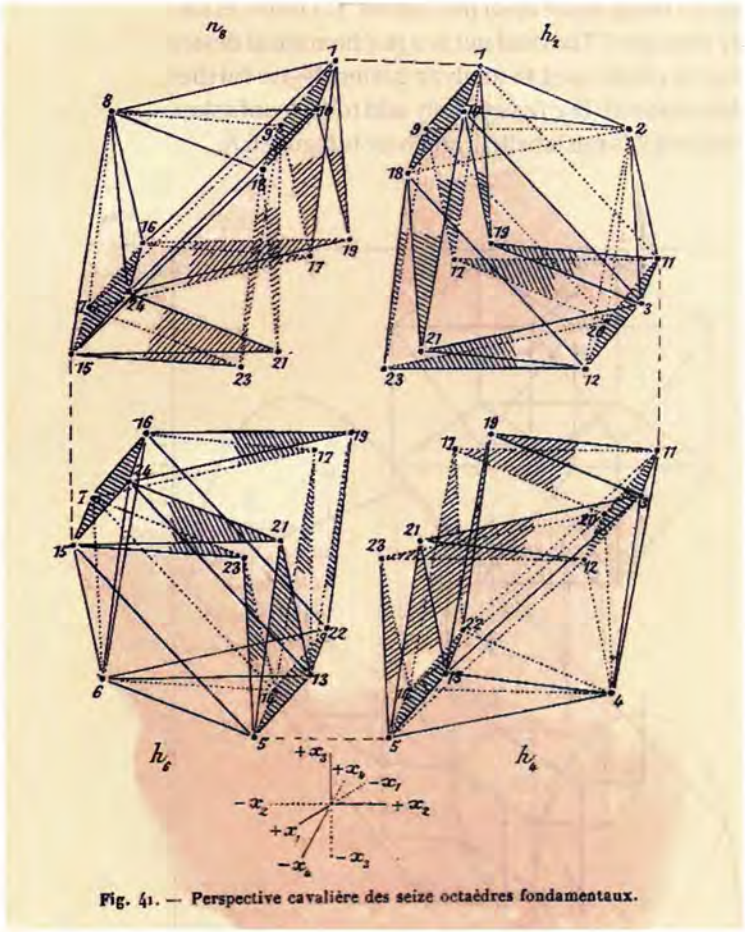

Figure 5. Jouffret's Figure 41, *perspective cavalière* of the 16 fundamental octahedra.

The caption ("The de Bruijn projection method is applied to a grid of hypercubes to produce a Penrose tiling. Only one hypercube is shown here.") does not mention the slicing plane, and the "projection method" is represented by a *four*-dimensional hypercube floating in the upper left-hand corner, along with vertical lines attached to the vertices of the Penrose tiling drawn in perspective in the center. I do not believe this figure is meant to be judged as a work of art; but I know that as a mathematical diagram, while it contains some suggestive truth, it is too impressionistic to be useful.

Conclusion

In sum, this is an attractive but very disappointing book. The topic has great potential, and Robbin's earlier work [Rob], both entertaining and enlightening, promised an idiosyncratic, artist's-eye look, with perhaps new insights, at these fascinating phenomena. In fact I am grateful to Robbin for introducing me to de Bruijn's work and for bringing to my attention Esprit Jouffret, a charming mathematical representative of the Belle Époque. But the book is seriously marred by the inadequacy and inaccuracy of its mathematical component: the author has been allowed to venture, alone, too far above the plane of his expertise. In his acknowledgments he singles out several "readers of the manuscript" and several Yale University Press staffers. He alone of course is responsible for the text and illustrations, but I am sure that some of those individuals wish now that they had done more to give this beautiful idea a more accurate and intelligible realization.

Appendix: Section, Projection, and *Perspective Cavalière*

Section and projection are mathematically dual operations. Any finite n-dimensional object can be completely reconstructed either from a 1-dimensional set of parallel $(n-1)$-dimensional slices or from a 1-dimensional set of $(n-1)$-dimensional projections, by stacking in one case or by a tomographic-style calculation in the other. The total information is the same, although packaged quite differently. When it comes to geometric objects, like regular polyhedra, the (innate or trained) ability of the human eye, which can resolve perspective problems almost automatically, means that a 2-dimensional projection can often be counted on to be "read" to give a mental image of its 3-dimensional antecedent. The "Necker cube" phenomenon referred to by Kemp is a familiar example of this human propensity.

But in fact neither of the natural interpretations is even plausible unless we *assume* that we are looking at the projection of a connected polyhedron. The configuration we see might be like one of the constellations in the night sky: our mind joining several completely unrelated objects.

Projection of more general objects is usually less informative. Knot theorists work comfortably with two-dimensional "projections" of three-dimensional knots, but these projections have been enhanced with clues: at each intersection a

graphical convention gives information (from "the third dimension") telling which strand goes over and which goes under.

To explore the limitations of projection as a means for studying four-dimensional polyhedra, let us look at an image from Jouffret, reproduced both in Henderson's book and in Robbin's (Figure 5).

Jouffret's Figure 41 is part of his effort to explain the structure of the polytope C_{24}, which is a 3-dimensional object, without boundary (as such it can only exist in 4-dimensional space), made up of 24 octahedra. This polytope has 24 vertices, 96 edges, and 96 triangular faces. It is the most interesting of the 3-dimensional regular polytopes because it is the only one of the 6 that has no analogue among our familiar 2-dimensional polyhedra (in fact its group of symmetries is the Weyl group of the exceptional Lie group F_4 [Cox]). Jouffret works rigorously, in the style of the descriptive geometry *épures* required of every French candidate to the École Polytechnique or the École Normale Supérieure between 1858 and 1960 [Asa]: he constructs a C_{24} of edge-length a in 4-dimensional space as a polytope with vertices at the 24 points $(\pm\alpha, \pm\alpha, 0, 0)$, $(\pm\alpha, 0, \pm\alpha, 0)$, $(\pm\alpha, 0, 0, \pm\alpha)$, $(0, \pm\alpha, \pm\alpha, 0)$, $(0, \pm\alpha, 0, \pm\alpha,)$, $(0, 0, \pm\alpha, \pm\alpha)$, where $\alpha = a\frac{\sqrt{3}}{2}$. Since each vertex is connected to eight others, when you put in all the edges and project the polytope onto a page of the book you get something like one of the two pictures in Figure 6.

Which one you get depends on the projection you choose from 4-space to 2-space. These pictures are attractive, but they do not convey much information about how 24 octahedra fit together to form an object with no boundary. Putting in any indication of where the 96 2-dimensional faces fit in would certainly not improve legibility. Jouffret credits Schoute with the idea of a *perspective cavalière*: a cavalier bending of the rules in the interest of intelligibility. In this case there are two violations. First, sixteen of the octahedra are *selected* to represent the surface. The sixteen are geometrically *projected* as before. That projected image is *dissected* into four pieces, with common edges and faces drawn twice, and common vertices two or four times. The shading, which, as Henderson notes, gives the image a "shimmering quality of iridescence", marks the faces that are duplicated. (Note here two small errors: the face <9, 18, 21> in quadrant h_8 should be shifted to become <10, 18, 21>, and the face <17, 13, 14> in quadrant h_6 should be moved to <17, 22, 14>).

Before trying to understand how these pieces all fit together, and then where the missing eight octahedra should be inserted, we should note how far this picture is, even when restricted to one of the quadrants, from a straightforward projection. When two lines meet in this figure, a heavy dot tells us whether their intersection is essential (they actually meet in four dimensions) or contingent (two of their points happen to project to the same place). The lines themselves come with two different values: full and dotted, according to a somewhat arbitrary convention of Jouffret's regarding which lines are "seen" and which are "hidden".

Finally, even with all the extra information, and despite the accuracy of the *épure*, the picture is hard to read. There is a clue for modern readers: the rectangular parallelipipeds in each quadrant become a torus when the quadrants are slid back together. The inside and the outside of this torus are partitioned into 24 half-octahedra each by coning the boundary of each inside parallelipiped from an "interior" vertex (one of 2, 4, 6, or 8) and by coning the boundary of each outside block from its appropriate middle vertex (one of 17, 19, 21, 23). Each solid torus contains a necklace of 4 complete octahedra. The remaining *seize octaèdres fondamentaux* are formed when the two solid tori are glued along their boundaries to give a topological 3-sphere. I write "modern readers" because I do not believe that in 1903 this decomposition of the 3-sphere, even though it is almost unavoidable when S^3 is written as $\{|z_1|^2 + |z_2|^2 = 1\}$ in complex coordinates, was anywhere nearly as well known as it is today.

Jouffret remarks that the 16 vertices that span our torus are the vertices of a hypercube (*octaédroïde*), but he does not use this information to clarify the combinatorial structure of C_{24}. He also remarks that the other 8 vertices define a 16-cell (*hexadécadroïde*) of side $a\sqrt{2}$, and that this partition into 16 + 8 can be done in three symmetric ways.

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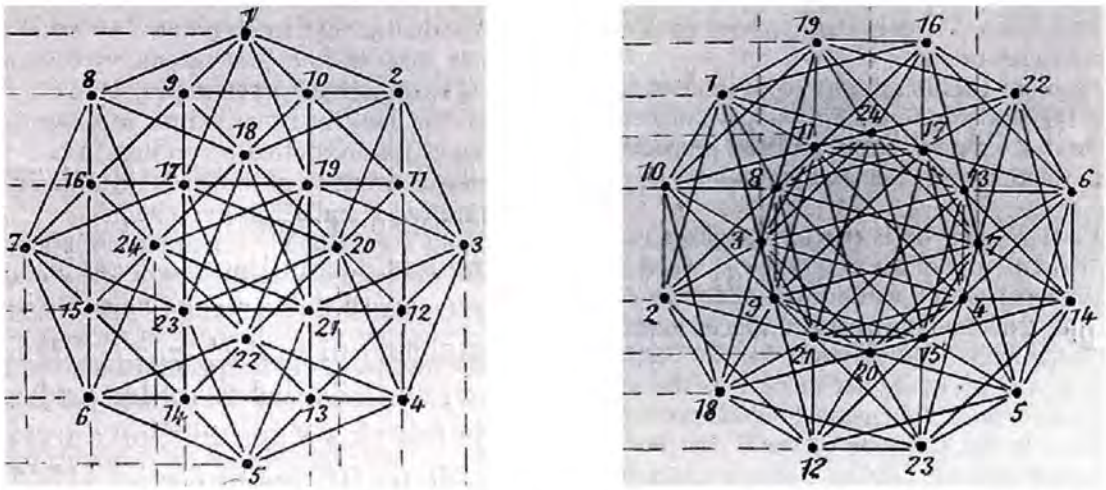

Figure 6. From Jouffret's Figure 36, two projections of the 24-cell.

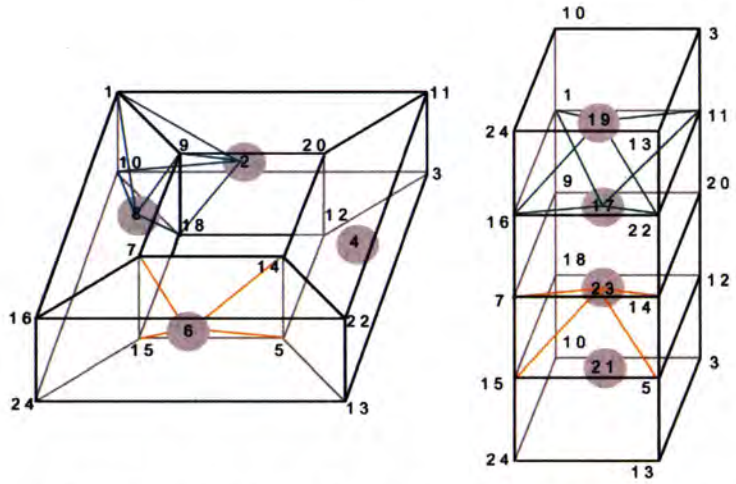

Figure 7. The two solid tori implicit in Jouffret's diagram, with their interior vertices. The extra edges for two of the "necklace" octahedra are shown in green; those for one of the *seize octaèdres fondamentaux* are shown in orange.

[Metz] JEAN METZINGER, Notes sur la peinture, *Pan*, Oct.-Nov. 1910, translated and partially reprinted in EDWARD FRY, *Cubism*, McGraw-Hill, New York 1966. Quoted in [Hen] page 64. Here is the whole paragraph: "Picasso ne nie pas l'objet, il l'illumine avec son intelligence et son sentiment. Aux perceptions visuelles, il joint les perceptions tactiles. Il éprouve, comprend, organise: le tableau ne sera transposition ni schema, nous y contemplerons l'équivalent sensible et vivant d'une idée, l'image totale. Thèse, antithèse, synthèse, la vieille formule subit une énergique interversion dans la substance des deux premiers termes: Picasso s'avoue réaliste. Cézanne nous montra les formes vivre dans la réalité de la lumière, Picasso nous apporte un compte-rendu matériel de leur vie réelle dans l'esprit, il fonde une perspective libre, mobile, telle que le sagace mathématicien Maurice Princet en déduit toute une géométrie."

[Pic] Picasso speaks, *The Arts*, New York, May 1923. Reproduced in ALFRED H. BARR JR., *Picasso: Fifty Years of his Art*, Museum of Modern Art, New York 1946, reprint edition published for the museum by Arno Press, 1966. Also reprinted in EDWARD FRY, *Cubism*, McGraw-Hill, New York 1966.

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[web2] `http://gregegan.customer.netspace.net.au/APPLETS/12/12.html`, Greg Egan—refers to Quasitiler documentation.

WHAT IS . . .

a Tropical Curve?

Grigory Mikhalkin

A tropical curve is an algebraic curve defined over the semifield $\mathbb{T}$ of tropical numbers. The goal of this note is to make sense out of this phrase.

Figure 1 depicts a union of two planar tropical curves (also known in physics as (p,q)-webs), namely a tropical conic and a tropical line. Each of them may look very different from its classical counterpart, but they do share many features, e.g., a line (the tripod graph in the lower part of the picture) intersects a conic (the rest of the picture) in two points. In can be shown that tropical curves come as limits of classical curves (Riemann surfaces) under a certain procedure degenerating their complex structure. Tropical curves proved to be useful *en lieu* of honest holomorphic curves in a range of classical problems.

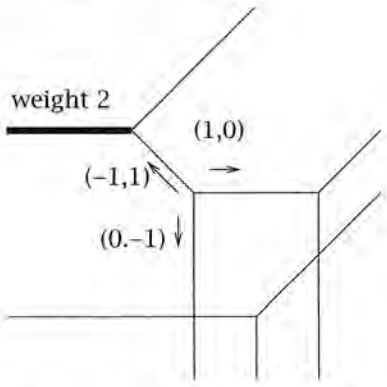

Figure 1. A conic and a line in the plane.

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We define the tropical semifield $\mathbb{T}$ to be the set $\mathbb{R} \cup \{-\infty\}$, and we equip it with the *"addition"* operation "$x + y$" $= \max\{x, y\}$ and with the "multiplication" operation "xy" $= x + y$. We use the quotation marks to distinguish between the standard and tropical arithmetic operations. Our "additive zero" is $-\infty$ while the "multiplicative unit" is 0. We have $\mathbb{T}^\times = \mathbb{T} \setminus \{-\infty\} = \mathbb{R}$.

The term *tropical* is taken from computer science, where it was coined to commemorate contributions of the Brazilian school. The term *semifield* refers to the properties of the tropical arithmetic operations: we have all the axioms required for a *field*, except for the existence of subtraction (as our addition is idempotent "$x + x$" $= x$). Luckily, one does not need subtraction to write down polynomials (they are *sums* of monomials)!

Consider a polynomial in two variables

$$p(x,y) = \text{“}\sum_{j,k} a_{jk} x^j y^k\text{”} = \max_{j,k}(jx + ky + a_{jk}).$$

The tropical curve C defined by p consists of those points $(x, y) \in \mathbb{R}^2$ where p is not differentiable. In other words, C is the locus where the maximum is assumed by more than one of the "monomials" of p. It is easy to see that $C \subset \mathbb{R}^2$ is a graph and its edges are straight intervals with rational slopes.

The edge E, where "$a_{j_1k_1}x^{j_1}y^{k_1}$" $=$ "$a_{j_2k_2}x^{j_2}y^{k_2}$", is perpendicular to the vector $(j_1 - j_2, k_1 - k_2)$. We can enhance E with a natural number $w(E)$ (called its *weight*) equal to $\mathrm{GCD}(j_1 - j_2, k_1 - k_2)$.

Take a vertex $A \in C$ and consider the edges $E_1, \dots, E_n$ adjacent to A. Let $v(E_j) \in \mathbb{Z}^2$ be the primitive integer vector from A in the direction of E_j. It is easy to see that we have the following *balancing* (or zero-tension) condition at each vertex

of C:

$$\sum_{j=1}^{n} w(E_j)v(E_j) = 0. \tag{1}$$

Furthermore, one can easily show that any weighted piecewise-linear graph in $\mathbb{R}^2$ with rational slopes of the edges and with the zero-tension condition at the vertices is given by some tropical polynomial.

The plane $\mathbb{R}^2$ can be thought of as a part of the tropical affine plane $\mathbb{T}^2 = [-\infty, +\infty)^2$. The regular functions on $\mathbb{T}^2$ are tropical polynomials, and the regular functions on $\mathbb{R}^2 = (\mathbb{T}^\times)^2$ are tropical Laurent polynomials. Note that a monomial is an affine-linear function with an integer slope and therefore the geometric structure on $\mathbb{R}^2$ encoding the tropical structure is the *$\mathbb{Z}$-affine structure*.

The plane $\mathbb{T}^2$ can be compactified to the projective plane $\mathbb{TP}^2$. To construct $\mathbb{TP}^2$ we take the quotient of $\mathbb{T}^3 \setminus \{(-\infty, -\infty, -\infty)\}$ by the usual equivalence relation $(x, y, z) \sim (\lambda x, \lambda y, \lambda z), \lambda \neq 0$. As in the classical case we have three affine charts, so $\mathbb{TP}^2$ can be obtained by gluing three copies of $\mathbb{T}^2$. Thus we may think of $\mathbb{TP}^2$ as a triangle-like compactification of $\mathbb{R}^2$ taken with the tautological $\mathbb{Z}$-affine structure. Each side of the triangle corresponds to a copy of $\mathbb{TP}^1$ (which is itself a compactification of $\mathbb{R}$ by two points). Similarly, we may define $\mathbb{TP}^n \supset \mathbb{R}^n$ as well as other tropical toric varieties.

We have compact tropical curves in $\mathbb{TP}^n$. Let Γ be a finite graph and $h : \Gamma \to \mathbb{TP}^n$ be a continuous map that takes the interior of every edge E to a straight (possibly unbounded) interval with a rational slope in $\mathbb{R}^n$. If we can prescribe a positive integer weight to each edge so that (1) holds at every vertex in $\mathbb{R}^n$, then we say that $h : \Gamma \to \mathbb{TP}^n$ is a tropical curve.

The *degree* d of $h(\Gamma)$ is the intersection number with any of the $(n+1)$ $\mathbb{TP}^{n-1}$-divisors at infinity. The degree can be calculated by examining the unbounded edges $E_1, \ldots, E_l$. For example the intersections with the "last" infinity divisor $D_\infty = \mathbb{T}P^n \setminus \mathbb{T}^n$ are given by those E_j whose outward primitive vectors $v(E_j)$ have at least one coordinate positive; the local intersection number with D_∞ is $w(E_j)$ times that coordinate (assuming that it is maximal), and d is the sum of these local intersection numbers. The balancing condition ensures that the total intersection number with all infinity divisors is the same. The *genus* of Γ is $g = \dim H_1(\Gamma)$. We see that both the line and the conic from Figure 1 have genus 0.

Note that tropical curves behave quite similarly to classical algebraic curves. Prove (as an exercise) that any two points in $\mathbb{TP}^n$ can be connected with a line. Curves in $\mathbb{TP}^n$ of degree d and genus g vary in a family of dimension at least $(n+1)d + (n-3)(1-g)$. In many cases this lower bound is exact, for instance if $g = 0$ (for any n) or if $n = 2$ (for any g) if h is an immersion.

For example if we fix a configuration C of $3d - 1 + g$ generic points in $\mathbb{TP}^2$ then only finitely many curves $h_j : \Gamma_j \to \mathbb{TP}^2$ of degree d and genus g will pass through C. In contrast to the case of complex coefficients, the actual number of such tropical curves h_j will depend on the choice of C. However, each such tropical curve comes with a combinatorial multiplicity so that the number of curves with multiplicity is invariant. And this invariant coincides with the number of complex curves of degree d and genus g passing through a generic configuration of $3d - 1 + g$ points in $\mathbb{CP}^2$ and gives an efficient way of computing that number.

There is also a different choice of multiplicities for h_j, responsible for enumeration over $\mathbb{R}$ (some real curves are counted with the sign $+1$ and some with -1, and their total number is different from the complex counterpart). The sum of the real multiplicities of $h_j(\Gamma_j)$ gives the answer for the corresponding real enumerative problem.

As an example consider the case $d = 3$ and $g = 0$ (see Figure 2). We fix a configuration C of 8 points in $\mathbb{R}^2 \subset \mathbb{TP}^2$. Depending on the choice of C there might be 9 or 10 tropical curves via C. However, the sum of their complex multiplicities is always 12 while the sum of their real multiplicities is always 8.

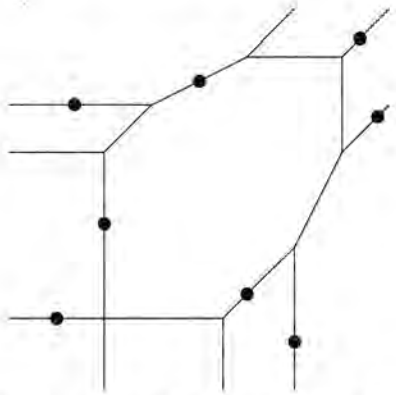

Figure 2. A cubic of multiplicity 1 via 8 points.

A map $h : \Gamma \to \mathbb{TP}^n$ can be used to induce tropical structure on Γ. Tropical monomials p in $\mathbb{R}^n = (\mathbb{T}^\times)^n$ give smooth functions on every edge $E \subset \Gamma$ and can be used to measure the length of E. Define the length of E to be the smallest of such lengths divided by the weight of E. This turns Γ into a metric graph: the leaves (i.e., the edges adjacent to 1-valent vertices) get infinite length while the inner edges are finite.

Conversely, a metric graph structure on Γ can be used to define *tropical* maps $\Gamma \to X$, where X is $\mathbb{T}^n$, $\mathbb{TP}^n$, or any other tropical variety (which can

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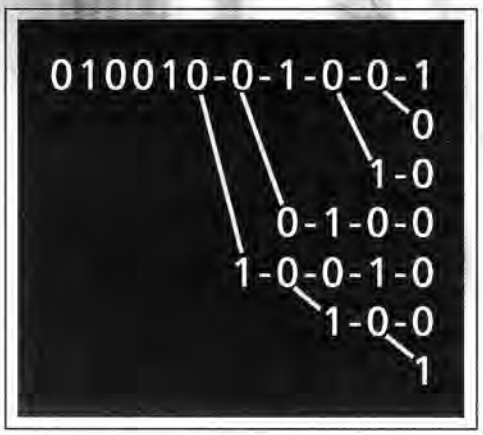

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be defined in higher dimension as a polyhedral complex equipped with an integer affine structure; only in dimension 1 we can hide the integer affine structure under the guise of a metric). Higher weight appears when h "stretches" the edges by an integer amount.

There is an equivalence relation between tropical curves generated by the following relation: at any point $x \in \Gamma$ we may introduce an infinite length interval connecting x with a new 1-valent vertex. This equivalence allows us to turn a map given by regular functions into a tropical morphism. Also it allows us to treat any marked point as a 1-valent vertex. This turns, e.g., the space $\mathcal{M}_{0,n}$ of trees with n marked points into an $(n-3)$-dimensional tropical variety.

Most classical theorems on Riemann surfaces have counterparts for tropical curves, in particular, the Abel-Jacobi theorem, the Riemann-Roch theorem, and the Riemann theorem on the θ-functions. Many features of complex and real curves become easily visible after tropicalization.

Further reading.

[1] G. MIKHALKIN, Enumerative tropical algebraic geometry in $\mathbb{R}^2$, *J. Amer. Math. Soc.* **18** (2005), no. 2, 313–377.

[2] G. MIKHALKIN and I. ZHARKOV, Tropical curves, their Jacobians and Theta-functions, `http://arxiv.org/abs/math.AG/0612267`.

[3] J. RICHTER-GEBERT, B. STURMFELS, and T. THEOBALD, *First steps in tropical geometry*, Contemporary Mathematics, vol. 377, Amer. Math. Soc., 2005, pp. 289–317.

The "WHAT IS...?" column carries short (one- or two-page) nontechnical articles aimed at graduate students. Each article focuses on a single mathematical object rather than a whole theory. The *Notices* welcomes feedback and suggestions for topics. Messages may be sent to `notices-whatis@ams.org`.

2007 Steele Prizes

The 2007 Leroy P. Steele Prizes were awarded at the 113th Annual Meeting of the AMS in New Orleans in January 2007.

The Steele Prizes were established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein. Osgood was president of the AMS during 1905–1906, and Birkhoff served in that capacity during 1925–1926. The prizes are endowed under the terms of a bequest from Leroy P. Steele. Up to three prizes are awarded each year in the following categories: (1) Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students; (2) Mathematical Exposition: for a book or substantial survey or expository-research paper; (3) Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research. Each Steele Prize carries a cash award of US$5,000.

The Steele Prizes are awarded by the AMS Council acting on the recommendation of a selection committee. For the 2007 prizes, the members of the selection committee were: Rodrigo Banuelos, Daniel S. Freed, John B. Garnett, Victor W. Guillemin, Nicholas M. Katz, Linda P. Rothschild (chair), Donald G. Saari, Julius L. Shaneson, and David A. Vogan.

The list of previous recipients of the Steele Prize may be found on the AMS website at `http://www.ams.org/prizes-awards`.

The 2007 Steele Prizes were awarded to DAVID B. MUMFORD for Mathematical Exposition, to KAREN K. UHLENBECK for a Seminal Contribution to Research, and to HENRY P. MCKEAN for Lifetime Achievement. The text that follows presents, for each awardee, the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Mathemamatical Exposition: David B. Mumford

Citation

The Leroy P. Steele Prize for Mathematical Exposition is awarded to David Mumford in recognition of his beautiful expository accounts of a host of aspects of algebraic geometry. His *Red Book of Varieties and Schemes*, which began life over forty years ago, introduced successive generations of beginning students to "modern" algebraic geometry and to how the "modern" theory clarifies classical problems. Students could then go on to his 1970 book *Abelian Varieties*, where the whole theory is developed "without the crutch of the Jacobian", and which remains the definitive account of the subject. Here again the classical theory is beautifully intertwined with the modern theory, in a way which sharply illuminates both. Students who wanted to learn about the crutch of the Jacobian had to wait for his 1974 Michigan lectures *Curves and their Jacobians*, now reprinted with the latest reedition of the *Red Book*. Two years later saw the appearance of *Complex Projective Varieties*. And the years 1983–1991 saw the appearance of his three-volume *Tata Lectures on Theta Functions*. In all of these books, there is constant interaction between modern methods and classical problems, leading the reader to a deeper appreciation of both. This modern-classical interaction also underlies, at the more abstruse level, his 1965 book *Geometric Invariant Theory* and his 1966 book *Lectures on Curves on an Algebraic Surface*, a pair of books which provided many advanced readers their baptism by fire into the world of moduli spaces. All of these books are, and will remain for the foreseeable future, classics to which the reader returns over and over.

Biographical Sketch

David Mumford was born in Sussex, England, in 1937, but grew up in the U.S. from 1940 on. He went to Harvard University as a freshman in 1953 and stayed there until 1996, working up through the ranks. He was awarded a Fields Medal in 1974, was

chair of the Department of Mathematics in 1981–84, and became also a member of the Division of Applied Sciences in 1985. In 1996, he moved to the Division of Applied Mathematics at Brown University, joining its strong interdisciplinary program. He delivered the AMS Gibbs Lecture entitled "The shape of objects in two and three dimensions" in 2003.

His research was in algebraic geometry from roughly 1960 to 1983. His focus was the construction and analysis of moduli spaces, especially that of curves and abelian varieties. From 1984 to the present, his research has concerned the construction of mathematical models for the understanding of perception, a field called Pattern Theory by its founder Ulf Grenander. Mumford's focus here has been the modeling of vision by computer and in the animal brain, especially statistical models.

Response

I am very honored by receiving this prize and also, as it is many years since I worked in algebraic geometry, very surprised to hear that people still read my books on the subject. The subject has grown in so many exciting and unexpected ways in the last few decades. It may be of some interest to recall what the state of that field was when I was a graduate student in the 1950s. Firstly, it was said that, between them, Zariski and Weil knew everything about the field and, if neither of them knew some fact, it was probably wrong or unimportant. But one thing they were both struggling with was finding a language in which they could express both characteristic p geometry and the arithmetic structures which bound it with characteristic 0, yet retain the geometric intuition which had so often driven the field. When pressed, all of his students had seen Zariski draw a small lemniscate on the corner of the blackboard, away from the mass of algebraic formulas, to revive his geometric intuition. Then Grothendieck arrived on the scene and with a simplicity that was pure genius defined the concept "spec", saying that all prime ideals were to be treated as points. About this time, I was reading, in Klein's history of nineteenthth century mathematics, how Kronecker had started on the same road of integrating number theory and geometry—"*Es bietet sich da ein ungeheurer Ausblick auf ein rein theoretisches Gebiet* [It offered an enormous view of a purely theoretical area]". Well, that was what we grad students thought too!

But I loved pictures. I drew cartoons like those in the accompanying figure in my *Red Book* showing how everyone probably thought about schemes. I was amused when a book on *Five Centuries of French Mathematics* asked to include these cartoons with the description: "*Par nature, la notion de schema est trop abstraite pour être reellement 'desinée'. Ces dessins sont dus à l'auteur d'un des rares livres de géométrie algébrique qui osa se lancer dans telle aventure* [By its nature, the notion of scheme is too abstract to really be 'drawn'. These drawings are by an author of one of those rare books of algebraic geometry that dared to fling itself into such an adventure]". After all, it was the French who started impressionist painting and isn't this just an impressionist scheme for rendering geometry?

David B. Mumford

The connections between traditional Italian algebraic geometry and Grothendieck's ideas continued to fascinate me. My book *Lectures on Curves on an Algebraic Surface* was written to show how wonderfully Grothendieck's ideas had completed one of the great quests of the Italian geometers. That was the problem of relating two ways of measuring the "irregularity" of an algebraic surface: could you find algebraic but not linear families of divisors whose dimension was H^1 of the structure sheaf (they called it $p_a - p_g$)? Over the complex numbers, the theory of harmonic forms had come to the rescue and proved this, but they sought an algebraic proof. Grothendieck, by representing functors defined on arbitrary schemes, had, *in passing*, solved this. All you needed at the end was the simple fact that characteristic 0 group schemes were reduced and out it pops. What a triumph for the great abstraction with which he formulated mathematics.

One of the most moving sequels for me was that these books were translated into Russian—several of them by Manin himself—and reached what was then the isolated school of Russian algebraic geometers. I want to thank both Manin and my many co-authors, Ash, Bergman, Fogarty, Kempf, Knudsen, Kirwan, Nori, Norman, Ramanujam, Rapaport, Saint-Donat, and Tai, who have added wonderful material. Writing books is often a team effort and working with all these collaborators has been a major stimulus for me. I am deeply grateful to them all and to the prize committee for this recognition.

Seminal Contribution to Research: Karen K. Uhlenbeck

Citation

The 2007 Steele Prize for a Seminal Contribution to Mathematical Research is awarded to Karen Uhlenbeck for her foundational contributions in analytic aspects of mathematical gauge theory. These results appeared in the two papers:

1) "Removable singularities in Yang-Mills fields", *Comm. Math. Phys.* **83** (1982), 11–29; and

2) "Connections with bounds on curvature", *Comm. Math. Phys.* **83** (1982), 31–42.

Connections are local objects in differential geometry, just as functions are local. But there are two crucial differences. First, connections admit automorphisms, called gauge transformations. Thus there are several different local representations of a connection. Second, the basic elliptic equation on functions—the Laplace equation—is linear whereas its counterpart on connections—the Yang-Mills equation—is nonlinear and not even elliptic as it stands. Its nonellipticity is tied up with the existence of automorphisms. One of Uhlenbeck's fundamental results proves the existence of good local representatives for connections, called Coulomb gauges. The Yang-Mills equations become elliptic when restricted to Coulomb gauges, and so Uhlenbeck deduces many basic theorems: smoothness of solutions, compactness of solutions with bounds on the curvature, etc. Uhlenbeck also proves that solutions to the Yang-Mills equations defined on a punctured ball with suitable boundedness on the curvature extend over the puncture. (Compare the much easier Riemann removable singularities theorem in complex analysis.) These theorems and the techniques Uhlenbeck introduced to prove them are the analytic foundation underlying the many applications of gauge theory to geometry and topology. The most immediate was Donaldson's work in the 1980s on smooth structures on 4-manifolds through invariants of the anti-self-dual equations, a system of first-order partial differential equations closely related to the Yang-Mills equations. Recall that Donaldson proved the existence of topological 4-manifolds which admit no smooth structure and topological 4-manifolds which admit inequivalent smooth structures. These equations have also advanced the theory of stable vector bundles in algebraic geometry. The analysis of various dimensional reductions of the anti-self-dual equations—the monopole and vortex equations and other closely related equations of gauge theory—begins with Uhlenbeck's theorems. More recently, these gauge theoretic ideas have yielded new insights in symplectic and contact geometry.

Karen K. Uhlenbeck

Biographical Sketch

Karen K. Uhlenbeck spent her early years in New Jersey, after which she attended the University of Michigan. She received her Ph.D. in 1968 under the direction of Richard Palais at Brandeis University. She has held posts at the Massachusetts Institute of Technology, the University of California at Berkeley, the University of Illinois in both Champaign-Urbana and Chicago, and the University of Chicago. Since 1988 she has held the Sid W. Richardson Foundation Regents Chair in Mathematics at the University of Texas in Austin.

Uhlenbeck is a member of the National Academy of Sciences and the American Academy of Arts and Sciences. She received a MacArthur Prize Fellowship in 1983, the Commonwealth Award for Science and Technology in 1995, and the National Medal of Science in 2000. Uhlenbeck is a co-founder of the IAS/Park City Mathematics Institute and the program for Women and Mathematics in Princeton.

Response

I thank the American Mathematical Society, its members and the Steele Prize committee for the honor and the award of the Steele Prize.

This honor confirms what I have been suspecting for quite some time. I am becoming an old mathematician, if I am not already there. It gives me cause to look back at my research and teaching. All in all, I have found great delight and pleasure in the pursuit of mathematics. Along the way I have made great friends and worked with a number of creative and interesting people. I have been saved from boredom, dourness, and self-absorption. One cannot ask for more.

My mathematical career has intersected some exciting mathematical changes. My thesis, written under Richard Palais, was written in the thick of the days of "Global Analysis", a period in which the tools and methods of differential topology were applied to analysis problems. This fell into disfavor, but it must be admitted that these ideas are today taken as a matter of course as part of the subject of analysis. During my days as an analyst, I wrote a paper on the regularity of elliptic systems, which I still think is the hardest paper I ever wrote.

The next revolution was single-handedly sponsored and spearheaded by S. T. Yau, who introduced techniques of analysis into the problems of topology, differential geometry, and algebraic geometry. Mind you, S. T. Yau was quite something in his younger days! I am quite proud of the paper I wrote with Jonathan Sachs on minimal spheres. Next we come to the introduction of gauge theory into topology, where I did the work which is cited in the award. I had started work on the analysis of gauge theory after hearing a lecture by Michael Atiyah on gauge theory at the University of Chicago, and was fully prepared to understand the thesis of his student Simon Donaldson, which used the two papers cited in this award. The work of Donaldson and Cliff

Taubes, whom I met when he was still a graduate student, was the start of a new era in four-manifold topology. Finally, due to what was now an addiction to intellectual excitement, I tried to follow the influence of physics on geometry which is associated with the name of Ed Witten. My work in integrable systems grew out of this connection with physics. This part of my career was not entirely successful. The more physics I learned, the less algebraic geometry I seemed to know.

Given that I started my academic career in the late 1960s at the University of California, Berkeley, during the Vietnam War, where protests and tear gas were commonplace, it must be said that I rarely found mathematics and the academic life boring.

The accomplishments of which I am most proud are not exactly mathematical theorems. One does mathematics because one has to, and if it is appreciated, all the better! However, encouraged by my young and enthusiastic colleague Dan Freed, I became involved in educational issues. We were among the founders of the IAS/Park City Mathematics Institute. The original intent was to bring mathematics researchers, students, and high school teachers together. This is now an ongoing institution with a yearly summer school, overseen by the Institute for Advanced Study in Princeton. The Women and Mathematics Program at IAS is an outgrowth of the Park City Institute. Founded by my collaborator Chuu-Lian Terng and me, the original purpose was to encourage and prepare more women to take part in the Park City Summer School. It has now grown to a self-sufficient two-week yearly program sponsored by IAS. I watch with real delight the emergence of our graduates into prominence in the mathematics community.

Another outcome of this involvement with education is our Saturday Morning Math Group at the University of Texas. We started this in conjunction with the beginnings of Park City. It is now an ongoing program which our graduate students organize for local high school students. It is often cited and much boasted of by our university. Finally, I would like to boast further of my department at the University of Texas. During the years that I have held an endowed chair in this department, we have become one of the leading departments of mathematics, admittedly below the top ranked, but still quite respectable. Certainly this is due mostly to my colleagues but I take a little credit. Our primary benefactor is also due some praise. We used to "thank Peter" after a particularly enjoyable colloquium talk and dinner and I do again now.

Starting from my days in Berkeley, the issue of women has never been far from my thoughts. I have undergone wide swings of feeling and opinion on the matter. I remain quite disappointed at the numbers of women doing mathematics and in leadership positions. This is, to my mind, primarily due to the culture of the mathematical community as well as harsh societal pressures from outside. Changing the culture is a momentous task in comparison to the other minor accomplishments I have mentioned.

I want to end by thanking my thesis advisor, Richard Palais, my two present collaborators Chuu-Lian Terng and Andrea Nahmod, my longtime friend and supporter, S. T. Yau, my colleagues, particularly Dan Freed and Lorenzo Sadun as well as all my collaborators, Ph.D. students, and assistants. My husband, Bob Williams, is due a share in this award.

Lifetime Achievement: Henry P. McKean

Citation

McKean launched his rich and magnificent mathematical career as an analytically oriented probabilist. After completing his thesis, which is motivated by, but makes essentially no explicit use of, probability theory, he began his collaboration with K. Itô. Together, he and Itô transformed Feller's analytic theory of one dimensional diffusions into probability theory, a heroic effort that is recorded in their famous treatise *Diffusion Processes and Their Sample Paths* [Die Grundlehren der mathematischen Wissenschaften, Band 125, Springer-Verlag, 1974]. After several years during which he delved into a variety of topics with probabilistic origins, spanning both Gaussian and Markov processes and including the first mathematically sound treatment of "American options", I. M. Singer deflected McKean's attention from probability and persuaded him to turn his powerful computational skills on a problem coming from Riemannian geometry. The resulting paper remains a milestone in the development of index theory.

Henry P. McKean

After moving to the Courant Institute, McKean played a central role in the creation of the analytic ideas which underpin our understanding of the KdV and related nonlinear evolution equations, and here again his computational prowess came to the fore. In recent years, McKean has returned to his probabilistic past, studying measures in pathspace, which are the "Gibbs" state for various nonlinear evolutions.

McKean has had profound influence on his own and succeeding generations of mathematicians. In addition to his papers and his book with Itô, he has authored several books that are simultaneously erudite and gems of mathematical exposition. Of particular importance is the little monograph in which he introduced Itô's theory of stochastic

integration to a wide audience. As his long list of students attests, he has also had enormous impact on the careers of people who have been fortunate enough to study under his direction.

Biographical Sketch

Henry McKean was born in Wenham, Massachusetts, in 1930. He graduated from Dartmouth College (A.B. 1952), spent a year at Cambridge University (1952–53), then went to Princeton University (Ph.D. 1955). He worked at the Massachusetts Institute of Technology (1958–66), at Rockefeller University (1966–70), and since then at the Courant Institute, of which he was director (1988–92). In the year 1979–80 he was George Eastman Professor at Balliol College, Oxford. He is a member of the American Academy of Arts and Sciences and of the National Academy of Sciences and received a Doctor Honoris Causa from the University of Paris in 2002.

Introduced to probability by M. Kac (summer school, MIT, 1949), McKean continued in this subject for some twenty-five years with W. Feller (1953–56) and in a long collaboration with K. Itô (1952–65), including a visit to Kyoto (1957–58). In 1974 his interests shifted to Hamiltonian mechanics, in particular, to the application of infinite-genus projective curves to KdV, on which he spoke at the International Congress of Mathematicians in Helsinki in 1978, in parallel to S. P. Novikoff's report on the same topic. Now he alternates between "KdV and all that" and his old affection for Brownian motion.

Response

I have been lucky in my mathematical life. Now comes this new piece of luck, the Steele Prize, something I never imagined I might receive and am very grateful for. In school, I really disliked mathematics with its tiresome triangles and its unintelligible x until I began to learn calculus from the amiable Dr. Conwell. Then I saw that you could do something with it, and that was exciting. Besides, I was better at it than the other kids and I liked that *very* much. Coming to Dartmouth (not so much as to learn anything particular, but to ski) I knew I liked mathematics pretty well but decided on it only little by little, thinking I might be an oceanographer. (A skiing accident helped concentrate my mind.) There I started to read P. Levy on Brownian motion, the first love of my mathematical life, and I worked on that and related things with Kac, Feller, Itô, and Levinson who taught me so much, and not just in mathematics. This went on from 1949 to 1972 or so when I began to look for something else to do.

One morning in 1974 Pierre Van Moerbeke came and told me that KdV could be solved by an elliptic function, and being an amateur of these, I sat up, took notice, and made a 90-degree turn into Hamiltonian mechanics and the (to me) very surprising use of infinite-genus projective curves for solving mechanical problems with an infinite number of commuting constants of motion. This led to delightful collaborations with van Moerbeke, Trubowitz, Moser and Airault, Ercolani, and others, building on Peter Lax's deep understanding of the question and paralleling the work of S. P. Novikoff and his school. At the beginning, we knew nothing of algebraic geometry. I remember a private seminar with Sarnak, Trubowitz, Varadhan, and others: how we would get the giggles at how little we understood—except for Sarnak, who was way ahead. Anyhow, a beautiful picture slowly emerged, though it is still a mystery to me what projective curves have to do with all those constants of motion. I mean, why is complex structure hidden there? I suppose it must come from the fact that the "n choose 2" system of vanishing brackets for n constants of motion is vastly over-determined. But that is just one of the queer things about "KdV and all that".

Well then: I have been lucky in my teachers, in my collaborations, and in my students. A few of those last are named above. The others know who they are. My thanks to them all: to those still present and to those present only to memory, of whom I count myself merely the representative in the receipt of this generous prize.

2007 Conant Prize

The 2007 Levi L. Conant Prize was awarded at the 113th Annual Meeting of the AMS in New Orleans in January 2007.

The Conant Prize is awarded annually to recognize an outstanding expository paper published in either the *Notices of the AMS* or the *Bulletin of the AMS* in the preceding five years. Established in 2001, the prize honors the memory of Levi L. Conant (1857–1916), who was a mathematician at Worcester Polytechnic University. The prize carries a cash award of US$1,000.

The Conant Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2007 prize, the members of the selection committee were: Noam D. Elkies, Carl R. Riehm, and Mary Beth Ruskai.

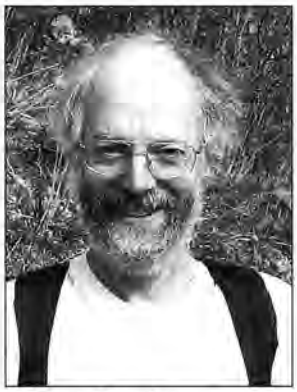
Previous recipients of the Conant Prize are: Carl Pomerance (2001), Elliott Lieb and Jakob Yngvason (2002), Nicholas Katz and Peter Sarnak (2003), Noam D. Elkies (2004), Allen Knutson and Terence Tao (2005), and Ronald M. Solomon (2006).

The 2007 Conant Prize was awarded to JEFFREY WEEKS. The text that follows presents the committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Citation

The Conant prize in 2007 is awarded to Jeffrey Weeks for his article "The Poincaré Dodecahedral Space and the Mystery of the Missing Fluctuations" [*Notices*, June/July 2004]. In this article, together with an earlier one "Measuring the Shape of the Universe" [*Notices*, December 1998], co-authored with Neil Cornish, Weeks explains how extremely sensitive measurements of microwave radiation across the sky provide information about the origins and shape of the universe.

After giving some physical background, Weeks summarizes the evidence for and against a universe that locally looks like a spherical, Euclidean, or hyperbolic 3-manifold. He then considers spherical universes in more detail, emphasizing the role of symmetry groups and making a case for a model based on the Poincaré dodecahedral space. Throughout the paper, he makes this material accessible by using analogies with more familiar structures in one and two dimensions. He gives a particularly elegant exposition of the free symmetry groups of the 3-sphere via an extension of rotation groups of the 2-sphere.



Jeffrey Weeks

Most accounts of the development of physical theories are presented after the dust has settled and the experimental evidence has convinced most scientists. Weeks has explained the mathematics behind models whose validity cosmologists debate while waiting for more experimental evidence. Whether or not the dodecahedral model turns out to be consistent with future observations remains to be seen. In either case, Weeks has given a rare glimpse into the role of mathematics in the development and testing of physical theories.

Biographical Sketch

Jeff Weeks fell in love with geometry in the 12th grade when he read *Flatland*. While an undergraduate at Dartmouth College he bounced back and forth between math and physics, eventually settling on math. He went on to study 3-manifolds under Bill Thurston at Princeton but maintained his interest in physics on the sly. After a few years teaching at Stockton State College and Ithaca College, Weeks resigned to be a full-time dad for a few years. From there he fell into the life of a freelance mathematician, at first part-time, then full-time. He has enjoyed extensive work with the Geometry Center and the National Science Foundation as well as smaller gigs for science museums and teaching at Middlebury College. In 1999 a telephone call from the MacArthur Foundation brought five years of work with zero administrative overhead. The timing could not have been better: 1999–2004 was an intense period for cosmic topology, as well as the time to finish the unit Exploring the Shape of Space for middle schools and high schools. Weeks

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currently splits his time between puzzling over the microwave background and writing Macintosh versions of his geometry and topology software.

Response

Mathematics, physics, and astronomy share a deep common history. It's been a pleasure working in this tradition, and I'm particularly pleased that *Notices* readers enjoyed hearing one small piece of the ongoing story.

As for the cosmic microwave background, new satellite data (WMAP 2006) and more detailed analysis (carefully excluding a neighborhood of the galactic plane) find the large-scale fluctuations to be even weaker than previously thought, implying that we're seeing something real and not just a statistical fluke. Nontrivial cosmic topology could account for the weakness, but so far no rigorous evidence exists for this or any other explanation. Mother Nature is keeping us guessing.

2007 Morgan Prize

The 2007 AMS-MAA-SIAM Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student was awarded at the Joint Mathematics Meetings in New Orleans in January 2007.

The Morgan Prize is awarded annually for outstanding research in mathematics by an undergraduate student (or students having submitted joint work). Students in Canada, Mexico, or the United States or its possessions are eligible for consideration for the prize. Established in 1995, the prize was endowed by Mrs. Frank Morgan of Allentown, Pennsylvania, and carries the name of her late husband. The prize is given jointly by the AMS, the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM) and carries a cash award of US$1,000.

Recipients of the Morgan Prize are chosen by a joint AMS-MAA-SIAM selection committee. For the 2007 prize, the members of the selection committee were: Kelly J. Black, James H. Curry, Herbert A. Medina (chair), Karen E. Smith, Judy L. Walker, and Paul Zorn.

Previous recipients of the Morgan Prize are: Kannan Soundararajan (1995), Manjul Bhargava (1996), Jade Vinson (1997), Daniel Biss (1998), Sean McLaughlin (1999), Jacob Lurie (2000), Ciprian Manolescu (2001), Joshua Greene (2002), Melanie Wood (2003), Reid Barton (2005), and Jacob Fox (2006).

The 2007 Morgan Prize was awarded to DANIEL KANE. The text that follows presents the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Daniel Kane

Citation

Daniel Kane is majoring in both mathematics and physics at the Massachusetts Institute of Technology (MIT) and expects to receive his bachelor's degree in June 2007. At this early stage of his mathematical career, Daniel has already established a research record that would be the envy of many professional mathematicians. Indeed, he has authored or co-authored ten articles that have appeared or will soon appear (have been accepted) in research journals including the *Proceedings of the American Mathematical Society, The Ramanujan Journal, The Journal of Number Theory, Foundations of Computer Science*, and *Integers: Electronic Journal of Combinatorial Number Theory*. In addition, he has six other research papers that have been submitted or are in preparation for a total of sixteen research papers! The specifics of his research are too long to detail, but we mention that it has been in fields as diverse as number theory, computer science, ergodic theory, combinatorics, computational geometry, and game theory.

Mr. Kane's mathematical talent is captured well in some of the comments/summaries contained in the letters supporting his nomination for the prize:

- "Daniel's first paper improves on a famous *Annals of Mathematics* paper by Paul Erdős."
- "He proved an open conjecture stated by a well-known number theorist several years before. It [Kane's paper] was written while Daniel was in 12th grade."
- "He is by far the sharpest and most productive math undergraduate I have come in contact with in my five years [at MIT]."

In addition to all of his mathematical research, Daniel is also a three-time Putnam Fellow and two-time International Mathematical Olympiad (IMO) Gold Medalist.

Biographical Sketch

Daniel Kane was born in Madison, Wisconsin, to professors of mathematics and of biochemistry. His schooling began at Wingra, a private school unusual in its noncompetitive policies and open classrooms. When it became clear (in about the third grade) that he was ready for high school math, he was allowed to do more advanced math assigned by his parents. Due to this head start, he was ready

to begin taking university classes at the beginning of high school. After graduating, Kane went to MIT to study mathematics and physics.

Kane first learned about mathematical problem solving through the University of Wisconsin's Van Vleck Talent search. This training helped immensely when he took the USA Mathematical Olympiad in the 7th grade, qualifying for the Mathematical Olympiad Program (MOP), which he continued to attend for the duration of high school, earning two IMO gold medals.

Kane became interested in research near the end of high school, when he did work on modular forms under the supervision of Ken Ono. In college his opportunities for research expanded greatly, including the summer programs at Williams College and University of Minnesota-Duluth, class projects, and competitions.

Response

In receiving this award I would like to extend my thanks to all of those who helped make it possible. Most importantly, I would like to thank my parents for giving me such a good environment to grow up in and in particular my dad for teaching me and helping me to develop my love of mathematics. Thanks are also due to those who helped me get started learning mathematical problem solving skills such as Marty Isaacs, who ran the Van Vleck Talent Search, and Titu Andreescu and the other MOP instructors. I would like to thank those who helped supervise parts of my research, namely, Ken Ono, Joe Gallian, Cesar Silva, and Erik Demaine. Lastly, I would like to thank my many teachers over the years who have provided me with the knowledge base to be able to conduct interesting research in so many areas.

2007 Satter Prize

The 2007 Ruth Lyttle Satter Prize in Mathematics was awarded at the 113th Annual Meeting of the AMS in New Orleans in January 2007.

The Satter Prize is awarded every two years to recognize an outstanding contribution to mathematics research by a woman in the previous five years. Established in 1990 with funds donated by Joan S. Birman, the prize honors the memory of Birman's sister, Ruth Lyttle Satter. Satter earned a bachelor's degree in mathematics and then joined the research staff at AT&T Bell Laboratories during World War II. After raising a family she received a Ph.D. in botany at the age of forty-three from the University of Connecticut at Storrs, where she later became a faculty member. Her research on the biological clocks in plants earned her recognition in the U.S. and abroad. Birman requested that the prize be established to honor her sister's commitment to research and to encouraging women in science. The prize carries a cash award of US$5,000.

The Satter Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2007 prize, the members of the selection committee were: Benedict H. Gross, Karen E. Smith, and Chuu-Lian Terng (chair).

Previous recipients of the Satter Prize are: Dusa McDuff (1991), Lai-Sang Young (1993), Sun-Yung Alice Chang (1995), Ingrid Daubechies (1997), Bernadette Perrin-Riou (1999), Karen E. Smith (2001), Sijue Wu (2001), Abigail Thompson (2003), and Svetlana Jitomirskaya (2005).

The 2007 Satter Prize was awarded to CLAIRE VOISIN. The text that follows presents the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Citation

The Ruth Lyttle Satter Prize is awarded to Claire Voisin of the Institut de Mathématiques de Jussieu for her deep contributions to algebraic geometry, and in particular for her recent solutions to two long-standing open problems. Voisin solved the Kodaira problem in her paper "On the homotopy types of compact Kähler and complex projective manifolds", *Invent. Math.* **157** (2004), no. 2, 329–343. There she shows that in every dimension greater than three, there exist compact Kähler manifolds not homotopy equivalent to any smooth projective variety. This problem has been open since the 1950s when Kodaira proved that every compact Kähler surface is diffeomorphic to (and hence homotopy equivalent to) some projective algebraic variety. Her idea is to start with the fact that certain endomorphisms can prevent a complex torus from being realized as a projective variety, and then to construct Kähler manifolds whose Albanese tori must carry such endomorphisms for homological reasons. In a completely different direction, Voisin also solves Green's Conjecture in her papers "Green's canonical syzygy conjecture for generic curves of odd genus", *Compos. Math.* **141** (2005), no. 5, 1163–1190, and "Green's generic syzygy conjecture for curves of even genus lying on a K3 surface", *J. Eur. Math. Soc.* **4** (2002), no. 4, 363–404.

Claire Voisin

A century ago, Hilbert saw that syzygies (relations among relations) were important invariants of varieties in projective space, and in the early 1980s, Mark Green conjectured that the syzygies of a general curve canonically embedded in projective space should be as simple as possible. This conjecture attracted a huge amount of effort by algebraic geometers over twenty years before finally being settled by Voisin. Her idea is to work with curves on a suitable K3 surface, where she executes

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deep calculations with vector bundles (at least in even genus) that lead to the required vanishing theorems.

Biographical Sketch

Claire Voisin defended her thesis in 1986 under the supervision of Arnaud Beauville. She began employment in the Centre National de la Recherche Scientifique as *chargée de recherche* in 1986 and since then pursued her career in this institution. She occasionally taught graduate courses but mainly does research and advises students. Her honors include the European Mathematical Society Prize (1992), the Servant Prize (1996) and the Sophie Germain Prize (2003) of the Académie des Sciences de Paris, and the silver medal of the CNRS (2006). She was an invited speaker at the International Congress of Mathematicians in 1994 in Zurich.

Response

I am deeply honored to have been chosen to receive the 2007 Ruth Lyttle Satter Prize. I feel of course very encouraged by this recognition of my work. I would like to thank the members of the prize committee for selecting me. I am also very grateful to my institution, the CNRS, which made it possible for me to do research in the best conditions.

2007 Wiener Prize

The 2007 AMS-SIAM Norbert Wiener Prize in Applied Mathematics was awarded at the Joint Mathematics Meetings in New Orleans in January 2007.

The Wiener Prize is awarded every three years to recognize outstanding contributions to applied mathematics in the highest and broadest sense (until 2001, the prize was awarded every five years). Established in 1967 in honor of Norbert Wiener (1894–1964), the prize was endowed by the Department of Mathematics of the Massachusetts Institute of Technology. The prize is given jointly by the AMS and the Society for Industrial and Applied Mathematics (SIAM). The recipient must be a member of one of these societies and a resident of the United States, Canada, or Mexico. The prize carries a cash award of US$5,000.

The recipient of the Wiener Prize is chosen by a joint AMS-SIAM selection committee. For the 2007 prize, the members of the selection committee were: Percy A. Deift (chair), David B. Mumford, and Stanley J. Osher.

The previous recipients of the Wiener Prize are: Richard E. Bellman (1970), Peter D. Lax (1975), Tosio Kato (1980), Gerald B. Whitham (1980), Clifford S. Gardner (1985), Michael Aizenman (1990), Jerrold E. Marsden (1990), Hermann Flaschka (1995), Ciprian Foias (1995), Alexandre J. Chorin (2000), Arthur T. Winfree (2000), and James Sethian (2004).

The 2007 Wiener Prize was awarded to CRAIG TRACY and HAROLD WIDOM. The text that follows presents the selection committee's citation, a brief biographical sketch of each awardee, and their response upon receiving the prize.

Craig Tracy

Harold Widom

Citation

Craig Tracy and Harold Widom have done deep and original work on Random Matrix Theory, a subject which has remarkable applications across the scientific spectrum, from the scattering of neutrons off large nuclei to the behavior of the zeros of the Riemann zeta-function.

The contributions of Tracy and Widom center around a connection between a class of Fredholm determinants associated with random matrix ensembles on the one hand, and Painlevé functions on the other. Most notably, they have introduced a new class of distributions, the so-called Tracy-Widom distributions, which have a universal character and which have applications, in particular, to Ulam's longest increasing subsequence problem in combinatorics, tiling problems, the airline boarding problem of Bachmat et al., various random walker and statistical mechanical growth models in the KPZ class, and principal component analysis in statistics when the number of variables is comparable to the sample size.

The committee also recognizes the earlier work of Craig Tracy with Wu, McCoy, and Barouch, in which Painlevé functions appeared for the first time in exactly solvable statistical mechanical models. In addition, the committee recognizes the seminal contributions of Harold Widom to the asymptotic analysis of Toeplitz determinants and their various operator theoretic generalizations.

Biographical Sketch: Craig Tracy

Craig Arnold Tracy was born in England on September 9, 1945, the son of Eileen Arnold, a British subject, and Robert C. Tracy, an American serving in the U.S. Army. After immigrating to the United States as an infant, Tracy grew up in Missouri where he attended the University of Missouri at Columbia, graduating in 1967 as an O. M. Stewart Fellow with a B.S. degree in physics. He began his graduate studies as a Woodrow Wilson Fellow at the State University of New York at Stony Brook, where he wrote his doctoral dissertation under the supervision of Barry M. McCoy. After postdoctoral positions at the University of Rochester (1973–75) and the C. N. Yang Institute for Theoretical Physics (1975–78), Tracy was at Dartmouth College for six years before joining the University of California, Davis, in 1984. He is currently Distinguished Professor of Mathematics at UC Davis. In 2002 Tracy was awarded, jointly with Harold Widom, the SIAM George Pólya Prize. He is a member of the American Academy of Arts and Sciences. Tracy has two daughters and three grandchildren. He is married to Barbara Nelson, and they reside in Sonoma, California.

Biographical Sketch: Harold Widom

Harold Widom is professor emeritus at the University of California, Santa Cruz. He grew up in New York City, where he attended Stuyvesant High School and the City College of New York. He did his graduate work at the University of Chicago, receiving his Ph.D. under the supervision of Irving Kaplansky. His first academic position was at Cornell University where, inspired by Mark Kac, he turned his attention to the study of Toeplitz and Wiener-Hopf operators. This influenced much of his subsequent research and led ultimately to his work (largely in collaboration with Craig Tracy) in integrable systems and random matrix theory.

He is a member of the American Academy of Arts and Sciences and in 2002 received, jointly with Tracy, the SIAM George Pólya Prize. He is an associate editor of *Asymptotic Analysis; Journal of Integral Equations and Applications*; and *Mathematical Physics, Analysis and Geometry*. He is an honorary editor of *Integral Equations and Operator Theory*.

Response

We are honored to be named the recipients of the 2007 AMS-SIAM Norbert Wiener Prize in Applied Mathematics. We thank the members of the Selection Committee for their consideration and in particular for their recognition of our field of random matrix theory and integrable systems. Underlying much of our own research have been Wiener-Hopf operators and Wiener processes, so it is especially gratifying to receive this prize, named for Norbert Wiener. We thank AMS and SIAM for this honor.

One of us (Tracy) would like to acknowledge the support, early in his career, from Barry M. McCoy, J. Laurie Snell, Tai Tsun Wu, and Chen Ning Yang. We both express our appreciation of Estelle L. Basor, with whom we wrote our first joint paper on random matrices.

And we thank the diverse group of researchers in random matrix theory and integrable systems for making this an exciting field in which to work.

2007 Veblen Prize

The 2007 Oswald Veblen Prize in Geometry was awarded at the 113th Annual Meeting of the AMS in New Orleans in January 2007.

The Veblen Prize is awarded every three years for a notable research memoir in geometry or topology that has appeared during the previous five years in a recognized North American journal (until 2001 the prize was usually awarded every five years). Established in 1964, the prize honors the memory of Oswald Veblen (1880–1960), who served as president of the AMS during 1923–1924. It carries a cash award of US$5,000.

The Veblen Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2007 prize, the members of the selection committee were: Cameron M. Gordon, Michael J. Hopkins (chair), and Ronald J. Stern.

Previous recipients of the Veblen Prize are: Christos D. Papakyriakopoulos (1964), Raoul H. Bott (1964), Stephen Smale (1966), Morton Brown and Barry Mazur (1966), Robion C. Kirby (1971), Dennis P. Sullivan (1971), William P. Thurston (1976), James Simons (1976), Mikhael Gromov (1981), Shing-Tung Yau (1981), Michael H. Freedman (1986), Andrew Casson (1991), Clifford H. Taubes (1991), Richard Hamilton (1996), Gang Tian (1996), Jeff Cheeger (2001), Yakov Eliashberg (2001), Michael J. Hopkins (2001), and David Gabai (2004).

The 2007 Veblen Prize was awarded to two pairs of collaborators: to PETER KRONHEIMER and TOMASZ MROWKA, and to PETER OZSVÁTH and ZOLTÁN SZABÓ. The text that follows presents the selection committee's citations, brief biographical sketches, and the awardees' responses upon receiving the prize.

Citation: Peter Kronheimer and Tomasz Mrowka

The 2007 Veblen Prize in Geometry is awarded to Peter Kronheimer and Tomasz Mrowka for their joint contributions to both three- and four-dimensional topology through the development of deep analytical techniques and applications. In particular the prize is awarded for their seminal papers:

Peter Kronheimer

Tomasz Mrowka

"Embedded surfaces and the structure of Donaldson's polynomial invariants", *J. Differential Geom.* **41** (1995), no. 3, 573–734.

Since 1982, most of the progress in four-dimensional differential topology has arisen from the applications of gauge theory pioneered by S. K. Donaldson. In particular, Donaldson's polynomial invariants have been used to prove a variety of results about the topology and geometry of four-manifolds. This paper is one of the pinnacles of this development. It gives a conceptual framework and an organizing principle for some of the disparate observations and calculations of Donaldson invariants that had been made earlier, it reveals a deep structure encoded in the Donaldson invariants which is related to embedded surfaces in four-manifolds, and it has been the point of departure and the motivating example for important further developments, most spectacularly for Witten's introduction of the so-called Seiberg-Witten invariants.

"The genus of embedded surfaces in the projective plane", *Math. Res. Lett.* **1** (1994), no. 6, 797–808.

This paper proves the Thom conjecture, which claims that if C is a smooth holomorphic curve in $\mathbf{CP}^2$, and C' is a smoothly embedded oriented two-manifold representing the same homology class as C, then the genus of C' satisfies $g(C') \geq g(C)$.

"Witten's conjecture and property P", *Geometry and Topology* **8** (2004), 295–310.

Here the authors use their earlier development of Seiberg-Witten monopole Floer homology to prove the Property P conjecture for knots. In other words, if $K \subset S^3$ is a nontrivial knot, and $K_{p/q}$ is the three-manifold obtained by p/q Dehn surgery along K with $q \neq 0$, then $\pi_1(K_{p/q})$ must be nontrivial. The proof is a beautiful work of synthesis which draws upon advances made in the fields of gauge theory, symplectic and contact geometry, and foliations over the past twenty years.

Biographical Sketch: Peter Kronheimer

Born in London, Peter Kronheimer was educated at the City of London School and Merton College, Oxford. He obtained his B.A. in 1984 and his D.Phil. in 1987 under the supervision of Michael Atiyah. After a year as a Junior Research Fellow at Balliol and two years at the Institute for Advanced Study, he returned to Merton as Fellow and Tutor in Mathematics. In 1995 he moved to Harvard University, where he is now William Caspar Graustein Professor of Mathematics. A recipient of the Förderpreis from the Mathematisches Forschungsinstitut Oberwolfach, and a Whitehead Prize from the London Mathematical Society, he was elected a Fellow of the Royal Society in 1997.

Next to mathematics, his main pastime has often been music—the horn in particular, which he studied as a pupil of Ifor James. Peter Kronheimer lives in Newton, Massachusetts, with his wife Jenny and two sons, Matthew and Jonathan.

Biographical Sketch: Tomasz Mrowka

Tom Mrowka is a professor at the Massachusetts Institute of Technology. He received his undergraduate degree from MIT in 1983 and attended graduate school at the University of California at Berkeley, receiving his Ph.D. under the direction of Clifford H. Taubes in 1989. After graduate school he held postdoctoral positions at the Mathematical Sciences Research Institute in Berkeley (1988–89), Stanford (1989–91) and Caltech (1991–92). He held a professorship at Caltech from 1992 until 1996 and was a visiting professor at Harvard (spring of 1995) and at MIT (fall of 1995) before returning to MIT permanently in the fall of 1996.

He received the National Young Investigator Grant of the NSF in 1993 and was a Sloan Foundation Fellow from 1993 to 1995. He gave an invited lecture in the topology section of the 1994 International Congress of Mathematicians in Zurich, the Marston Morse lectures at the Institute for Advanced Study in Princeton in 1999, the Stanford Distinguished Visiting Lecture Series in 2000, the Joseph Fels Ritt Lectures at Columbia University in 2004, and the 23rd Friends of Mathematics Lecture at Kansas State University in 2005.

He works mainly on the analytic aspects of gauge theories and applications of gauge theory to problems in low-dimensional topology.

Response: Peter Kronheimer and Tomasz Mrowka

We are honored, surprised, and delighted to be selected, together with Peter Ozsváth and Zoltán Szabó, as recipients of the Oswald Veblen Prize in Geometry.

The Thom conjecture, and other related questions concerning the genus of embedded 2-manifolds in 4-manifolds, are natural and central questions in 4-dimensional differential topology. After Simon Donaldson's work in gauge theory opened up this field, these problems became tempting targets for the newly available techniques. In the summer of 1989, we were both at MSRI and discussed the idea of using "singular instantons" to prove such conjectures. But it was not until two years later, when we spent a month together at Oberwolfach, that a proof began to emerge of a version of the Thom conjecture for embedded 2-manifolds in $K3$ surfaces. This theorem and its proof filled our first two joint papers, and provided the first truly sharp results for the genus problem.

In March 1993, we met at Columbia, at the invitation of John Morgan. We understood that the singular instanton techniques that we had used for the genus problem should lead to universal relations among the values of Donaldson's polynomial invariants for 4-manifolds. At the time, calculating Donaldson's invariants in special cases was a challenging occupation. Although we could see how to prove relations, no coherent picture was emerging; and at the end of this visit, Peter headed to LaGuardia with the forest still not visible for the trees. New York's "Blizzard of the Century" closed the airport, and we worked together for another day, during which we noticed that our relations implied a simple linear recurrence relation for certain values of Donaldson's invariants. This soon led to a beautiful structure theorem for the polynomial invariants in terms of "basic classes", intricately entwined with the genus question through an "adjunction inequality". These developments provided a proof of the Thom conjecture for a large class of algebraic surfaces, though the original version for the complex projective plane had to wait until 1994 and the introduction of the Seiberg-Witten equations.

While techniques from gauge theory revolutionized the field of 4-manifolds, providing answers to many important questions, these ideas had no comparable impact in 3-dimensional topology, where the questions and tools remained very different. Around 1986, Andreas Floer used Yang-Mills gauge theory to define his "instanton homology" groups for 3-manifolds. The Euler number of these

homology groups recaptured an integer invariant introduced by Andrew Casson which had already been seen to imply results about surgery on 3-manifolds, including partial results in the direction of the "Property P" conjecture. It seems likely that Floer himself foresaw the possibility of using his homology theory to prove stronger results in the same direction; in particular, he established an "exact triangle" relating the Floer homology groups of the 3-manifolds obtained by three different surgeries on a knot. But a missing ingredient at this time was any very general result stating that these homology groups were not trivial. In 1995 Yasha Eliashberg visited Harvard and lectured on his work with Bill Thurston on foliations and contact structures. It was apparent that these results could be combined with work of Cliff Taubes and Dave Gabai to give a non-vanishing theorem for a version of Floer's homology groups defined using the Seiberg-Witten equations, and to show, for example, that these versions of Floer homology encode sharp information about the genus of embedded surfaces in 3-manifolds. This was the first strong indication that, by combining non-vanishing theorems with surgery exact triangles, one would be able to use Floer groups to obtain significant new results about Dehn surgery. This hope was eventually realized in our joint paper with the other co-recipients of this prize, Peter Ozsváth and Zoltán Szabó, on lens space surgeries, and in the eventual resolution of the Property P conjecture using instanton homology. In the meantime, Ozsváth and Szabó's "Heegaard Floer theory" has transformed the field once again: it has led to a wealth of new results, and open problems to attract a new generation of researchers.

Peter would like to thank his wife Jenny and all his family for their love and support; and his mathematical mentors, Michael Atiyah, Simon Donaldson, and Dominic Welsh, for their guidance.

Tom thanks Gigliola, Mario, and Sofia for the joy they bring. Tom also thanks his teachers Victor Guillemin, Richard Melrose, Raoul Bott, Stephen Smale, John Stallings, and Rob Kirby for lighting up very different directions and ways of thinking at the beginning of his mathematical journey. Special thanks to his advisor Cliff Taubes and to John Morgan whose confidence and interest at the beginning of his career were crucial.

Finally we would both like to thank the American Mathematical Society for recognizing the field of gauge theory and low-dimensional topology with this year's Veblen Prize. We feel privileged to be chosen, together with our co-recipients, as representatives for an area of research that has seen so many exciting developments.

Peter Ozsváth

Zoltán Szabó

Citation: Peter Ozsváth and Zoltán Szabó

The 2007 Veblen Prize is awarded to Peter Ozsváth and Zoltán Szabó in recognition of the contributions they have made to 3- and 4-dimensional topology through their Heegaard Floer homology theory.

Ozsváth and Szabó have developed this theory in a highly influential series of over twenty papers produced in the last five years, and in doing so have generated a remarkable amount of activity in 3- and 4-dimensional topology. Specifically, they are cited for their papers

"Holomorphic disks and topological invariants for closed three-manifolds", *Ann. of Math.* (2) **159** (2004), 1027-1158;

"Holomorphic disks and three-manifold invariants: properties and applications", *Ann. of Math.* (2) **159** (2004), 1159-1245;

"Holomorphic disks and genus bounds", *Geometry and Topology* **8** (2004), 311-334.

The Heegaard Floer homology of a 3-manifold plays a role, in the context of the Seiberg-Witten invariants of 4-manifolds, analogous to that played by Lagrangian Floer homology in the context of the Donaldson invariants. There is also a version for knots, whose Euler characteristic is the Alexander polynomial. It detects the genus of a knot and also whether or not a knot is fibered. The combinatorial nature of these invariants has led to many deep applications in 3-dimensional topology. Among these are results about Dehn surgery on knots, such as the Dehn surgery characterization of the unknot, strong restrictions on lens space and other Seifert fiber space surgeries, and dramatic new results on unknotting numbers. Ozsváth and Szabó have used Heegaard Floer homology to define a contact structure invariant, which has led to new results in 3-dimensional contact topology. They have also defined a new concordance invariant of knots, which gives a lower bound on the 4-ball genus. The 4-dimensional version of Heegaard Floer homology has enabled them to give gauge-theory-free proofs of many of the results in 4-dimensional

topology obtained in the last decade using Donaldson and Seiberg-Witten theory, such as the Thom Conjecture on the minimal genus of a smooth representative of the homology class of a curve of degree d in CP^2, and the Milnor Conjecture on the unknotting number of a torus knot.

Biographical Sketch: Peter Ozsváth

Peter Ozsváth was born on October 20, 1967, in Dallas, Texas. He received his B.S. from Stanford University (1989) and his Ph.D. from Princeton University (1994) under the direction of John W. Morgan. He held postdoctoral positions at Caltech, the Max-Planck-Institut in Bonn, the Mathematical Sciences Research Institute in Berkeley, and the Institute for Advanced Study in Princeton. He held faculty positions at Princeton University, Michigan State University, and the University of California, Berkeley. He has been on the faculty at Columbia University since 2002.

Ozsváth received a National Science Foundation Postdoctoral Fellowship and an Alfred P. Sloan Research Fellowship. His invited lectures include an Abraham Robinson Lecture at Yale University (2003), a William H. Roever Lecture at Washington University St. Louis (2004), a Kuwait Foundation Lecture at Cambridge University (2006), and a lecture in the topology section of the International Congress of Mathematicians (2006).

Response: Peter Ozsváth

I am greatly honored to be a co-recipient of of the Oswald Veblen Prize, along with my long-time collaborator Zoltán Szabó, and also Peter Kronheimer and Tomasz Mrowka, whose work has always been a profound source of inspiration for me.

Heegaard Floer homology grew out of our efforts at understanding gauge theory from a more combinatorial point of view. The mathematical starting point was Yang-Mills theory and then Seiberg-Witten theory, which started with the work of S. K. Donaldson, A. Floer, and C. H. Taubes. But we have also been fortunate to be able to draw on the work of many interlocking neighboring fields, including contact and symplectic geometry, where at various times the work of Y. Eliashberg and E. Giroux provided answers to fundamental questions, and also three-manifold topology, where the questions raised by C. Gordon serve as a guiding light and the work of D. Gabai provides powerful tools which fit very neatly into the context of Floer homology.

I would like to thank my family and friends for their support throughout the years, and I also owe a great debt of gratitude to my teachers, coauthors, and students. In particular, I thank Zoltán for those many caffeinated mathematical sessions. I also thank my thesis advisor J. W. Morgan for introducing me to gauge theory and T. S. Mrowka, to whom I have turned many times for insight and advice. I would also like to thank R. Fintushel, R. Kirby, and R. Stern for helping to make the field so pleasant socially and so rich mathematically. I am deeply grateful to my undergraduate teachers P. J. Cohen, R. L. Cohen, R. J. Milgram, and fellow student D. B. Karagueuzian, for introducing me to mathematics. I would also like to thank my more recent collaborators C. Manolescu, S. Sarkar, A. Stipsicz, D. P. Thurston, and my former students E. Grigsby, M. Hedden, P. Sepanski, and the many additional members of the Columbia mathematics department who are constantly bringing new insights to an exciting and ever-developing field.

Biographical Sketch: Zoltán Szabó

Zoltán Szabó was born in Budapest, Hungary, in 1965. He did his undergraduate studies at Eötvös Loránd University in Budapest, and then his graduate studies at Rutgers University with John Morgan and Ted Petrie. He has worked at Princeton University since graduating in 1994. He also spent a year at the University of Michigan in 1999–2000. He has been a professor at Princeton University since 2002. He received a Sloan Research Fellowship and a Packard Fellowship. He was an invited lecturer at the 2006 International Congress of Mathematicians in Madrid, and a plenary speaker at the 2004 European Congress of Mathematics in Stockholm. Szabó's main research interests are smooth 4-manifolds, 3-manifolds, knots, Heegaard Floer homology, symplectic geometry, and gauge theory.

Response: Zoltán Szabó

I am greatly honored to be named, along with Peter Kronheimer, Tom Mrowka, and Peter Ozsváth, as a recipient of the Oswald Veblen Prize. The joint work with Peter Ozsváth which is noted here grew out of our attempts to understand Seiberg-Witten moduli spaces over three-manifolds where the metric degenerates along a surface. This led to the construction of Heegaard Floer homology that involved both topological tools, such as Heegaard diagrams, and tools from symplectic geometry, such as holomorphic disks with Lagrangian boundary constraints. The time spent on investigating Heegaard Floer homology and its relationship with problems in low-dimensional topology was rather interesting. I am very glad that this effort was rewarded by the prize committee.

First of all I would like to thank my wife, Piroska, for her support over the years. I also owe a lot to my co-author Peter Ozsváth whose boundless energy made this work possible, and to my thesis advisor, John Morgan, who introduced me to the world of gauge theory.

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2007 E. H. Moore Prize

The 2007 E. H. Moore Research Article Prize was awarded at the 113th Annual Meeting of the AMS in New Orleans in January 2007.

The prize is awarded every three years for an outstanding research article that appeared in one of the primary AMS research journals: *Journal of the AMS, Proceedings of the AMS, Transactions of the AMS, AMS Memoirs, Mathematics of Computation, Electronic Journal of Conformal Geometry and Dynamics,* or *Electronic Journal of Representation Theory.* The article must have appeared during the six calendar years ending a full year before the meeting at which the prize is awarded. The prize carries a cash award of US$5,000.

The prize honors the extensive contributions of E. H. Moore (1862–1932) to the AMS. Moore founded the Chicago section of the AMS, served as the Society's sixth president (1901–1902), delivered the Colloquium Lectures in 1906, and founded and nurtured the *Transactions of the AMS.*

The previous recipient of the Moore Prize is Mark Haiman (2004).

The Moore Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2007 prize, the members of the selection committee were: Lawrence Craig Evans, Carolyn S. Gordon (chair), Grigorii A. Margulis, George C. Papanicolaou, and Efim I. Zelmanov.

The 2007 Moore Prize was awarded to IVAN SHESTAKOV and UALBAI UMIRBAEV. The text that follows presents the selection committee's citation, brief biographical sketches, and the awardees' responses upon receiving the prize.

Citation

In two groundbreaking papers published in the *Journal of the American Mathematical Society* ("The tame and the wild automorphisms of polynomial rings in three variables", **17** (2004), no. 1, 197–227; and "Poisson brackets and two-generated subalgebras of rings of polynomials", **17** (2004), no. 1, 181–196), Ivan Shestakov and Ualbai Umirbaev develop powerful new techniques to address the structure of automorphism groups of polynomial algebras. Their dramatic results include a proof of the longstanding Nagata Conjecture, establishing the existence of a wild automorphism of a polynomial algebra in three variables.

Of particular importance is their novel use of Poisson structures and their universal quantizations to obtain a criterion of tameness. This innovation is already resulting in further major applications.

Biographical Sketch: Ivan Shestakov

Ivan Shestakov was born on August 13, 1947, in the Irkutsk region in Russia. After graduating from the Physical-Mathematical School in Novosibirsk, he entered Novosibirsk University in 1965. There he obtained his first results in algebra, under the guidance of professors K. Zhevlakov and A. Shirshov. His master's thesis "On a Class of Non-commutative Jordan Rings" was awarded the Medal of the Academy of Sciences of USSR for students.

In 1970 Shestakov graduated from Novosibirsk University and entered the Sobolev Institute of Mathematics as a researcher. In 1973 he received his Ph.D. from Novosibirsk University, and in 1978 he earned the Doctor of Sciences from the Sobolev Institute of Mathematics for the work "Free Alternative Algebras". The book *Rings That Are Nearly Associative*, written by Shestakov jointly with K. Zhevlakov, A. Slinko, and A. Shirshov, was published in 1978.

In 1974 Shestakov became a professor of the Novosibirk State University. Since 1999 he has held the position of full professor at the University of São Paulo.

Shestakov's interests lie in ring theory and combinatorial algebra. He has focused on the structure and representations of nonassociative algebras and superalgebras, PI-algebras, free algebras and their automorphisms.

Ivan Shestakov

Ualbai Umirbaev

Response: Ivan Shestakov

It is a great honor for me to receive the E. H. Moore Research Article Prize, and I would like to thank the AMS and the selection committee for awarding this prize. I am especially happy to share it with my former student Ualbai Umirbaev. During my mathematical career, I experienced help and support from my friends and colleagues in different countries. I would also like to use this opportunity to thank all of them, especially my colleagues from the Sobolev Institute of Mathematics, where I grew up as a mathematician, and from the University of São Paulo, where I have been working during the last several years.

Biographical Sketch: Ualbai Umirbaev

Ualbai Umirbaev was born in Turtkul, Shymkent region, Kazakhstan, in 1960. He studied mathematics at Novosibirsk State University. He got his Ph.D. in mathematics from the Sobolev Institute of Mathematics of the Siberian branch of the Soviet Academy of Sciences with Ivan Shestakov in 1986, and from the same Institute he got his Doctor of Science degree in 1995.

During 1986–1995 Umirbaev taught at the Kazakh State University, Almaty, first as an assistant professor, then as a senior lecturer and then as an associate professor. In 1995 he moved to the South-Kazakhstan State University in Shymkent as a full professor and Chair of Informatics. In 2001 Umirbaev moved to the Eurasian National University in the new capital Astana, where he became a professor and Chair of Algebra and Geometry. His main research interests are in the areas of combinatorial algebra, subalgebras and automorphisms of free algebras, and affine algebraic geometry.

Response: Ualbai Umirbaev

I am deeply honored to have been chosen to receive the 2007 E. H. Moore Research Article Prize together with Ivan Shestakov.

It is very interesting to recall that I met the Nagata automorphism for the first time in a survey by Vladimir Popov in 1989. It was really an amazing and a concrete problem! Later I studied two very interesting papers related to the Nagata automorphism: by Hyman Bass in 1984 on nontriangular actions and by Martha Smith in 1989 on stably tame automorphisms. I was studying subalgebras of free algebras with a view towards to algorithmic problems. Since then I related these investigations with the study of automorphisms of free algebras.

I am very glad that the two cited papers were published in the *Journal of the American Mathematical Society*. I am very glad that the committee recognized the significance of these results. Many challenging problems of affine algebraic geometry and combinatorial algebra are still open. I hope that the recognition by the Moore Prize will spur further activity in this area.

I would like to thank my friends, colleagues, and collaborators with whom discussions of mathematics were very important and useful. Also I would like to thank my father Utmakhanbet Umirbaev (1922–2001), who was a teacher of mathematics, thanks to whom I started to study math.

2007 Robbins Prize

The David P. Robbins Prize prize was awarded at the 113th Annual Meeting of the AMS in New Orleans in January 2007.

The Robbins Prize was established in 2005 in memory of David P. Robbins by members of his family. Robbins, who died in 2003, received his Ph.D. in 1970 from the Massachusetts Institute of Technology. He was a long-time member of the Institute for Defense Analysis Center for Communications Research and a prolific mathematician whose work (much of it classified) was in discrete mathematics. The prize is given for a paper that (1) reports on novel research in algebra, combinatorics, or discrete mathematics, (2) has a significant experimental component, (3) is on a topic broadly accessible, and (4) provides a simple statement of the problem and clear exposition of the work. The US$5,000 prize is awarded every three years. This is the first time the prize was awarded.

The Robbins Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2007 prize, the members of the selection committee were: Jonathan M. Borwein, Jeffrey C. Lagarias (chair), David I. Lieberman, Richard P. Stanley, and Robin Thomas.

The 2007 Robbins Prize was awarded to SAMUEL P. FERGUSON and THOMAS C. HALES. The text that follows presents the selection committee's citation, brief biographical sketches, and the awardees' responses upon receiving the prize.

Citation

This Robbins Prize is presented to Thomas C. Hales and Samuel P. Ferguson for the paper: Thomas C. Hales, "A proof of the Kepler conjecture", *Ann. Math.* **162** (2005), 1065–1185. Section 5 of this paper is jointly authored with Samuel P. Ferguson.

The Kepler conjecture asserts that the densest three-dimensional sphere packing is attained by the cannonball packing. This 400-year-old problem, going back to Kepler in 1611, was mentioned as part of Hilbert's eighteenth problem. The proof of this result is a landmark achievement.

These two authors used experimental methods to formulate a local density inequality that would both establish the result and be provable by a computation of feasible length. Laszlo Fejes Tóth suggested in the 1950s that it might be possible to prove Kepler's conjecture by establishing a local inequality that would simultaneously be maximized at every sphere center in the cannonball packing. Local inequalities at a single sphere center can in principle be proved by maximizing a nonlinear function over a compact set, but in practice the resulting problems are too large to be computationally feasible. One of the contributions of this work was to find a way to obtain a computationally feasible problem. In the early 1990s Hales began an approach to formulating suitable local density inequalities that combined information from both the Voronoi and Delaunay triangulations associated to the sphere centers. A very delicate balance is needed between their contributions to obtain a suitable inequality, which was arrived at by computer experiments. Although Samuel Ferguson is credited only with one section of the cited paper, he made essential contributions on the theoretical and experimental side, in formulating the local density inequality used and in proving the most difficult special case of it.

The cited paper elegantly describes the main theoretical structure of the proof. It formulates a novel and complicated local density inequality and shows that its proof would establish Kepler's conjecture. The proof of the local inequality reduces to a very large nonlinear optimization problem

of minimizing a function over a compact region consisting of many connected components of high dimension. The authors introduce decomposition methods that simplify the optimization. The optimization is checked analytically in neighborhoods of the two global minima, and after many reductions the remainder is checked by computer; there are thousands of cases. The cited paper presents an extensive road map of this proof and includes motivation for the truth of the given local inequality. A more detailed version appears in six papers published in *Discrete and Computational Geometry* **36** (2006), 5–265, four authored by T. C. Hales, one by S. P. Ferguson alone, and one joint paper which formulates the precise local inequality.

Some controversy has surrounded this proof, with its large computer component, concerning its reliable checkability by humans. Addressing this issue, Hales has an ongoing project, called the "Flyspeck" project, whose object is to construct a "second-generation" proof which is entirely checkable by computer in a formal logic system.

Biographical Sketch: Thomas C. Hales

Thomas C. Hales received his master's degree from Stanford University in the School of Engineering and his Ph.D. from Princeton University in mathematics in 1986 under Robert Langlands. He has held positions at Harvard University, the University of Chicago, and the University of Michigan. He is currently the Andrew Mellon Professor of Mathematics at the University of Pittsburgh. His honors include the Chauvenet Prize (2003) of the Mathematical Association of America and the R. E. Moore Prize (2004) for applications of interval analysis. His research interests include representation theory, motivic integration, discrete geometry, and formal proof theory.

Response: Thomas C. Hales

It is an honor to be a recipient of the David P. Robbins Prize. Without the fundamental contributions of my collaborator, Samuel P. Ferguson, the Kepler conjecture would still be unsolved. He made essential contributions to the formulation of the local density inequality and to the computer algorithms that were used. He solved the most difficult case that arises in the proof. I am proud to share the prize with him.

The solution to the Kepler conjecture relies on fundamental advances by many researchers in several domains. It is a pleasure to acknowledge the many researchers who developed algorithms that permit the rapid solution of largescale linear programs, those who developed the tools of interval computations, and L. Fejes Tóth, who had the original vision about how the Kepler conjecture might be solved by computer. Finally, I wish to thank my colleagues in the formal theorem-proving community for elevating computer proofs to unprecedented levels of mathematical rigor.

Thomas C. Hales

Samuel Ferguson

Biographical Sketch: Samuel Ferguson

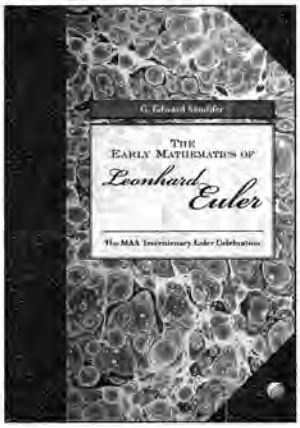
Samuel Ferguson earned a B.S. in mathematics at Brigham Young University in 1991. A Research Experience for Undergraduates program at the College of William and Mary provided support for his interest in pursuing graduate studies in mathematics. He earned his Ph.D. in 1997 at the University of Michigan, working with Tom Hales. He is currently employed by the National Security Agency.

Response: Samuel Ferguson

I am honored to have been selected for this award. Having met David Robbins and being familiar with some of his remarkable work makes this award all the more meaningful. I wish to express my gratitude to everyone who has helped me along the way, from my parents and siblings, to teachers, mentors, and friends. Thank you.

The MAA is Celebrating the 300th Anniversary of Euler's Birth

C. Edward Sandifer



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The Early Mathematics of *Leonhard Euler*

Volume 1—The MAA Tercentenary Euler Celebration

The Early Mathematics of Leonhard Euler gives an article-by-article description of Leonhard Euler's early mathematical works, the 50 or so mathematical articles he wrote before he left St. Petersburg in 1741 to join the Academy of Frederick the Great in Berlin. These early pieces contain some of Euler's greatest work, the Königsberg bridge problem, his solution to the Basel problem, and his first proof of the Euler-Fermat theorem. It also presents important results that we seldom realize are due to Euler; that mixed partial derivatives are (usually) equal, our $f(x)$ notation, and the integrating factor in differential equations.

The book shows how contributions in diverse fields are related, how number theory relates to series, which, in turn, relate to elliptic integrals and then to differential equations. There are dozens of such strands in this beautiful web of mathematics. At the same time, we see Euler grow in power and sophistication, from a young student when at 18 he published his first work on differential equations (a paper with a serious flaw) to the most celebrated mathematician and scientist of his time.

It is a portrait of the world's most exciting mathematics between 1725 and 1741, rich in technical detail, woven with connections within Euler's work and with the work of other mathematicians in other times and places, laced with historical context.

The Genius of Euler

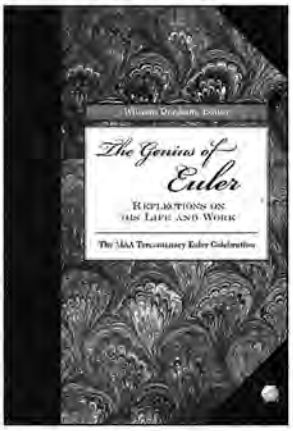
Reflections on his Life and Work

Volume 2—The MAA Tercentenary Euler Celebration

This book celebrates the 300th birthday of Leonhard Euler (1707–1783), one of the brightest stars in the mathematical firmament. The book stands as a testimonial to a mathematician of unsurpassed insight, industry, and ingenuity—one who has been rightly called "the master of us all." The collected articles, aimed at a mathematically literate audience, address aspects of Euler's life and work, from the biographical to the historical to the mathematical. The oldest of these was written in 1872, and the most recent dates to 2006.

Some of the papers focus on Euler and his world, others describe a specific Eulerian achievement, and still others survey a branch of mathematics to which Euler contributed significantly. Along the way, the reader will encounter the Königsberg bridges, the 36 officers, Euler's constant, and the zeta function. There are papers on Euler's number theory, his calculus of variations, and his polyhedral formula. Of special note are the number and quality of authors represented here. Among the 34 contributors are some of the most illustrious mathematicians and mathematics historians of the past century—including Florian Cajori, Carl Boyer, George Pólya, André Weil, and Paul Erdős. And there are a few poems and a mnemonic just for fun.

William Dunham, Editor



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Just published

Rüdiger Göbel / Jan Trlifaj

■ Approximations and Endomorphism Algebras of Modules

2006. xxiv, 640 pages. Cloth.
€ [D] 128.00 / sFr 205.00 / *US$ 148.00
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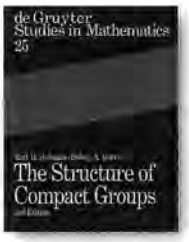
(de Gruyter Expositions in Mathematics 41)

This monograph provides a thorough treatment of module theory, a subfield of algebra. The authors develop an approximation theory as well as realization theorems and present some of its recent applications, notably to infinite-dimensional combinatorics and model theory. The book is devoted to graduate students interested in algebra as well as to experts in module theory.



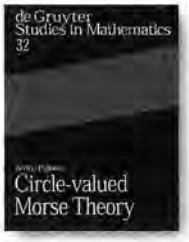
Karl H. Hofmann / Sidney A. Morris

■ The Structure of Compact Groups

A Primer for Students - A Handbook for the Expert

2nd rev. and augmented ed. 2006.
xvii, 858 pages. Cloth.
€ [D] 128.00 / sFr 205.00 / *US$ 148.00
ISBN 978-3-11-019006-9
(de Gruyter Studies in Mathematics 25)

Dealing with the subject matter of compact groups that is frequently cited in fields like algebra, topology, functional analysis, and theoretical physics, this book has been conceived with the dual purpose of providing a text book for upper level graduate courses or seminars, and of serving as a source book for research specialists who need to apply the structure and representation theory of compact groups. For the present new edition the text has been improved in various sections. New material has been added in order to reflect ongoing research.

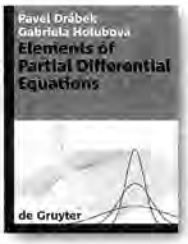


Andrei Pajitnov

■ Circle-valued Morse Theory

2006. ix, 454 pages. Cloth.
€ [D] 98.00 / sFr 157.00 / *US$ 128.00
ISBN 978-3-11-015807-6
(de Gruyter Studies in Mathematics 32)

In 1927 M. Morse discovered that the number of critical points of a smooth function on a manifold is closely related to the topology of the manifold. This became a starting point of the Morse theory which is now one of the basic parts of differential topology. It is a large and actively developing domain of differential topology, with applications and connections to many geometrical problems. The aim of the present book is to give a systematic treatment of the geometric foundations of a subfield of that topic, the circle-valued Morse functions, a subfield of Morse theory.

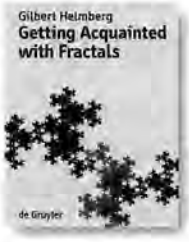


Pavel Drábek / Gabriela Holubová

■ Elements of Partial Differential Equations

2007. Approx. 290 pages. Paperback.
€ [D] 34.95 / sFr 56.00 / *US$ 45.00
ISBN 978-3-11-019124-0
(de Gruyter Textbook)

This textbook presents a first introduction to PDEs on an elementary level, enabling the reader to understand what partial differential equations are, where they come from and how they can be solved. The intention is that the reader understands the basic principles which are valid for particular types of PDEs, and to acquire some classical methods to solve them, thus the authors restrict their considerations to fundamental types of equations and basic methods. Only basic facts from calculus and linear ordinary differential equations of first and second order are needed as a prerequisite.



Gilbert Helmberg

■ Getting Acquainted with Fractals

2007. Approx. 192 pages. Cloth.
€ [D] 78.00 / sFr 125.00 / *US$ 98.00
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This well-written book provides a mathematically oriented introduction to fractals, with a focus upon three types of fractals: fractals of curves, attractors for iterative function systems in the plane, and Julia sets. The presentation is on an undergraduate level, with an ample presentation of the corresponding mathematical background, e.g., linear algebra, calculus, algebra, geometry, topology, measure theory and complex analysis. The book contains over 100 color illustrations.

■ Combinatorial Number Theory

Proceedings of the ‚Integers Conference 2005' in Celebration of the 70th Birthday of Ronald Graham, Carrollton, Georgia, October 27-30, 2005

Ed. by Bruce Landman, Melvyn Nathanson, Jaroslav Nesetril, Richard Nowakowski, and Carl Pomerance

2007. Approx. ix, 489 pages. Cloth.
€ [D] 168.00 / sFr 269.00 / *US$ 186.00
ISBN 978-3-11-019029-8 (de Gruyter Proceedings in Mathematics)

This carefully edited volume contains selected refereed papers based on lectures presented by many distinguished speakers at the „Integers Conference 2005", an international conference in combinatorial number theory. The conference was held in celebration of the 70th birthday of Ronald Graham, a leader in several fields of mathematics.

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Mathematics People

Feldman Awarded CRM-Fields-PIMS Prize

JOEL S. FELDMAN of the University of British Columbia has been awarded the 2007 CRM-Fields-PIMS Prize. The prize, awarded annually by the Centre de Recherches Mathématiques (CRM), the Fields Institute, and the Pacific Institute for the Mathematical Sciences (PIMS), recognizes exceptional contributions by a mathematician working in Canada. The prize carries a cash award of CA$10,000 (approximately US$8,500) and an invitation to give a lecture at each institute.

Feldman was chosen "in recognition of his exceptional achievement and work in mathematical physics." According to the prize citation, he "has risen to a position of international prominence in the world of mathematical physics, with a thirty-year record of sustained output of the highest caliber. He has made important contributions to quantum field theory, many-body theory, Schrödinger operator theory, and the theory of infinite genus Riemann surfaces. Many of Professor Feldman's recent results on quantum many-body systems at positive densities and on Fermi liquids and superconductivity have been classed as some of the best research in mathematical physics in the last decade."

Feldman received his bachelor's degree from the University of Toronto in 1970 and his master's (1971) and Ph.D. (1974) degrees from Harvard University. He was a research fellow at Harvard University from 1974 to 1975 and C. L. E. Moore Instructor at the Massachusetts Institute of Technology (MIT) from 1975 to 1977. He has been teaching at the University of British Columbia since 1977. He was an invited speaker at the International Congress of Mathematicians in Kyoto in 1990. He was a plenary speaker at the International Congress on Mathematical Physics in Brisbane in 1997 and an invited speaker at the International Congress on Mathematical Physics in Lisbon in 2003. He is a fellow of the Royal Society of Canada (RSC) and was awarded the 1996 John L. Synge Award of the RSC, the Aisenstadt Chair Lectureship of the CRM (1999–2000), and the 2004 Jeffery-Williams Prize of the Canadian Mathematical Society (CMS) for outstanding contributions to mathematical research.

The CRM and the Fields Institute established the CRM-Fields prize in 1994 to recognize exceptional research in the mathematical sciences. In 2005 PIMS became an equal partner, and the name was changed to the CRM-Fields-PIMS prize. Previous recipients of the prize are H. S. M. (Donald) Coxeter, George A. Elliott, James Arthur, Robert V. Moody, Stephen A. Cook, Israel Michael Sigal, William T. Tutte, John B. Friedlander, John McKay, Edwin Perkins, Donald A. Dawson, David Boyd, and Nicole Tomczak-Jaegermann.

—From a Fields Institute announcement

Smith and Holroyd Awarded Aisenstadt Prize

GREGORY D. SMITH of Queen's University and ALEXANDER E. HOLROYD of the University of British Columbia are the recipients of the 2007 André Aisenstadt Prize of the Centre de Recherches Mathématiques (CRM) of the University of Montreal. Smith was honored for his work in algebraic geometry and computational algebra, and Holroyd was chosen for his work in probability theory, with emphasis on discrete spatial models, including cellular automata, percolation, matching, and coupling.

The André Aisenstadt Mathematics Prize consists of CA$3,000 (approximately US$2,500) and a medal. The prize recognizes talented young Canadian mathematicians

in pure and applied mathematics who have held a Ph.D. for no longer than seven years.

—From a CRM announcement

Viale Awarded ASL Sacks Prize

MATTEO VIALE of the University of Torino and the University of Paris 7 is the recipient of the 2006 Sacks Prize of the Association for Symbolic Logic (ASL). The prize is awarded for the most outstanding doctoral dissertation in mathematical logic. Viale received his Ph.D. in 2006 from the University of Torino and the University of Paris 7. According to the prize citation, his thesis "makes fundamental contributions to our understanding of the consequences of forcing axioms in the combinatorics of singular cardinals. In particular, it solves a well-known problem, by showing that the proper forcing axiom implies the singular cardinals hypothesis."

The Sacks Prize was established in honor of Gerald Sacks for his unique contribution to mathematical logic. It consists of a cash award and five years' free membership in the ASL.

—From an ASL announcement

Mustata Receives Packard Fellowship

The David and Lucile Packard Foundation has awarded twenty Fellowships for Science and Engineering for the year 2006. MIRCEA MUSTATA, a mathematician at the University of Michigan, has received an unrestricted research grant of US$625,000 over five consecutive years. He will pursue research in algebraic geometry, particularly on singularities of algebraic varieties.

The fellowships are awarded to researchers in mathematics, natural sciences, computer science, and engineering who are in the first three years of a faculty appointment.

—From a Packard Foundation announcement

AWM Essay Contest Winners Announced

The Association for Women in Mathematics (AWM) has announced the winners of its 2006 essay contest, "Biographies of Contemporary Women in Mathematics". The grand prizes were awarded to ANNIE DAVIS of the Solomon Schechter Day School of Greater Boston, Newton, Massachusetts, for her essay, "Margo Levine, Mathematician"; and to STEPHANIE HIGGINS of Bates College for her essay, "Dr. Bonnie Shulman: A Different Kind of Story". Davis's essay won first place in the middle school (grades 6–8) category, and Higgins's essay won first place in the college category. As grand prize winners, these essays will be published in the *AWM Newsletter.* The first-place winner in the grade 9–12 category was MARGARITE BECHIS of Mount Saint Joseph Academy, Flourtown, Pennsylvania, for an essay titled "Splendor of the Heavens: Dr. Knapp's Astronomical Odyssey". A complete list of the winners and copies of their essays can be found on the AWM website, `http://www.awm-math.org/biographies/contest/2006.html`.

—From an AWM announcement

Correction

The February 2007 issue of the *Notices* carried a list of doctoral degrees conferred in 2005–2006. Because of incorrect information supplied by his institution, David S. Torain II was listed as having received a doctorate in mathematics. His degree is in systems science, with a specialization in mathematics.

—Allyn Jackson

About the Cover *(continued from page 502)*

whereas nonexperts use only short-term memory. Episodic memory is used to store memory of our own life's events, and is one of several types of long-term memory recognized by neuroscientists. As Pesenti writes in a survey article in the *Handbook of Mathematical Cognition*, the work supports the suggestion that "high-level expertise is not only accounted for by an acceleration of existing processes and by local modulation of activations, but" ..."also involves new processes involving new brain areas." Of course mental arithmetic is not the same as higher mathematics, but sometime in the not too distant future it should be possible to analyze what is involved in discovering and proving theorems!

The important role of long-term memory should make us wary of referring disparagingly to "mere memorization". But neither should it make us pessimistic about the value of attempting to teach our students how to reason. In a brief note by Brian Butterworth about this work (in the January 2001, issue of *nature neuroscience* at `http://neuroscience.nature.com`), Gamm is quoted as saying that at school he was "very bad at arithmetic" because the teachers never explained the concepts in a way he could understand. It was only later in life that he worked these things out for himself.

Pesenti's home page is at `http://www.nesc.ucl.ac.be/mp/pesentiHomepage.htm`.

—Bill Casselman, Graphics Editor
(`notices-covers@ams.org`)

Mathematics Opportunities

NSF Integrative Graduate Education and Research Training

The Integrative Graduate Education and Research Training (IGERT) program was initiated by the National Science Foundation (NSF) to meet the challenges of educating Ph.D. scientists and engineers with the interdisciplinary backgrounds and the technical, professional, and personal skills needed for the career demands of the future. The program is intended to catalyze a cultural change in graduate education for students, faculty, and universities by establishing innovative models for graduate education in a fertile environment for collaborative research that transcends traditional disciplinary boundaries. It is also intended to facilitate greater diversity in student participation and to contribute to the development of a diverse, globally aware science and engineering workforce. Supported projects must be based on a multidisciplinary research theme and administered by a diverse group of investigators from U.S. Ph.D.-granting institutions with appropriate research and teaching interests and expertise.

The preliminary proposal deadline for the 2007 IGERT competition is **April 5, 2007.** The deadline for full proposals is **October 5, 2007.** Further information may be found at the website `http://www.nsf.gov/pubs/2007/nsf07540/nsf07540.htm`.

—From an NSF announcement

Interdisciplinary Training for Undergraduates in Biological and Mathematical Sciences

The National Science Foundation's (NSF) Division of Mathematical and Physical Sciences (MPS) and directorates for Education and Human Resources (EHR) and Biological Sciences (BIO) invite proposals for the Undergraduate Biology and Mathematics (UBM) project. This project aims to enhance interdisciplinary training for undergraduates in biological and mathematical sciences and to better prepare undergraduate biology or mathematics students to pursue graduate study and careers in fields that integrate the mathematical and biological sciences.

The project provides long-term research experiences for interdisciplinary balanced teams of at least two undergraduates. Projects should focus on research that integrates the mathematical and biological sciences and that provides students with exposure to contemporary mathematics and biology, addressed with modern research tools and methods. Projects must involve students from both areas in collaborative research experiences and include joint mentorship by faculty in both fields.

The deadline for full proposals is **April 4, 2007.** For more information, see the website `http://nsf.gov/pubs/2007/nsf07539/nsf07539.txt`.

—From an NSF announcement

AP Calculus Readers Sought

The Educational Testing Service and the College Board invite interested college faculty to apply to be Readers for the Advanced Placement Calculus Exam. In June, AP high school and college faculty members from around the world gather in the United States for the annual AP Reading. There they evaluate and score the free-response sections of the AP Exams. AP Exam Readers are led by a Chief Reader, a college professor who has the responsibility of ensuring that students receive grades that accurately reflect college-level achievement. Readers find the experience an intensive collegial exchange in which they can receive professional support and training.

To learn more about this opportunity or to apply for a position as a Reader, see the website `http://apcentral.collegeboard.com/apc/public/teachers/opportunities/4137.html`; email: `apreader@ets.org`; telephone: 609-406-5384.

—Caren L. Diefenderfer, Hollins University

Center for Women in Mathematics at Smith College

The Center for Women in Mathematics is a place for women to get intensive training in mathematics at the advanced undergraduate level. Participants take courses, work on research projects, and enjoy a rich mathematical environment in the supportive, dynamic company of other women serious about mathematics. The center features two programs for visiting students.

The Junior Year for Women at Smith College is for undergraduate women mathematics majors. Financial aid is available to U.S. citizens and permanent residents. Applicants should be majoring, or intending to major, in mathematics.

The Post-Baccalaureate Program is for women with bachelor's degrees who did not major in mathematics or whose mathematics major did not sufficiently prepare them for graduate school. The program is designed to improve a student's preparation and motivation to help them determine if they want to continue to graduate school in the mathematical sciences. This program is open to all women who have graduated college with some course work in mathematics above the level of calculus and an interest in pursuing it further. Full tuition and a living stipend is available to U.S. citizens and permanent residents who are admitted to the program.

Applications are reviewed on a rolling basis. The preferred deadline for January entrance is **October 15**, but applications are accepted through December 15. For September entrance, the preferred deadline is **March 15**, but applications are accepted through July 1. Students applying for financial aid are encouraged to apply by the preferred deadlines, as funds are limited.

For more information or to request application materials, please visit the website `http://www.math.smith.edu/center`, or contact: Ruth Haas, Chair, Department of Mathematics and Statistics, Smith College, Northampton, MA 01063; email `mathchair@email.smith.edu`; telephone 413-585-3872. The center is supported by the National Science Foundation and Smith College.

—Center for Women in Mathematics announcement

MSRI-UP

The Mathematical Sciences Research Institute Undergraduate Program (MSRI-UP) is a comprehensive program for undergraduates that aims at increasing the number of students from underrepresented groups in mathematics graduate programs. MSRI-UP includes summer research opportunities, mentoring, workshops on the graduate school application process, and follow-up support.

For further information, visit the webpage at `http://www.msri.org/up/description`. Review of applications will begin on **March 2, 2007**.

—From an MSRI announcement

News from BIRS

In the past two years, the Banff International Research Station (BIRS) in Banff, Alberta, Canada, has received substantial continued funding for its operations. In 2005, BIRS received an award of 3.3 million Canadian dollars (about US$2.8 million) from the Alberta Ministry of Innovation and Science, as well as a five-year grant of US$2.6 million from the U.S. National Science Foundation. In 2006 BIRS also received a five-year grant of 2.9 million Canadian dollars from the National Science and Engineering Research Council of Canada. BIRS also has a pledge of support from the National Council for Science and Technology (CONACYT) of Mexico.

BIRS is a joint Canada-U.S.-Mexico initiative to provide an environment for creative interaction and the exchange of ideas, knowledge, and methods within the mathematical sciences and with related sciences and industry. In 2007 BIRS is running a 48-week program of activities, including five-day workshops, two-day workshops, focused research group activities, research in teams, and summer schools. For further information, including information on submitting a proposal to organize an activity at BIRS, visit the website `http://www.pims.math.ca/birs`.

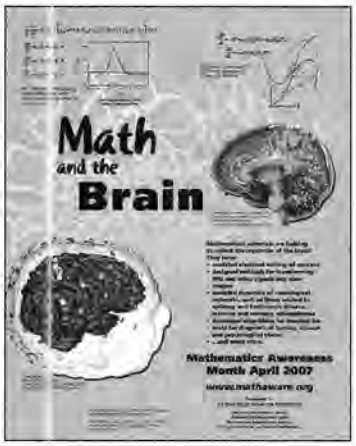
—From BIRS announcements

For Your Information

Mathematics Awareness Month 2007

The AMS, the American Statistical Association, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics announce that the theme for Mathematics Awareness Month 2007 is *Mathematics and the Brain.*

One of the most exciting challenges in modern science is to fully understand the human brain and its mechanisms. Mathematics plays a vital role in this research to understand the mechanisms and function of the human brain from its smallest components to the whole brain.



Mathematical models continue to play a central role in understanding brain cells, their interaction, and their function. The 1963 Nobel Prize was awarded to Alan Lloyd Hodgkin and Andrew Huxley for a model that uses differential equations to approximate the electrical characteristics of excitable cells. Their original model described the ionic mechanisms underlying the initiation and propagation of action potentials in the squid giant axon and led to many later developments to model brain activity at the single neuron level.

Modeling and computational simulation have complemented laboratory experiments to understand how the brain functions at many levels from single cells and clusters to large networks of interacting cells.

Theoretical models and computational methods, along with experiments guided by these models, are used to unravel the circuitry of the cerebral cortex, the rules by which this circuitry develops and self-organizes, and its computational functions.

Research in the dynamics of networks is helping to understand, at a cellular and network level, how widespread synchronous patterns arise in large nonhomogeneous networks such as the brain. Such widespread synchronization of rhythmic activity among networks of neurons that normally function to produce distinct behavior may help explain disorders such as generalized epilepsy and Parkinson's disease, for example.

Imaging provides a noninvasive method for gathering information on brain activity and form and provides information used to infer function. Imaging methods depend heavily on mathematics and computational tools. While each imaging modality uses different mathematical methods, each must combine and process raw sensor data to form coherent and meaningful images. Many new imaging methods are used to study properties of the brain. For example, diffusion tensor MRI (magnetic resonance imaging) is used to analyze the structure of white matter in the brain. Imaging methods like diffusion MRI offer means to noninvasively study the microstructure of biological tissues, enabling better understanding of the anatomical relationships between various functional areas of the brain. This allows scientists to map and quantify the connectivity between different brain regions.

The imaging itself also raises mathematical and computational challenges. A typical fMRI (functional MRI) experiment may collect as much as or more than one gigabyte per hour, calling for new statistical analysis techniques to analyze large volumes of data. The complexity and sheer volume of the data require new mathematical tools for processing and analyzing the information, while the data points to the need for new models.

Mathematics plays a central role in modeling, analyzing, and understanding the underlying structure and function of various regions of the brain and the interaction among these regions. Further research in the underlying mathematics of dynamical systems and networks, statistical methods, and mathematical tools for enhancing imaging will continue to help advance our understanding of our closest of all frontiers.

For more information, visit the website `http://www.mathaware.org`.

—Mathematics Awareness Month announcement

Inside the AMS

Joint Meetings Donations Benefit New Orleans

During the Joint Mathematics Meetings in New Orleans in January 2007, the AMS and the Mathematical Association of America (MAA) sponsored a raffle and T-shirt sale to benefit survivors of Hurricane Katrina. The hurricane struck the southern United States in the fall of 2005, causing a state of emergency and extensive damage, especially to the New Orleans area, which is still struggling to recover from the disaster.

The raffle, T-shirt sales, and other donations brought in US$10,000 for the Second Harvest of New Orleans Food Bank BackPack Program. This program provides to children at risk of hunger a backpack of nutritious, child-friendly food for weekends and out-of-school times.

Winners of the raffle took home a range of prizes: Mackichan Scientific Workplace software, a Dell notebook computer, a four-night stay at the Marriott in San Diego for the 2008 Joint Meetings, an Apple iPod, AMS and MAA bookstore credits, a New England Legal Seafoods lobster bake, a Texas Instruments graphing calculator, and a Bose tri-port acoustic headphone.

The raffle winners are: Michael Barry, Allegheny College; M. M. Bogacz, St. Xavier University; Karen Bolinger, Clarion University; Deanna Caveny, College of Charleston; Carolyn Cuff, Westminster College; Michael Falk, Northern Arizona University; Fred J. Hickernell, Illinois Institute of Technology; Stephen Hilbert, Ithaca College; Barbara Jur, Macomb Community College; Catherine Murphy, Purdue University Calumet; Emily Moore, Grinnell College; Richard Neidinger, Davidson College; Andrew Shallue, University of Wisconsin; Max Tran, Kingsborough Community College; and Kimberly Vincent, Washington State University.

Many other Joint Meetings attendees made donations to help New Orleans. One especially noteworthy example is Jennifer Quinn, executive director of the Association for Women in Mathematics, who received the MAA's Haimo Award for Distinguished College or University Teaching of Mathematics. The award came with a cash prize of US$1,000, which Quinn donated to the New Orleans Area Habitat for Humanity.

—Allyn Jackson

2007 AMS Department Chairs' Workshop

The AMS hosted a one-day workshop for mathematical sciences department chairs at the 2007 Joint Mathematics Meetings in New Orleans, Louisiana. This year's workshop was designed around the theme of "growing a mathematics department". Growing a department, not just in terms of size, can affect a variety of departmental areas including students and programs, faculty and staff, relationships within and/or outside the institution, departmental leadership, and the culture or the environment of the department.

Based on input from those attending, the workshop focused on four areas: personnel, educational programs, research and teaching, and outside relationships and external perceptions. Over forty-five department chairs and leaders participated in the program sharing ideas and experiences. Workshop leaders included: Krishnaswami Alladi, department chair of mathematics, University of Florida; Deanna Caveny, department chair of mathematics, College of Charleston; Guillermo Ferreyra, dean of the College of Arts and Sciences, Louisiana State University; and David Manderscheid, department chair of mathematics, University of Iowa.

The Department Chairs Workshop is an annual event hosted by the AMS prior to the start of the Joint Meetings. Past workshop sessions have focused on a range of issues facing departments today, including personnel issues (staff and faculty), long range planning, hiring, promotion and tenure, budget management, assessments, outreach, stewardship, junior faculty development, communication, and departmental leadership.

Those interested in attending a future workshop should look for registration information sent out in advance of the Joint Meetings or contact the AMS Washington Office at amsdc@ams.org.

—Anita Benjamin, AMS Washington Office

Deaths of AMS Members

ARIENNE S. BALSER, of Bayside, WI, died on April 21, 2006. She was a member of the Society for 55 years.

ROBERT C. F. BARTELS, professor emeritus, University of Michigan, Ann Arbor, died on September 9, 2006. Born on October 24, 1911, he was a member of the Society for 69 years.

EDUARD BELINSKY, professor, University of the West Indies, Barbados, died on October 7, 2004. Born on September 26, 1947, he was a member of the Society for 8 years.

RONALD C. BIGGERS, professor, Kennesaw State University, died on April 23, 2005. Born on October 6, 1945, he was a member of the Society for 28 years.

PETER BOD, professor, Hungarian Academy of Sciences, died on December 29, 2005. Born on June 9, 1924, he was a member of the Society for 26 years.

ALEXANDER I. BUDKIN, professor, Altai State University, Russia, died on August 6, 2006. He was a member of the Society for 12 years.

BOHUMIL CENKL, professor, Northeastern University, died on May 4, 2006. He was a member of the Society for 35 years.

ANDRE DABROWSKI, assistant professor, University of Ottawa, died on October 7, 2006. Born on August 10, 1955, he was a member of the Society for 29 years.

JERRY DONATO, from Timberville, VA, died on October 18, 2006. He was a member of the Society for 16 years.

PHILIP DWINGER, dean emeritus and professor emeritus, University of Illinois at Chicago, died on November 2, 2006. Born on September 25, 1914, he was a member of the Society for 53 years.

LEON A. HENKIN, professor emeritus, University of California, Berkeley, died on November 1, 2006. Born on April 19, 1921, he was a member of the Society for 62 years.

M. GWENETH HUMPHREYS, professor emerita, Randolph-Macon Woman's College, died on October 5, 2006. Born on October 22, 1911, she was a member of the Society for 70 years.

SYED A. HUQ, from Castle Hill, Australia, died on June 30, 2002. Born on January 25, 1938, he was a member of the Society for 31 years.

EDWARD L. KAPLAN, from Corvallis, OR, died on September 26, 2006. Born on May 11, 1920, he was a member of the Society for 59 years.

DONALD L. KREIDER, of Sugar Hill, NH, died on December 7, 2006. He was a member of the Society for 50 years.

MARTIN D. KRUSKAL, professor, Rutgers University, New Brunswick, died on December 26, 2006. He was a member of the Society for 55 years.

ROY B. LEIPNIK, professor, University of California, Santa Barbara, died on October 10, 2006. He was a member of the Society for 59 years.

XIAO-SONG LIN, professor, University of California, Riverside, died on January 13, 2007. Born on July 27, 1957, he was a member of the Society for 22 years.

KJARTAN G. MAGNUSSON, professor, University of Iceland, died on January 13, 2006. Born on March 20, 1952, he was a member of the Society for 16 years.

DANIEL MARTIN, from Lanarkshire, Scotland, died on September 15, 2006. Born on April 16, 1915, he was a member of the Society for 44 years.

JANET MCDONALD, professor, Vassar College, died on October 29, 2006. Born in September 1905, she was a member of the Society for 63 years.

MORRIS NEWMAN, professor, University of California, Santa Barbara, died on January 4, 2007. Born on February 25, 1924, he was a member of the Society for 61 years.

MORIO OBATA, professor, Keio University, Japan, died on December 21, 2006. He was a member of the Society for 47 years.

WILLIAM PARRY, professor emeritus, University of Warwick, England, died on August 20, 2006. Born on July 3, 1934, he was a member of the Society for 43 years.

EVERETT PITCHER, professor, Lehigh University, died on December 4, 2006. Born in 1912, he was a member of the Society for 71 years.

G. BALEY PRICE, professor emeritus, University of Kansas, died on November 7, 2006. Born on March 14, 1905, he was a member of the Society for 77 years.

VICTOR A. PLOTNIKOV, professor, Odessa State University, Ukraine, died on September 4, 2006. Born on January 5, 1938, he was a member of the Society for one year.

ANATOL RAPOPORT, from Toronto, Canada, died on January 20, 2007. Born on May 22, 1911, he was a member of the Society for 60 years.

DANIEL A. ROBINSON, professor, Georgia Institute of Technology, died on January 31, 2007. Born on April 9, 1932, he was a member of the Society for 50 years.

ALEXANDER M. RUBINOV, professor, University of Ballarat, Australia, died on October 9, 2006. Born on March 28, 1940, he was a member of the Society for 9 years.

LEON W. RUTLAND JR., professor emeritus, Virginia Polytechnic Institute and State University, died on August 28, 2006. Born on August 24, 1919, he was a member of the Society for 47 years.

YURI TIHONOVICH SILCHENKO, professor, Voronezh State University, Russia, died on June 12, 2005. He was a member of the Society for 12 years.

SUMANGALI KIDAMBI SRINIVASAN, professor, Indian Institute of Technology, India, died on October 21, 2006. Born on December 16, 1930, he was a member of the Society for 6 years.

MATHUKUMALLI V. SUBBARAO, professor, University of Alberta, Canada, died on February 15, 2006. He was a member of the Society for 46 years.

GEORGE B. THOMAS JR., professor emeritus, MIT, died on October 31, 2006. Born on January 11, 1914, he was a member of the Society for 68 years.

ROBERT N. TOMPSON, professor, University of Nevada, died on January 6, 2007. Born on January 7, 1920, he was a member of the Society for 55 years.

STEVEN MARK TURADEK, from Minden, NV, died on August 4, 2006. Born on October 8, 1956, he was a member of the Society for 3 years.

BOB VAN ROOTSELAAR, retired, from Amsterdam, The Netherlands, died on October 10, 2006. He was a member of the Society for 45 years.

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are notices@math.ou.edu in the case of the editor and notices@ams.org in the case of the managing editor. The fax numbers are 405-325-7484 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

March 15, 2007: Preferred deadline for September entrance in junior-year program at the Smith College Center for Women in Mathematics. See "Mathematics Opportunities" in this issue.

March 31, 2007: Nominations for Third World Academy of Sciences Prizes. See http://www.twas.org/.

April 4, 2007: Full proposals for NSF Undergraduate Biology and Mathematics (UBM) project. See "Mathematics Opportunities" in this issue.

April 5, 2007: Preliminary proposals for NSF IGERT competition. See "Mathematics Opportunities" in this issue.

April 15, 2007: Applications for AMS "Math in Moscow" Scholarships for fall 2007. See http://www.mccme.ru/mathinmoscow or contact Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax +7095-291-65-01; email: mim@mccme.ru. For information and application forms for the AMS scholarships see http://www.ams.org/outreach/mimoscow.html or contact Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2294; email: student-serv@ams.org.

April 16, 2007: Applications for Project NExT: New Experiences in Teaching. See http://archives.math.utk.edu/projnext/.

May 1, 2007: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone 703-934-0163; email: awm@math.umd.edu; or contact

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

AMS Bylaws—*November 2005, p. 1239*

AMS Email Addresses—*February 2007, p. 271*

AMS Ethical Guidelines—*June/July 2006, p. 701*

AMS Officers 2005 and 2006 (Council, Executive Committee, Publications Committees, Board of Trustees)—*May 2006, p. 604*

AMS Officers and Committee Members—*October 2006, p. 1076*

Conference Board of the Mathematical Sciences—*September 2006, p. 911*

Information for *Notices* Authors—*June/July 2006, p. 696*

Mathematics Research Institutes Contact Information—*August 2006, p. 798*

National Science Board—*January 2007, p. 57*

New Journals for 2004—*June/July 2006, p. 697*

NRC Board on Mathematical Sciences and Their Applications—*March 2007, p. 426*

NRC Mathematical Sciences Education Board—*April 2007, p. 546*

NSF Mathematical and Physical Sciences Advisory Committee—*February 2007, p. 274*

Program Officers for Federal Funding Agencies—*October 2006, p. 1072 (DoD, DoE); December 2006 p. 1369 (NSF)*

Stipends for Study and Travel—*September 2006, p. 913*

Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

June 1, 2007: Applications for National Academies Christine Mirzayan Graduate Fellowships for the fall program. See http://www7.nationalacademies.org/policyfellows or contact The National Academies Christine Mirzayan Science and Technology Policy Graduate Fellowship Program, 500 5th Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667; email: policyfellows@nas.edu.

June 5, 2007: Proposals for Enhancing the Mathematical Sciences Workforce in the Twenty-First Century. See http://www.nsf.gov/publications/pub_summ.jsp?ods_key=nsf05595.

June 30, 2007: Nominations for 2007 Fermat Prize. See http://www.math.ups-tlse.fr/Fermat/.

October 1, 2007: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone 703-934-0163; email: awm@math.umd.edu; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

October 5, 2007: Full proposals for NSF IGERT competition. See "Mathematics Opportunities" in this issue.

October 15, 2007: Preferred deadline for January entrance in junior-year program at the Smith College Center for Women in Mathematics. See "Mathematics Opportunities" in this issue.

Mathematical Sciences Education Board, National Research Council

Jan de Lange, Freudenthal Institute, The Netherlands

Keisha M. Ferguson, Pattengill Elementary School, Ann Arbor, MI

Louis Gomez, Northwestern University

Javier Gonzalez, Pioneer High School, Whittier, CA

Sharon Griffin, Clark University

Phillip A. Griffiths (chair), Institute for Advanced Study

Arthur Jaffe, Harvard University

Jeremy Kilpatrick, University of Georgia

Julie Legler, St. Olaf College

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Marge Petit (vice chair), Consultant, Fayston, VT

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MSEB Staff

Martin Orland, Acting Director

Terry Holmer, Senior Program Assistant

The contact information is: Mathematical Sciences Education Board, National Academy of Sciences, 500 Fifth Street, NW, 11th Floor, Washington, DC 20001; telephone 202-334-2353; fax 202-344-1294; email: mseb@nas.edu; World Wide Web http://www7.nationalacademies.org/mseb/1MSEB_Membership.html.

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, *a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to* notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

An Abundance of Katherines, by John Green. Dutton Juvenile Books, September 2006. ISBN 0-525-47688-1.

Alan Turing's Automatic Computing Engine: The Master Codebreaker's Struggle to Build the Modern Computer, edited by B. Jack Copeland. Oxford University Press, June 2005. ISBN 0-198-56593-3.

Analysis and Probability: Wavelets, Signals, Fractals by Palle E. T. Jorgensen. Springer, September 2006. ISBN 0-387-29519-4.

Arthur Cayley: Mathematician Laureate of the Victorian Age, by Tony Crilly. Johns Hopkins University Press, December 2005. ISBN 0-801-88011-4.

The Artist and the Mathematician: The Story of Nicolas Bourbaki, the Genius Mathematician Who Never Existed, by Amir D. Aczel. Thunder's Mouth Press, August 2006. ISBN 1-560-25931-0.

A Beautiful Math: John Nash, Game Theory, and the Modern Quest for a Code of Nature, by Tom Siegfried. Joseph Henry Press, October 2006. ISBN 0-309-10192-1.

**The Best of All Possible Worlds: Mathematics and Destiny*, by Ivar Ekeland. University Of Chicago Press, October 2006. ISBN-13: 978-0-226-19994-8.

Bourbaki, A Secret Society of Mathematicians, Maurice Mashaal. AMS, June 2006. ISBN 0-8218-3967-5.

The Coxeter Legacy: Reflections and Projections, edited by Chandler Davis and Erich W. Ellers. AMS, March 2006. ISBN 0-8218-3722-2.

Dark Hero of the Information Age: In Search of Norbert Wiener, by Flo Conway and Jim Siegelman. Basic Books, December 2004. ISBN 0-738-20368-8. (Reviewed May 2006.)

Descartes: A Biography, by Desmond Clarke. Cambridge University Press, March 2006. ISBN 0-521-82301-3.

Descartes: The Life and Times of a Genius, by A. C. Grayling. Walker & Company, November 2006. ISBN 0-8027-1501-X.

**Einstein's Heroes: Imagining the World through the Language of Mathematics*, by Robyn Arianrhod. Oxford University Press, July 2006. ISBN-13: 978-0-195-30890-7.

The Equations: Icons of Knowledge, by Sander Bais. Harvard University Press, November 2005. ISBN 0-674-01967-9.

The Essential Turing, edited by B. Jack Copeland. Oxford University Press, September 2004. ISBN

0-198-25080-0. (Reviewed November 2006.)

Euclid in the Rainforest: Discovering Universal Truths in Logic and Math, by Joseph Mazur. Pi Press, October 2004. ISBN 0-131-47994-6.

Euler through Time: A New Look at Old Themes, by V. S. Varadarajan. AMS, June 2006. ISBN 0-8218-3580-7.

Evolutionary Dynamics: Exploring the Equations of Life, by Martin Nowak. Belknap Press, September 2006. ISBN 0-674-02338-2.

The Fabulous Fibonacci Numbers, by Alfred S. Posamentier and Ingmar Lehmann. Prometheus Books, February 2007. ISBN 1-591-02475-7.

Fearless Symmetry: Exposing the Hidden Patterns of Numbers, by Avner Ash and Robert Gross. Princeton University Press, May 2006. ISBN 0-691-12492-2. (Reviewed January 2007.)

From Cosmos to Chaos: The Science of Unpredictability, by Peter Coles. Oxford University Press, August 2006. ISBN 0-198-56762-6.

From Zero to Infinity: What Makes Numbers Interesting, by Constance Reid. Fiftieth anniversary edition, A K Peters, February 2006. ISBN 1-568-81273-6. (Reviewed February 2007.)

Gödel's Theorem: An Incomplete Guide to Its Use and Abuse, by Torkel Franzen. A K Peters, May 2005. ISBN 1-568-81238-8. (Reviewed March 2007.)

Great Feuds in Mathematics: Ten of the Liveliest Disputes Ever, by Hal Hellman. Wiley, September 2006. ISBN 0-471-64877-9.

How Mathematics Happened, by Peter S. Rudman. Prometheus Books, October 2006. ISBN 1-591-02477-3.

How to Cut a Cake: And Other Mathematical Conundrums, by Ian Stewart. Oxford University Press, November 2006. ISBN 0-199-20590-6.

Infinite Ascent: A Short History of Mathematics, by David Berlinski. Modern Library, September 2005. ISBN 0-679-64234-X.

It's about Time: Understanding Einstein's Relativity, by N. David Mermin. Princeton University Press, September 2005. ISBN 0-691-12201-6.

John von Neumann: Selected Letters, edited by Miklós Rédei. AMS, November 2005. ISBN 0-8218-3776-1.

King of Infinite Space: Donald Coxeter, the Man Who Saved Geometry, by Siobhan Roberts. Walker & Company, September 2006. ISBN 0-802-71499-4.

The Lifebox, the Seashell, and the Soul: What Gnarly Computation Taught Me about Ultimate Reality, the Meaning of Life, and How to Be Happy, by Rudy Rucker. Thunder's Mouth Press, October 2005. ISBN 1-560-25722-9.

A Madman Dreams of Turing Machines, by Janna Levin. Knopf, August 2006. ISBN 1-400-04030-2.

The Man Who Knew Too Much: Alan Turing and the Invention of the Computer, by David Leavitt. Great Discoveries series, W. W. Norton, December 2005. ISBN 0-393-05236-2. (Reviewed November 2006.)

The Math Instinct: Why You're a Mathematical Genius (along with Lobsters, Birds, Cats, and Dogs), by Keith Devlin. Thunder's Mouth Press, March 2005. ISBN 1-56025-672-9.

Mathematical Illustrations: A Manual of Geometry and PostScript, by Bill Casselman. Cambridge University Press, December 2004. ISBN 0-521-54788-1. (Reviewed January 2007.)

Mathematics and Common Sense: A Case of Creative Tension, by Philip J. Davis. A K Peters, October 2006. ISBN 1-568-81270-1.

Measuring the World, by Daniel Kehlmann. Pantheon, November 2006. ISBN 0-375-42446-6.

More Mathematical Astronomy Morsels, by Jean Meeus. Willmann-Bell, 2002. ISBN 0-943396-743.

Musimathics: The Mathematical Foundations of Music, Volume 1, by Gareth Loy. MIT Press, September 2006. ISBN 0-262-12282-0.

Mystic, Geometer, and Intuitionist: The Life of L. E. J. Brouwer. Volume 2: Hope and Disillusion, by Dirk van Dalen. Oxford University Press, October 2005. ISBN 0-198-51620-7.

The Newtonian Moment: Isaac Newton and the Making of Modern Culture, by Mordechai Feingold. New York Library and Oxford University Press, December 2004. ISBN 0-195-17735-5.

Not Even Wrong: The Failure of String Theory and the Continuing Challenge to Unify the Laws of Physics, by Peter Woit. Jonathan Cape, April 2006. ISBN 0-224-07605-1.

Once upon Einstein, by Thibault D'Amour. A K Peters, March 2006. ISBN 1-568-81289-2.

The Pea and the Sun: A Mathematical Paradox, by Leonard M. Wapner. A K Peters, April 2005. ISBN 1-568-81213-2. (Reviewed October 2006.)

Piano Hinged Dissections: Time to Fold!, by Greg Frederickson. A K Peters, October 2006. ISBN 1-568-81299-X.

Piero della Francesca: A Mathematician's Art, by J. V. Field. Yale University Press, August 2005. ISBN 0-300-10342-5. (Reviewed March 2007.)

Prince of Mathematics: Carl Friedrich Gauss, by M. B. W. Tent. A K Peters, January 2006. ISBN 1-568-81261-2.

Pursuit of Genius: Flexner, Einstein, and the Early Faculty at the Institute for Advanced Study, by Steve Batterson. A K Peters, June 2006. ISBN 1-568-81259-0.

Reality Conditions: Short Mathematical Fiction, by Alex Kasman. Mathematical Association of America, May 2005. ISBN 0-88385-552-6. (Reviewed August 2006.)

The Road to Reality: A Complete Guide to the Laws of the Universe, by Roger Penrose. Knopf, February 2005. ISBN 0-679-45443-8. (Reviewed June/July 2006.)

The Secret Life of Numbers: 50 Easy Pieces on How Mathematicians Work and Think, by George G. Szpiro. Joseph Henry Press, March 2006. ISBN 0-309-09658-8.

Shadows of Reality: The Fourth Dimension in Relativity, Cubism, and Modern Thought, by Tony Robbin. Yale University Press, March 2006. ISBN 0-300-11039-1. (Reviewed in this issue.)

The Shoelace Book: A Mathematical Guide to the Best (and Worst) Ways to Lace Your Shoes, by Burkard Polster. AMS, June 2006. ISBN 0-8218-3933-0. (Reviewed December 2006.)

Stalking the Riemann Hypothesis: The Quest to Find the Hidden Law of Prime Numbers, by Dan Rockmore. Pantheon, April 2005. ISBN 0-375-42136-X. (Reviewed September 2006.)

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Reference and Book List

Omniscience, Immortality, and Incomprehensibility, by Steven Brams. Springer, second edition, November 2007. ISBN-13: 978-0-387-48065-7.

Symmetry and the Monster: The Story of One of the Greatest Quests of Mathematics, by Mark Ronan. Oxford University Press, May 2006. ISBN 0-192-80722-6. (Reviewed February 2007.)

The Three Body Problem, by Catherine Shaw. Allison and Busby, March 2005. ISBN 0-749-08347-6. (Reviewed October 2006.)

**The Triumph of Numbers: How Counting Shaped Modern Life*, by I. B. Cohen. W. W. Norton, July 2006. ISBN-13: 978-0-393-32870-7.

The Trouble with Physics: The Rise of String Theory, the Fall of a Science, and What Comes Next, by Lee Smolin. Joseph Henry Press, October 2006. ISBN 0-309-10192-1.

Unknown Quantity: A Real and Imaginary History of Algebra, by John Derbyshire. Joseph Henry Press, May 2006. ISBN 0-309-09657-X.

Yearning for the Impossible: The Surprising Truths of Mathematics, by John Stillwell. A K Peters, May 2006. ISBN 1-568-81254-X.

AMERICAN MATHEMATICAL SOCIETY

PUBLISHER

Applications and nominations are invited for the position of Publisher of the American Mathematical Society.

The publisher oversees all editorial and acquisitions activities of the Society's publishing program for journals and books (but not Mathematical Reviews), which includes coordinating the activities of eight editorial boards for journals, nine editorial boards for books, three acquisitions editors, and several governing bodies related to AMS publishing. The publisher is the primary representative of the AMS publication program to the scholarly and publishing community and should normally hold a doctorate in the mathematical sciences with a record of research.

Responsibilities of the publisher include:

- Direction of the Acquisitions Department (for books)
- Leadership in setting scientific and editorial standards for books and journals
- Development and implementation of long-range plans for the AMS book and journal programs
- Budgetary planning and control for all scientific aspects of AMS books and journals
- Representing publications to the Council, the Board of Trustees, the Committee on Publications, and the Editorial Boards Committee. (The publisher serves on the two latter committees, ex officio.)

The publisher reports directly to the executive director, but works closely with the associate executive director for publishing, who oversees all non-scientific aspects of publishing (production, printing, distribution, promotion, and marketing).

The publishing departments are in the AMS headquarters, which are located in Providence, Rhode Island, not far from the Brown University campus. A printing and warehouse facility is nearby in Pawtucket, Rhode Island. The Rhode Island offices of the AMS employ approximately 150 staff.

The appointment will be for three to five years, with possible renewal, and will commence in 2008. The starting date and length of term are negotiable. The publisher position is full time, but applications are welcome from individuals taking leaves of absence from another position. Salary is negotiable and will be commensurate with experience.

Nominations and applications should be sent on or before August 1, 2007, to:

Dr. John Ewing, Executive Director
American Mathematical Society
201 Charles Street
Providence, RI 02904
jhe@ams.org

Applications should include a curriculum vitae, information on editorial and administrative experience, and the names and addresses of at least three references. The American Mathematical Society is an Equal Opportunity Employer.

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MATHEMATICAL REVIEWS EXECUTIVE EDITOR

Applications and nominations are invited for the position of Executive Editor of Mathematical Reviews (MR).

The executive editor heads the MR Division of the AMS and is responsible for all phases of its operations. The major activity of the division is the creation of the MR database along with its derived products, MathSciNet and the paper journal. The executive editor is the primary representative of Mathematical Reviews to the mathematics community and should normally hold a doctorate in the mathematical sciences with a record of research.

Responsibilities of the executive editor include:

- Direction of all staff in the Ann Arbor office of the American Mathematical Society
- Leadership in setting scientific and editorial standards for MR
- Development and implementation of long-range plans for the MR Division
- Budgetary planning and control for the MR Division
- Relations with reviewers and authors

The managing editor and administrative staff assist the executive editor in carrying out day-to-day tasks and non-editorial aspects of MR production. The MR Editorial Committee provides high-level scientific advice about the editorial standards of MR. The executive editor reports to the executive director.

The MR Division is located in Ann Arbor, Michigan, near the campus of the University of Michigan, and editors enjoy many faculty privileges at the university. MR employs approximately 70 personnel, including associate editors, copyeditors, bibliographic specialists, information technology staff, and clerical support.

The appointment will be for three to five years, with possible renewal, and will commence in the summer of 2008. The starting date and length of term are negotiable. The executive editor position is full time, but applications are welcome from individuals taking leaves of absence from another position. Salary is negotiable and will be commensurate with experience.

Nominations and applications should be sent on or before August 1, 2007, to:

Dr. John Ewing, Executive Director
American Mathematical Society
201 Charles Street
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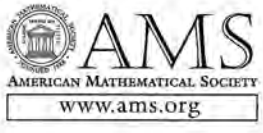
Applications should include a curriculum vitae, information on editorial and administrative experience, and the names and addresses of at least three references. The American Mathematical Society is an Equal Opportunity Employer.



▸Call for Nominations

Leonard Eisenbud Prize for Mathematics and Physics

This prize was established in 2006 in memory of the mathematical physicist, Leonard Eisenbud (1913–2004), by his son and daughter-in-law, David and Monika Eisenbud. Leonard Eisenbud was a student of Eugene Wigner. He was particularly known for the book, *Nuclear Structure* (1958), which he co-authored with Wigner. He was one of the founders of the Physics Department at SUNY Stony Brook, where he taught from 1957 until his retirement in 1983. In later years he became interested in the foundations of quantum mechanics and in the interaction of physics with culture and politics, teaching courses on the anti-science movement.

The prize will honor a work or group of works that brings the fields of mathematics and physics closer together. Thus, for example, the prize might be given for a contribution to mathematics inspired by modern developments in physics or for the development of a physical theory exploiting modern mathematics in a novel way.

The US$5,000 prize will be awarded every three years for a work published in the preceding six years. The first award will be made in January 2008.

All nominations should be submitted to the AMS Secretary, Robert J. Daverman, American Mathematical Society, 312D Ayres Hall, University of Tennessee, Knoxville TN 37996-1330. Include a short description of the work that is the basis of the nomination, with complete bibliographic citations when appropriate. A brief curriculum vitae should be included for the nominee. The nominations will be forwarded by the Secretary to the Prize Selection Committee, which will make the final decision on the award.

Deadline for nominations is June 30, 2007.

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Call for

NOMINATIONS

The selection committees for these prizes request nominations for consideration for the 2008 awards, which will be presented at the Joint Mathematics Meetings in San Diego, CA, in January 2008. Information about these prizes may be found in the November 2005 issue of the *Notices*, pp. 1243–1260, and at http://www.ams.org/prizes-awards.

LEVI L. CONANT PRIZE

This prize was established in 2000 in honor of Levi L. Conant to recognize the best expository paper published in either the *Notices of the AMS* or the *Bulletin of the AMS* in the preceding five years. Levi L. Conant was a mathematician at Worcester Polytechnic Institute. His will provided for funds to be donated to the AMS upon his wife's death. The US$1,000 prize is awarded annually.

JOSEPH L. DOOB PRIZE

This prize was established by the AMS in 2003 and endowed in 2005 by Paul and Virginia Halmos in honor of Joseph L. Doob. The prize recognizes a single, relatively recent, outstanding research book that makes a seminal contribution to the research literature, reflects the highest standards of research exposition, and promises to have a deep and long-term impact in its area. The book must have been published within the six calendar years preceding the year in which it is nominated. Books may be nominated by members of the Society, by members of the selection committee, by members of AMS editorial committees, or by publishers. The US$5,000 prize is awarded every three years.

DISTINGUISHED PUBLIC SERVICE AWARD

This award was established by the AMS Council in response to a recommendation from their Committee on Science Policy. The award is presented every two years to a research mathematician who has made a distinguished contribution to the mathematics profession during the preceding five years.

Nominations for all prizes should be submitted to the secretary, Robert J. Daverman, American Mathematical Society, 312D Ayres Hall, University of Tennessee, Knoxville, TN 37996-1330. Include a short description of the work that is the basis of the nomination, with complete bibliographic citations when appropriate. A brief curriculum vitae should be included for the nominee. The nominations will be forwarded by the secretary to the appropriate prize selection commttee, which, as in the past, will make final decisions on the awarding of these prizes.

Deadline for nominations is June 30, 2007.

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Call For NOMINATIONS

The selection committees for these prizes request nominations for consideration for the 2008 awards, which will be presented at the Joint Mathematics Meetings in San Diego, CA, in January 2008. Information about these prizes may be found in the November 2005 issue of the *Notices*, pp. 1243–1260, and at http://www.ams.org/prizes-awards.

BÔCHER MEMORIAL PRIZE

This prize, the first to be offered by the AMS, was founded in memory of Professor Maxime Bôcher, who served as president of the AMS 1909–1910. The original endowment was contributed by members of the Society. It is awarded for a notable paper in analysis published during the preceding six years. To be eligible, the author should be a member of the American Mathematical Society or the paper should have been published in a recognized North American journal. The US$5,000 prize is awarded every three years.

FRANK NELSON COLE PRIZE IN NUMBER THEORY

This prize (and the Frank Nelson Cole Prize in Algebra) was founded in honor of Professor Frank Nelson Cole on the occasion of his retirement as secretary of the American Mathematical Society after twenty-five years of service and as editor-in-chief of the *Bulletin* for twenty-one years. The original fund was donated by Professor Cole from moneys presented to him on his retirement, and was augmented by contributions from members of the Society. The fund was later doubled by his son, Charles A. Cole. The prize is for a notable paper in number theory published during the preceding six years. To be eligible, the author should be a member of the American Mathematical Society or the paper should have been published in a recognized North American journal. The US$5,000 prize is awarded every three years.

Nominations should be submitted to the secretary, Robert J. Daverman, American Mathematical Society, 312D Ayres Hall, University of Tennessee, Knoxville, TN 37996-1330. Include a short description of the work that is the basis of the nomination, with complete bibliographic citations when appropriate. A brief curriculum vitae should be included for the nominee. The nominations will be forwarded by the secretary to the appropriate prize selection commttee, which, as in the past, will make final decisions on the awarding of these prizes.

Deadline for nominations is June 30, 2007.

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Call for Nominations

2008

Frank and Brennie Morgan AMS-MAA-SIAM Prize

for Oustanding Research in Mathematics by an Undergraduate Student

The prize is awarded each year to an undergraduate student (or students having submitted joint work) for outstanding research in mathematics. Any student who is an undergraduate in a college or university in the United States or its possessions, or Canada or Mexico, is eligible to be considered for this prize.

The prize recipient's research need not be confined to a single paper; it may be contained in several papers. However, the paper (or papers) to be considered for the prize must be submitted while the student is an undergraduate; they cannot be submitted after the student's graduation. The research paper (or papers) may be submitted for consideration by the student or a nominator. All submissions for the prize must include at least one letter of support from a person, usually a faculty member, familiar with the student's research. Publication of research is not required.

The recipients of the prize are to be selected by a standing joint committee of the AMS, MAA, and SIAM. The decisions of this committee are final. The 2008 prize will be awarded for papers submitted for consideration no later than June 30, 2007, by (or on behalf of) students who were undergraduates in December 2006.

Questions may be directed:

Dr. Martha J. Siegel, MAA Secretary
Mathematics Department
Stephens Hall 302
Towson University
8000 York Road
Towson, MD 21252-0001
telephone: 410-704-2980
e-mail: siegel@towson.edu

Nominations and submissions should be sent to:

Morgan Prize Committee
c/o Robert J. Daverman, Secretary
American Mathematical Society
312D Ayres Hall
University of Tennessee
Knoxville, TN 37996-1330

Mathematics Calendar

The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at http://www.ams.org/mathcal/.

April 2007

*12-13 **Second International Workshop on Differential Algebra and Related Topics**, Rutgers University at Newark, Newark, New Jersey.

Description: The Kolchin Seminar in Differential Algebra of The City University of New York and the Department of Mathematics at Rutgers University at Newark are pleased to announce the second joint International Workshop and AMS Special Session on Differential Algebra and Related Topics. The joint conferences will bring together experts from different areas related to differential algebra. The purpose is to disseminate the methods and results of differential algebra to other areas, to encourage potential collaborations, and to attract graduate students and new researchers. During the workshop, invited speakers will give expository or survey talks on their fields. The Special Session at the AMS Eastern Section Meeting for Spring 2007 will bring the participants further up to date on the most current research through invited research reports.

Topics: The topics include but not limited to: differential and difference algebra, differential Galois theory, differential algebraic geometry, differential algebraic groups, computational differential algebra, Rota-Baxter type algebras, and applications to combinatorics, arithmetic geometry, control theory, dynamical systems, and integrability theories.

Information: http://newark.rutgers.edu/~liguo/DARTII/diffalg.html.

*13-15 **Midwest Workshop on Complex Analysis and Geometry**, University of Illinois at Chicago, Chicago, Illinois.

Speakers: David Barrett (Michigan), Salah Baouendi (UCSD), John D'Angelo (UIUC), Charles Fefferman (Princeton), Eric Fornaess (Michigan), Joseph J. Kohn (Princeton), Madhav Nori (University of Chicago), Linda Rothschild (UCSD), Yum-Tong Siu (Harvard), Mei-Chi Shaw (Notre Dame), Sidney Webster (University of Chicago).

Information: For information, contact the Organizing Committee: Jeff Lewis; jlewis@uic.edu, David Tartakoff; dst@uic.edu, Stephen S. T. You; yau@uic.edu.

Information: http://www.math.uic.edu/~lewis/scv2007/.

*16-July 13 **Dynamical Systems and Number Theory**, Centro di Ricerca Matematica Ennio de Giorgi, Pisa, Italy.

Tentative Schedule: (I) April 16-April 27, Renormalization, small divisors, continued fractions and geodesic flows; (II) April 30-May 11, Continued fractions, modular symbols, periods of automorphic forms and theta functions; (III) May 14-June 1, Teichmuller dynamics; (IV) May 28-June 8, TBA; (V) June 11-July 6, Clay Mathematics Institute summer school on Homogeneous flows, moduli spaces, and arithmetic.

Organizing Committee: Manfred Einsiedler (Ohio State University), David Ellwood (Clay Mathematics Institute), Alex Eskin (University of Chicago), Dmitry Kleinboc (Brandeis University), Elon LINDENSTRAUSS (chair, Princeton University), Gregory Margulis (Yale University), Stefano Marmi (Scuola Normale Superiore Pisa), Peter Sarnak (Princeton University), Jean-Christophe Yoccoz (CollÃ¤ge de France) and Don Zagier (Max Planck Institut, Bonn and CollÃ¤ge de France)

Information: More detailed information will be soon available on the website http://www.crm.sns.it/. Detailed information about the Clay School (June 11-July 6) can be retrieved from the website of the Clay Institute http://www.claymath.org/programs/summer_school/2007/.

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences held in North America carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. Meetings held outside the North American area may carry more detailed information. In any case, if there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http://www.ams.org/.

May 2007

*18-19 **Groups in Galway 2007**, National University of Ireland, Galway, Ireland.
Scope: The scope of the conference covers all areas of group theory, applications, and related fields. This year the conference will once again include a poster session. All interested persons, especially postgraduate students, are invited to contribute to this session.
Speakers: P. Campbell (University of Bristol, UK), Bettina Eick (TU Braunschweig, Germany), Murray Elder (Stevens Institute of Technology, USA), Charles Leedham-Green (Queen Mary, University of London, UK), C. Roney-Dougal (University of St Andrews, UK), H. Smith (Bucknell University, USA).
Information: Prospective participants should register their intention to attend the conference by contacting Dane Flannery (dane.flannery@nuigalway.ie) or Rachel Quinlan (rachel.quinlan@nuigalway.ie). Further information will be posted on the conference website: http://www.maths.nuigalway.ie/conferences/gig07/ as it becomes available.

*18-20 **International Conference on Special Functions & their Applications (7th Annual Conference of SSFA, India)**, St. Thomas College, Arunapuram P.O., Pala, Kottayam, Pin 686574, Kerala, India.
Information: http://www.ssfa.gq.nu/conf.htm.

*20-24 **Canadian Applied and Industrial Mathematics Society (CAIMS) Annual Meeting**, Banff, Alberta, Canada.
Themes: Applied linear Algebra, Computational PDE's, Cryptography, Dynamical Systems, Fluid Dynamics, Mathematics of Finance, Mathematical Biology, Geophysical Inversion. Each of these themes features a plenary speaker and 6-8 invited speakers. There is also a poster session.
Deadlines: Accommodation at the Banff Centre: February 23, 2007, for guaranteed rooms.
Organizers: Head of Scientific Committee is Peter Lancaster, University of Calgary. Co-chairs of Local Organizing Committee are Ben Aggarwala of the University of Calgary and Jack Macki of the University of Alberta.
Information: For a list of plenary speakers and other information, please visit: http://www.math.ualberta.ca/ami/caims2007.html.

*21-24 **Cornell Topology Festival**, Cornell, Ithaca, New York.
Description: This year's Festival includes a number of talks in group theory and low-dimensional topology. There will be two introductory workshops in those areas as well as a more general panel discussion.
Speakers: Danny Calegari, Ralph Cohen, Cornelia Drutu, Alex Eskin, Mark Feighn, Ilya Kapovich, Christopher Leininger, Tim Riley, Juan Souto, Gang Tian.
Financial Support: Is available. Young researchers are especially encouraged to apply.
Information: For registration and further details see the Festival web page: http://www.math.cornell.edu/~festival/.

*21-25 **International Conference on Mathematical Fluid Mechanics**, Hotel Estoril Eden, Estoril, Portugal.
Description: Conference in honor of Giovanni Paolo Galdi (Univ. Pittsburgh) on the occasion of Professor Galdi's 60th birthday.
Topics: Navier-Stokes equations, non-Newtonian models, hydrodynamic stability and fluid-particle interactions.
Information: http://www.math.ist.utl.pt/cemat/gpgaldi.

*23-25 **Combinatorial and Additive Number Theory (CANT 2007)**, CUNY Graduate Center, New York, New York.
Description: This is the fifth in a series of annual workshops sponsored by the New York Number Theory Seminar on problems in combinatorial and additive number theory and related parts of mathematics. Some funding is available to support graduate students and junior faculty. For more information about attending or speaking at the workshop, please contact the organizer, Mel Nathanson, at melvyn.nathanson@lehman.cuny.edu.
Information: The program and a list of speakers will be posted on the conference Website: http://www.theoryofnumbers.com.

*28-June 1 **Complex Analysis and Geometry XVIII**, G. Hotel Bellavista, Levico Terme (Trento), Italy.
Scientific Organizers: V. Ancona (Firenze), C. Arezzo (Parma), F. Bracci(Roma II) and A. Silva (Roma I).
Confirmed Participants: E. Amar (Bordeaux I), D. Barlet (Nancy), E. Bedford (Bloomington), B. Berndtsson (Chalmers), F. Berteloot (Tolouse), Z. Blocki (Krakow), F. Campana (Nancy), M. Contreras (Sevilla), B. Coupet (Marseille), S. Diaz-Madrigal (Sevilla), T.-C. Dinh (Paris), G. Dloussky (Marseille), B. Drinovec Drnovsek (Ljubljana), F. Forstneric (Ljubljana), B. Gilligan (Regina), J. Globevnik (Ljubljana), V. Guedj (Tolouse), C.D. Hill (Stony Brook), A. Huckleberry (Bochum), A. Iordan (Paris), S.M. Ivashkovich (Lille I), B. Juhl-Joricke (Uppsala), L. Lanzani (Fayetteville), C. Laurent-Thiebaut (S. Martin d'Heres), J. Leiterer (Berlin), L. Lempert (West Lafayette), J.J. Loeb (Angers), Z. Lu (Irvine), J.I. Magnusson (Reykjavik), N. Mir (Rouen), S. Nivoche (Tolouse), K. Oeljeklaus (Marseille), F. Pacard (Paris XII), I. Patyi (Atlanta), D.H. Phong (Columbia), D. Popovici (Tolouse), N. Shcherbina (Wuppertal), G. Schumacher (Marburg), D. Shoikhet (Karmiel), N. Sibony (Orsay), B. Stensones (Ann Arbor), T. Suwa (Niigata), A. Teleman (Marseille), G. Trautmann (Kaiserslautern), J.-P. Viguè (Poitiers), D. Zaitsev (Dublin), S. Zelditch (Johns Hopkins), X. Zhu (Beijing).
Registration deadline: April 30, 2007.
Information: http://www.science.unitn.it/cirm/.
Contact Information: A. Micheletti, Secretary of CIRM, Centro Internazionale per la Ricerca Matematica, Istituto Trentino di Cultura, 38050 Povo (Trento); Tel. +39-0461-881628; Telefax +39-0461-810629; email: michelet@science.unitn.it.

*30-June 2 **ACMS 16th Biennial Conference**, Messiah College, Grantham, Pennsylvania.
Description: The Association of Christians in the Mathematical Sciences will sponsor their 16th summer conference since 1977.
Featured Speakers: Francis Su from Harvey Mudd College and Jennifer Wiseman who is NASA's Hubble program scientist.
Conference Chair: Angela Hare, Dept of Mathematics, Messiah College; email: ahare@messiah.edu.
Deadline: Abstracts of proposed papers may be sent to the conference chair before April 6, or until the program is filled.
Information: General information and registration is at http://www.acmsonline.org.

June 2007

*2007 **International Conference on Learning**, Johannesburg, South Africa.
Information: http://www.LearningConference.com.

*2 **Developments in Algebraic Geometry**, Brown University, Providence, Rhode Island.
Description: On the occasion of David Mumford's 70th birthday there will be a one-day conference, in algebraic geometry. The Clay Mathematics Institute is kindly sponsoring the event. The conference will feature five one-hour talks, on recent developments in algebraic geometry following from Mumford's work. The algebraic geometry conference is part of a two-day event. On June 1, 2007 there will be a sister meeting on computer vision. The computer vision component will be held in Newport, Rhode Island.
Speakers: Valery Alexeev, Igor Krichever, Michael Rapoport, Vyacheslav Shokurov, Ulrike Tillmann.
Organizing committee: Ching-Li Chai, Amnon Neeman, Takahiro Shiota.
Funding: We have applied for funding to support graduate students, postdocs and early-career researchers. If the funding materializes, we will be happy to invite as many as we can.

Information: Further details, about both conferences, can be found from the joint webpage: http://www.dam.brown.edu/mumford-conf/.

* 4-9 **Algebraic Geometry in Higher Dimensions**, G. Hotel Bellavista, Levico Terme (Trento), Italy.
Scientific Committee: M. Andreatta (Trento), E. Ballico (Trento), C. Ciliberto (Roma II), Y. Kawamata (Tokyo) and J. Kollár (Princeton).
Local Organizers: G. Occhetta (Trento) and R. Pignatelli (Trento).
Invited Speakers (* to be confirmed): V. Alexeev* (Georgia), L. Badescu (Genova), I. Bauer (Bayreuth), F. Campana (Nancy I), L. Caporaso* (Roma III), F. Catanese (Bayreuth), L. Chiantini (Siena), B. Fantechi* (SISSA Trieste), S. Kebekus (Koeln), S. Kovács (Washington), A. Lopez (Roma III), J. McKernan (S. Barbara), M. Mella (Ferrara), R. Miranda (Colorado), Y. Miyaoka* (Tokyo), S. Mukai (Kyoto), G. Ottaviani (Firenze), R. Pardini (Pisa), T. Peternell (Bayreuth), G.P. Pirola (Pavia), M. Reid (Warwick), F. Russo (Recife), E. Sernesi (Roma III), V. Shokurov* (Johns Hopkins), A. Sommese (Notre Dame), J. Starr (Stony Brook), B. van Geemen (Milano), A. Verra (Roma III), G. Vezzosi* (Firenze), A. Vistoli (Bologna), J. Wisniewski (Warsaw).
Registration deadline: April 30, 2007.
Information: http://www.science.unitn.it/~occhetta/aghd/index.html;email: aghdtn@gmail.com.

* 7-10 **74th Workshop on General Algebra**, Tampere University of Technology, Tampere, Finland.
Topics: The topics of the conference are the traditional topics of the AAA conference series: Universal Algebra and Lattice Theory, Classical Algebra, Applications of Algebra. In the "applications of algebra" area contributions related to Clifford algebras and possible applications of semilattice and tree structures are welcome in particular. In the "classical algebra" area the organizers hope that there will be contributions related to rings, associative algebras, and semigroups. Contributions related to logic are invited in the "universal algebra and lattice theory" area.
Information: http://math.tut.fi/aaa74/.
Organizers: From Tampere University of Technology; Stephan Foldes, Miguel Couceiro; email: miguel.couceiro@uta.fi; Lauri Hella, email: lauri.hella@uta.fi; Erkko Lehtonen, email: erkko.lehtonen@tut.fi; Jenõ Szigeti, University of Miskolc; email: jeno.szigeti@uni-miskolc.hu.

* 11-12 **SPEED: Software Performance Enhancement for Encryption and Decryption**, Amsterdam, the Netherlands.
Important dates: April 6, 2007: Submission of papers. May 11, 2007: Notification of acceptance or rejection. May 25, 2007: Revised version of accepted papers due. June 11-12, 2007: SPEED workshop in Amsterdam.
Information: More details on http://www.hyperelliptic.org/SPEED/.

* 24-26 **Mathematical Modelling in Sport**, Salford Quays Conference Centre, Salford, Manchester, UK.
Description: The term Sport is interpreted liberally here and includes: Games and Pastimes; Gambling and On-line Gaming; Lotteries; and General Fitness and Health-related Activities.
Topics: Analysis of sporting technologies; Analysis of rules and adjudication; Performance measures and models; Optimisation of sports performance; Mathematics education and sport, Econometrics in Sport; Optimal tournament design and scheduling; Competitive strategy; Match outcome models; Decision support systems, Computationally intensive methods.
Keynote Speakers: Stephen R. Clarke (Swinburne Sports Statistics, Australia), John Haigh (Sussex University), Rudd Koning (University of Groningen, The Netherlands).
Information: email: pam.bye@ima.org.uk.

* 24-29 **Fifth School on Analysis and Geometry in Metric Spaces**, Grand Hotel Bellavista, Levico Terme (Trento), Italy.
Aim: The aim of the school is to offer an outline of the present state of research concerning geometric measure theory on Carnot-Caratheodory groups and on more general metric spaces. Analysis and Geometry on these structures has been object of extensive research in the last few years, with applications ranging from degenerate elliptic equations to optimal control theory and differential geometry. It is intention of the organizers to put together young researchers and well-known researchers active in the field and to encourage informal discussion on current research trends and developments in the area. Young researchers are encouraged to participate: it is possible to support some of them upon request.
Organizers: L. Ambrosio (SNS Pisa), B. Franchi (Bologna), R. Serapioni (Trento) and F. Serra Cassano (Trento).
Short courses by: G. Citti (Bologna), M. Cowling (Sydney), J. Heinonen (Ann Arbor), M. Ritore' (Granada).
Registration deadline: May 31, 2007.
Information: Visit http://www.science.unitn.it/cirm/MeSpa07.html.

* 25-29 **First International Summer School on Geometry, Mechanics and Control**, Castro Urdiales, Cantabria, Spain.
Description: The International Summer School on Geometry, Mechanics and Control is oriented to Ph.D. students and to postdoctoral students with undergraduate studies in Mathematics, Physics or Engineering, in particular to those who want to begin its research in geometrical aspects of mechanics, numerical integration, field theory and control theory. In this sense, the courses could be a complement to Ph.D. programs of different Universities. The School is also open to professionals who want to attend advanced and specialized courses oriented to geometrical techniques in those fields.
Support: The Summer School is supported by: Mathematica-Consolider; SIMUMAT; CSIC; ULL; Network: Geometry, Mechanics and Control.
Funds: A limited number of scholarships for Ph.D. students and advanced undergraduate students, that may cover registration fees, lodging expenses or/and travel expenses, will be provided by the organizers. Applications, including a CV (for Ph.D. students) and CV and a certificate with grades of subjects (for undergraduate students), should be sent before April 30, 2007, to: gmcnet@ull.es.
Organizing committee: Manuel de León (CSIC), David Martín de Diego (CSIC), Edith Padrón (ULL).
Information: For more information and online registration, see the official webpage of Summer School at: http://webpages.ull.es/users/gmcnet/Summer_School.

* 28-30 **Third Statistical Days at the University of Luxembourg**, University of Luxembourg, Luxembourg, Luxembourg.
Program: The conference is meant for statisticians as well as researchers or postgraduate students working in research conditions not allowing the use of parametric statistics or for researchers handling qualitative variables. The theme of the seminar addresses different topics, such as the evaluation of medical, psychotherapeutic or pedagogical interventions or the study of factors fluctuating over time in economy. The mathematical foundations and the conditions of application of some multi-dimensional non parametric procedures will be discussed, and the current evolution in this realm will be outlined. Configural frequency analysis (CFA), partial least squares (PLS) methods and dynamic factor analysis (DFA) will especially be stressed.
Main Topics: Research strategies for small samples respectively non metric data, dynamic factor analysis (DFA), partial least square models (PLS), configural frequency analysis (CFA).
Invited Speakers: Ellen Hamaker (Utrecht University), Erwin Lautsch (Universität Kassel), Peter C. M. Molenaar (Pennsylvania State University), Christian Preda (Universite. de Lille 2), Michel Tenenhaus (HEC Paris), Alexander von Eye (Michigan State University).
Organizers: Prof. Dr. Jang Schiltz (University of Luxembourg), Dr. Lony Schiltz (Crp-Santé, Luxembourg).

Deadlines: March 31, 2007: Deadline for application for young researchers grants. April 15, 2007: Notification of attribution of the grants. April 30, 2007: Deadline for submission of abstracts. May 15, 2007: Notification of acceptance of contributed papers.
Information: http://sma.uni.lu/stat3.

July 2007

*2-20 **Geometric and Topological Methods for Quantum Field Theory**, Villa de Leyva, Colombia.
Description: This summer school is the fifth of a series of schools to take place in Colombia. During the school the lecturers will give both mini-courses and specialized talks on their fields of expertise for students at the masters and Ph.D. levels. The length of each course will range from 4 hours to 6 hours and they will serve as introductions to active areas of research these days. The courses are addressed to both physicists and mathematicians, there will be the opportunity for participants to present their research work.
Topics: The border line between geometry, topology and quantum field theory.
Deadline for Registration: April 15, 2007.
Information:http://matematicas.uniandes.edu.co/summer2007.

*5-8 **XVIth Oporto Meeting on Geometry, Topology and Physics**, University of the Algarve, Faro, Portugal.
Main Theme: The combinatorics, geometry, topology and physics of knot homology.
Main speakers and invited speakers: Sergei Gukov (three 1-hour lectures), Mikhail Khovanov (three 1-hour lectures), Peter Ozsvath (three 1-hour lectures), Paul Seidel (three 1-hour lectures), Jacob Rasmussen (1 hour lecture), Paul Turner (1 hour lecture).
Deadline: Registration: March 31, 2007.
Information: http://www.ualg.pt/fct/omgtp/.

*8-21 **37th Probability Summer School**, Saint-Flour, France.
Speakers: J. Buzzi, F. den Hollander, J. Mattingly.
Information: http://math.univ-bpclermont.fr/stflour/.

*16-19 **Communicating Mathematics**, Duluth, Minnesota.
Description: A conference to inspire research productivity, collaborations, and enthusiasm among mathematicians in all areas, at all stages of their careers. This conference recognizes the 30th anniversary of the Duluth Research Experience for Undergraduates in mathematics and Joseph Gallian's 65th birthday.
Invited Speakers: Manjul Bhargava (Princeton University), Tim Chow (Center for Communications Research, Princeton), Ron Graham (UC, San Diego), Aparna Higgins (University of Dayton), Tara Holm (Cornell University), Nathan Kaplan (Princeton University), Dave Witte Morris (University of Lethbridge), David Moulton (Center for Communications Research, Princeton), Lenny Ng (Duke University), Francis Edward Su (Harvey Mudd College), Tad White (National Security Agency), Amie Wilkinson (Northwestern University).
Information: Please visit http://events.olin.edu/CommunicatingMathematics/index.cfm.

*19-22 **Applications of Computer Algebra (ACA 2007)**, Oakland University, Rochester, Michigan.
Theme: The ACA series of conferences is devoted to promoting the applications and development of Computer Algebra and Symbolic Computation. Topics include computer algebra and symbolic computation in engineering, the sciences, medicine, pure and applied mathematics, education, communication and computer science.
Topics: Applications of computer algebra in enumerative and algebraic combinatorics, A. Tefera, M. Apagodu and M. Zeleke; Non-Standard Applications of Computer Algebra, E. Roanes-Lozano, M. Wester; Coding theory, D. Joyner, T. Shaska, C. Shor; Computer algebra in education, M. Beaudin, M. Wester, A. Akritas, B. Pletsch; Computational algebraic geometry, T. Shaska; Symbolic Symmetry Analysis and Its Applications. N. Bila; Algebraic and Numerical Computation for Engineering and Optimization Problems, D. Chibisov, V. Ganzha, E. W. Mayr; Numerical Algebraic Geometry, C. Wampler, A. Sommese.
Call for Sessions: We are still accepting proposals to organize sessions at the conference. Session chairs are expected to organize 4 or more speakers on a theme consistent with that of the conference.
Organizer: Proposals for organizing a session should be directed to the Program Chair: Tony Shaska; shaska@oakland.edu.
Local Organizers: J. Nachman, T. Shaska; http://www.oakland.edu/~shaska/aca07.html. Note: This replaces the first call on http://www.ams.org/mathcal/info/2007_aug3-6_rochester.html.

*21-23 **5th International Summer School and Workshop on Pattern Recognition: Call for Participation and Papers**, Plymouth, United Kingdom.
Description: The International Summer School on Pattern Recognition is the premier event on research training in the area of pattern recognition and machine learning. The school is fully residential and its registration includes all attendance, and living costs. The vision of the summer school is to empower its participants with the state-of-the-art techniques in pattern recognition and machine learning; to provide a deep understanding of how techniques work, their strengths and limitations, and the future of things to come in this field. Call for Papers and Participation: Starting from 2007, the summer school will include the International Workshop on Advances in Pattern Recognition (IWAPR). IWAPR will have the following characteristics: (a) The workshop is limited to summer school participants who must be one of the authors of the submitted research paper. (b) All submitted papers will be fully reviewed by 2 to 3 referees. All accepted papers will be published online and the best rated accepted papers will be published by Springer-Verlag in a book titled, 'Novel Advances in Pattern Recognition Theory and Applications': ISSPR/IWAPR Proceedings, 2007. (c) IWAPR accepted papers will be presented in poster sessions.
Topics: Statistical pattern recognition, Non- and semi-parametric pattern recognition, Evolutionary Computation and Optimisation, Classifiers and their Evaluation, Information Fusion, Applications.
Deadlines: Submission Deadline: March 15, 2007.
Information: For Summer school: http://www.patternrecognitionschool.com. Workshop: http://www.iwapr.org.

*22-August 4 **Biomedical Modeling and Cardiovascular-Respiratory Control: Theory and Practice**, Schloss Seggau, Leibnitz, (near Graz) Austria.
Description: To explore mathematical modeling techniques essential for studying human biomedical and clinical problems at primarily the organ and system level with an emphasis on control mechanisms and clinical problems arising from deficiencies in these control mechanisms. The event includes a 14 day summer school including 11 days of courses (July 22 to Aug. 1) after which the summer school participants attend a 3 day scientific workshop (Aug. 2 to Aug. 4) on the same scientific theme. The summer school course component is aimed primarily toward Ph.D. students and new Post-Docs. The three day workshop will be a small scientific workshop which include presentations from approximately 16 additional scientists actively involved in research in the focus theme of the school allowing school participants to apply what they have learned and meet current researchers. Teachers of the summer school and other auditors at the workshop may attend as well.
Organizers: Mostafa Bachar, Jerry Batzel, and Franz Kappel, Institute for Mathematics and Scientific Computing, University of Graz, Austria.
Information: http://www.uni-graz.at/mc_training_schools/graz/index.html.

*30-August 4 **On Certain L-functions**, Purdue University, West Lafayette, Indiana.
Organizers: James Arthur (Toronto), James Coggell (Ohio State), Stephen Gelbart (Weizmann Institute), Stephen Kudla (Toronto),

Dinakar Ramakrshnan (Calthech), and Peter Sarnak (Princeton). Local Organnizers: David Goldberg (Purdue) and Jiu Kang Yu (Purdue).
Information: email: goldberg@math.purdue.edu.

August 2007

*5-11 **Equadiff 2007**, Vienna University of Technology, Vienna, Austria.
Topics: The conference is devoted to all mathematical aspects of differential equations. Central themes are: ordinary differential equations, partial differential equations, delay equations, stochastic differential equations, dynamical systems, numerical analysis, computation and applications.
Information: http://equadiff07.tuwien.ac.at; email: info@equadiff07.tuwien.ac.at.

*6-10 **Enhancing the problem authoring capabilities of WeBWorK**, AIM Research Conference Center, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to refining and enhancing the WeBWorK problem authoring language by providing better macros, tools and examples for writing problems and at the same time making the language easier to use, learn, document, and maintain.
Organizers: David Cervone, Michael Gage, and Arnold Pizer.
Deadline: April 6, 2007.
Information: http://aimath.org/ARCC/workshops/webwork.html.

*12-16 **Summer Symposium in Real Analysis XXXI**, Trinity College, Oxford, United Kingdom.
Description: The Mathematical Institute and Trinity College, Oxford and the *Real Analysis Exchange* will host the Summer Symposium in Real Analysis XXXI. An important aspect of the conference will be the interplay between classical real analysis and geometric measure theory. The program includes hour long talks by Giovanni Alberti (Pisa), Alexander Olevskii (Tel Aviv) and David Preiss (Warwick). In addition to the principal lectures we have a number of slots set aside for twenty minute talks. Young researchers areparticularly encouraged to present their work.
Information: For additional information and online registration see: http://www.stolaf.edu/people/analysis. Individual questions concerning conference details may be addressed to the Conference Director, Bernd Kirchheim at kirchhei@maths.ox.ac.uk.

*19-25 **Loops'07**, Charles University, Prague, Czech Republic.
Description: Loops '07 aspires to become a forum for all aspects of loops and quasigroups. We intend not only to highlight new relevant results in algebra and geometry, but also to foster connections to combinatorics, cryptography and group theory. All manifestations of the quasigroup concept are welcome at the conference. There will be a Special Session on Cryptography.
Workshops: A series of lectures and seminars, called Workshops Loops '07, will be held immediately before the conference and is aimed at both active researchers and graduate students.
Information: http://www.karlin.mff.cuni.cz/~loops07.

*20-24 **Fourier analytic methods in convex geometry**, AIM Research Conference Center, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, concerns the interface between convex geometry and harmonic analysis. Particular attention will be given to applications of Fourier analysis to the study of sections and projections of convex bodies.
Organizers: Alexander Koldobsky, Dmitry Ryabogin, and Artem Zvavitch.
Deadline: April 20, 2007.
Information: Visit http://aimath.org/ARCC/workshops/fourierconvex.html.

*28-30 **2nd Regional Conference on Ecological and Environmental Modelling (ECOMOD 2007)**, Gurney Hotel, Penang, Malaysia.
Objectives: The objective of the conference is to give an opportunity to academicians, scientists, engineers and post-graduates to present the results of their research and development work as well as to share their experience in the use and development of ecological and environmental models. The scope of the conference encompasses the use of mathematical, statistical, computing, science and engineering techniques and concepts in ecological and environmental modelling. Topics to be addressed are: Environmental pollution and treatment; Flora & fauna diversity, population and habitat; Artificial intelligence-techniques and systems; Environmental engineering and technology; GIS, remote sensing and image processing; Mathematical modelling; other related fields.
Important Dates: Early Registration Deadline: May 30, 2007. Abstract Submission Deadline: April 30, 2007. Full paper Submission Deadline: August 28, 2007.
Information: For more information, please contact: The Secretariat (Att: Assoc. Prof. Dr. Ahmad Sofi man Othman), Ecological and Environmental Modelling (ECOMOD 2007), c/o School of Biological Sciences, Universiti Sains Malaysia, 11800 Pulau Pinang, Malaysia; email: ecomod2007@usm.my; fax: 604-6565125.

September 2007

*3-14 **Epidemiology & Control of Infectious Diseases: Introduction to mathematical models of global and emerging infections**, Imperial College, London, England.
Description: Our understanding of infectious-disease epidemiology and control has been greatly increased through mathematical modelling. Insights from this increasingly-important and exciting field are now informing policymaking at the highest levels, for pandemic influenza, SARS, HIV/AIDS, TB, malaria, foot-and-mouth disease and other infections. Participants need only a basic mathematical ability (high school level is more than sufficient): most course participants do not use maths regularly, and calculation is done using Excel and the user- friendly modelling package, Berkeley Madonna; hence manipulation of equations is not required. We offer an optional 'maths refresher' day on Sunday 2 September, free of charge.
Information: For further details visit the course website: http://www.imperial.ac.uk/cpd/epidemiology/ which includes full information on how to apply for the course or contact Ulrika Wernmark, Centre for Professional Development, Imperial College London, South Kensington Campus, London SW7 2AZ, Tel: +44 (0)20 7594 6886.

*7-11 **KIAS Conference on Geometric Analysis**, KIAS, Seoul, Korea.
Topics: Minimal surfaces, Constant mean curvature surfaces, Lagrangian submanifolds, Isoperimetric problems, Geometric measure theory, Geometric analysis and PDE's.
Organizers: Jaigyoung Choe (KIAS) and Richard Schoen (Stanford).
Invited Speakers: Richard Schoen (Stanford), Robert Hardt (Rice), Motoko Kotani (Tohoku), Conan Leung (CUHK), Rafe Mazzeo (Stanford), Michael Wolf (Rice), Jon Wolfson (Michigan State), and more to be invited.
Information: http://newton.kias.re.kr/~choe/cga.html.

*10-14 **5th Symposium on Nonlinear Analysis**, Nicolaus Copernicus University, Torun, Poland.
Scientific Committee: Lech Górniewicz (chair), Jan Andres, Jurgen Appell, Thomas Bartsch, Kazimierz Goebel, Russel Johnson Wojciech Kryszewski, Wacaw Marzantowicz, Jean Mawhin Paolo Nistri, Zbigniew Peradzyński, Krzysztof Rybakowski Roman Srzednicki, Andrzej Szulkin, Andrzej Wieczorek.
Topics: Topics in the topological and metric fixed point theory, Topological and variational methods in nonlinear analysis, Qualitiative theory of ordinary and partial differential equations and inclusions, Nonsmooth and convex analysis, Critical point theory, Optimal control theory, Applications.

Information: http://www-users.mat.uni.torun.pl/~sna2007/; email: sna2007@mat.uni.torun.pl.

*10-15 **10th Quantum Mathematics International Conference: QMath10**, Moeciu, Romania.
Workshop Topics: Schroedinger Operators & Inverse Scattering, Random Schroedinger Operators & Random Matrices, Quantum Field Theory & Relativistic Quantum Mechanics, Condensed Matter & Open Systems, Pseudodifferential Operators & Semiclassical Analysis, Quantum Information.
Scientific Committee: Yosi Avron (Haifa), Pavel Exner (Prague), Bernard Helffer (Paris), Ari Laptev (Stockholm), Gheorghe Nenciu (Bucharest), Heinz Siedentop (Muenchen).
Organizers: The SPECT Program of ESF: The Institute of Mathematics "Simion Stoilow" of the Romanian Academy.
Plenary Speakers: M. Esteban, F. Germinet, G.M. Graf, Ch. Hainzl, A. Joye, R. Killip, M. Loss, Ch. Maes, S. Popescu, R. Purice, P. Seba, R. Seiringer, S. Serfaty, R. Wedder, M. Zworski.
Information: Contact: Radu.Purice@imar.ro; tel: +40-21-319.65.09; Institute of Mathematics "Simion Stoilow" of the Romanian Academy; P. O. Box 1-764, RO-014700 Bucharest; http://www.imar.ro/~purice/QM10/QM10.html.

*24-29 **International Algebraic Conference dedicated to the 100th anniversary of D. K. Faddeev**, Euler International Mathematical Institute, St. Petersburg, Russia.
Organizers: St. Petersburg State University, St. Petersburg Dept. of the V. A. Steklov Inst. of Math. of the Russian Acad. of Sci., Euler International Math. Inst., Euler Foundation, St. Petersburg Math. Soc.
Program Committee: A. V. Yakovlev, S. V. Vostokov, N. A. Vavilov, A. I. Generalov, N. L. Gordeev.
Information: email: novikova@pdmi.ras.ru; http://www.pdmi.ras.ru/EIMI/2007/DKF.

October 2007

*24-27 **Integers Conference 2007**, University of West Georgia, Carrollton, Georgia.
Purpose: The Integers Conference 2007 is an international conference in combinatorial number theory, held for the purpose of bringing together mathematicians, students, and others interested in combinatorics and number theory.
Organizers: B. Landman, M. Nathanson, J. Nesetril, R. Nowakowski, C. Pomerance.
Plenary Speakers: George Andrews, Vitaly Bergelson, Bryna Kra, Florian Luca, Ken Ono, Van Vu.
Information: See http://www.westga.edu/~math/IntegersConference2007.pdf.

December 2007

*10-12 **SIAM Conference on Analysis of Partial Differential Equations (PD07)**, Hilton Phoenix East/Mesa, Mesa, Arizona.
Information: The Call for Presentations for this conference is available at: http://www.siam.org/meetings/pd07/.
Deadlines: Minisymposium proposals: May 11, 2007. Abstracts for all contributed and minisymposium presentations: June 11, 2007.
Information: For additional information, contact SIAM Conference Department at email: meetings@siam.org.

*10-14 **Triangulations, Heegaard splittings and hyperbolic geometry**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to the study of 3-manifolds via two new methods of decomposing them: efficient triangulations and Mom technology. These are reminiscent of the classical tools of triangulations and Heegaard splittings and one goal of the workshop is to draw connections amongst these four structures. Furthermore, we aim to make connections between these decompositions and the geometry of the manifold.
Organizers: Jennifer Schultens and Maggy Tomova.
Deadline: August 1, 2007.
Information: http://aimath.org/ARCC/workshops/triangulations.html.

*17-22 **Transformation Groups 2007**, Independent University of Moscow, Moscow, Russia.
Description: The conference is aimed to review the development of Transformation Group Theory during last decades, to present the most bright recent achievements in this area, and to discuss the perspectives of further research. The motivations for the conference are to gather researchers working on transformation groups in various areas of mathematics, in order to extend their ideas and methods to a broader context and to provide an opportunity for a broad discussion and exchange of ideas and experience between mathematicians from various countries and of different generations. The conference will be dedicated to the anniversary of Ernest Vinberg, who will become seventy in the summer of 2007.
Invited speakers: Dmitri N. Akhiezer, Dmitri V. Alexeevsky, Leonid A. Bokut, Michel Brion, Victor M. Buchstaber, Corrado De Concini, Michel Duflo, Alexander G. Elashvili, Victor M. Gichev, Simon G. Gindikin, Victor A. Ginzburg, Victor G. Kac, Alexandre A. Kirillov, Friedrich Knop, Bertram Kostant, Hanspeter Kraft, Shrawan Kumar, Peter Littelmann, Alex Lubotzky, Dominique Luna, Alexander D. Mednykh, Vladimir F. Molchanov, Grigori I. Olshanski, Arkadiy L. Onishchik, Alexander N. Panov, Dmitri I. Panyushev, Alexey N. Parshin, Vladimir L. Popov, M. S. Raghunathan, Osip V. Shvartsman, Tonny A. Springer, Irina D. Suprunenko, Jacques L. Tits, Nikolai A. Vavilov, Anatoly M. Vershik, Andrei Yu. Vesnin, Valentin E. Voskresensky.
Deadlines: For registration: September 15, 2007. For abstracts submission: October 15, 2007.
Information: Visit http://www.mccme.ru/tg2007/; email: tg2007@mccme.ru.

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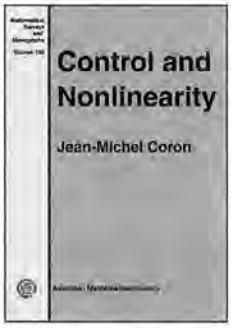

Control and Nonlinearity

Jean-Michel Coron, *Université de Paris-Sud, Orsay, France*

This book presents methods to study the controllability and the stabilization of nonlinear control systems in finite and infinite dimensions. The emphasis is put on specific phenomena due to nonlinearities. In particular, many examples are given where nonlinearities turn out to be essential to get controllability or stabilization. Various methods are presented to study the controllability or to construct stabilizing feedback laws. The power of these methods is illustrated by numerous examples coming from such areas as celestial mechanics, fluid mechanics, and quantum mechanics.

The book is addressed to graduate students in mathematics or control theory and to mathematicians or engineers with an interest in nonlinear control systems governed by ordinary or partial differential equations.

This item will also be of interest to those working in differential equations.

Contents: *Controllability of linear control systems:* Finite-dimensional linear control systems; Linear partial differential equations; *Controllability of nonlinear control systems:* Controllability of nonlinear systems in finite dimension; Linearized control systems and fixed-point methods; Iterated Lie brackets; Return method; Quasi-static deformations; Power series expansion; Previous methods applied to a Schrödinger equation; *Stabilization:* Linear control systems in finite dimension and applications to nonlinear control systems; Stabilization of nonlinear control systems in finite dimension; Feedback design tools; Applications to some partial differential equations; Elementary results on semigroups of linear operators; Degree theory; Bibliography; List of symbols; Index.

Mathematical Surveys and Monographs, Volume 136

April 2007, approximately 427 pages, Hardcover, ISBN-10: 0-8218-3668-4, ISBN-13: 978-0-8218-3668-2, 2000 *Mathematics Subject Classification:* 93B05, 93B52, 93C10, 93C20, 93D15; 35K50, 35L50, 35L60, 35Q30, 35Q53, 35Q55, 93D09, 93D20, **All AMS members US$79**, List US$99, Order code SURV/136

Discrete Mathematics and Combinatorics

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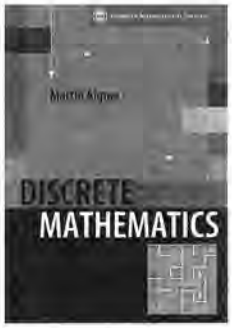

Discrete Mathematics

Martin Aigner, *Freie Universität Berlin, Germany*

The advent of fast computers and the search for efficient algorithms revolutionized combinatorics and brought about the field of discrete mathematics. This book is an introduction to the main ideas and results of discrete mathematics, and with its emphasis on algorithms it should be interesting to mathematicians and computer scientists alike. The book is organized into three parts: enumeration, graphs and algorithms, and algebraic systems. There are 600 exercises with hints and solutions to about half of them. The only prerequisites for understanding everything in the book are linear algebra and calculus at the undergraduate level.

Praise for the German edition...

This book is a well-written introduction to discrete mathematics and is highly recommended to every student of mathematics and computer science as well as to teachers of these topics.

—Konrad Engel for MathSciNet

Martin Aigner is a professor of mathematics at the Free University of Berlin. He received his PhD at the University of Vienna and has held a number of positions in the USA and Germany before moving to Berlin. He is the author of several books on discrete mathematics, graph theory, and the theory of search. The Monthly article *Turan's graph theorem* earned him a 1995 Lester R. Ford Prize of the MAA for expository writing, and his book *Proofs from the BOOK* with Günter M. Ziegler has been an international success with translations into 12 languages.

Contents: *Counting:* Fundamentals; Summation; Generating functions; Counting patterns; Asymptotic analysis; Bibliography for Part 1; *Graphs and algorithms:* Graphs; Trees; Matchings and networks; Searching and sorting; General optimization methods; Bibliography for Part 2; *Algebraic systems:* Boolean algebras; Modular arithmetic; Coding; Cryptography; Linear optimization; Bibliography for Part 3; Solutions to selected exercises; Index.

April 2007, 388 pages, Hardcover, ISBN-10: 0-8218-4151-3, ISBN-13: 978-0-8218-4151-8, LC 2006052285, 2000 *Mathematics Subject Classification:* 05A10, 05A15, 05B05, 05B15, 05C70, 05C85, 06E30, 11A07, 90C05, 94B05, **All AMS members US$47**, List US$59, Order code DISCMAT

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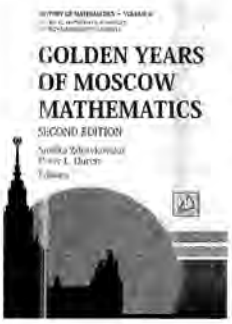


Golden Years of Moscow Mathematics

Second Edition

Smilka Zdravkovska, *Mathematical Reviews, Ann Arbor, MI,* and **Peter L. Duren,** *University of Michigan, Ann Arbor, MI,* Editors

This volume contains articles on the history of Soviet mathematics, many of which are personal accounts by mathematicians who witnessed and contributed to the turbulent and glorious years of Moscow mathematics. The articles in the book focus on mathematical developments in that era, the personal lives of Russian mathematicians, and political events that shaped the course of scientific work in the Soviet Union. Important contributions include an article about Luzin and his school, based in part on documents that were released only after perestroika, and two articles on Kolmogorov. The volume concludes with annotated bibliographies in English and Russian for further reading.

The revised edition is appended by an article of Tikhomirov, which provides an update and general overview of 20th-century Moscow mathematics, and it also includes an Index of Names.

This book should appeal to mathematicians, historians, and anyone else interested in Soviet mathematical history.

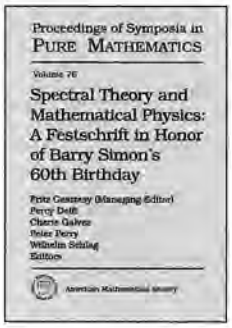
Co-published with the London Mathematical Society beginning with Volume 4. Members of the LMS may order directly from the AMS at the AMS member price. The LMS is registered with the Charity Commissioners.

Contents: **A. P. Yushkevich**, Encounters with mathematicians; **S. S. Demidov**, The Moscow school of the theory of functions in the 1930s; **E. M. Landis**, About mathematics at Moscow State University in the late 1940s and early 1950s; **B. A. Rosenfeld**, Reminiscences of Soviet mathematicians; **V. M. Tikhomirov**, A. N. Kolmogorov; **V. I. Arnol'd**, On A. N. Kolmogorov; **M. M. Postnikov**, Pages of a mathematical autobiography (1942–1953); **B. A. Kushner**, Markov and Bishop: An essay in memory of A. A. Markov (1903–1979) and E. Bishop (1928–1983); **I. Piatetski-Shapiro**, Étude on life and automorphic forms in the Soviet Union; **D. B. Fuchs**, On Soviet mathematics of the 1950s and 1960s; **A. B. Sossinsky**, In the other direction; **S. S. Demidov**, A brief survey of the literature on the development of mathematics in the USSR; **S. S. Demidov**, Russian bibliography; **V. M. Tikhomirov**, Moscow mathematics—Then and now; Errata; Index of names.

History of Mathematics, Volume 6

May 2007, approximately 300 pages, Hardcover, ISBN-10: 0-8218-4261-7, ISBN-13: 978-0-8218-4261-4, LC 2006052223, 2000 *Mathematics Subject Classification:* 01-06, 01A60, 01A65, 01A70, **All AMS members US$47**, List US$59, Order code HMATH/6.R

Mathematical Physics



Spectral Theory and Mathematical Physics: A Festschrift in Honor of Barry Simon's 60th Birthday

Fritz Gesztesy, Managing Editor, *University of Missouri, Columbia, MO,* **Percy Deift,** *New York University, Courant Institute, NY,* **Cherie Galvez,** *California Institute of Technology, Pasadena, CA,* **Peter Perry,** *University of Kentucky, Lexington, KY,* and **Wilhelm Schlag,** *University of Chicago, IL,* Editors

This Festschrift had its origins in a conference called SimonFest held at Caltech, March 27–31, 2006, to honor Barry Simon's 60th birthday. It is not a proceedings volume in the usual sense since the emphasis of the majority of the contributions is on reviews of the state of the art of certain fields, with particular focus on recent developments and open problems. The bulk of the articles in this Festschrift are of this survey form, and a few review

Simon's contributions to a particular area. Part 1 contains surveys in the areas of Quantum Field Theory, Statistical Mechanics, Nonrelativistic Two-Body and N-Body Quantum Systems, Resonances, Quantum Mechanics with Electric and Magnetic Fields, and the Semiclassical Limit. Part 2 contains surveys in the areas of Random and Ergodic Schrödinger Operators, Singular Continuous Spectrum, Orthogonal Polynomials, and Inverse Spectral Theory. In several cases, this collection of surveys portrays both the history of a subject and its current state of the art. Exhaustive lists of references enhance the presentation offered in these surveys.

A substantial part of the contributions to this Festschrift are survey articles on the state of the art of certain areas with special emphasis on open problems. This will benefit graduate students as well as researchers who want to get a quick yet comprehensive introduction into an area covered in this volume.

This item will also be of interest to those working in analysis.

Contents: Part I: *Quantum field theory, statistical mechanics:* **M. Aizenman**, Perspectives in statistical mechanics; **T. Chen** and **J. Fröhlich**, Coherent infrared representations in non-relativistic QED; **B. Nachtergaele**, Quantum spin systems after DLS 1978; **L. Rosen**, Barry Simon's contributions to quantum field theory; *Nonrelativistic two-body and N-body quantum systems, resonances:* **M. S. Ashbaugh** and **R. D. Benguria**, Isoperimetric inequalities for eigenvalues of the Laplacian; **E. B. Davies**, Non-self-adjoint operators and pseudospectra; **R. Froese**, Barry Simon's contribution to non-relativistic quantum mechanics: Two-body and N-body Schrödinger operators and resonances; **C. Gérard**, N-body quantum scattering and quantum resonances: An overview; **G. A. Hagedorn** and **A. Joye**, Mathematical analysis of Born-Oppenheimer approximations; **E. M. Harrell II**, Perturbation theory and atomic resonances since Schrödinger's time; **A. Lenard**, On a theorem for quantum mechanical scattering theory; **V. Maz'ya**, Analytic criteria in the qualitative spectral analysis of the Schrödinger operator; **P. Perry**, The spectral geometry of geometrically finite hyperbolic manifolds; **Y. Pinchover**, Topics in the theory of positive solutions of second-order elliptic and parabolic partial differential equations; **W. P. Reinhardt**, Complex scaling in atomic physics: A staging ground for experimental mathematics and for extracting physics from otherwise impossible computations; **A. V. Sobolev**, Recent results on the Bethe-Sommerfeld conjecture; *Electric and magnetic fields, semiclassical limit:* **L. Erdős**, Recent developments in quantum mechanics with magnetic fields; **G. M. Graf**, Aspects of the integer quantum Hall effect; **I. W. Herbst**, Barry Simon's work on electric and magnetic fields and the semi-classical limit; **D. Hundertmark**, Some bound state problems in quantum mechanics; **Part II:** *Random and ergodic Schrödinger operators, singular continuous spectrum:* **J. Bourgain**, A new approach to spectral gap problems; **D. Damanik**, Strictly ergodic subshifts and associated operators; **D. Damanik**, Lyapunov exponents and spectral analysis of ergodic Schrödinger operators: A survey of Kotani theory and its applications; **S. A. Denisov** and **A. Kiselev**, Spectral properties of Schrödinger operators with decaying potentials; **M. Goldstein** and **W. Schlag**, On the formation of gaps in the spectrum of Schrödinger operators with quasi-periodic potentials; **S. Jitomirskaya**, Ergodic Schrödinger operators (on one foot); **W. Kirsch** and **B. Metzger**, The integrated density of states for random Schrödinger operators; **Y. Last**, Exotic spectra: A review of Barry Simon's central contributions; *Orthogonal polynomials, inverse spectral theory:* **P. Deift**, Riemann-Hilbert methods in the theory of orthogonal polynomials; **F. Gesztesy**, Inverse spectral theory as influenced by Barry Simon; **L. Golinskii** and **V. Totik**, Orthogonal polynomials: From Jacobi to Simon; **S. Khrushchev**, Orthogonal polynomials: The first minutes; **R. Killip**, Spectral theory via sum rules; **F. Gesztesy, P. Deift, C. Galvez, P. Perry**, and **W. Schlag**, Barry Simon's list of publications.

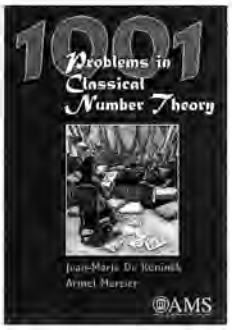
Proceedings of Symposia in Pure Mathematics, Volume 76

Part 1: March 2007, 496 pages, Hardcover, ISBN-10: 0-8218-4248-X, ISBN-13: 978-0-8218-4248-5, 2000 *Mathematics Subject Classification:* 00B30, 34-XX, 35-XX, 47-XX, 81-XX, 82-XX, **All AMS members US$89**, List US$111, Order code PSPUM/76.1

Part 2: March 2007, 464 pages, Hardcover, ISBN-10: 0-8218-4249-8, ISBN-13: 978-0-8218-4249-2, 2000 *Mathematics Subject Classification:* 00B30, 34-XX, 35-XX, 47-XX, 81-XX, 82-XX, **All AMS members US$78**, List US$97, Order code PSPUM/76.2

Set: March 2007, 960 pages, Hardcover, ISBN-10: 0-8218-3783-4, ISBN-13: 978-0-8218-3783-2, 2000 *Mathematics Subject Classification:* 00B30, 34-XX, 35-XX, 47-XX, 81-XX, 82-XX, **All AMS members US$135**, List US$169, Order code PSPUM/76

Number Theory



1001 Problems in Classical Number Theory

Jean-Marie De Koninck, *Université Laval, Quebec, QC, Canada*, and **Armel Mercier**, *Université du Québec à Chicoutimi, QC, Canada*

In the spirit of *The Book of the One Thousand and One Nights*, the authors offer 1001 problems in number theory in a way that entices the reader to immediately attack the next problem. Whether a novice or an experienced mathematician, anyone fascinated by numbers will find a great variety of problems—some simple, others more complex—that will provide them with a wonderful mathematical experience.

Contents: Key elements from the theory; Statements of the problems; Solutions; Bibliography; Terminology index; Index of authors.

May 2007, 336 pages, Hardcover, ISBN-10: 0-8218-4224-2, ISBN-13: 978-0-8218-4224-9, 2000 *Mathematics Subject Classification:* 11A05, 11A07, 11A15, 11A25, 11A41, 11A51, 11A55, 11D04, 11N05, 11N25, **All AMS members US$39**, List US$49, Order code PINT

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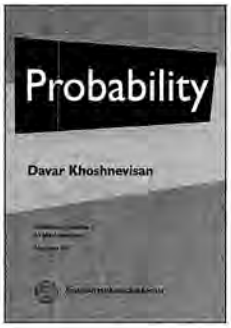

Probability

Davar Khoshnevisan, *University of Utah, Salt Lake City, UT*

This is a textbook for a one-semester graduate course in measure-theoretic probability theory, but with ample material to cover an ordinary year-long course at a more leisurely pace. Khoshnevisan's approach is to develop the ideas that are absolutely central to modern probability theory, and to showcase them by presenting their various applications. As a result, a few of the familiar topics are replaced by interesting non-standard ones.

The topics range from undergraduate probability and classical limit theorems to Brownian motion and elements of stochastic calculus. Throughout, the reader will find many exciting applications of probability theory and probabilistic reasoning. There are numerous exercises, ranging from the routine to the very difficult. Each chapter concludes with historical notes.

Contents: Classical probability; Bernoulli trials; Measure theory; Integration; Product spaces; Independence; The central limit theorem; Martingales; Brownian motion; Terminus: Stochastic integration; Background material; Bibliography; Index.

Graduate Studies in Mathematics, Volume 80

April 2007, 224 pages, Hardcover, ISBN-10: 0-8218-4215-3, ISBN-13: 978-0-8218-4215-7, LC 2006052603, 2000 *Mathematics Subject Classification:* 60-01; 60-03, 28-01, 28-03, **All AMS members US$36,** List US$45, Order code GSM/80

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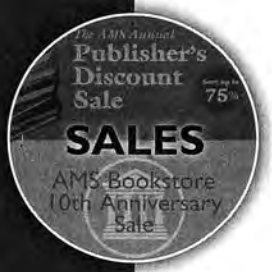

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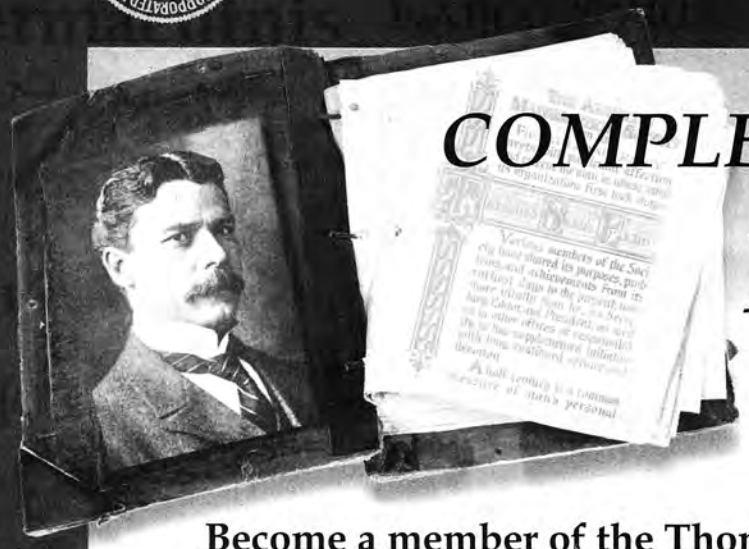

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> A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas.
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> —G. H. Hardy, A Mathematician's Apology

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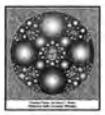
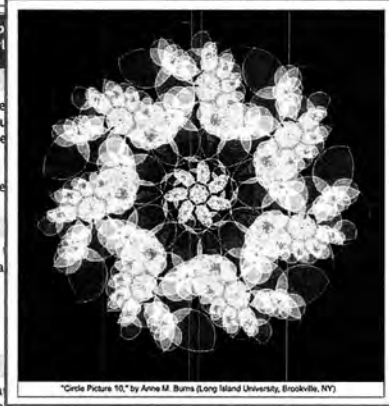
--- Anne M. Burns

Notices of the American Mathematical Society :: Cover Art



People have long been fascinated with repea
symmetries. The discovery of hyperbolic geo
greater wealth of patterns, some popularized by Dutch artist M. C. Escher in his Circle Limit
series of works. The cover illustration on this issue of the Notices portrays a pattern which is symmetric under a group generated by two Möbius transformations. These are not distance-preserving, but they do preserve angles between curves and they map circles to circles. See Double Cusp Group by David J. Wright in Notices of the American Mathematical Society (December 2004, p. 1322).



"Circle Picture 10," by Anne M. Burns (Long Island University, Brookville, NY)

Dear Peter,

Here's one of the e-postcards from the site.

Nancy

GALLERIES & MUSEUMS

M.C. Escher: the Official Website
The KnotPlot Site
Bridges: Mathematical Connections in Art, Music, and Science
Images and Mathematics, MathArchives
Mathematics Museum (Japan)
Kalendar, by Herwig Hauser

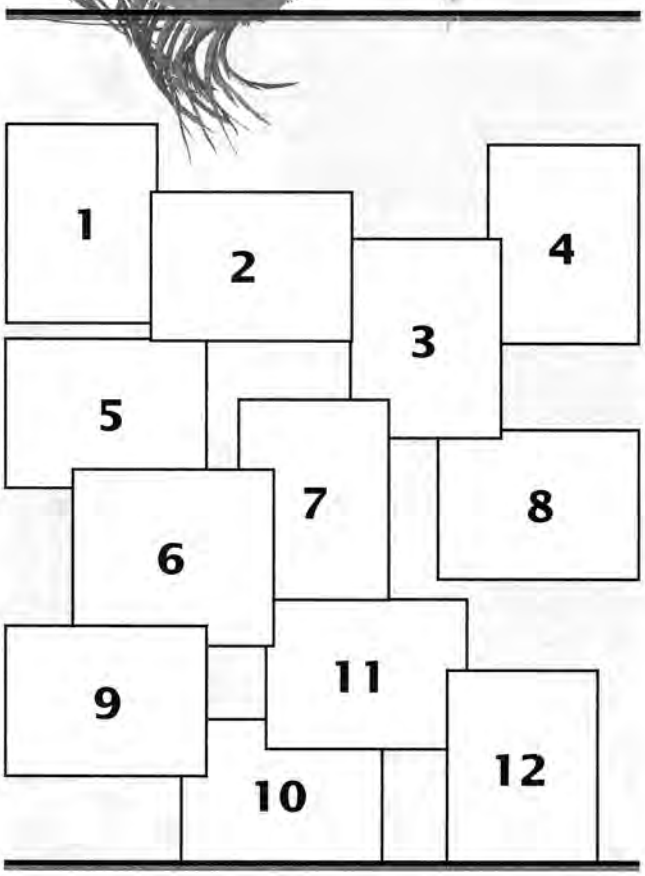
ARTICLES & RESOURCES

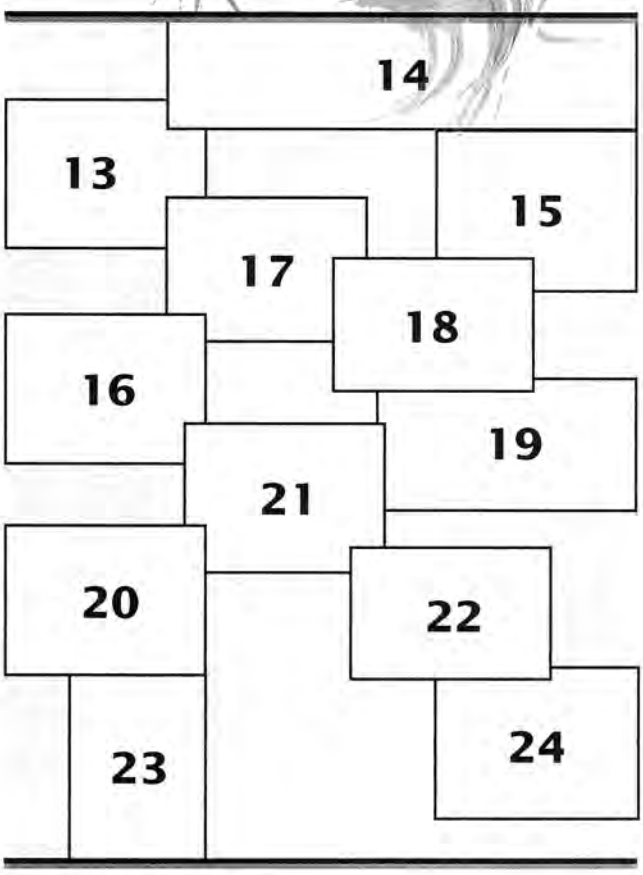
Mathematics and Art, (The theme for Mathematics Awareness Month in 2003)
International Society of the Arts, Mathematics and Architecture
Maths and Art: the whistlestop tour, by Lewis Dartnell
Geometry in Art & Architecture, by Paul Calter (Dartmouth College)
Harmony and Proportion, by John Boyd-Brent
Viewpoints: Mathematics and Art, by Annalisa Crannell (Franklin & Marshall College) and Marc Frantz (Indiana University)
Mathematics and Art, the April 2003 Feature Column by Joe Malkevitch
Art & Music, MathArchives

Explore the world of mathematics and art, send an e-postcard, and bookmark this page to see new featured works.

www.ams.org/mathimagery

2007 Joint Mathematics Meetings in New Orleans Montage Key

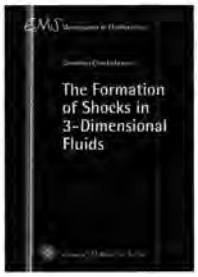




1. MAA Minicourse speaker, **Donald Saari**.
2. Exhibits ribbon-cutting ceremony. Left to right:
   John Ewing (AMS), **Penny Pina** (AMS), **Martha Siegel** (MAA), **Carl Cowen** (MAA), **James Arthur** (AMS), **Tina Straley** (MAA), **Robert Daverman** (AMS).
3. Invited Speaker, **Manjul Bhargava**.
4. *Who Wants To Be a Mathematician* host, **Mike Breen**.
5. Email Center.
6. Networking.
7. Steele Prize winner, **David Mumford**.
8. Opening of Exhibits.
9. Poster Session.
10. AMS booth in exhibits area.
11. Glass & geometry exhibit.
12. Robbins Prize winner, **Samuel Ferguson**.
13. AMS book sales booth.
14. Employment Center.
15. Morgan Prize winner, **Daniel Kane**.
16. AMS president, **James Arthur**.
17. Veblen Prize winner, **Peter Oszváth**.
18. MAA booth in exhibits area.
19. *Who Wants to Be a Mathematician* contestants (prizewinner, **Samaneh Khoshini**, at right)
20. Employment Center.
21. Poster Session.
22. **James Arthur** (left) presents Veblen Prize to **Peter Kronheimer** (center) and **Tomasz Mrowka**. Seated at front is MAA president **Carl Cowen**.
23. Satter Prize winner, **Claire Voisin**.
24. Audience in lecture hall.

New books from the

European Mathematical Society



EMS Monographs in Mathematics

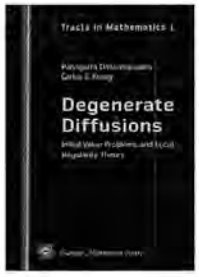
Demetrios Christodoulou

The Formation of Shocks in 3-Dimensional Fluids

ISBN 978-3-03719-031-9. January 2007, 1000 pages, hardcover, 16.5 cm x 23.5 cm. 198.00 US$

This monograph considers the relativistic Euler equations in three space dimensions for a perfect fluid with an arbitrary equation of state. We consider initial data for these equations which outside a sphere coincide with the data corresponding to a constant state. Under suitable restriction on the size of the initial departure from the constant state, we establish theorems that give a complete description of the maximal classical development. In particular, it is shown that the boundary of the domain of the maximal classical development has a singular part where the inverse density of the wave fronts vanishes, signalling shock formation. The theorems give a detailed description of the geometry of this singular boundary and a detailed analysis of the behavior of the solution there. A complete picture of shock formation in three-dimensional fluids is thereby obtained. The approach is geometric, the central concept being that of the acoustical spacetime manifold.

The monograph will be of interest to people working in partial differential equations in general and in fluid mechanics in particular.



EMS Tracts in Mathematics 1

Panagiota Daskalopoulos
Carlos E. Kenig

Degenerate Diffusions
Initial Value Problems and Local Regularity Theory

ISBN 978-3-03719-033-3. April 2007, approx. 200 pages, 17 x 24 cm, hardcover. 62.00 US$

The book deals with existence, uniqueness, regularity and asymptotic behavior of solutions to the initial value problem (Cauchy problem) and the initial-Dirichlet problem for a class of degenerate diffusions modeled on the porous medium type equation $u_t = \Delta u^m$, $m \geq 0$, $u \geq 0$. Such models arise in plasma physics, diffusion through porous media, thin liquid film dynamics as well as in geometric flows such as the Ricci flow on surfaces and the Yamabe flow. The approach presented to these problems is through the use of local regularity estimates and Harnack type inequalities, which yield compactness for families of solutions. The theory is quite complete in the slow diffusion case ($m > 1$) and in the supercritical fast diffusion case ($m_c < m < 1$, $m_c = (n-2)_+/n$) while many problems remain in the range $m \leq m_c$. All of these aspects of the theory are discussed in the book.

The book is addressed to both researchers and graduate students with a good background in analysis and some previous exposure to partial differential equations.

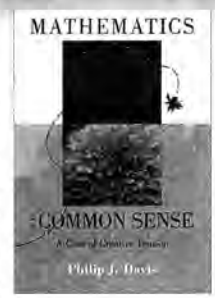
EMS Publishing House
www.ems-ph.org
orders@ems-ph.org

Orders from the Americas:
American Mathematical Society
www.ams.org/bookstore
cust-serv@ams.org

MATHEMATICS *Then and Now*

Mathematics and Common Sense
A Case of Creative Tension

Philip J. Davis
$29.95; Hardcover; 270 pp.

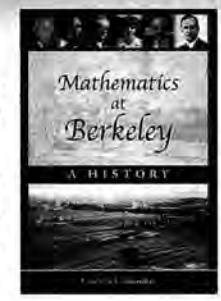


"There is hardly an aspect of human existence to which mathematics cannot contribute understanding ... In bringing some of these crucial issues to a wider audience, Davis has made a valuable contribution to the public understanding of the importance of the subject."

—**John Ball,** ***The New Scientist***

Mathematics at Berkeley
A History

Calvin C. Moore
$39.00; Hardcover; 376 pp.

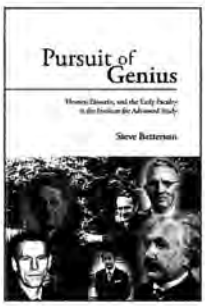


"Cal Moore has been the ultimate insider in the modern history of mathematics at Berkeley. He combines his insider's knowledge of the events in our own times with an unsurpassed curiosity about the history of how this venerable institution came into being and reached its present state. ... a fascinating account ..."

—**Elwyn Berlekamp**

Pursuit of Genius
Flexner, Einstein, and the Early Faculty at the Institute for Advanced Study

Steve Batterson
$39.00; Hardcover; 314 pp.



"The book is based primarily on a meticulous study of the institute's documentary records ... Batterson tells a fascinating story ..."

— **John Stachel,** ***Nature 445***

"Batterson's detailed rendition ... will delight anyone who has ever served on a faculty search committee."

— ***MAA Reviews***

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Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Hoboken, New Jersey

Stevens Institute of Technology

April 14–15, 2007
Saturday – Sunday

Meeting #1026
Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: February 2007
Program first available on AMS website: March 8, 2007
Program issue of electronic *Notices*: April 2007
Issue of *Abstracts*: Volume 28, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses

Neal Koblitz, University of Washington, *Stormy marriage—A periodization of the history of the relationship between mathematics and cryptography.*

Florian Luca, Universidad Nacional Autónoma de México, *Values of arithmetic functions.*

Natasa Pavlovic, Princeton University, *The enigma of the equations of fluid motion: A survey of existence and regularity results.*

Elisabeth Werner, Case Western Reserve University, *Convex bodies: Best and random approximation.*

Special Sessions

Affine Invariants, Randomness, and Approximation in Convex Geometry, **Elisabeth Werner**, Case Western Reserve University, and **Artem Zvavitch**, Kent State University.

Automorphic Forms and Arithmetic Geometry, **Gautam Chinta**, City College of New York, and **Paul E. Gunnells**, University of Massachusetts, Amherst.

Combinatorial Algebraic Geometry, **Angela C. Gibney**, University of Pennsylvania, and **Diane Maclagan**, Rutgers University.

Convex Sets, **David Larman**, University College London, and **Valeriu Soltan**, George Mason University.

Differential Algebra, **Phyllis J. Cassidy**, Smith College and The City College of CUNY, **Richard C. Churchill**, Hunter College and The Graduate Center of CUNY, **Li Guo** and **William F. Keigher**, Rutgers University at Newark, and **Jerald J. Kovacic** and **William Sit**, The City College of CUNY.

Fourier Analysis and Convexity, **Alexander Koldobsky**, University of Missouri Columbia, and **Dmitry Ryabogin**, Kansas State University.

Graph Theory and Combinatorics, **Daniel J. Gross**, **Nathan W. Kahl**, and **John T. Saccoman**, Seton Hall University, and **Charles L. Suffel**, Stevens Institute of Technology.

History of Mathematics on Leonhard Euler's Tercentenary, **Patricia R. Allaire**, Queensborough Community College, CUNY, and **Robert E. Bradley** and **Lee J. Stemkoski**, Adelphi University.

Languages and Groups, **Sean Cleary**, The City College of New York and CUNY Graduate Center, **Murray J. Elder**, Stevens Institute of Technology, and **Gretchen Ostheimer**, Hofstra University.

Mathematical Aspects of Cryptography, **Robert H. Gilman**, Stevens Institute of Technology, **Neal I. Koblitz**, University of Washington, and **Susanne Wetzel**, Stevens Institute of Technology.

Nonlinear Waves in Dissipative/Dispersive Media, **Keith S. Promislow**, Michigan State University, and **Yi Li**, Stevens Institute of Technology.

Number Theory, **Florian Luca**, Universidad Nacional Autónoma de México, and **Allison M. Pacelli**, Williams College.

Optimization of Stochastic Systems, **Darinka Dentcheva**, Stevens Institute of Technology, and **Andrzej Ruszczynski**, Rutgers University.

Tucson, Arizona

University of Arizona

April 21–22, 2007

Saturday - Sunday

Meeting #1027

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: February 2007
Program first available on AMS website: March 8, 2007
Program issue of electronic *Notices*: April 2007
Issue of *Abstracts*: Volume 28, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses

Liliana Borcea, Rice University, *Array imaging in random media.*

James Cushing, University of Arizona, Tucson, *Matrix population models & semelparity.*

Hans Lindblad, University of California, San Diego, *The weak null condition and global existence for Einstein's equations.*

Vinayak Vatsal, University of British Columbia, Vancouver, *Local splitting of ordinary Galois representations.*

Special Sessions

Advances in Spectral Theory of Operators, **Roger Roybal**, California State University, Channel Islands, and **Michael D. Wills**, Weber State University.

Algebraic Combinatorics, **Helene Barcelo** and **Susanna Fishel**, Arizona State University.

Automorphisms of Curves, **Aaron D. Wootton**, University of Portland, **Anthony Weaver**, Bronx Community College, and **S. Allen Broughton**, Rose-Hulman Institute of Technology.

Graph Theory and Combinatorics, **Sebastian M. Cioaba**, University of California at San Diego, and **Joshua Cooper**, University of South Carolina.

Inverse Problems for Wave Propagation, **Liliana Borcea**, Rice University.

Mathematical Modeling in Biology and Medicine, **Carlos Castillo-Chavez**, **Yang Kuang**, **Hal L. Smith**, and **Horst R. Thieme**, Arizona State University.

Moduli Spaces and Invariant Theory, **Philip Foth** and **Yi Hu**, University of Arizona.

New Developments and Directions in Random Matrix Theory, **Peter David Miller**, University of Michigan, and **Estelle Basor**, California Polytechnic State University.

Number Theory in the Southwest, **Dinesh S. Thakur** and **Douglas L. Ulmer**, University of Arizona.

Operator Algebras, **Steven P. Kaliszewski**, **Jack Spielberg**, and **John C. Quigg**, Arizona State University.

Partial Differential Equations and Geometric Analysis, **Sunhi Choi**, **Lennie Friedlander**, and **David Alan Glickenstein**, University of Arizona.

Representations of Algebras, **Frauke Maria Bleher**, University of Iowa, **Birge K. Huisgen-Zimmermann**, University of California Santa Barbara, and **Dan Zacharia**, Syracuse University.

Special Functions and Orthogonal Polynomials, **Diego Dominici**, State University of New York at New Paltz, and **Robert S. Maier**, University of Arizona.

Spectral Analysis on Singular and Noncompact Manifolds, **Juan Bautista Gil**, Pennyslvania State University, and **Thomas Krainer**, Pennsylvania State University.

Subjects in and around Fluid Dynamics, **Robert Owczarek**, Los Alamos National Laboratory, and **Mikhail Stepanov**, University of Arizona.

Zacatecas, Mexico

Universidad Autónoma de Zacatecas

May 23–26, 2007

Wednesday - Saturday

Meeting #1028

Seventh Joint International Meeting of the AMS and the Sociedad Matematica Mexicana.
Associate secretary: Matthew Miller
Announcement issue of *Notices*: April 2007
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: Expired
For abstracts: April 20, 2007

Invited Addresses

Monica Clapp, Universidad Nacional Autónoma de México, *Multiple solutions to an elliptic equation with critical nonlinearity.*

Edward L. Green, Virginia Polytechnic Institute & State University, *Koszul Algebras: A very special class of algebras.*

Kiran S. Kedlaya, Massachusetts Institute of Technology, *Title to be announced.*

John Lott, University of Michigan, Ann Arbor, *Curvature of metric spaces.*

Gelasio Salazar, Universidad Autónoma de San Luis Potosí, *Crossing numbers of graphs in surfaces.*

Petr Zhevandrov, Universidad Michoacana de San Nicolás de Hidalgo, *Title to be announced.*

Special Sessions

Applied Category Theory: Graph-Operad Logic, **Zbigniew Oziewicz**, UNAM, and **Hanna Makaruk**, Los Alamos National Laboratory.

Applied Mathematics in Petroleum Industry Problems, **Jorge Velasco-Hernandez**, Instituto Mexicano del Petroleo, and **William Fitzgibbon**, University of Houston.

Convexity, **Luis Montejano**, UNAM, and **Paul Goodey**, University of Oklahoma.

Differential Geometry, **Rafael Herrera Guzman**, CIMAT, Guanajuato, and **Haydee Herrera**, Rutgers University.

Functional and Harmonic Analysis (in honor of Mischa Cotlar), **Salvador Perez Esteva**, UNAM, Cuernavaca, and **Josefina Alvarez**, New Mexico State University.

Holomorphic Dynamics in the Riemann Sphere (in memory of Adrien Douady), **Monica Moreno Rocha**, CIMAT, Guanajuato, and **Araceli Medina-Bonifant**, University of Rhode Island.

Low Dimensional Topology, **Mario Eudave-Munoz**, UNAM, and **Jennifer Schultens**, University of California Davis.

Mathematical Physics, **Jaime Cruz Sampedro**, Universidad Autonoma del Estado de Hidalgo, and **Daniel Tataru**, University of California Berkeley.

Metric Differential Geometry, **Catherine Searle**, UNAM, Cuernavaca, and **Gerard Walschap**, University of Oklahoma.

Nonlinear Boundary Value Problems, **Monica Clapp**, UNAM, and **Alfonso Castro**, Harvey Mudd College.

Nonlinear Waves, **Petr Zhevandrov**, Universidad Michoacana de San Nicolás de Hidalgo, and **Arturo Vargas** and **Gustavo Cruz**, UNAM.

Operator Theory and Complex Analysis, **Enrique Ramirez de Arellano** and **Nikolai Vasilevski**, CINVESTAV, and **Raul Curto**, University of Iowa, Iowa City.

Representation Theory (in honor of Edward Green's 60th birthday), **Christof Geiss**, UNAM, and **Gordana Todorov**, Northeastern University.

Rings and Modules, **Maria Jose Arroyo**, UAM-Iztapalapa, and **Sergio R. Lopez-Permouth**, Ohio University.

Conference Web Site

Most of the information in this announcement is taken from the website maintained by the local organizers. Watch `http://matematicas.reduaz.mx/JointMeeting/DefaultI.htm` for additional program details; links to sites for hotels, tours, other local information; and continuous updates.

Abstracts

The deadline for abstract submission is April 20, 2007. Abstracts will be accepted for the Invited Addresses, Special Sessions, and a Session for Contributed Talks. Watch the conference website for the form and instructions for your electronic submission.

Accommodations

Participants should make their own arrangements for accommodations.

Hotel Emporio Zacatecas (SEDE), Av. Hidalgo No. 703, Centro Histórico, C.P. 98000 Zacatecas, telephone: 52-492-925-65-12; fax: 52-492-922-62-45; email: `zacatecas.grupos@hotelesemporio.com` is the headquarters hotel. See `http://www.hotelesemporio.com/eng/idt/66/emporio-zacatecas//` for more information. The conference rate is Mex$1,053 or about US$96 (including tax) per room. Participants should request the Joint Meeting rate when making reservations.

Other hotels in the area:

Hotel Santa Rita, Av. Hidalgo No. 507, Centro Zacatecas, 52-492-925-41-41; `www.hotelsantarita.com`.

Hotel Casona de los Vitrales, Callejón del Espejo No. 104, Centro Zacatecas; 52-492-925-00-96; email: `ventas1@lasasonadelosvitrales.com.mx`; `www.lacasonadelosvitrales.com.mx`.

Hotel Mesón del Jobito, Jardin Juárez No. 143, Centro Colonial, Zacatecas; 52-492-924-17-22; email: `gruos@mesondegobito.com`; `www.mesondejobito.com/`.

Hotel Casa Torres, Calle 1° de Mayo No. 325, Centro Histórico, Zacatecas; 52-492-925-32-66, email: `ventas@hotelcasatorres.com`; `www.hotelcasatorres.com`.

Hotel del Bosque Teleférico, Periférico Díaz Ordaz No. 602, Centro, Zacatecas; 52-492-922-07-45; email: `ventasdelbosqueteleferico@yahoo.com`; `hotelesdelbosque.com.mx`.

Hotel Don Miguel, Prolongación Calle del Plomo s/n, Zacatecas; email: `dircomercial.grupoinguanzo@yahoo.com`; `www.donmiguel.com/mx/DM/Index_DM.htm`.

Registration and Meeting Information

Sessions will be held at the University of Zacatecas. See the conference website to register on line. There is a registration fee of US$65.

Social Events and Tours

All participants are invited to the Opening Ceremonies at 6:00 p.m. and Reception at 8:00 p.m. on Wednesday.

There are plans for a concert by the state band and a folk dancing demonstration on Thursday and Friday evenings. Also, there will be Closing Ceremonies on Saturday

at 1:30 p.m. and "Callejoneada" and dinner on Saturday evening beginning at 7:00 p.m. Please watch the conference website for ticket prices and further details.

Travel

By Air: In a valley surrounded by steep hills is the city of Zacatecas, capital of the state of the same name, founded in 1546. Its majestic buildings and colonial churches were made possible by the wealth of earlier gold and silver miners, who amassed great fortunes from its hills. The city is home to some of the country's most important art and history museums, and is renowned for its beautifully preserved and elegant pink limestone buildings. It is a true colonial gem and one of UNESCO's World Heritage Sites. Zacatecas is approximately 651 km northwest of Mexico City. Its international airport (ZCL) is served by Mexicana, Líneas Aéreas Azteca, and Aeromar airlines. Rental cars are not recommended.

All U.S. citizens traveling by air must have a valid passport to re-enter the United States after traveling out of the country. See these pages maintained by the Bureau of Consular Affairs, U.S. Department of State for specific information about travel to Mexico at `http://travel.state.gov/travel/tips/regional/regional_1174.html`.

Warsaw, Poland

University of Warsaw

July 31 – August 3, 2007

Tuesday – Friday

Meeting #1029

First Joint International Meeting between the AMS and the Polish Mathematical Society
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Invited Addresses

Henryk Iwaniec, Rutgers University, *Title to be announced.*

Tomasz J. Luczak, Adam Mickiewicz University, *Title to be announced.*

Tomasz Mrowka, Massachusetts Institute of Technology, *Title to be announced.*

Ludomir Newelski, University of Wroclaw, *Title to be announced.*

Madhu Sudan, Massachusetts Institute of Technology, *Title to be announced.*

Anna Zdunik, Warsaw University, *Title to be announced.*

Special Sessions

Arithmetic Algebraic Geometry, **Grzegorz Banaszak**, Adam Mickiewicz University, **Eric Friedlander**, Northwestern University, **Wojciech Gajda**, Adam Mickiewicz University, **Piotr Krason**, Szczecin University, and **Wieslawa Nizio**.

Complex Analysis, **Zeljko Cuckovic**, University of Toledo, **Zbigniew Blocki**, Jagiellonian University, and **Marek Ptak**, University of Agriculture.

Complex Dynamics, **Robert Devaney**, Boston University, **Jane N. Hawkins**, University of North Carolina, and **Janina Kotus**, Warsaw University of Technology.

Complexity of Multivariate Problems, **Joseph F. Traub**, Columbia University, **Grzegoorz W. Wasilkowski**, University of Kentucky, and **Henryk Wozniakowski**, Columbia University.

Control and Optimization of Non-linear PDE Systems, **Irena Lasiecka**, University of Virginia, and **Jan Sokolowski**, Systems Research institute.

Dynamical Systems, **Steven Hurder**, University of Illinois at Chicago, **Michal Misiurewicz**, Indiana University-Purdue University Indianapolis, and **Pawel Walczak**, University of Lodz.

Dynamics, Control and Optimization of Finite Dimensional Systems: Theory and Applications to Biomedicine, **Urszula Forys**, Warsaw University, **Urszula Ledzewicz**, Southern Illinois University, and **Heinz Schaettler**, Washington University.

Ergodic Theory and Topological Dynamics, **Dan Rudolph**, Colorado State University, and **Mariusz Lemanczyk**, Nicholas Copernicus University.

Extremal and Probabilistic Combinatorics, **Joel Spencer**, New York University-Courant Institute, and **Michal Karonski** and **Andrzej Rucinski**, Adam Mickiewicz University.

Function Spaces, Theory of Operators and Geometry of Banach Spaces, **Henryk Hudzik**, Adam Mickiewicz University, **Anna Kaminska**, University of Memphis, and **Mieczyslaw Mastylo**.

Geometric Applications of Homotopy Theory, **Yuli B. Rudyak**, University of Florida, **Boguslaw Hajduk**, Warsaw University, **Jaroslaw Kedra**, University of Aberdeen, and **Aleksy Tralle**, The College of Economics & Comp Science.

Geometric Function Theory, **Michael Dorff**, Brigham Young University, **Piotr Liczberski**, University of Lodz, **Maria Nowak**, Biblioteka Instytutu Matematyki, and **Ted Suffridge**, University of Kentucky.

Geometric Group Theory, **Mladen Bestvina**, University of Utah, **Tadeusz Januszkiewicz**, Ohio State University, and **Jacek Swiatkowski**, University of Wroclaw.

Geometric Topology, **Jerzy Dydak**, University of Tennessee, **Slawomir Nowak**, and **Stanislaw Spiez**, University of Warsaw.

Invariants of Links and 3-manifolds, **Mieczyslaw Dabkowski**, University of Texas at Dallas, **Jozef H. Przytycki**, George Washington University, **Adam S. Siroka**, State

University of New York at Buffalo, and **Pawel Traczyk**, Warsaw University.

Issues in Reforming Mathematics Education, **Jeremy Kilpatrick**, University of Georgia, and **Zbigniew Semadeni**, University of Warsaw.

Mathematics of Large Quantum Systems, **Michael Loss**, Georgia Institute of Technology, **Jan Philip Solovej**, University of Copenhagen, and **Jan Derezinski**, University of Warsaw.

Noncommutative Geometry and Quantum Groups, **Paul Baum**, Pennsylvania State University, and **Ulrich Kraehmer** and **Tomasz Maszczyk**.

Partial Differential Equations of Evolution Type, **Susan J. Friedlander**, University of Illinois at Chicago, and **Grzegorz A. Karch**, University of Wroclaw.

Quantum Information Theory, **Robert Alicki**, University of Gdansk, and **Mary Beth Ruskai**, Tufts University.

Topological Fixed Point Theory and Related Topics, **Jerzy Jezierski**, University of Agriculture, **Wojciech Kryszewski**, Nicholas Copernicus University, and **Peter Wong**, Bates College.

Topology of Manifolds and Transformation Groups, **Slawomir Kwasik**, Tulane University, **Krzysztof Pawalowski**, Adam Mickiewicz University, and **Dariusz Wilczynski**, Utah State University.

Chicago, Illinois

DePaul University

October 5–6, 2007

Friday - Saturday

Meeting #1030

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: August 2007
Program first available on AMS website: August 16, 2007
Program issue of electronic *Notices*: October 2007
Issue of *Abstracts*: Volume 28, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 19, 2007
For abstracts: August 7, 2007

Invited Addresses

Martin Golubitsky, University of Houston, *Title to be announced.*

Matthew J. Gursky, University of Notre Dame, *Title to be announced.*

Alex Iosevich, University of Missouri, *Title to be announced.*

David E. Radford, University of Illinois at Chicago, *Title to be announced.*

Special Sessions

Algebraic Combinatorics: Association Schemes and Related Topics (Code: SS 1A), **Sung Y. Song**, Iowa State University, and **Paul Terwilliger**, University of Wisconsin.

Algebraic Geometry (Code: SS 5A), **Lawrence Man Hou Ein** and **Anatoly S. Libgober**, University of Illinois at Chicago.

Extremal and Probabilistic Combinatorics (Code: SS 3A), **Jozsef Balogh**, University of Illinois at Urbana-Champaign, and **Dhruv Mubayi**, University of Illinois at Chicago.

Hopf Algebras and Related Areas (Code: SS 2A), **Yevgenia Kashina** and **Leonid Krop**, DePaul University, **M. Susan Montgomery**, University of Southern California, and **David E. Radford**, University of Illinois at Chicago.

The Euler and Navier-Stokes Equations (Code: SS 4A), **Alexey Cheskidov**, University of Michigan, and **Susan J. Friedlander** and **Roman Shvydkoy**, University of Illinois at Chicago.

New Brunswick, New Jersey

Rutgers University-New Brunswick, Busch Campus

October 6–7, 2007

Saturday - Sunday

Meeting #1031

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: August 2007
Program first available on AMS website: August 16, 2007
Program issue of electronic *Notices*: October 2007
Issue of *Abstracts*: Volume 28, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 19, 2007
For abstracts: August 7, 2007

Invited Addresses

Satyan L. Devadoss, Williams College, *Title to be announced.*

Tara S. Holm, University of Connecticut, *Title to be announced.*

Sir Roger Penrose, University of Oxford, *Title to be announced* (Einstein Public Lecture in Mathematics).

Scott Sheffield, Courant Institute and Institute for Advanced Study, *Title to be announced.*

Mu-Tao Wang, Columbia University, *Title to be announced.*

Special Sessions

Commutative Algebra (Code: SS 4A), **Jooyoun Hong**, University of California Riverside, and **Wolmer V. Vasconcelos**, Rutgers University.

Mathematical and Physical Problems in the Foundations of Quantum Mechanics (in honor of Shelly Goldstein's 60th birthday) (Code: SS 3A), **Roderich Tumulka** and **Detlef Dürr**, München University, and **Nino Zanghi**, University of Genova.

Partial Differential Equations in Mathematical Physics (in honor of Shelly Goldstein's 60th birthday) (Code: SS 2A), **Sagun Chanillo**, **Michael K.-H. Kiessling**, and **Avy Soffer**, Rutgers University.

Partial Differential Equations of Mathematical Physics, I (dedicated to the memory of Tom Branson) (Code: SS 7A), **Sagun Chanillo**, **Michael K.-H. Kiessling**, and **Avy Soffer**, Rutgers University.

Probability and Combinatorics (Code: SS 1A), **Jeffry N. Kahn** and **Van Ha Vu**, Rutgers University.

Set Theory of the Continuum (Code: SS 5A), **Simon R. Thomas**, Rutgers University.

Toric Varieties (Code: SS 6A), **Milena S. Hering**, Institute for Mathematics and Its Applications, and **Diane Maclagan**, Rutgers University.

Albuquerque, New Mexico

University of New Mexico

October 13–14, 2007

Saturday - Sunday

Meeting #1032

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2007
Program first available on AMS website: August 30, 2007
Program issue of electronic *Notices*: October 2007
Issue of *Abstracts*: Volume 28, Issue 4

Deadlines

For organizers: March 13, 2007
For consideration of contributed papers in Special Sessions: June 26, 2007
For abstracts: August 21, 2007

Invited Addresses

Emmanuel Candes, California Institute of Technology, *Title to be announced.*

Alexander Polischuk, Univerity of Oregon, *Title to be announced.*

Eric Raines, University of California Davis, *Title to be announced.*

William E. Stein, University of California San Diego, *SAGE: Software for Algebra and Geometry Experimentation.*

Special Sessions

Affine Algebraic Geometry (Code: SS 2A), **Dennis Robert Finston**, New Mexico State University.

Computational Methods in Harmonic Analysis and Signal Processing (Code: SS 1A), **Emmanuel Candes**, California Institute of Technology, and **Joseph D. Lakey**, New Mexico State University.

Recent Developments in 2-D Turbulence (Code: SS 3A), **Michael S. Jolly**, Indiana University, and **Greg Eyink**, Johns Hopkins University.

Murfreesboro, Tennessee

Middle Tennessee State University

November 3–4, 2007

Saturday - Sunday

Meeting #1033

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: September 2007
Program first available on AMS website: September 20, 2007
Program issue of electronic *Notices*: November 2007
Issue of *Abstracts*: Volume 28, Issue 4

Deadlines

For organizers: April 3, 2007
For consideration of contributed papers in Special Sessions: July 17, 2007
For abstracts: September 11, 2007

Invited Addresses

Sergey Gavrilets, University of Tennessee, *Title to be announced.*

Daniel K. Nakano, University of Georgia, *Title to be announced.*

Carla D. Savage, North Carolina State University, *Title to be announced.*

Sergei Tabachnikov, Pennsylvania State University, *Ubiquitous billiards.*

Special Sessions

Advances in Algorithmic Methods for Algebraic Structures (Code: SS 3A), **James B. Hart**, Middle Tennessee State University.

Applied Partial Differential Equations (Code: SS 4A), **Yuri A. Melnikov**, Middle Tennessee State University, and **Alain J. Kassab**, University of Central Florida.

Billiards and Related Topics (Code: SS 6A), **Sergei Tabachnikov**, Pennsylvania State University, and **Richard Schwartz**, Brown University.

Differential Equations and Dynamical Systems (Code: SS 1A), **Wenzhang Huang** and **Jia Li**, University of Alabama,

Huntsville, and **Zachariah Sinkala**, Middle Tennessee State University.

Graph Theory (Code: SS 2A), **Rong Luo**, **Don Nelson**, **Chris Stephens**, and **Xiaoya Zha**, Middle Tennessee State University.

Splines and Wavelets with Applications (Code: SS 5A), **Don Hong**, Middle Tennessee State University, and **Qingtang Jiang**, University of Missouri-St. Louis.

Wellington, New Zealand

Victoria University of Wellington

December 12–15, 2007

Wednesday - Saturday

Meeting #1034

First Joint International Meeting between the AMS and the New Zealand Mathematical Society (NZMS).
Associate secretary: Matthew Miller
Announcement issue of *Notices*: June 2007
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: March 31, 2007
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

AMS Invited Addresses

Marston Conder, University of Auckland, *Title to be announced.*

Rodney G. Downey, Victoria University of Wellington, *Title to be announced.*

Michael H. Freedman, Microsoft Research/University of California Santa Barbara, *Title to be announced.*

Gaven J. Martin, Massey University, *Title to be announced.*

Assaf Naor, Microsoft Research/Courant Institute, *Title to be announced.*

Theodore A. Slaman, University of California Berkeley, *Title to be announced.*

Matthew J. Visser, Victoria University of Wellington, *Title to be announced.*

AMS Special Sessions

Computability Theory, **Rodney G. Downey** and **Noam Greenberg**, Victoria University of Wellington.

Dynamical Systems: Probabilistic and Semigroup Methods, **Arno Berger**, University of Canterbury, **Rua Murray**, University of Waikato, and **Matthew J. Nicol**, University of Houston.

Hopf Algebras and Quantum Groups, **M. Susan Montgomery**, University of Southern California, and **Yinhuo Zhang**, Victoria University of Wellington.

Infinite-Dimensional Groups and Their Actions, **Christopher Atkin**, Victoria University of Wellington, **Greg Hjorth**, University of California Los Angeles/University of Melbourne, **Alica Miller**, University of Louisville, and **Vladimir Pestov**, University of Ottawa.

Matroids, Graphs, and Complexity, **Dillon Mayhew**, Victoria University of Wellington, and **James G. Oxley**, Louisiana State University.

New Trends in Spectral Analysis and Partial Differential Equations, **Boris P. Belinskiy**, University of Tennessee, Chattanooga, **Anjan Biswas**, Delaware State University, and **Boris Pavlov**, University of Auckland.

Quantum Topology, **David B. Gauld**, University of Auckland, and **Scott E. Morrison**, University of California Berkeley.

Special Functions and Orthogonal Polynomials, **Shaun Cooper**, Massey University, **Diego Dominici**, SUNY New Paltz, and **Sven Ole Warnaar**, University of Melbourne.

San Diego, California

San Diego Convention Center

January 6–9, 2008

Sunday - Wednesday

Meeting #1035

Joint Mathematics Meetings, including the 114th Annual Meeting of the AMS, 91st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2007
Program first available on AMS website: November 1, 2007
Program issue of electronic *Notices*: January 2008
Issue of *Abstracts*: Volume 29, Issue 1

Deadlines

For organizers: April 1, 2007
For consideration of contributed papers in Special Sessions: July 26, 2007
For abstracts: September 20, 2007

New York, New York

Courant Institute of New York University

March 15–16, 2008
Saturday – Sunday

Meeting #1036
Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: August 15, 2007
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Special Sessions
L-Functions and Automorphic Forms (Code: SS 1A), **Alina Bucur**, Institute for Advanced Study, **Ashay Venkatesh**, Courant Institute of Mathematical Sciences, **Stephen D. Miller**, Rutgers University, and **Steven J. Miller**, Brown University.

Baton Rouge, Louisiana

Louisiana State University, Baton Rouge

March 28–30, 2008
Friday – Sunday

Meeting #1037
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: August 28, 2007
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Bloomington, Indiana

Indiana University

April 4–6, 2008
Friday – Sunday

Meeting #1038
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: September 4, 2007
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Special Sessions
Combinatorial and Geometric Aspects of Commutative Algebra (Code: SS 1A), **Juan Migliore**, University of Notre Dame, and **Uwe Nagel**, University of Kentucky.

Claremont, California

Claremont McKenna College

May 3–4, 2008
Saturday – Sunday

Meeting #1039
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: October 4, 2007
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Rio de Janeiro, Brazil

Instituto Nacional de Matemática Pura e Aplicada (IMPA)

June 4–7, 2008
Wednesday - Saturday

Meeting #1040
First Joint International Meeting between the AMS and the Sociedade Brasileira de Matemática.
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Vancouver, Canada

University of British Columbia and the Pacific Institute of Mathematical Sciences (PIMS)

October 4–5, 2008
Saturday - Sunday

Meeting #1041
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: March 9, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Middletown, Connecticut

Wesleyan University

October 11–12, 2008
Saturday - Sunday

Meeting #1042
Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: March 11, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Kalamazoo, Michigan

Western Michigan University

October 17–19, 2008
Friday - Sunday

Meeting #1043
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: March 17, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Huntsville, Alabama

University of Alabama, Huntsville

October 24–26, 2008
Friday - Sunday

Meeting #1044
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 24, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Shanghai, People's Republic of China

Fudan University

December 17–21, 2008

Wednesday – Sunday

Meeting #1045

First Joint Interntional Meeting Between the AMS and the Shanghai Mathematical Society
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Invited Addresses

L. Craig Evans, University of California Berkeley, *Title to be announced.*

Zhi-Ming Ma, Chinese Academy of Sciences, *Title to be announced.*

Richard Schoen, Stanford University, *Title to be announced.*

Richard Taylor, Harvard University, *Title to be announced.*

Xiaoping Yuan, Fudan University, *Title to be announced.*

Weiping Zhang, Chern Institute, *Title to be announced.*

Washington, District of Columbia

Marriott Wardman Park Hotel and Omni Shoreham Hotel

January 7–10, 2009

Wednesday – Saturday
Joint Mathematics Meetings, including the 115th Annual Meeting of the AMS, 92nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: October 2008
Program first available on AMS website: November 1, 2008
Program issue of electronic *Notices*: January 2009
Issue of *Abstracts*: Volume 30, Issue 1

Deadlines

For organizers: April 1, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Urbana, Illinois

University of Illinois at Urbana-Champaign

March 27–29, 2009

Friday – Sunday
Southeastern Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 29, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Raleigh, North Carolina

North Carolina State University

April 4–5, 2009

Saturday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 4, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Francisco, California

San Francisco State University

April 25–26, 2009

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 25, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Boca Raton, Florida

Florida Atlantic University

October 30 – November 1, 2009

Friday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 30, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Francisco, California

Moscone Center West and the San Francisco Marriott

January 6–9, 2010

Wednesday – Saturday
Joint Mathematics Meetings, including the 116th Annual Meeting of the AMS, 93rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society of Industrial and Applied Mathematics (SIAM).
Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2009
Program first available on AMS website: November 1, 2009
Program issue of electronic *Notices*: January 2010
Issue of *Abstracts*: Volume 31, Issue 1

Deadlines

For organizers: April 1, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

New Orleans, Louisiana

New Orleans Marriott and Sheraton New Orleans Hotel

January 5–8, 2011

Wednesday – Saturday
Joint Mathematics Meetings, including the 117th Annual Meeting of the AMS, 94th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: October 2010

Program first available on AMS website: November 1, 2010
Program issue of electronic *Notices*: January 2011
Issue of *Abstracts*: Volume 32, Issue 1

Deadlines

For organizers: April 1, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Boston, Massachusetts

John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel

January 4–7, 2012

Wednesday - Saturday
Joint Mathematics Meetings, including the 118th Annual Meeting of the AMS, 95th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2011
Program first available on AMS website: November 1, 2011
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 33, Issue 1

Deadlines

For organizers: April 1, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013

Wednesday - Saturday
Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center

January 15–18, 2014

Wednesday - Saturday
Joint Mathematics Meetings, including the 120th Anual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Sproul Hall, Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

Central Section: Susan-J. Friedlander, Department of Mathematics, University of Illinois at Chicago, 851 S. Morgan (M/C 249), Chicago, IL 60607-7045; e-mail: susan@math.nwu.edu; telephone: 312-996-3041.

Eastern Section: Lesley-M. Sibner, Department of Mathematics, Polytechnic University, Brooklyn, NY 11201-2990; e-mail: lsibner@duke.poly.edu; telephone: 718-260-3505.

Southeastern Section: Matthew Miller, Department of Mathematics, University of South Carolina, Columbia, SC 29208-0001, e-mail: miller@math.sc.edu; telephone: 803-777-3690.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** www.ams.org/meetings/.

Meetings:

2007

April 14–15	Hoboken, New Jersey	p. 571
April 21–22	Tucson, Arizona	p. 572
May 23–26	Zacatecas, Mexico	p. 572
July 31–August 3	Warsaw, Poland	p. 574
October 5–6	Chicago, Illinois	p. 575
October 6–7	New Brunswick, New Jersey	p. 575
October 13–14	Albuquerque, New Mexico	p. 576
November 3–4	Murfreesboro, Tennessee	p. 576
December 12–15	Wellington, New Zealand	p. 577

2008

January 6–9	San Diego, California Annual Meeting	p. 577
March 22–23	New York, New York	p. 578
March 28–30	Baton Rouge, Louisiana	p. 578
April 4–6	Bloomington, Indiana	p. 578
May 3–4	Claremont, California	p. 578
June 4–7	Rio de Janeiro, Brazil	p. 579
October 4–5	Vancouver, Canada	p. 579
October 11–12	Middletown, Connecticut	p. 579
October 17–19	Kalamazoo, Michigan	p. 579
October 24–26	Huntsville, Alabama	p. 579
December 17–21	Shanghai, People's Republic of China	p. 580

2009

January 7–10	Washington, DC Annual Meeting	p. 580
March 27–29	Urbana, Illinois	p. 580
April 4–5	Raleigh, North Carolina	p. 581
April 25–26	San Franciso, California	p. 581
Oct. 30–Nov. 1	Boca Raton, Florida	p. 581

2010

January 6–9	San Franciso, California Annual Meeting	p. 581

2011

January 5–8	New Orleans, Louisiana Annual Meeting	p. 581

2012

January 4–7	Boston, Massachusetts Annual Meeting	p. 582

2013

January 9–12	San Diego, California Annual Meeting	p. 582

2014

January 15–18	Baltimore, Maryland Annual Meeting	p. 582

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 78 in the the January 2007 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit http://www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)

June 16–July 6, 2007: Joint Summer Research Conferences, Snowbird, Utah.

July 8–July 12, 2007: von Neumann Symposium on Sparse Representation and High-Dimensional Geometry, Snowbird, Utah.

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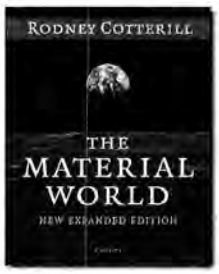

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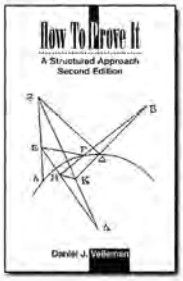

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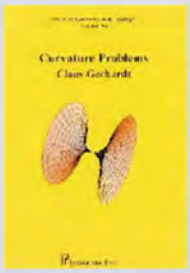

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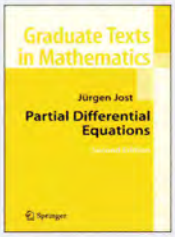

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