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AMERICAN MATHEMATICAL SOCIETY

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A WORD FROM...

Robin Wilson, *Notices* Associate Editor

Author photo courtesy of Tom Zasadzinski/Cal Poly Pomona.

In this issue of the *Notices*, we reflect on the sacrifices and accomplishments made by generations of African Americans to the mathematical sciences. This year marks the 100th birthday of David Blackwell, who was born in Illinois in 1919 and went on to become the first Black professor at the University of California at Berkeley and one of America's greatest statisticians. Six years after Blackwell was born, in 1925, Frank Elbert Cox was to become the first Black mathematician when he earned his PhD from Cornell University, and eighteen years later, in 1943, Euphemia Lofton Haynes would become the first Black woman to earn a mathematics PhD. By the late 1960s, there were close to 70 Black men and women with PhDs in mathematics. However, this first generation of Black mathematicians was forced to overcome many obstacles. As a Black researcher in America, segregation in the South and de facto segregation elsewhere provided little access to research universities and made it difficult to even participate in professional societies. Even David Blackwell, despite his contributions, had the experience of being turned away from American Mathematical Society (AMS) meetings because of exclusive housing practices.[1] Change did not occur in a vacuum, and it took the courageous leadership of mathematicians Etta Falconer and Lee Lorch, among others, to urge the AMS and the Mathematical Association of America (MAA) to take steps to prevent non-discriminatory practices at their meetings.[2]

The above examples remind us that the history of our American mathematics community is a complicated one, and we should take lessons from it in order to build a more successful future. In this spirit, we hope to give a voice to stories of individuals that often go untold, as we celebrate 94 years of Black mathematicians making contributions in a diverse range of research areas at many different types of institutions.

The articles in this issue honor the accomplishments and sacrifices of the previous generations of Black mathematicians by highlighting the work of four individuals who represent a generation of scholars that are standing on the shoulders of those who came before them. University of Central Florida professor Talea Mayo describes her work in storm surge modeling involving interdisciplinary teams of mathematicians, computer scientists, engineers, architects, and public policy experts. Statistician Jacqueline Hughes-Oliver from North Carolina State University shares her work on assessment of ranking algorithms in the context of drug discovery. Ryan Hynd from the University of Pennsylvania explains his work on the dynamics of inelastic collisions and its applications to the study of the large-scale structure of the universe. And Pomona College professor and National Association of Mathematicians (NAM) President Edray Goins writes about applications of elliptic curves.

Offering historical perspective, one of the founders of NAM, Johnny Houston, shares the story of David Blackwell on the year of his 100th birthday and reflects on his remarkable life and accomplishments. Mathematician and Associate Dean for Curricular Affairs at Occidental College Ron Buckmire discusses the Blackwell–Tapia Award, which is the only research award for minority mathematicians. His article highlights its most recent winner Ronald Mickens, whose work in applied mathematics has spanned over 50 years. Finally, we include a memorial article remembering the life of Morehouse professor Rudy Horne, who became well known for his work as mathematical consultant on the film *Hidden Figures*, and whose impact at Morehouse College and throughout the mathematics community will be long-lasting.

Robin Wilson is a professor of mathematics at Cal Poly Pomona and an associate editor of Notices. *His email address is* `robinwilson@cpp.edu`.

[1] *David Blackwell, "An Oral History with David Blackwell," conducted by Nadine Wilmot in 2002 and 2003, Regional Oral History Office University of California, The Bancroft Library, University of California, Berkeley, 2003.*

[2] *Parshall, Karen Hunger. Mathematics and the Politics of Race: The Case of William Claytor (PhD, University of Pennsylvania, 1933),* The American Mathematical Monthly, *Vol. 123, No. 3 (March 2016), pp. 214-240, Mathematics Association of America.*

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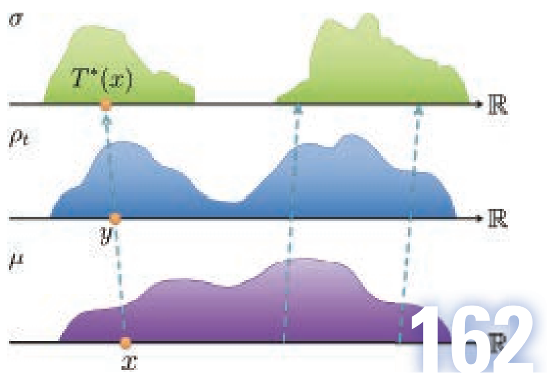

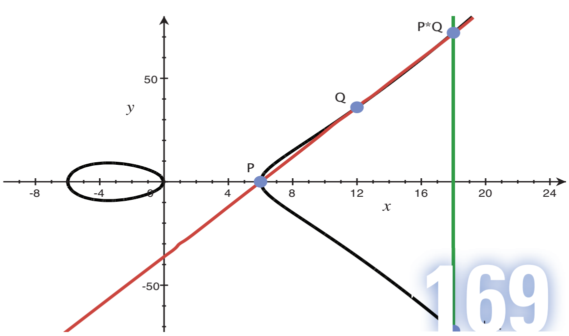

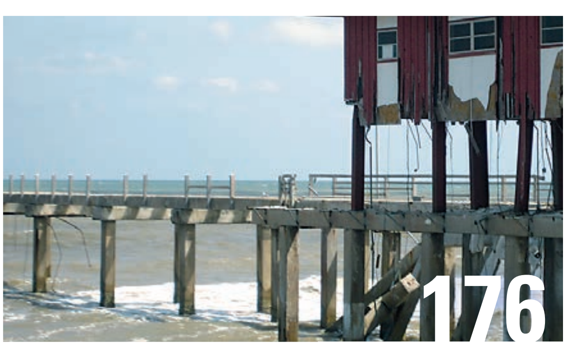

ALSO IN THIS ISSUE

Notices
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FROM THE AMS SECRETARY

ALSO IN THIS ISSUE CONT'D

IN EVERY ISSUE

CURRENT INVITATIONS FROM THE AMS

ERRATA AND CORRIGENDA

Erratum. In the January issue, an erroneous credit line ran on page 59 of an AMS Communication. The credit read, “Photo of Sergei Gelfand by Eric Buck.” The credit should have read, “Photo of Sergei Gelfand by Erin Buck.” *Notices* apologizes to our colleague for the misprint.

Corrigendum. Author Alladi Krishnaswami wishes to direct readers to the following correction for The Origins section of his January Communication, page 64. The sentence regarding Srinivasa Ramanujan's birth and early life should read: "Ramanujan was born in Erode on December 22, 1887, but it is in the Tanjore region steeped in culture that he lived in the town of Kumbakonam until he completed high school."

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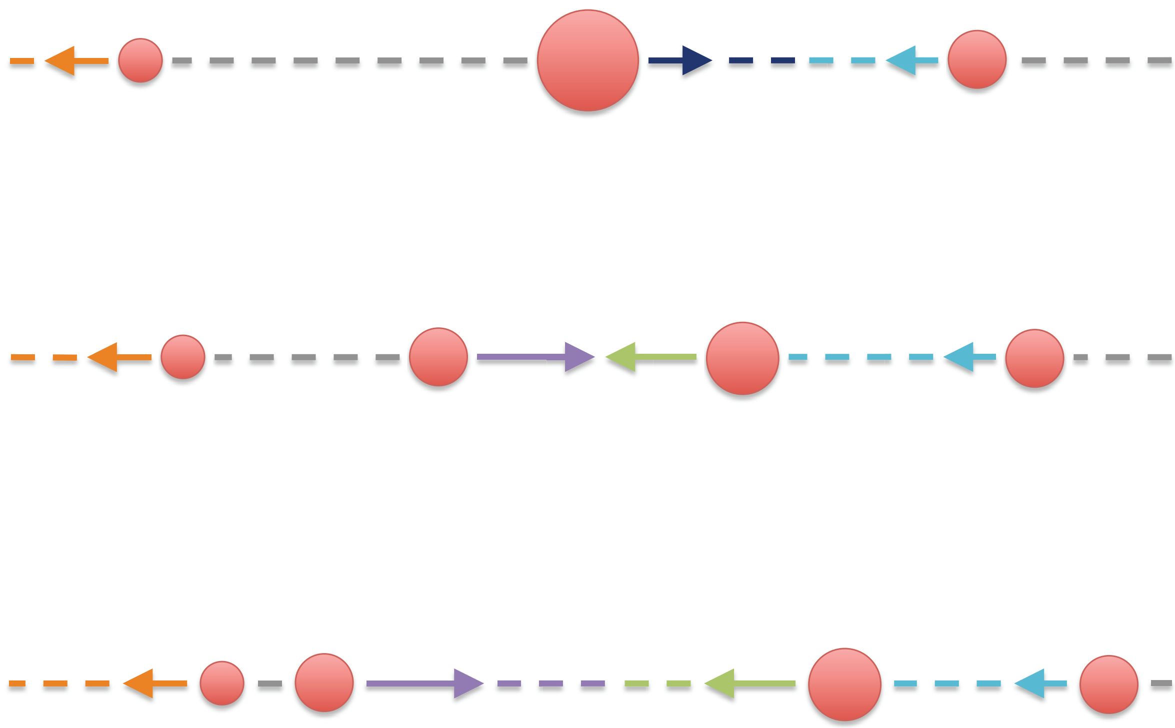
Sticky Particle Dynamics on the Real Line



Ryan Hynd

Introduction

At the simplest level, the physics of adhesion can be modeled with perfectly inelastic collisions. Let us recall this notion by considering the example of two point masses in $\mathbb{R}$ that each move with constant velocity until they collide. To fix ideas, we assume these particles have respective masses $m_1, m_2 > 0$ and velocities $v_1, v_2 \in \mathbb{R}$. Upon colliding, these particles undergo a perfectly inelastic collision provided they merge to form a single particle of mass $m_1 + m_2$ and the resulting particle travels with velocity v chosen to satisfy

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v.$$

Refer to Figure 1. Note in particular that mass and momentum are conserved during the collision. Moreover, this example is easily generalized to any number of colliding particles.

It turns out that the elementary mechanics of perfectly inelastic collisions can be used to model the evolution of matter that is not subject to pressure. This observation is usually attributed to Zel'dovich in his work on the early stages of galaxy formation [18]. His theory was further developed by other cosmologists who studied the role of adhesion in the dynamics of large-scale structures in the universe [7, 10, 15, 16].

In this article, we will discuss some of the mathematics involved in Zel'dovich's theory. A system of equations which embodies the conservation of mass and momentum

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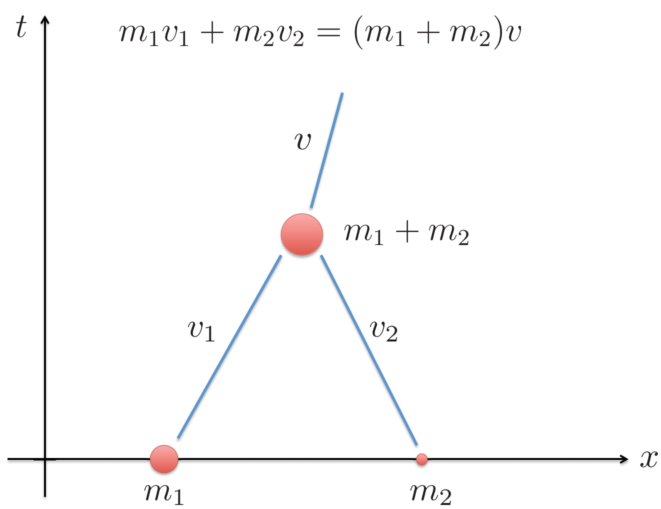

Figure 1. A perfectly inelastic collision between two point masses in $\mathbb{R}$. Here x denotes a variable on the real line and t denotes time. The particles are sketched to be larger than points in order to highlight the possibility that they may have different masses. Further, the particle resulting from the collision is sketched even larger to highlight that it is more massive.

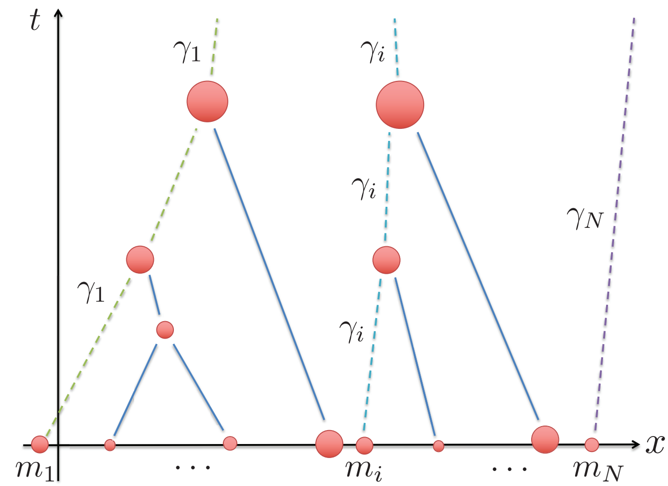

Figure 2. Sticky particle trajectories $\gamma_1, \dots, \gamma_N$ in $\mathbb{R} \times (0, \infty)$ that track N particles moving freely in $\mathbb{R}$ that only interact via perfectly inelastic collisions. The corresponding masses $m_1, \dots, m_N$ are drawn with different sizes to indicate that they are not assumed to be identical. The three dashed trajectories γ_1, γ_i, and γ_N emphasize that these trajectories track m_1, m_i, and m_N, respectively.

of interacting particles will play an important role. This system also describes how to best move from one probability distribution to another in the theory of optimal mass transportation. First, we will build on our example above and examine finite collections of point masses on the real line that interact only via perfectly inelastic collisions.

Finitely Many Interacting Particles

Let us consider N particles on the real line that have masses $m_1, \dots, m_N \geq 0$. We suppose $m_1 + \cdots + m_N = 1$ and that these point masses move freely unless they collide. When any sub-collection of these particles collide, they will undergo a perfectly inelastic collision. We will write $\gamma_1, \dots, \gamma_N : [0, \infty) \to \mathbb{R}$ for the trajectories that are associated with the respective masses $m_1, \dots, m_N$. That is, $\gamma_i(t)$ is the location of point mass m_i at time $t \geq 0$, which could be by itself or part of a larger mass if it has collided with other particles prior to time t. A schematic of the graphs of $\gamma_1, \dots, \gamma_N$ is displayed in Figure 2.

These sticky particle trajectories are continuous and piecewise linear, and they have two important properties. The first is that they satisfy the inequality:

$$\frac{1}{t}|\gamma_i(t) - \gamma_j(t)| \leq \frac{1}{s}|\gamma_i(s) - \gamma_j(s)| \qquad (1)$$

for all $i, j = 1, \dots, N$ and $0 < s \leq t$. Observe that this inequality quantifies the fact that if two particles collide at time s, they will remain stuck together for all times $t \geq s$.

The second feature is

$$\sum_{\gamma_i(t)=\gamma_j(t)} m_i \dot{\gamma}_i(t-) = \left(\sum_{\gamma_i(t)=\gamma_j(t)} m_i \right) \dot{\gamma}_j(t+) \qquad (2)$$

for $j = 1, \dots, N$ and $t > 0$. Here $\dot{\gamma}_i(t-)$ and $\dot{\gamma}_i(t+)$ denote the limits from the left and right of the slope of γ_i at t, respectively; and each summation is over $i = 1, \dots, N$ for which $\gamma_i(t) = \gamma_j(t)$. We note that this identity encodes the conservation of momentum that occurs in between and during collisions. Both properties are discussed in section 3 of [9].

Since the total mass of this physical system is conserved, it makes sense for us to consider the space $\mathcal{P}(\mathbb{R})$ of (Borel) probability measures on $\mathbb{R}$. We can then regard the mass distribution of particles as the map $\rho : [0, \infty) \to \mathcal{P}(\mathbb{R}); t \mapsto \rho_t$, where

$$\rho_t := \sum_{i=1}^{N} m_i \delta_{\gamma_i(t)}. \qquad (3)$$

Here δ_z is the Dirac measure at $z \in \mathbb{R}$. That is,

$$\delta_z(A) = \begin{cases} 1, & z \in A \\ 0, & z \notin A \end{cases}$$

for $A \subset \mathbb{R}$. As a result, $\rho_t(A)$ is the fraction of mass within A at time t. A corresponding velocity is a (Borel) function $v : \mathbb{R} \times [0, \infty) \to \mathbb{R}$ for which

$$v(x, t) = \dot{\gamma}_i(t+) \quad \text{when} \quad x = \gamma_i(t). \qquad (4)$$

The Sticky Particle System (SPS)

As it turns out, the mass distribution ρ defined in (3) and velocity v specified in (4) satisfy

$$\int_0^\infty \int_{\mathbb{R}} (\partial_t \phi + v\partial_x \phi) d\rho_t dt + \int_{\mathbb{R}} \phi(\cdot, 0) d\rho_0 = 0 \quad (5)$$

and

$$\int_0^\infty \int_{\mathbb{R}} (\partial_t \phi + v\partial_x \phi) v d\rho_t dt + \int_{\mathbb{R}} \phi(\cdot, 0) v_0 d\rho_0 = 0 \quad (6)$$

for each smooth $\phi : \mathbb{R} \times [0, \infty) \to \mathbb{R}$ with compact support. Here

$$\rho_0 = \sum_{i=1}^{N} m_i \delta_{\gamma_i(0)} \quad (7)$$

is the initial mass distribution and $v_0 : \mathbb{R} \to \mathbb{R}$ is any function such that $v_0(\gamma_i(0)) = \dot{\gamma}_i(0)$ for $i = 1, \dots, N$. We note that the averaging property (2) is an important ingredient in the derivation of (6).

We interpret (5) and (6) to mean that the pair ρ and v is a solution of the conservation of mass equation

$$\partial_t \rho + \partial_x(\rho v) = 0 \quad (8)$$

and the conservation of momentum equation

$$\partial_t(\rho v) + \partial_x(\rho v^2) = 0 \quad (9)$$

in $\mathbb{R} \times (0, \infty)$, respectively, that satisfies the initial conditions

$$\rho|_{t=0} = \rho_0 \quad \text{and} \quad v|_{t=0} = v_0. \quad (10)$$

The partial differential equations (8) and (9) together are known as the *sticky particle system* (SPS), and they describe the dynamics of one dimensional matter that is not subject to pressure. Our notion of solution is consistent in the sense that if the pair ρ and v were a conventional solution, we could multiply the equations (8) and (9) by ϕ and integrate by parts to derive (5) and (6), respectively. We will also say that ρ and v is a solution pair of the SPS on $\mathbb{R} \times (0, \tau)$ with initial conditions (10) provided (5) and (6) hold for all ϕ that are supported in $\mathbb{R} \times [0, \tau)$.

We now know that we can always find a solution of the SPS when the initial mass density arises from a finite collection of point masses and is specifically of the form (7). We would like to extend these considerations to general initial mass distributions $\rho_0 \in \mathcal{P}(\mathbb{R})$. Our goal is to prescribe the mass distribution and velocity at time 0 and then use the SPS to describe the evolution of the mass distribution and velocity at all later times. We will make a few observations before further exploring this fundamental existence question.

The Method of Characteristics

The SPS always has a solution for a given initial mass distribution $\rho_0 \in \mathcal{P}(\mathbb{R})$ provided that the initial velocity v_0 is continuous and nondecreasing. In this case, the trajectories $t \mapsto x + tv_0(x)$ and $t \mapsto y + tv_0(y)$ do not coincide for distinct $x, y \in \mathbb{R}$ since the function $\mathrm{id}_{\mathbb{R}} + tv_0$ is increasing. As a result, the initial mass distribution is simply translated along linear trajectories that do not intersect. With this observation, we define ρ_t by

$$\int_{\mathbb{R}} g(y) d\rho_t(y) := \int_{\mathbb{R}} g(x + tv_0(x)) d\rho_0(x)$$

for each g belonging to the space $C_b(\mathbb{R})$ of bounded continuous functions on $\mathbb{R}$. This definition can be written more concisely as

$$\rho_t := (\mathrm{id}_{\mathbb{R}} + tv_0)_{\#}\rho_0, \quad t \geq 0. \quad (11)$$

Since the trajectories are linear, any velocity $v : \mathbb{R} \times [0, \infty) \to \mathbb{R}$ associated with ρ must satisfy

$$v(x + tv_0(x), t) = v_0(x), \quad x \in \mathbb{R}, \ t \in [0, \infty)$$

as shown in Figure 3. That is,

$$v(\cdot, t) := v_0 \circ (\mathrm{id}_{\mathbb{R}} + tv_0)^{-1}. \quad (12)$$

We leave it as an exercise to check that the pair ρ and v is indeed a solution of SPS with initial conditions ρ_0 and v_0.

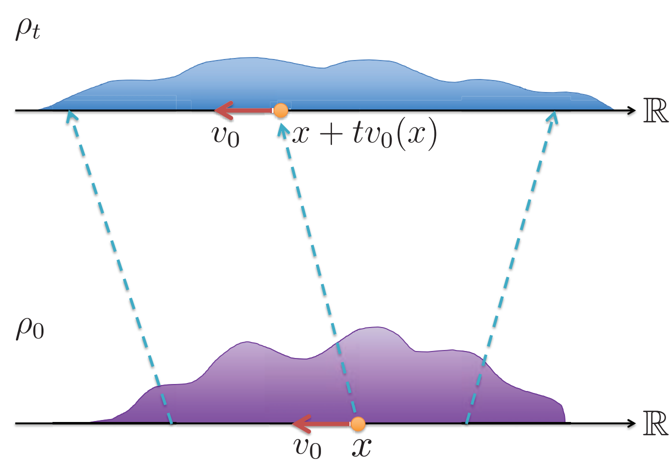

Figure 3. The evolution of the mass ρ_0 with nondecreasing initial velocity v_0 at time $t \geq 0$. The purple shape is a schematic of the mass distribution of ρ_0; the blue shape is a schematic of the evolved mass ρ_t; the dashed lines represent the linear trajectories which do not cross. The initial velocity of the particle starting at x is $v_0(x)$; its velocity v at position $x + tv_0(x)$ and time t is also equal to $v_0(x)$.

When v_0 isn't nondecreasing, we can sometimes build on this example to obtain a solution of the SPS on $\mathbb{R} \times (0, \tau)$ for some $\tau > 0$ and sufficiently small. In particular, if $\mathrm{id}_{\mathbb{R}} + t v_0$ is increasing for each $t \in [0, \tau)$, ρ defined in (11) and v specified in (12) is a solution pair of the SPS on $\mathbb{R} \times (0, \tau)$ with initial conditions ρ_0 and v_0.

Optimal Mass Transportation on the Real Line

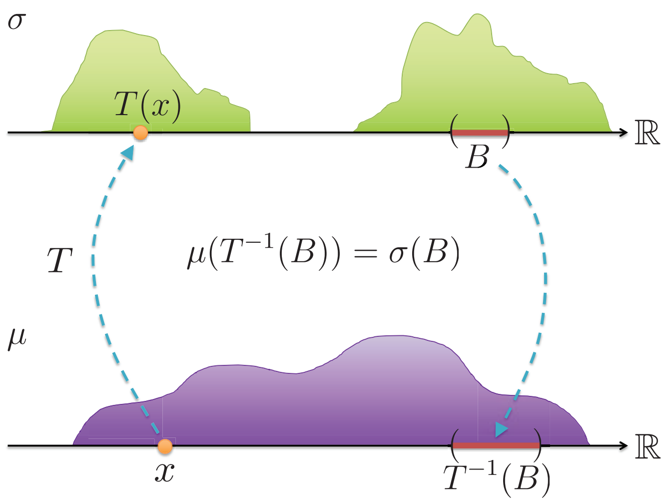

Figure 4. A function T that transports mass in a measure preserving way from a distribution μ (in purple) which represents a pile of sand to another distribution σ (in green) representing a hole to be filled.

In a 1781 memoir [12], Monge famously asked: how do you best move a given pile of sand to fill up a given hole of the same total volume? We will model the sand and hole as measures belonging to

$$\mathcal{P}_2(\mathbb{R}) := \left\{ \mu \in \mathcal{P}(\mathbb{R}) : \int_{\mathbb{R}} x^2 d\mu(x) < \infty \right\}$$

and consider the following version of Monge's problem on the real line. For given $\mu, \sigma \in \mathcal{P}_2(\mathbb{R})$, find $T : \mathbb{R} \to \mathbb{R}$ that minimizes the total cost of transportation

$$\int_{\mathbb{R}} (x - T(x))^2 d\mu(x)$$

among all T satisfying the measure preserving constraint $T_{\#}\mu = \sigma$. This means

$$\int_{\mathbb{R}} g(y) d\sigma(y) = \int_{\mathbb{R}} g(T(x)) d\mu(x)$$

for each $g \in C_b(\mathbb{R})$ or equivalently that $\sigma(B) = \mu(T^{-1}(B))$ for each $B \subset \mathbb{R}$. Refer to Figure 4.

It turns out that if μ does not charge mass to points, our version of Monge's problem has a solution given by a nondecreasing function $T^* : \mathbb{R} \to \mathbb{R}$. In particular, this function is known as the monotone rearrangement of μ onto σ and it can be written rather explicitly in terms of the distribution functions of μ and σ (as detailed in section 2.2 of [17]). Using this optimal T^*, McCann introduced the *displacement interpolation* between μ and σ as

$$\rho_t = ((1 - t)\mathrm{id}_{\mathbb{R}} + tT^*)_{\#}\mu$$

for $0 \le t \le 1$ [11]. Note that $\rho_0 = \mu$, $\rho_1 = \sigma$, and $(1 - t)\mathrm{id}_{\mathbb{R}} + tT^* = \mathrm{id}_{\mathbb{R}} + t(T^* - \mathrm{id}_{\mathbb{R}})$ is an increasing function for $0 \le t < 1$.

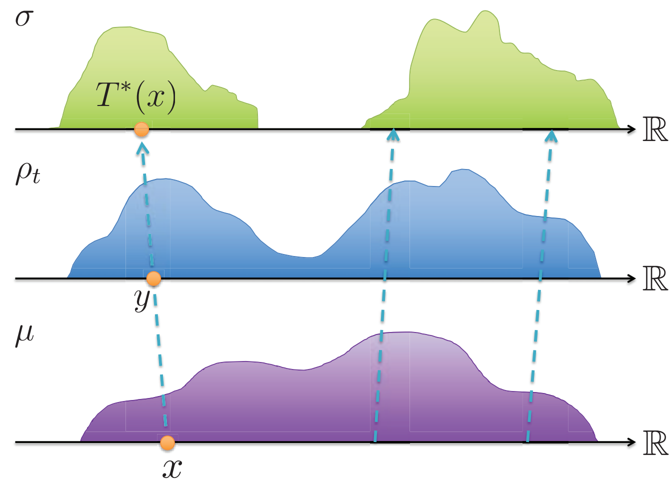

Figure 5. The optimal transportation function T^* that rearranges μ onto σ shown with the displacement interpolation ρ_t between μ and σ for some $t \in [0, 1]$. Here $y = (1 - t)x + tT^*(x)$, and we emphasize that mass is optimally transported along the linear trajectories between the support of μ and the support of σ.

If T^* is additionally continuous, we can set

$$v(\cdot, t) = (T^* - \mathrm{id}_{\mathbb{R}}) \circ ((1 - t)\mathrm{id}_{\mathbb{R}} + tT^*)^{-1}$$

for $t \in [0, 1)$. In view of (11) and (12), the pair $\rho : t \mapsto \rho_t$ and v is a solution of the SPS on $\mathbb{R} \times (0, 1)$ with initial conditions $\rho_0 = \mu$ and $v_0 = T^* - \mathrm{id}_{\mathbb{R}}$. This solution corresponds to intersection-less trajectories and it means that mass is optimally transferred from the support of μ to the support of σ along straight lines. See Figure 5.

The square root of the minimal cost in Monge's problem is related to a quantity known as the *Wasserstein distance*

$$W_2(\mu, \sigma) := \inf_{\pi} \left(\int_{\mathbb{R}\times\mathbb{R}} (x - y)^2 d\pi(x, y) \right)^{1/2}$$

between μ and σ. Here the infimum is taken over (Borel) probability measures π on $\mathbb{R} \times \mathbb{R}$ satisfying

$$\pi(A \times \mathbb{R}) = \mu(A) \quad \text{and} \quad \pi(\mathbb{R} \times A) = \sigma(A)$$

for each $A \subset \mathbb{R}$. We note that the *Wasserstein space* $(\mathcal{P}_2(\mathbb{R}), W_2)$ is a complete, separable metric space.

In the case that μ does not charge mass to points, the displacement interpolation ρ between μ and σ satisfies

$$W_2(\rho_t, \rho_s) = (t - s)W_2(\mu, \sigma), \quad 0 \le s \le t \le 1.$$

As a result, ρ is a constant speed geodesic. These ideas extend to general $\mu \in \mathcal{P}(\mathbb{R})$ which may charge mass to points. Indeed, the Benamou-Brenier formula [1] and the computations performed on action minimizing paths in the Wasserstein space by Gangbo, Nguyen, and Tudorascu [6] together imply that all constant speed geodesics in the Wasserstein space correspond to solutions of the SPS.

Solution for Given Initial Conditions

Now let us return to the initial value problem for the SPS, which is posed as follows.

Initial value problem: For a given $\rho_0 \in \mathcal{P}(\mathbb{R})$ and $v_0 \in C_b(\mathbb{R})$, find $\rho : [0,\infty) \to \mathcal{P}(\mathbb{R})$ and $v : \mathbb{R}\times[0,\infty) \to \mathbb{R}$ that satisfy (5) and (6).

The existence of a solution to this initial value problem was first established by Weinan, Rykov, and Sinai [5] and by Brenier and Grenier [2] using novel methods based on Hamilton-Jacobi equations and conservations laws, respectively. Natile and Savaré [13] subsequently extended and merged both of these approaches by employing a flow in Lagrangian variables. We also mention that Dermoune [4] solved this problem using probabilistic techniques.

All of these approaches rely on the fact that convex combinations of Dirac measures are dense in $\mathcal{P}(\mathbb{R})$. That is, for a given $\rho_0 \in \mathcal{P}(\mathbb{R})$, there is a sequence $(\rho_0^k)_{k\in\mathbb{N}}$ such that each ρ_0^k is of the form (7) and

$$\int_{\mathbb{R}} g(x)d\rho_0(x) = \lim_{k\to\infty}\int_{\mathbb{R}} g(x)d\rho_0^k(x)$$

for each $g \in C_b(\mathbb{R})$. Recall that when the initial mass distribution is of the form (7), we can solve the initial value problem using sticky particle trajectories. Therefore, for a given initial velocity $v_0 \in C_b(\mathbb{R})$ and $k \in \mathbb{N}$, we can find a solution pair ρ^k and v^k of the SPS with initial conditions

$$\rho^k|_{t=0} = \rho_0^k \quad \text{and} \quad v^k|_{t=0} = v_0.$$

It turns out that we can send $k \to \infty$ along a subsequence to obtain a solution pair ρ and v of the SPS with the desired initial mass distribution ρ_0 and velocity v_0 [9]. The two key features which allow for this type of compactness are as follows. The first is

$$(v^k(x,t) - v^k(y,t))(x-y) \le \frac{1}{t}(x-y)^2 \qquad (13)$$

for each $t > 0$ and x, y in the support of ρ_t^k; this inequality follows directly from (1). The second feature is

$$\int_{\mathbb{R}} \frac{1}{2}(v^k(x,t))^2 d\rho_t^k(x) \le \int_{\mathbb{R}} \frac{1}{2}(v_0(x))^2 d\rho_0^k(x)$$

for $t \ge 0$, which can be derived from (2) using Jensen's inequality. This solution pair ρ and v will also satisfy a version of inequality (13), and Huang and Wang showed that it is the unique solution pair of the SPS for given initial conditions with this property [8].

Instability in Higher Dimensions

Many of the ideas that we have discussed so far apply to sticky particle dynamics in $\mathbb{R}^d$. However, there is a fundamental difference between the associated dynamics when $d = 1$ and when $d > 1$. To see this disparity, let us consider a perfectly inelastic collision of two point masses in $\mathbb{R}^2$. We will suppose these particles have masses $m_1, m_2 > 0$ and move with constant velocity $v_1, v_2 \in \mathbb{R}^2$, respectively. When they collide, they form a single particle of mass $m_1 + m_2$ that has constant velocity v given by

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)v$$

as shown in Figure 6a.

Let's further assume that these particles are not restricted to move on the same line. In this case, we can replace v_2 with $w \in \mathbb{R}^2$ so that the particles initially located at the same positions with respective masses m_1, m_2 and velocities v_1, w do not collide. Moreover, our choice in w can be made so that v_2 and w are as close as we desire; see Figure 6b. As w tends to v_2, the nonintersecting trajectories in $\mathbb{R}^2 \times [0,\infty)$ associated with the particles in Figure 6b tend to two lines that intersect and *not* the piecewise linear trajectories that describe the perfectly inelastic collision in Figure 6a.

This type of sensitivity to small changes in initial conditions poses a challenge to understanding the evolution of the mass distribution and velocity of a collection of particles in space that interact primarily through inelastic collisions. Specifically, it prevents us from using the approximation method we discussed for $d = 1$ to construct a solution pair ρ and v of the conservation of mass equation

$$\partial_t \rho + \nabla \cdot (\rho v) = 0$$

and the conservation of momentum equation

$$\partial_t(\rho v) + \nabla \cdot (\rho v \otimes v) = 0$$

in $\mathbb{R}^d \times (0,\infty)$ for a given initial mass distribution $\rho|_{t=0} = \rho_0$ and velocity $v|_{t=0} = v_0$ when $d > 1$. While there have been a few notable works on this higher dimensional initial value problem [3, 14], we believe this topic is still primed for mathematical innovation. Of course we hope that any new ideas which arise in the study of sticky particles dynamics in space will also provide us with a better grasp on Zel'dovich's model for the origin of galaxies.

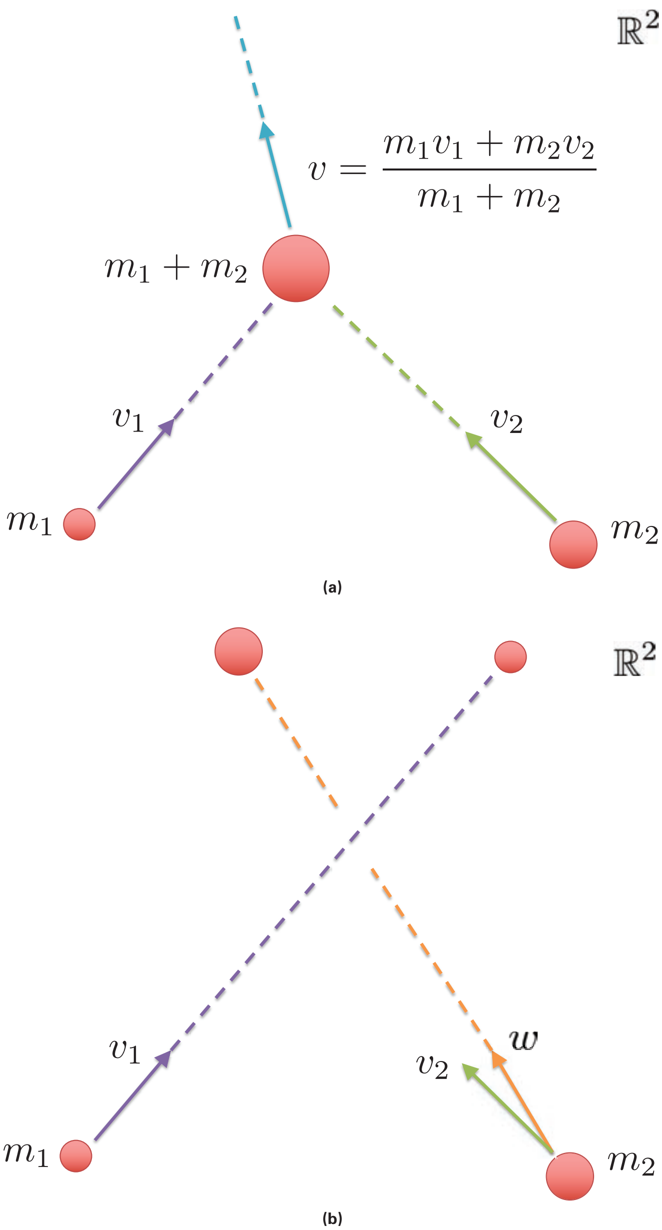

Figure 6. (a) A perfectly inelastic collision between two particles in the plane with masses m_1, m_2 and initial velocities v_1, v_2, respectively. The velocities are indicated with arrows and the corresponding paths are indicated with dashed line segments. (b) An initial velocity $w \neq v_2$ for the particle with mass m_2 which is selected so that the particles do not collide.

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Ryan Hynd

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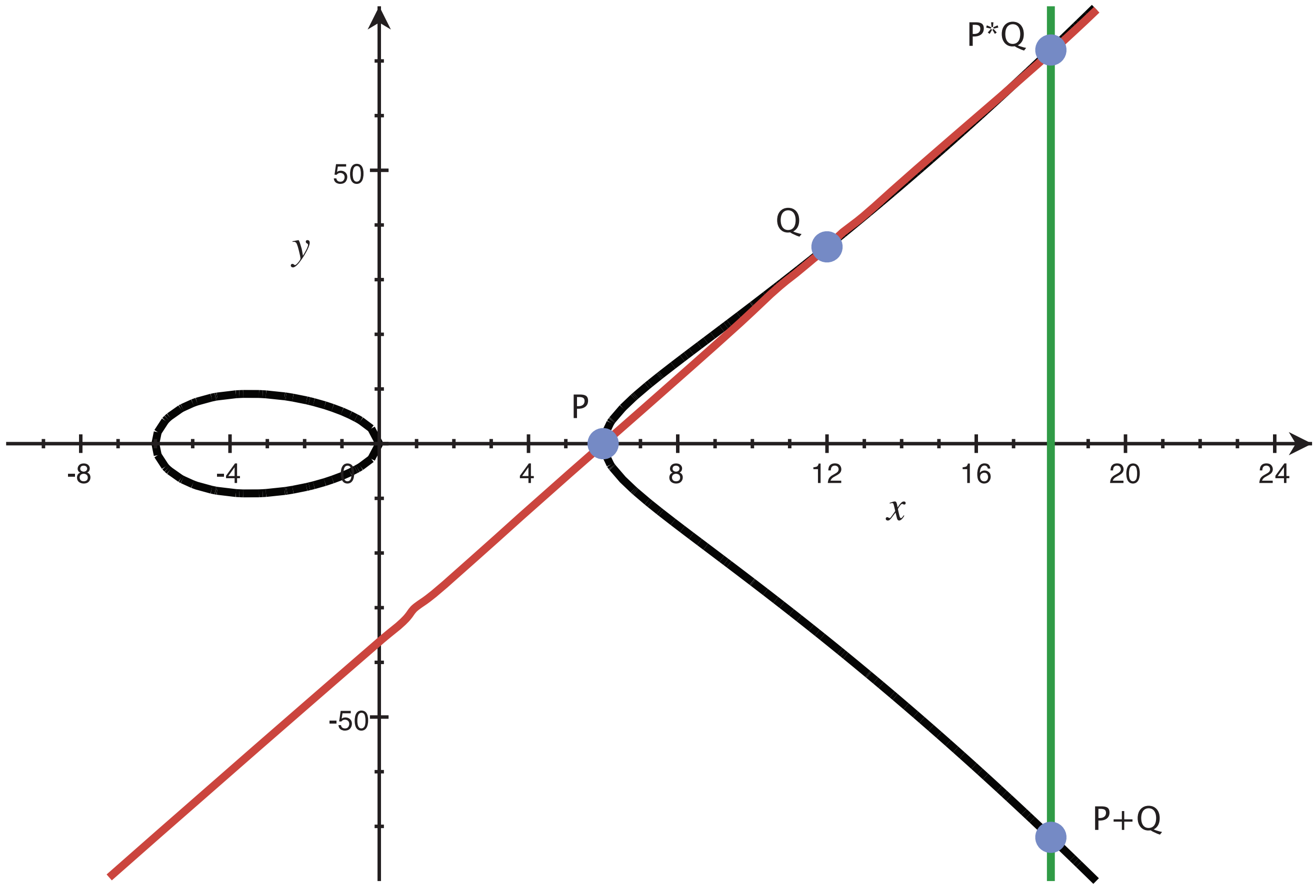
The Ubiquity of Elliptic Curves



Edray Herber Goins

Elliptic Curves appear in many branches of mathematics and science: Algebraic Geometry, Abstract Algebra, and even Computer Science. In this article, I provide a gentle introduction to the subject, and explore the many ways elliptic curves are used to answer questions from a variety of fields.

An elliptic curve is a non-singular projective curve of genus one having a specified base point. For the inspired, we can say this in fancier terms: Fix a field k, such as the rational numbers $\mathbb{Q}$ or a finite field $\mathbb{F}_q$ or even a function field $\mathbb{C}(t)$. We say that *an elliptic curve E defined over k* is that functor which associates fields K containing k to an algebraic set of the form

$$E(K) = \left\{ (x : y : z) \in \mathbb{P}^2(K) \;\middle|\; \begin{aligned} y^2 z + a_1\, x\, y\, z + a_3\, y\, z^2 \\ = x^3 + a_2\, x^2\, z + a_4\, x\, z^2 + a_6\, z^3 \end{aligned} \right\}$$

where (i) the coefficients $a_1, \ldots, a_6$ lie in k and (ii) each point $P = (x : y : z)$ in $E(K)$ has a well-defined tangent line. The so-called *point at infinity* $\mathcal{O} = (0 : 1 : 0)$ is our specified base point; it is always an element of $E(K)$. Here we employ notation which allows us to express our points in *projective space*: we write $(x : y : z)$ to denote the equivalence class of points in the form $(\lambda\, x,\ \lambda\, y,\ \lambda\, z)$ for nonzero scalars λ. We say that elements P of $E(K)$ are *K-rational points on E*.

If k is not of characteristic 2 or 3, it is equivalent to say that an elliptic curve is represented by a cubic equation of the form $y^2 = x^3 + A\,x + B$ for some $A, B \in k$ satisfying $4\,A^3 + 27\,B^2 \neq 0$ because we can make a linear change of variables. Indeed, $\mathcal{O} = (0 : 1 : 0)$ is the only projective point $P = (x : y : z)$ on the curve with $z = 0$, so for

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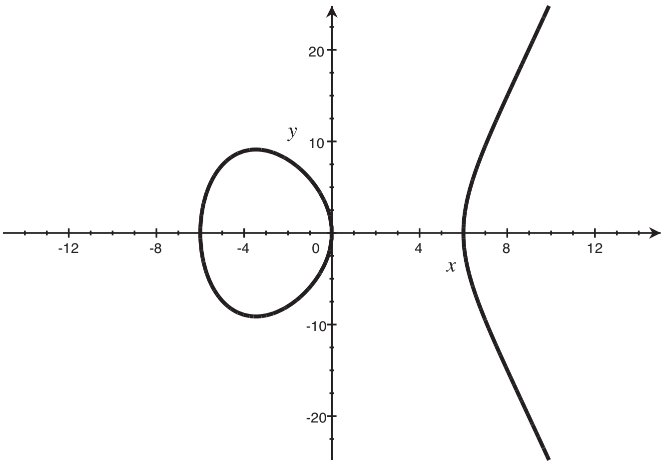


Figure 1. Graph of an elliptic curve over $K = \mathbb{R}$.

all other projective points we can scale by $\lambda = 1/z$ and assume that $P = (x : y : 1)$. We typically identify this as an affine point (x, y) in the usual sense. An example of a graph of the set $E(K)$ when $K = \mathbb{R}$ can be found in Figure 1. More information can be found in the standard texts [7] and [32].

Elliptic Curves in Algebraic Geometry

It is not always obvious that a given set of equations defines an elliptic curve. Here are some examples.

Fermat curves. First, consider the curve $a^3 + b^3 = c^3$ over the field $k = \mathbb{Q}$. Using the substitutions $a = 32 - 8y$, $b = 40 + 8y$, and $c = 24x$, we find the elliptic curve $y^2 + y = x^3 - 7$. On the other hand, the curve $3a^3 + 4b^3 = 5c^3$ is not an elliptic curve over $\mathbb{Q}$ because this equation has no $\mathbb{Q}$-rational solutions other than the degenerate one, namely $a = b = c = 0$. This equation does however define a projective curve of genus one. This example and a larger class are discussed in some detail in [31].

There has been considerable interest in the curve $a^n + b^n = c^n$ over $\mathbb{Q}$ for various exponents n. Such an equation defines a *Fermat curve* because Pierre de Fermat wondered whether there were any nonzero $\mathbb{Q}$-rational solutions when n is a sufficiently large integer. When $n = 2$, it is clear that any *Pythagorean Triple*, such as $(a, b, c) = (3, 4, 5)$, will suffice as a desired solution. We have seen that elliptic curves appear when $n = 3$, but this is not the only exponent where this happens. Curiously, when $n = 7$, any solution (a, b, c) yields a point $P = (x : y : 1)$ on the elliptic curve $E : y^2 + xy = x^3 - x^2 - 107x + 552$ through the unwieldly – and nontrivial – substitution

$$x = 49\,\frac{(a^2 + b^2 + c^2 + ab + ac + bc)^2 + abc\,(a + b + c)}{(a + b + c)^4} - 12,$$

$$y = \frac{7\,(x + 12)\,(a^2 + b^2 + c^2) - x\,(a + b + c)^2}{2\,(a + b + c)^2}.$$

Hence one can find all $\mathbb{Q}$-rational solutions to $a^7 + b^7 = c^7$ by focusing on $\mathbb{Q}$-rational points P on E. The reader may wish to compare with the discussion of elliptic curves with conductor 49 in [12].

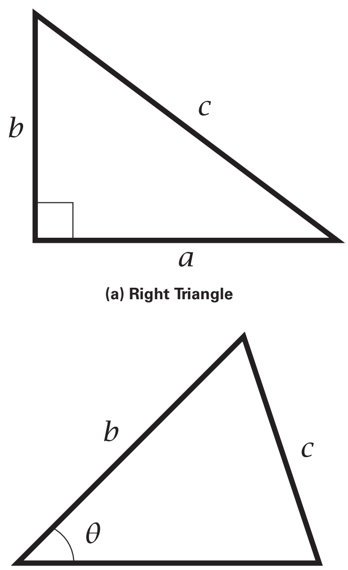
Congruent numbers. As another example, say that we are given a positive integer n. When can this integer be expressed as the area of a right triangle having rational sides of lengths a, b, and c? (See Figure 2.) Such n is said to be *congruent* because there exists a rational number d such that the arithmetic progression $\{d - n,\ d,\ d + n\}$ consists of three squares, namely $\{(a-b)^2/4,\ c^2/4,\ (a+b)^2/4\}$; see [33]. This question is equivalent to asking whether there is a $\mathbb{Q}$-rational solution (a, b, c) to the simultaneous equations $a^2 + b^2 = c^2$ and $(1/2)\,ab = n$. Using the substitutions $a = (x^2 - n^2)/y$, $b = 2nx/y$ and $c = (x^2 + n^2)/y$, we find the elliptic curve $y^2 = x^3 - n^2 x$. We see immediately that $n = 6$ is a congruent number because the elliptic curve has a $\mathbb{Q}$-rational point $(x : y : 1) = (12 : 36 : 1)$ which corresponds to the familiar 3–4–5 triangle. More interestingly, $n = 5$ is also a congruent number because the elliptic curve has a $\mathbb{Q}$-rational point $(x : y : 1) = (50 : 75 : 8)$ which corresponds to the less obvious $(9/6)$–$(40/6)$–$(41/6)$ triangle.

Quadric intersections. It may seem surprising that two quadratic equations—such as $a^2 + b^2 = c^2$ and $(1/2)\,ab = n$—yield an elliptic curve, but this is part of a more general phenomenon. A typical number theory course discusses how to find $\mathbb{Q}$-rational solutions (v, w) to a quadratic equation, such as the *Pell equation* $v^2 - d\,w^2 = 1$. More generally, say that d_1 and d_2 are distinct nonzero $\mathbb{Q}$-rational numbers. The collection of simultaneous Pell equations $u^2 - d_1 w^2 = 1$ and $v^2 - d_2 w^2 = 1$ is known as a *quadric intersection*. (These are closely related to *concordant quadratic forms*; see [2] and [26].) Using the substitutions

$$\begin{aligned} u &= (x^2 + 2\,d_1\,x + d_1\,d_2)/(x^2 - d_1\,d_2) \\ v &= (x^2 + 2\,d_2\,x + d_1\,d_2)/(x^2 - d_1\,d_2) \\ w &= 2\,y/(x^2 - d_1\,d_2) \end{aligned}$$

we find the elliptic curve $y^2 = x\,(x + d_1)\,(x + d_2)$. In general, given a set of equations, it is easy to determine whether it has genus one, but it is difficult to determine whether there is a K-rational solution which we could assign as our base point.

Heron triangles and θ-congruent numbers. It is only natural to wonder whether the geometric construction of congruent numbers as the area of a $\mathbb{Q}$-rational right triangle can be generalized to other types of triangles. For example, if a, b, and c are the lengths of a triangle with an angle



(a) Right Triangle

(b) Triangle with Angle θ

Figure 2. Triangles with sides of lengths a, b, and c.

$\theta = 90^\circ$, then $a^2 + b^2 = c^2$; but if we have a triangle with an angle $\theta = 106.26^\circ$, then $a^2 + (14/25)\, a\, b + b^2 = c^2$. Indeed, this concept can be generalized.

Say that we are given a positive integer n. When can this integer be expressed as the area of some triangle having rational sides of lengths a, b, and c? (See Figure 2.) Such a triangle is called a *Heron triangle* because Heron of Alexandria found a formula which relates these quantities, although a slightly more general formula was known much earlier to Brahmagupta: $n^2 = s\,(s-a)\,(s-b)\,(s-c)$ where $s = (a+b+c)/2$ is the *semi-perimeter*. If θ is the angle opposite of the side of length c, the Law of Cosines and the Law of Sines together assert that

$$\cos\theta = \frac{m^2-1}{m^2+1} \quad \text{and} \quad \sin\theta = \frac{2\,m}{m^2+1}$$
$$\text{where} \quad m = \frac{(a+b)^2 - c^2}{4\,n}.$$

Using the substitutions $a = y/x$, $b = (d_1 - d_2)\,x/y$, and $c = (x^2 - d_1\, d_2)/y$, we find the elliptic curve $E : y^2 = x\,(x+d_1)\,(x+d_2)$ in terms of $d_1 = n\,m$ and $d_2 = -n/m$. For example, if $\theta = 90^\circ$ then $m = 1$ and we find the curve $y^2 = x^3 - n^2\, x$; and if $\theta = 106.26^\circ$ then $m = 3/4$ and we find the curve $y^2 = x^3 - (7/12)\, n\, x - n^2\, x$. In other words, once we fix an area n and an angle θ in terms of m as above, then we can use $\mathbb{Q}$-rational points $P = (x : y : 1)$ to find Heron triangles with sides of lengths a, b, and c. As a generalization of congruent numbers, we say n is a *θ-congruent number*; see [14].

Elliptic Curves in Abstract Algebra

Chord tangent construction. We have seen that there are many questions one can ask about $\mathbb{Q}$-rational solutions to systems of polynomial equations. Such questions were of primary interest to Diophantus of Alexandria—the namesake of the so-called *Diophantine Equations*. In fact, Diophantus gave a general geometric trick to finding $\mathbb{Q}$-rational solutions.

The non-singularity of elliptic curves allows us to start with a few known K-rational points and construct more. For example, let $y = \lambda\, x + \nu$ be a line through two affine points $P = (x_1 : y_1 : 1)$ and $Q = (x_2 : y_2 : 1)$ in $E(K)$; we choose this to be the line tangent to E at P if $P = Q$. This line must intersect the curve as a third point in $E(K)$ which we denote by $P * Q = (x_3 : y_3 : 1)$. Rather explicitly, $x_3 = \lambda^2 + a_1\,\lambda - a_2 - x_1 - x_2$ and $y_3 = \lambda\, x_3 + \nu$. This is known as the *chord tangent construction*.

As an example, consider the $\mathbb{Q}$-rational point $P = (12 : 36 : 1)$ on the curve $E : y^2 = x^3 - 36\,x$. The line tangent to E at P is $y = (11/2)\,x - 30$, so we find the new $\mathbb{Q}$-rational point $P * P = (50 : 35 : 8)$. That is, the triangle $(49/70)$–$(1200/70)$–$(1201/70)$ also has area $n = 6$; hopefully you can see how to construct many more. As another example, consider the curve $y^2 + y = x^3 - 7$. Some $\mathbb{Q}$-rational points are $P = (3 : 4 : 1)$ and $Q = (3 : -5 : 1)$. The line tangent to E at P is $y = 3\,x - 5$, so we find that $P * P = P$. The line through P and Q is the vertical line $x = 3$, so we find that $P * Q = \mathcal{O}$. That is, the chord-tangent construction does not help much in this case to find more $\mathbb{Q}$-rational points.

Group law. The chord tangent construction yields a way to turn $E(K)$ into a group. Indeed, given two points P and Q in $E(K)$, define $P \oplus Q = (P * Q) * \mathcal{O}$, where the point $P * Q = (x : y : z)$ is found as described above, and the point $P \oplus Q = (x : -y - a_1\,x - a_3\,z : z)$ is that reflection of $P * Q$ about the line $2\,y + a_1\,x + a_3 = 0$. A graph of these lines and their intersections can be found in Figure 3. It is easy to check that $E(K)$ forms an abelian group under $\oplus$, where the point at infinity $\mathcal{O} = (0 : 1 : 0)$ is the identity and $[-1]P = P * \mathcal{O}$ is the inverse of a given point P. The difficulty in proving that this is indeed an abelian group comes down to showing associativity, namely that $(P \oplus Q) \oplus R = P \oplus (Q \oplus R)$. One typically does this via the famous Riemann-Roch Theorem; see [32].

The Mordell–Weil group. A natural question is to ask about the structure of the abelian group $E(K)$. A celebrated theorem of Louis Mordell [24], for $K = \mathbb{Q}$, generalized by André Weil [34] for finite extensions K of $\mathbb{Q}$, states that $E(K)$ is finitely generated, that is $E(K) = E(K)_{\text{tors}} \oplus \mathbb{Z}^r$ for some finite group $E(K)_{\text{tors}}$ consisting of the torsion elements, and some nonnegative integer r called the

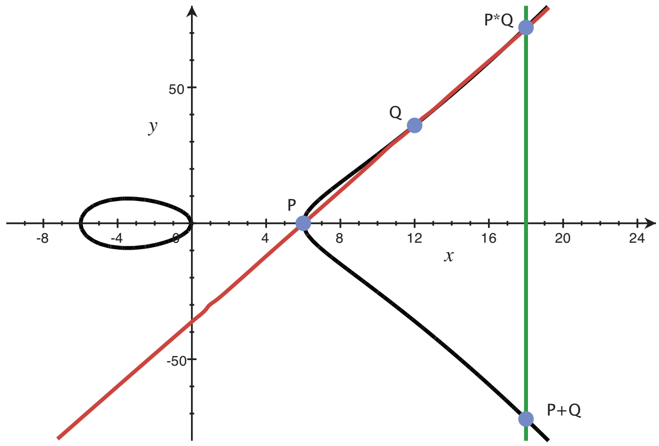


Figure 3. The group law of an elliptic curve.

rank. The group $E(K)$ is called the Mordell–Weil group in these cases. This result asserts there is a finite set of generators $\{T_1, \ldots, T_s, P_1, \ldots, P_r\} \subseteq E(K)$ such that any K-rational point $P \in E(K)$ can be expressed in the form $P = [m_1]T_1 \oplus \cdots \oplus [m_s]T_s \oplus [n_1]P_1 \oplus \cdots \oplus [n_r]P_r$ for some integers m_i and n_j, where we employ the notation $[n]P = P \oplus P \oplus \cdots \oplus P$ as a sum n times. For example, when $y^2 + y = x^3 - 7$ we have $E(\mathbb{Q}) \simeq (\mathbb{Z}/3\,\mathbb{Z})$ as generated by $T_1 = (3 : 4 : 1)$ because $T_1 * T_1 = T_1$, and so $T_1 \oplus T_1 = [-1]T$. Incidentally, this also explains why the there are only three rational solutions $(a : b : c)$ to $a^3 + b^3 = c^3$, and they each satisfy $a\,b\,c = 0$.

There has been a lot of work done in understanding the group $E(K)$ when $K = \mathbb{Q}$. Barry Mazur [22] [23], proving conjectures of Beppo Levi [30] and Andrew Ogg [25], showed that there are only 15 possible types of torsion subgroup, namely either $E(\mathbb{Q})_{\text{tors}} \simeq (\mathbb{Z}/n\,\mathbb{Z})$ for $n = 1, 2, \ldots, 10, 12$; or $E(\mathbb{Q})_{\text{tors}} \simeq (\mathbb{Z}/2\,\mathbb{Z}) \oplus (\mathbb{Z}/2\,m\,\mathbb{Z})$ for $m = 1, 2, 3, 4$. (There are similar classifications when K is a finite extension of $\mathbb{Q}$; see [16], [17], [18], and [19].) Not a lot is known about the rank r. Work of Manjul Bhargava [4] and others [1] suggest that the average value of r is $1/2$— meaning roughly half of elliptic curves have rank $r = 0$ while the other half have rank $r = 1$. An example of Noam Elkies shows that the rank can be as large as $r = 28$, but recent work of Bjorn Poonen et al. [27] [28] suggests that there is a uniform upper bound on r.

Computing the Mordell–Weil group. Computing $E(K)$ is a difficult task—even when $K = \mathbb{Q}$. One can ask two questions for a given elliptic curve E defined over $\mathbb{Q}$: (i) What are the torsion subgroup $E(K)_{\text{tors}}$ and the rank r? (ii) What is a generating set $\{T_1, \ldots, T_s, P_1, \ldots, P_r\}$ for the Mordell–Weil group? We give a method for determining the answers to these questions by focusing on a specific family of elliptic curves. Fix distinct nonzero $\mathbb{Q}$-rational numbers d_1 and d_2, and consider the elliptic curve $E : y^2 = x\,(x + d_1)\,(x + d_2)$. The torsion subgroup is relatively easy to compute: Mazur's Theorem states that $E(\mathbb{Q})_{\text{tors}} \simeq (\mathbb{Z}/2\,\mathbb{Z}) \oplus (\mathbb{Z}/2\,m\,\mathbb{Z})$ for some $m = 1, 2, 3,$ or 4. We can choose $T_1 = (0 : 0 : 1)$ as a generator of order 2, so it remains to find some $\mathbb{Q}$-rational point T_2 as a generator of order $2\,m$. The rank is considerably more difficult to determine: Following an idea of Mordell, there is an injective group homomorphism

$$\begin{array}{ccc} \dfrac{E(\mathbb{Q})}{2\,E(\mathbb{Q})} \simeq \left(\dfrac{\mathbb{Z}}{2\,\mathbb{Z}}\right)^{r+2} & \hookrightarrow & \dfrac{\mathbb{Q}^\times}{(\mathbb{Q}^\times)^2} \times \dfrac{\mathbb{Q}^\times}{(\mathbb{Q}^\times)^2} \\ P = (x : y : 1) & \longmapsto & (x + d_1,\ x + d_2) \end{array}$$

One can determine the image of this map by other means—such as looking at certain homogeneous spaces as torsors for E—then use this to determine the rank r. See [8], [9], and [10] for the state of the art on this topic.

Heron triangles revisited. Recall earlier that we introduced the elliptic curve $y^2 = x\,(x + d_1)\,(x + d_2)$ in terms of the distinct nonzero $\mathbb{Q}$-rational numbers $d_1 = n\,m$ and $d_2 = -n/m$; $\mathbb{Q}$-rational points on this curve correspond to a triangle with area n and rational sides of lengths a, b, and c. What can we say about the Mordell–Weil group of this elliptic curve? For example, take $n = 12$. This is the area of an isosceles triangle with sides of lengths $a = b = 5$ and $c = 8$. We find that $m = ((a + b)^2 - c^2)/(4\,n) = 3/4$, and hence the elliptic curve $E : y^2 = x^3 - (7/12)\,n\,x - n^2\,x$. One shows that $E(\mathbb{Q}) \simeq (\mathbb{Z}/2\,\mathbb{Z}) \oplus (\mathbb{Z}/4\,\mathbb{Z})$ as generated by $T_1 = (0 : 0 : 1)$ and $T_2 = (18 : 45 : 2)$. In general for $d_1 = n\,m$ and $d_2 = -n/m$, the torsion subgroup of the elliptic curve $E : y^2 = x\,(x + d_1)\,(x + d_2)$ contains $(\mathbb{Z}/2\,\mathbb{Z}) \oplus (\mathbb{Z}/4\,\mathbb{Z})$ if and only if n is the area of an isosceles triangle; and $E(\mathbb{Q})_{\text{tors}} \simeq (\mathbb{Z}/2\,\mathbb{Z}) \oplus (\mathbb{Z}/2\,\mathbb{Z})$ otherwise. Note that the torsion subgroup can never be $(\mathbb{Z}/2\,\mathbb{Z}) \oplus (\mathbb{Z}/6\,\mathbb{Z})$! For more information, see [15] and [29].

This elliptic curve is not the only one which yields Heron triangles. Fix a $\mathbb{Q}$-rational number t different from 0 or ± 1. It is easy to check that $\mathbb{Q}$-rational points $P = (x : y : 1)$ on the elliptic curve $E : y^2 = x\,(x + d_1)\,(x + d_2)$ yield a Heron triangle with area n and sides of lengths a, b, and c, all in terms of

$$\begin{aligned} a &= \left[\frac{1}{2}\,\frac{x-1}{x+1} + \frac{t^3 - t}{t^4 - 6\,t^2 + 1}\,\frac{y}{x}\right] c \\ b &= \left[\frac{1}{2}\,\frac{x-1}{x+1} - \frac{t^3 - t}{t^4 - 6\,t^2 + 1}\,\frac{y}{x}\right] c \qquad \text{and} \qquad d_1 = \left(\frac{t^2 - 1}{2\,t}\right)^2, \quad d_2 = \left(\frac{2\,t}{t^2 - 1}\right)^2 \\ n &= \frac{t^3 - t}{t^4 - 6\,t^2 + 1}\,c^2 \end{aligned}$$

It is also easy to check that this elliptic curve has torsion subgroup $E(\mathbb{Q})_{\text{tors}} \simeq (\mathbb{Z}/2\,\mathbb{Z}) \oplus (\mathbb{Z}/8\,\mathbb{Z})$. In fact, every elliptic curve defined over $\mathbb{Q}$ with this torsion subgroup is in the form of this E for some $\mathbb{Q}$-rational number t. We find an elliptic curve with rank $r = 3$ if we choose $t = 15/76$. This is the largest known rank among all elliptic curves E defined over $\mathbb{Q}$ having this torsion subgroup! For more information, see [6] and [11].

Elliptic Curves in Computer Science

We have seen how elliptic curves play a large role in finding K-rational solutions to collections of polynomial equations. Surprisingly, elliptic curves can be used to factor very large numbers, or even make it difficult for people to decode secret messages.

Elliptic curve factorization methods. Say that n is a large integer which we know is the product of two large primes roughly equal in size, but we don't know what those primes are. We outline a method using elliptic curves to determine these primes.

Pick an elliptic curve E defined over $K = k = \mathbb{Z}/n\mathbb{Z}$—even though we know that k is certainly not a field because it contains zero divisors. We will use this to our advantage. Also pick two points $P = (x_1 : y_1 : 1)$ and $Q = (x_2 : y_2 : 1)$ in $E(K)$; we will try to compute $P \oplus Q$. This would involve first constructing a line $y = \lambda x + \nu$. Since the slope $\lambda = (y_1 - y_2)/(x_1 - x_2)$ and the y-intercept is $\nu = (y_2 x_1 - y_1 x_2)/(x_1 - x_2)$, we consider the greatest common divisor $d = \gcd(n, x_1 - x_2)$. If $d = 1$, then we can compute $P \oplus Q$; but if $d \neq 1$ then d should be a nontrivial divisor of n. In this way, we expect to eventually find enough points to be able to factor n. This is known as the *elliptic curve factorization method (ECM)*; see [21].

The largest factor d found to date using ECM corresponds to the integer $n = 7^{337} + 1$; this divisor d has 83 digits! See [35].

Elliptic curve discrete logarithm problem. Say that we have two individuals, Shuri and T'Challa, who want to send each other a private message. First they must make sure that each is who they claim. Here is a protocol to explain how to do this.

Both individuals agree upon three items publicly: (i) a finite field $K = k = \mathbb{F}_q$, (ii) an elliptic curve E defined over k, and (iii) a point $P \in E(K)$ with a large order n, that is $[n]P = \mathcal{O}$. Shuri chooses some private information—such as a PIN—as a positive integer s less than n; T'Challa chooses the same as a positive integer t. Both Shuri and T'Challa publicly list $[s]P$ and $[t]P$ as their *public keys*—perhaps as the signature in an e-mail. If Shuri and T'Challa compute the same *shared key* $[s\,t]P = [s]([t]P) = [t]([s]P)$, then Shuri and T'Challa can feel confident that they are indeed who they say they are. This public key agreement exchange is known as the *elliptic curve Diffie–Hellman (ECDH)* protocol; see [5].

The question becomes this: If an eavesdropper, say Killmonger, sees the public keys P, $[s]P$, $[t]P$, and $[s\,t]P$, can he recover the private keys s and t? There is widespread belief that the answer is "no"—at least in a world where quantum computers do not exist. This is known as the *elliptic curve discrete logarithm problem (ECDLP)*; see [20].

Apple HomeKit and Curve25519. We give one last application of elliptic curves which is causing something of a controversy. Apple Computers has software which developers are slow to use because the software uses elliptic curves. Apple has created *HomeKit*, a platform for connecting your smartphones with WiFi and Bluetooth enabled accessories such as lights, cameras, and thermostats. Apple wishes to have strong security in this platform, so it has decided to employ 3072-bit encryption—much, much stronger than the 256-bit key Advanced Encryption Standard (AES).

To this end, Apple has asked developers to use elliptic curve cryptography for digital signatures and encrypted keys; the most secure seems to be an elliptic curve called `Curve25519`. Rather concretely, this is the curve $E : y^2 = x^3 + 486662\,x^2 + x$ defined over the field $k = \mathbb{F}_q$, where $q = \left(2^{255} - 19\right)^2$ is the square of a prime number. Daniel J. Bernstein et al. [3] showed that the abelian group $E(\mathbb{F}_q)$ has a subgroup $(\mathbb{Z}/n\mathbb{Z})$ of order

$$n = 2^{252} + 27742317777372353535851937790883648493$$

as generated by a K-rational point $P = (x_1 : y_1 : 1)$ having coordinate $x_1 = 9$. It is thought that elliptic curve cryptography is to blame for the slow rollout of *HomeKit*-ready devices for the market: developers are finding the mathematics behind this implementation to be…unusual.

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Edray Herber Goins

Credits

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How do AMS Graduate Student Chapters support the mathematical community and beyond?

Photos courtesy of Sam Houston State University.

Sam Houston State University | Hurricane Harvey Goods Drive

As fall Semester began, Hurricane Harvey hit South Texas. To help with recovery, our student-led chapter hosted a goods drive. We provided a list of most-needed items, posted flyers, sent out emails letting faculty know, and put out collection boxes. For the goods drive, we partnered with the Good Shepherd Mission. A professor's wife, working with the Mission, delivered and distributed our goods. We had a great collection of donated items, including food, toiletries, and other household goods. We were very proud with the outcome of the drive.

Photo courtesy of Wesleyan University.

Wesleyan University | Teaching Panels

As in past fall semesters, we organized a teaching panel for graduate students, who will teach for the first time this coming semester. The event featured our more-experienced graduate students sharing their know-how teaching at Wesleyan, and their advice for classroom success.

University of Kansas | Community Workshops

Each year for Math Awareness Month, the KU math department hosts a math competition and workshops for elementary school students. These workshops bring in local elementary school classes to participate in fun interactive activities designed to encourage mathematical thinking.

Photo courtesy of University of Kansas.

Photo courtesy of University of New Orleans.

University of New Orleans | "PI Talks!" Seminars

Seminars held every semester provide a venue for current graduate students to present their work on an interesting topic to other students. Faculty and staff from the mathematics department and other related fields are also invited to present their research to students who are looking for an advisor and seeking to learn about research in the department.

AMS American Mathematical Society

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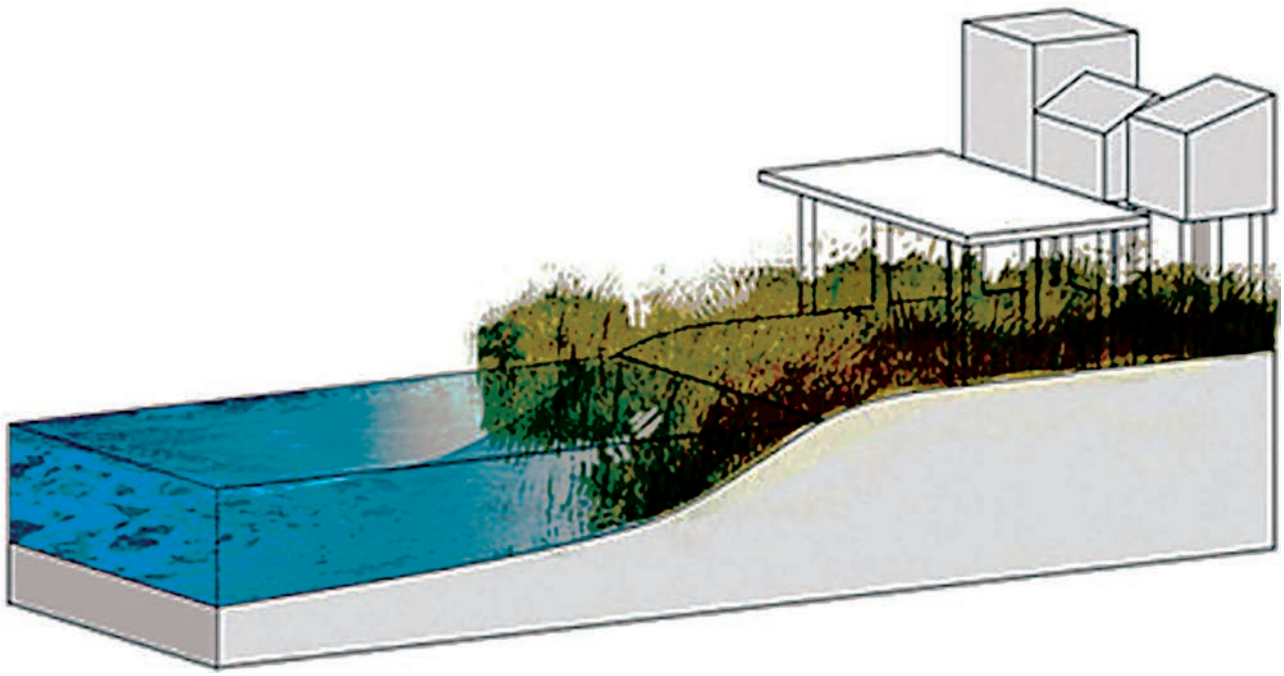
Predicting the 100-Year Flood to Improve Hurricane Storm Surge Resilience



Talea L. Mayo

If you ask a room full of fifty people to explain what is meant by the "100-year flood" you will probably get as many different answers. In fact, I know you will get at least three because that is the number of responses I got when I asked the question of my undergraduate fluid mechanics course on the first day of class last semester. The first student wondered if it described the largest flood level that had occurred in the past 100 years. The second student supposed it was the largest flood event that could be expected in the *next* 100 years. The third student guessed that it was the chance that flooding would occur at all over the next century.

I was mostly just excited that these students were brave enough to engage with me so early in the semester, so early in the morning, but I also appreciated having confirmation of something I already knew: there is much work to be done in the way scholars communicate math and science, especially to public audiences. The students I teach in the Civil, Environmental, and Construction Engineering Department at the University of Central Florida are incredibly bright. The answers they gave were informed by both their experiences as residents of coastal Florida and their access to higher education, and were completely reasonable suppositions.

However, our discussion made it clear that they did not understand the meaning of the 100-year flood level. One can only imagine the misconceptions by members of the

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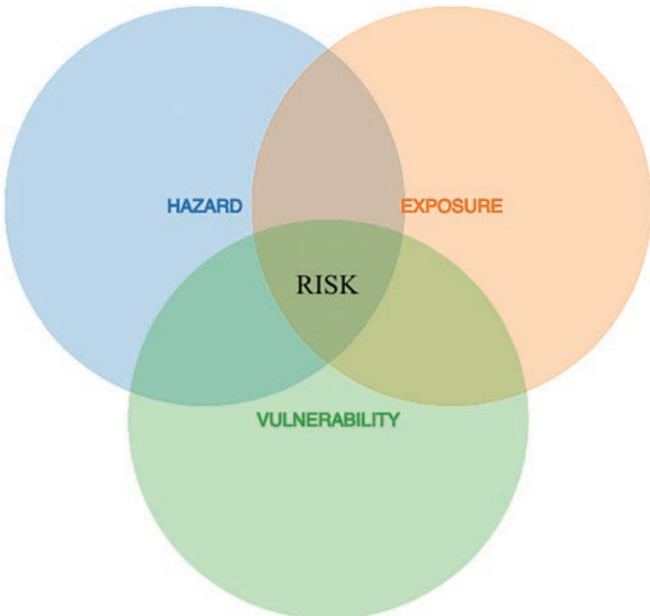

Figure 1. Risk is the product of hazard, exposure, and vulnerability.

Figure 2. Elevating assets reduces their vulnerability to storm surges, however if the hazard and/or exposure are great enough, the risk may still be large.

public who do not have coastal exposure or college educations. In fact, this ignorance has been well documented [1, 7].

This is problematic because the risk of storm surge is increasing, posing an inevitable threat to coastal communities. Innovative solutions will be required as these communities work to improve their resilience to storm surge over the coming decades. Diversity often fuels innovation, as the unique backgrounds and experiences of individuals from multiple groups can aid in the development of creative solutions to challenging problems. Thus, it is important that science be inclusive, and an important step toward inclusivity is making it accessible by explaining field-specific jargon so that larger audiences can engage in scientific conversations. A major component of the work in predicting the 100-year flood is communicating to broad audiences what this even actually means.

So what is the 100-year flood? And why does it matter so much? My third student was on the right track when he explained that it is used to describe the chance, or rather the *risk*, of flooding. The term "risk" is used rather loosely both within and outside of the scientific community. "You risked your life when you dived into the ocean after me." "The risk of heart disease decreases with regular exercise." "Forest fires pose a great risk to this neighborhood." In the context of natural hazards, however, risk is defined as the product of hazard, exposure, and vulnerability (Figure 1):

$$\text{risk} = \text{hazard} \times \text{exposure} \times \text{vulnerability}.$$

The hazard is the threatening climate or weather phenomenon of interest and its probability of occurrence. Exposure describes assessment of the values that are present in the area where the hazard may occur. Vulnerability describes the potential of the hazard to negatively impact the exposed community [3, 8]. If any of these components is reduced, the overall risk of the natural hazard is reduced as well. Clearly, if there is no storm surge, there is no (imminent) risk of flooding due to storm surge. The Atlantic hurricane season begins on July 1 and ends on November 30 of each year. The risk of storm surge is essentially zero the rest of the year, since there are generally no hurricanes and thus no storm surge hazard from December through June. More subtly, if there is no exposure to the hazard, there is no (imminent) risk of storm surge. In 2017, I was home alone and six months pregnant during Hurricane Irma. I was able to remain quite calm as the storm approached central Florida because I understood that my risk of storm surge was quite low, since exposure to storm surge is low in Orlando where I live. (For the record, I probably will not stay home during a major hurricane now that my daughter is outside of my body; we were out of power, i.e., air conditioning, for four days after the storm!) Finally, storm surge risk can be greatly reduced by reducing vulnerability. An elevated home is much less vulnerable to storm surge than one whose foundation rests on the ground (Figure 2). Similarly, communities with accurate knowledge of their storm surge risk can properly prepare for the hazard, and are thus less vulnerable to it.

To most effectively reduce the risk of a natural hazard, it must first be adequately assessed. Adequate assessment of risk requires adequate assessment of each of its components. Specifically, it requires the assessment of the storm

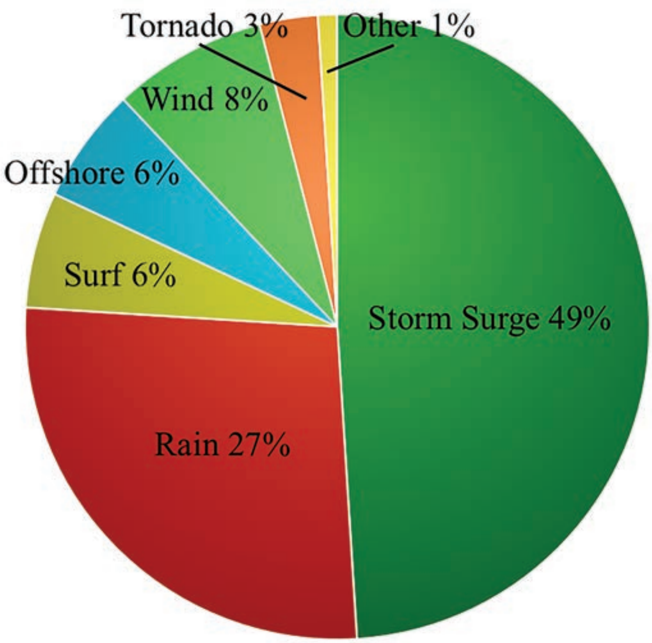

Figure 3. Storm surge is the primary cause of death during severe hurricanes in the United States.

surge hazard; we must understand the 100-year flood. **The 100-year flood is the flood height that has a $\frac{1}{100}$ = 1% probability of occurring *each year*.** (Similarly, the 1-year flood is the flood height with a $\frac{1}{1}$ = 100% probability of occurring each year, the 500-year flood is the flood height with a $\frac{1}{500}$ = 0.2% probability of occurring each year, etc.) The flood height is called the *return level* and the denominator in the fraction defining its probability is called the *return period*. One-hundred-year flood levels and other return levels are important because structures are often designed with these values in mind. The average home must be able to withstand the 100-year flood + 1 ft., while more critical infrastructure, e.g., hospitals and protective levees, is built to withstand flood levels with even higher return periods. These values are also used to inform flood insurance rates [2]. While several hazards can cause flooding, flooding due to hurricane storm surges is especially important for coastal communities. Storm surge is the primary cause of life and property loss during severe hurricanes in the United States (Figure 3) [11]. Understanding the 100-year flood due to storm surge plays a central role in saving lives and property.

A return curve can be used to graphically depict multiple return levels and their associated probabilities (Figure 4). Assuming these *annual exceedance probabilities* do not change, they can be used to determine exceedance probabilities over time using concepts from elementary probability theory. For example, we can compute the probability that the 100-year flood will occur at least once in 100 years. To do so, we must first calculate the probability that it will *not*. The probability that the 100-year flood will *not* occur in a given year is 99%, the probability that it will *not* occur in two years is 99%×99% = 98%, and the probability that it will *not* occur in 100 years is $(99\%)^{100} = 37\%$. Thus, the probability that the 100-year flood *will* occur at least once in 100 years is $1 - (99\%)^{100} = 1 - 37\% = 63\%$, i.e., it is quite likely (though it is not 100%, as its name might imply).

Typically, the computation of a desired return period requires data collected over lengthy time periods. This poses challenges in storm surge hazard assessment due to 1) the rarity of hurricane and storm surge events, and 2) the limited amount of data describing those events. This is especially problematic for regions with historically low cyclonic activity, such as the North Atlantic coast of the United States, and regions where data collection has historically been scarce, such as the northwestern Pacific Ocean. Furthermore, available data must be carefully modeled in order to extrapolate high return levels from lower impact, higher probability events. Return periods have been gravely underestimated when, e.g., linear extrapolation is used in place of extreme value analysis. Additionally, computing return periods in this way relies on an assumption of stationarity, i.e., that the risk of storm surge is not changing and storm surges in future climates will follow the patterns of those in the past. There is increasing evidence to refute this assumption. Variations in local mean sea levels are evident and directly impact the 100-year flood level. This should be accounted for, in the most elementary way, by adding the expected value of sea level rise to computed storm surge return levels, or with more sophisticated methods, e.g., including sea level rise probability distributions in the calculation of flood probability distributions:

$$\begin{aligned} P\{H_{max} + S \leq x\} &= \int_{-\infty}^{x} f_{H+S}(h)\, dh \\ &= \int_{-\infty}^{x} \int_{-\infty}^{\infty} f_H(h-s) f_S(s)\, ds\, dh. \end{aligned}$$

(Here, H_{max} and S are random variables that describe maximum storm surge height and mean sea level height. Their probability density functions are f_H and f_S, respectively, and x is a specified value used to define the probabilities of their sum, which has the probability density function f_{H+S}.) However, this in itself is insufficient for accounting for the impacts of impending climate changes. There is a general consensus among climate scientists that tropical cyclones are becoming more intense, and though there is some divergence regarding the direction, most climate models predict changes in tropical cyclone frequency [9]. This has direct implications for storm surges, which are

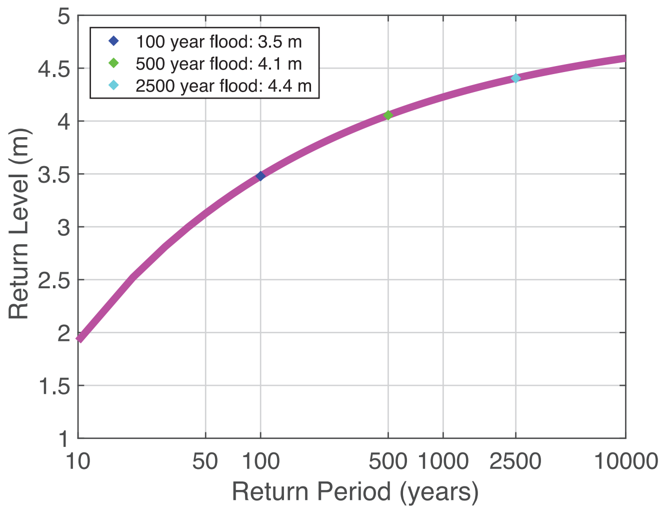

Figure 4. Return curves are used to describe multiple levels of flood risk.

not accounted for in the more traditional approaches to risk assessment described above.

Recently, new methods of computing storm surge risk that more comprehensively account for the impacts of climate change have been developed. Thousands of synthetic tropical cyclones can be generated using downscaled data from climate models in order to simulate representative hurricanes, i.e., hurricanes from the modeled climate, including those climates for which there is insufficient hurricane and/or storm surge data and those climate conditions which have not yet occurred [5]. To estimate the storm surges associated with these hurricanes, the storms can be used as the meteorological forcing in numerical models used to simulate coastal hydrodynamics (e.g., the Sea, Lake, and Overland Surges from Hurricanes (SLOSH) model [6] and the ADvanced Circulation (ADCIRC) model [10]). These models numerically solve the shallow water equations, a depth-averaged approximation of the Navier–Stokes equations, to estimate water elevations and currents. The numerical models can estimate the maximum storm surge height that would have occurred over the duration of each synthetic storm at any specified location. In the end, the thousands of synthetic hurricanes yield thousands of synthetic maximum storm surges, producing a large sample of storm surges from which extreme value statistics can be computed. Specifically, the generalized Pareto distribution (GPD) can be used to compute return curves for the synthetic storm surges of a given climate. The probability density function of the GPD,

$$f(h) = \frac{\zeta_u}{\sigma}\left[1 + \xi\left(\frac{h-u}{\sigma}\right)\right]^{-1-\frac{1}{\xi}},$$

is defined by fitting the extremes of the synthetic storm surge data. Here, h is storm surge height, u is a threshold parameter defining which storm surge heights are classified as "extreme," ζ_u is the probability that any given storm surge exceeds it, and σ and ξ are scale and shape parameters used to fit the GPD to the data. We can use the GPD to estimate the 100-year flood level for past, present, and, most importantly, future climate scenarios.

Applications of this approach to risk assessment are vast. Not only can the 100-year flood level be used to more accurately assess flood risk and design sound infrastructure, it can inform strategies for climate adaptation. As the climate changes, communities can implement various strategies to increase their resilience to storm surges. Strategies include natural, nature-based, structural, and non-structural approaches, such as preservation of wetlands, introduction of engineered sand dunes, construction of levees, and development of policies mandating retreat, respectively. These types of proactive approaches are more effective than reactive approaches. In the long run, they are more cost effective and save more lives and property [4]. Despite the large return on investment should a storm surge event occur, reactive approaches, such as the controlled release of two reservoirs during Hurricane Harvey to protect downtown Houston at the expense of homes in smaller communities [12], dominate emergency management practices because proactive approaches often come at a higher initial cost of implementation. Adopting resilience measures incrementally, as the climate changes and the risk of storm surge increases, is a more practical means of implementation for small communities and their local governments. These types of adaptation strategies were developed and studied for several communities in the Northeast United States as part of the project Structures of Coastal Resilience (SCR), which was developed with the goal of addressing the flood risks to vulnerable coastal populations in the wake of Hurricane Sandy (Figure 5).

Like the goals of many high impact projects, the goal of SCR was an ambitious one. As described above, comprehensive risk assessment is much more involved than e.g., computing statistics of historical flood data. The flood hazard, exposure, vulnerability, and all that each of the three entail must also be considered. The challenges we face as a society are growing more and more complex in a similar fashion. Effective, innovative solutions to these difficult problems require diverse perspectives and interdisciplinary teams. This proved to be instrumental to the success of SCR. The team that united to undertake its lofty aims included mathematicians, computational scientists, structural engineers, geoscientists, architects, and experts in public policy, among others. Additionally, several team members had expertise in geographic information systems

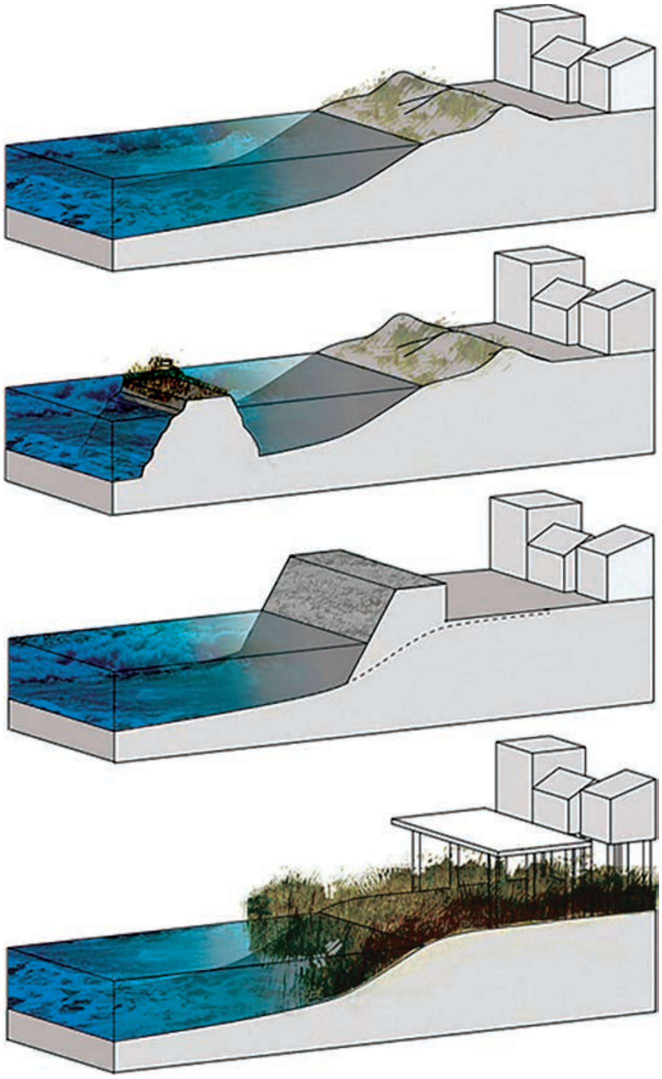

Figure 5. Layered adaptation strategies for improving storm surge resilience can be implemented incrementally and may be more practical for coastal communities than traditional approaches.

(i.e., ArcGIS), a platform that facilitates the creation, management, sharing, and analysis of spatial data. This greatly supported communication between team members from differing academic disciplines, and aided in presenting the research findings of the project in a mode more accessible and understandable to public audiences than more traditional academic formats (Figure 6). The team also included members from several organizations, including four universities, and several groups historically underrepresented in STEM fields, including both women and minorities (and women minorities, like me!). In the end, the diversity of the team played an important role in the development of creative solutions to a major research challenge.

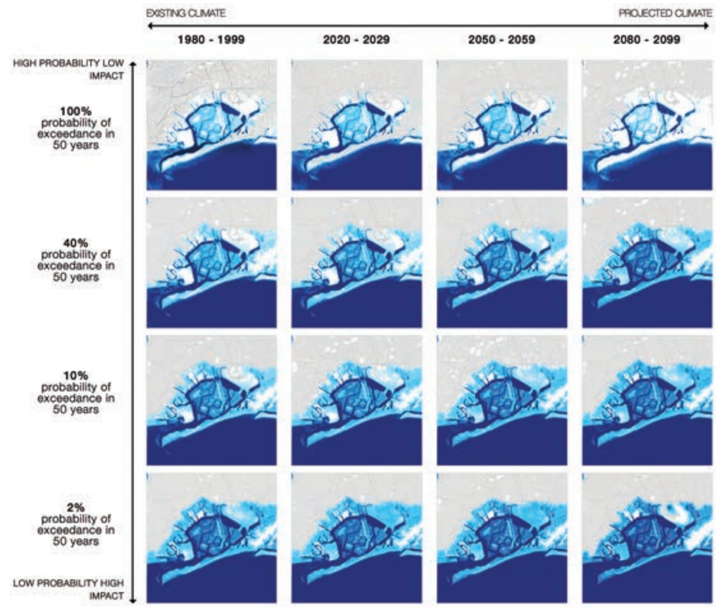

Figure 6. The project Structures of Coastal Resilience used storm surge risk assessment to develop adaptation strategies for vulnerable coastal populations in the wake of Hurricane Sandy. More information can be found at `www.structuresofcoastalresilience.org`.

The project was successful because of the unique contributions of each team member. So perhaps what is equally as important as knowing the meaning of the 100-year flood is ensuring the diversity in the groups of people working to increase our resilience to it.

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Talea L. Mayo

Credits

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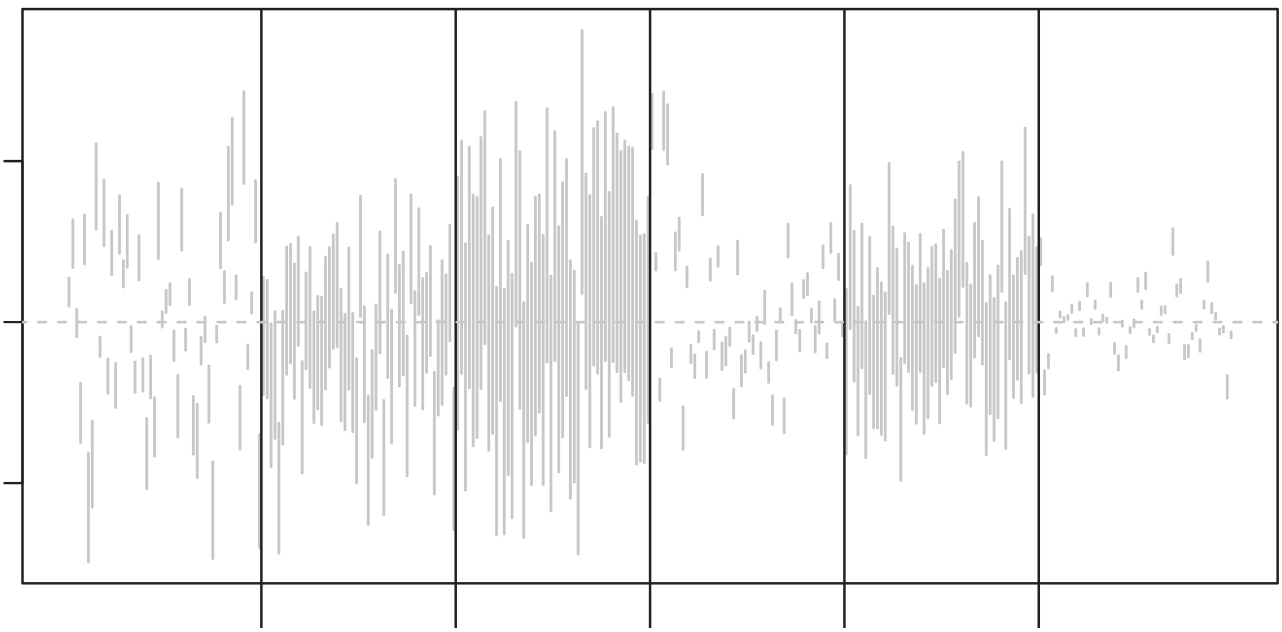
Assessment of Prediction Algorithms for Ranking Objects



Jacqueline M. Hughes-Oliver

Prediction algorithms are everywhere [11]. Based on known or observable features of a set of objects, these algorithms provide informed guesses about unknown or unobserved outcomes of the same set of objects. For this article, interest is limited to a single unknown outcome that can take one of two states: positive or negative. In the context of credit scoring, positive may indicate that an applicant for a loan is deemed likely to default on the loan. In the context of screening biomarkers, positive may indicate that a biomarker is strongly associated with a particular disease. In the context of information retrieval, positive may indicate that the returned item is relevant for the request. And in the context of drug discovery in the pharmaceutical industry, positive may indicate that a compound has scored well in a biological assay and has been cleared for follow-up testing as an active or hit compound. For these and other applications, prediction algorithms typically provide a **score** (S) that represents the belief that an object is in the positive class; this score is based on attributes of the object. Without loss of generality, assume that larger scores indicate strong belief for the positive class.

Prediction algorithms for qualitative or binary outcomes are sometimes referred to as **classification algorithms**. By applying a threshold (t) to the score, $S > t$ may be used to classify the object as a positive, while $S \leq t$ classifies the object as a negative. On the other hand, the task of ranking goes beyond the task of classification. Prediction algorithms are referred to as **ranking algorithms** when the returned scores are used to provide an ordered arrangement of objects. Consider an example in credit scoring where five applicants receive scores of 0.9, 0.2, 0.8, 0.4, and 0.7, respectively. Using a threshold of 0.5, applicants one, three, and five are classified as likely to default on the loan, while applicants two and four are classified as likely to repay the loan. Classification offers equal treatment to applicants two and four, because we believe they will both repay. Ranking, on the other hand, regards customer two as less risky due to their lower score. If money were available to fund only one applicant, ranking suggests

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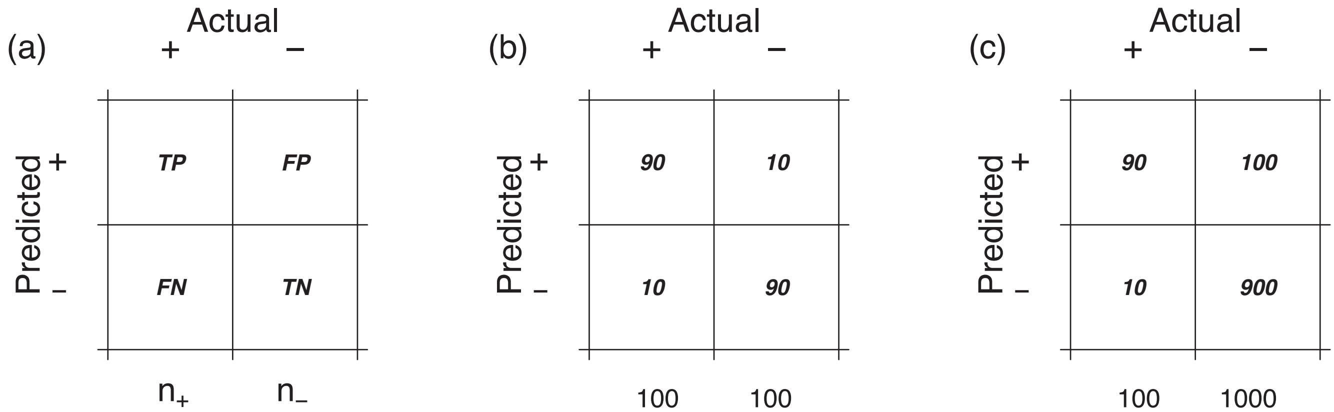

Figure 1. Cross-tabulation of actual and predicted classes, based on a specified threshold. (a) gives standard notation. (b) displays a numerical example where there are equal numbers of positive and negative objects. (c) displays a numerical example where there are ten times as many negative objects as there are positive objects. According to acc, acc^+, and acc^-, the examples in (b) and (c) are equivalent in quality. According to $prec$, the example in (c) has much worse performance than that in (b).

funding applicant two, while classification suggests that applicants two and four are equally worthy. Classification may occur after the ranking task is complete, but ranking needs more information than that provided by classification alone. This article is primarily concerned with the task of ranking objects. Assessment for ranking performance requires extra investigations beyond classification performance. Two popular assessment measures are compared, along with their uncertainty under perturbation of data.

Consider a collection of N objects with algorithm scores $S_1, S_2, \ldots, S_N$. Suppose these objects are classified by applying a threshold t. If the true membership classes for the objects could be determined, a natural check on the effectiveness of the classification task is a cross-tabulation according to **predicted class** and **actual class**, as shown in Figure 1(a). The counts TP and TN represent correct classifications and are referred to as the number of **true positives** and the number of **true negatives**. FP is the count of objects that are classified as positive but are actually negative, i.e., the number of **false positives**. FN is the number of **false negatives**. In any collection where class membership is static, the number of actual positives, n_+, does not change, and $TP + FN = n_+$. Similarly, $n_- = FP + TN$ is the number of actual negatives. A goal of classification is to maximize $TP + TN$ or maximize **accuracy** (acc), defined as $(TP + TN)/N$. This goal may be subdivided as maximizing **accuracy-for-positive-objects** (acc^+), defined as TP/n_+, and maximizing **accuracy-for-negative-objects** (acc^-), defined as TN/n_-. acc^+ is commonly referred to as the **true positive rate** (tpr) or **sensitivity** or **recall**. acc^- is very commonly referred to as **specificity** or $1-$ the **false positive rate** (fpr). Note that acc is a weighted average of acc^+ and acc^-: $acc = w \cdot acc^+ + (1-w) \cdot acc^-$ where $w = n_+/N$ is the observed fraction of positive objects.

Accuracy, while desirable even for the task of ranking, can mask undesirable performance when one class has far more members than the other class; this is the situation of **class imbalance** or **low prevalence**. Figures 1(b) and 1(c) show two classifiers with high values of 0.9 for acc, acc^+, and acc^-. But the classifier in Figure 1(c) is actually much worse than the one in Figure 1(b) in that 90 percent of the predicted positives are actually positive in Figure 1(b), while only 47 percent of the predicted positives are actually positive in Figure 1(c). This illustration demonstrates the usefulness of **precision** $(prec)$ as a measure of assessment. Precision, also known as **positive predictive value**, is the fraction of predicted positives that are actually positive, and is obtained as $TP/(TP + FP)$. The classifiers in Figures 1(b) and 1(c) have $prec = 0.9$ and $prec = 0.47$, respectively.

It is important to realize that acc^+, acc^-, and $prec$ can change according to changing the threshold, because changing the threshold can then lead to different tables as in Figure 1(a). To stress this dependence, we use notation acc_t^+, acc_t^-, and $prec_t$. Ideally, assessment for both classification and ranking should investigate the extent to which all of acc_t^+, acc_t^-, and $prec_t$ are large, for some ideal threshold or range of thresholds. The most popular approaches for assessment, however, unfortunately only focus on two of the three measures.

The **receiver operating characteristic** (ROC) curve focuses on acc^+ and acc^-. It plots $1 - acc_t^-$ as the abscissa and acc_t^+ as the ordinate, for different values of t. For largest t, the ROC curve starts at (0,0). As t decreases, the ROC curve is nondecreasing, ending at (1,1) for the smallest t. The ideal location on the ROC curve is the

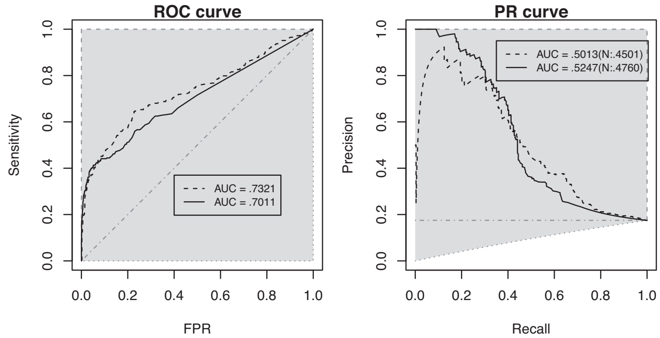

Figure 2. Estimated ROC and PR curves for the MAO dataset. Curves resulting from the random forest prediction algorithm are solid while curves from the logistic prediction algorithm are dashed and black. Shaded areas reflect the maximum and minimum possible curves, and the dot-dashed lines correspond to random guessing. Areas under each curve (AUC) are also shown. For the PR curves, normalized areas under the curve are preceded by "N:". Using ROC, logistic regression is better. Using PR, random forest is better. For ranking, random forest is better because three of the four largest logistic scores are assigned to inactive compounds while the 28 largest random forest scores are all assigned to active compounds.

point (0,1), but in practice this is rarely achieved. There is a large body of literature on methods for determining optimal or near-optimal thresholds, all based on the idea of getting as close to (0,1) as possible; see, for example, [15]. An alternative strategy (one that this article uses) is to avoid selecting a single threshold and instead assess the ROC curve over all thresholds by obtaining the **area under the ROC curve**, denoted ROC (note difference in font) hereafter. Ideal classification results in $ROC = 1$, while random guessing results in $ROC = 0.5$. The ROC curve and resulting ROC have been extensively used and studied in many fields, including medical testing, artificial intelligence, machine learning, and banking. See, for example, [14], [8], and [13].

Because it does not monitor $prec$, which can be appallingly low when classes are imbalanced (as implied in Figure 1), the ROC curve has lost prominence in the presence of imbalance. Its replacement, the **precision-recall** (PR) curve, plots acc_t^+ as the abscissa and $prec_t$ as the ordinate, for different values of t. The ideal location on the PR curve is the point (1,1). **Area under the PR curve**, denoted PR hereafter, has become a popular means of summary. However, while ROC can take values from zero to one for N sufficiently large and irrespective of class imbalance, PR cannot ([2], [3], [12]). PR curves are more affected than ROC curves by characteristics of the underlying population of objects to be ranked. Specifically, PR curves created from populations with balanced classes (i.e., prevalence near 0.5) will look very different from PR curves created from populations with class imbalance. PR curves directly reflect the degree of imbalance in the data in two primary ways ([6], [1],[10]). First, as t approaches its lower limit, $(acc_t^+, prec_t)$ often approaches $(1, w)$, where recall that w is the fraction of objects that are positive. Second, the PR curve is bounded below, thus preventing PR from taking the value zero. A consequence is that ROC values may be directly compared across different datasets, but PR values must be normalized before being compared across different datasets. A reasonable normalization is $(PR - PR_{min})/(1 - PR_{min})$, where PR_{min} is the smallest possible value of PR and it has a closed-form expression ([2], [3], [12]). If, however, different datasets are forced to have the same frequencies of positive and negative objects, then PR many be compared without normalization.

While the PR curve gained popularity to address issues regarding class imbalance, it is also very useful for assessing ranking algorithms. Ranking algorithms produce an ordered arrangement of objects where objects earlier in the ranking are expected to have a greater chance of being positive. By its very definition, $prec$ rewards this type of ranking so that only effective ranking algorithms will have high PR curves.

Monoamine Oxidase Inhibition

[4] was one of the first papers to make a pharmaceutical-quality drug discovery dataset publicly available. A collection of 1646 compounds were tested for their monoamine oxidase (MAO) inhibition ability. MAO inhibitors gained popularity in the 1950s for treatment of depression. Today they are used to treat Parkinson's disease [17]. Compounds were placed in four categories: 1358 have potency value zero (inactive, i.e., negative); 114 have potency value one (mildly active); 86 have potency value two (moderately active); and 88 have the highest potency value of three (highly active). Many papers studied these groups and found that the high potency group is dominated by

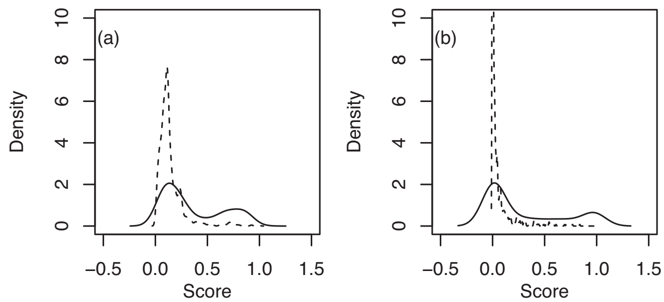

Figure 3. Estimated density curves of prediction scores for the MAO dataset: (a) for the logistic model and (b) for the random forest model. Curves corresponding to prediction score for the negative class are dashed while curves for the positive class are solid.

two mechanisms that are relatively easy to uncover [20]. On the other hand, the mild and moderate potency groups yield no clear structure or pattern. As a result, prediction algorithms tend to have extreme difficulty identifying the mild and moderate potency groups. This extra difficulty is a major motivation for selecting this dataset for illustrating subtleties between ranking and classification.

To convert the MAO data to a binary problem, the three potency groups are combined to give a single class of actives or positives. This results in $w = 0.175$ as the fraction of positive objects. While this level of imbalance is not as extreme as commonly seen in drug discovery prediction tasks (where w can easily be 0.001 or smaller), it is imbalanced enough to be noticeable. Analysis begins by developing prediction models that use a compound's chemical structure to predict activity. Many approaches are possible, but here I present output from just two models: **logistic regression** and **random forest**. Both logistic regression and random forest provide scores that represent the estimated probability that a compound (given its particular chemical structure) is active.

While some authors use the same data to build their prediction models and to create their ROC and/or PR curves, I believe this practice gives an unrealistically optimistic view of the effectiveness of the model, in much the same way that in-sample measures are frowned upon in model selection. Instead, I conduct **stratified five-fold cross validation** to obtain scores for both the logistic and random forest models. In other words, five sets of (training dataset, test dataset) pairs are created such that each compound appears in exactly one test dataset, and all training-test pairs have approximately the same degree of imbalance. A logistic or random forest model is built using data in a particular training set, then this model is applied to make predictions on the associated test set. The process is repeated for all training-test pairs. The overall result is a single prediction (i.e., score) for each compound, where that compound was not used in estimating the model that provided its prediction. (The idea is similar to pre-validation as described in [18].) These are the scores used to create estimated ROC and PR curves as shown in Figure 2. All computing was done in R [16], and ROC and PR curves and their corresponding ROC and PR were produced using package PRROC [12], [9].

According to the ROC curve, the logistic model is better with $ROC = 0.7321$, compared to $ROC = 0.7011$ for the random forest model. But a closer look at these curves reveal that the models appear equivalent up to a false positive rate of around 0.1 (acc^- around 0.9), yielding sensitivity (acc^+) over 0.4. Pharmaceutical companies have compound libraries that commonly contain over one million compounds. If the fraction of truly active compounds is tiny, *a false positive rate of just 0.1 could result in over 100,000 compounds being predicted as active when they are not*. Because compounds predicted as active early in the ranking receive additional testing at great expense, this would be a big waste of resources. For this reason, drug discovery applications are interested in regions of very low false positive rates. The fact that the logistic model has a much higher ROC curve for higher false positive rates is of no practical value.

According to the PR curve, the random forest model is better with unnormalized $PR = 0.5247$ and normalized $PR = 0.4760$, compared to 0.5013 and 0.4501 for the logistic model. First, the fact that PR and ROC can give different orderings, even though PR-space and ROC-space have duality, is well known [7]. Second, because $prec$ directly addresses the effect of ranking (while acc^+ and acc^- do not), the PR curve is better able to show that three of the four largest logistic scores are assigned to inactive compounds while the 28 largest random forest scores are assigned to active compounds. For ranking algorithms, it is critical that objects with the largest predicted scores actually turn out to be positive, because these are the objects that will be acted on first; correct predictions later in the ranking have less relevance. The logistic PR curve eventually exceeds the random forest PR curve, but the damage from poor early rankings cannot be overcome.

For this dataset, the random forest PR is better than the logistic PR, but is this difference significant and important? If the MAO dataset were slightly perturbed, e.g., from swapping a few of the investigated compounds, would we reach the same conclusion? What if I repeated my five-fold cross validation but used a different way of assigning compounds to the training-test pairs? The bottom-line is that this entire activity is reasonably viewed as an experiment, and we have observed one of many possible outcomes. Any decision about the quality of the logistic model versus the random forest model must address uncertainties that have not yet been explored.

To begin this exploration, I first attempt to estimate the underlying score distributions (see the following section for more details). Figure 3(a) shows estimated score densities for the logistic model, while Figure 3(b) shows estimated score densities for the random forest model. In both cases, the negatives have an estimated density with scores close to zero, but a long right tail results in scores that can be very close to one. The positives have a bimodal density that appears to be reasonably modeled as a mixture of normals. One mode coexists with the mode for the negatives; this mostly represents the 200 active compounds from the mild and moderate potency groups that we expected would be difficult to identify. The other mode is far away and represents the high potency group that we expected would be relatively easy to find. The best algorithms are able to completely separate the F_- and F_+ distributions. Clearly, neither the logistic nor the random forest algorithms have achieved separation. In fact, separation was not expected for the MAO data because of previous studies explaining extreme difficulty of identifying the mild and moderate potency groups.

Probabilistic Definitions of ROC and PR Curves

To formalize the probabilistic basis of ROC and PR curves, we consider S, the score produced by a prediction algorithm, to be a random variable. An object is predicted as positive if $S > t$ and negative otherwise, for some specified threshold t. Conditioned on the object being truly positive, the distribution function for S is given as $F_+(\cdot)$. Similarly, $F_-(\cdot)$ is the distribution function for S conditioned on the object being truly negative. Let π_+ denote the true probability of membership in the positive class. [6] and [10] are good references for additional details of what follows.

The **ROC curve plots** $(x_{\text{ROC}}, y_{\text{ROC}})$, where

$$\begin{aligned} x_{\text{ROC}} &= \Pr(S > t|-) &&= 1 - F_-(t) \\ y_{\text{ROC}} &= \Pr(S > t|+) &&= 1 - F_+(t) \end{aligned}$$

for all $t \in \mathbb{R}$, with functional representation as

$$y_{\text{ROC}}(x_{\text{ROC}}) = 1 - F_+\left(F_-^{-1}(1 - x_{\text{ROC}})\right), \qquad 0 < x_{\text{ROC}} < 1, \tag{1}$$

where $F_-^{-1}(\cdot)$ is a generalized inverse of distribution function $F_-(\cdot)$.

The **PR curve plots** $(x_{\text{PR}}, y_{\text{PR}})$, where

$$\begin{aligned} x_{\text{PR}} &= \Pr(S > t|+) &&= y_{\text{ROC}} \\ y_{\text{PR}} &= \Pr(+|S > t) &&= \frac{\pi_+}{\pi_+ + (1 - \pi_+)(x_{\text{ROC}}/y_{\text{ROC}})} \end{aligned}$$

for all $t \in \mathbb{R}$, with functional representation as

$$y_{\text{PR}}(x_{\text{PR}}) = \frac{\pi_+}{\pi_+ + (1 - \pi_+)\left[1 - F_-\left(F_+^{-1}(1 - x_{\text{PR}})\right)\right]/x_{\text{PR}}}, \qquad 0 < x_{\text{PR}} < 1. \tag{2}$$

Of course, one never knows $F_+(\cdot)$, $F_-(\cdot)$, or π_+, so they all must be estimated using data. Suppose we observe independent random samples of scores $\{S_i^+; i = 1, \ldots, n_+\}$ from the positive class and $\{S_i^-; i = 1, \ldots, n_-\}$ from the negative class. This data can be used to produce estimates of π_+ (call it $\hat{\pi}_+ = n_+/(n_+ + n_-)$), F_+ (call it $\hat{F}_+$), and F_- (call it $\hat{F}_-$) that may be directly inserted into equations (1) and (2) to give estimated ROC and PR curves. Varying levels of assumptions may be applied to obtain $\hat{F}_+$ and $\hat{F}_-$, ranging from fully parametric, to semiparametric, to nonparametric ([14], [13]). For example, $\hat{F}_+$ and $\hat{F}_-$ could denote the nonparametric class-level empirical distribution functions, i.e., $\hat{F}_+(t) = \frac{1}{n_+}\sum_{i=1}^{n_+} I(S_i^+ \le t)$ and $\hat{F}_-(t) = \frac{1}{n_-}\sum_{i=1}^{n_-} I(S_i^- \le t)$, where $I(\cdot)$ is the indicator function. Other, simpler but often less desirable, estimation strategies lead to the formulas given in the introduction. To see this, it may be helpful to note that the terms TP, FP, FN, and TN can all be represented using notation from this section:

$$TP = \sum_{i=1}^{n_+} I(S_i^+ > t) \qquad FP = \sum_{i=1}^{n_-} I(S_i^- > t)$$

$$FN = \sum_{i=1}^{n_+} I(S_i^+ \le t) \qquad TN = \sum_{i=1}^{n_-} I(S_i^- \le t).$$

To visualize the impact of different probabilistic models on ROC and PR curves, and to help understand difficulties that can arise from bimodal distributions as evidenced in the MAO example, I consider three population-level algorithms. Cases are fully described in Table 1. Each algorithm consists of distributions $F_-(\cdot)$ and $F_+(\cdot)$, and in all cases π_+ is set to 0.175 as observed in the MAO example. $F_-(\cdot)$ is always normally distributed with mean 0.3 and standard deviation 0.05. The cases differ only in terms of

Case	F_+ distribution: mixture of normals with 0.5 mixing proportion	PR	ROC
1	component 1: mean is 0.3, standard deviation is 0.25 component 2: mean is 0.7, standard deviation is 0.05	0.749	0.750
2	component 1: mean is 0.3, standard deviation is 0.05 component 2: mean is 0.7, standard deviation is 0.05	0.650	0.750
3	component 1: mean is 0.3, standard deviation is 0.005 component 2: mean is 0.7, standard deviation is 0.05	0.616	0.750

Table 1. Three cases of population-level algorithms. Objects in the negative class have scores that follow a normal distribution with mean 0.3 and standard deviation 0.05. Objects in the positive class have scores that follow a two-component mixture of normal distributions, with details of the components given in the table. PR and ROC values are also given for each algorithm. While PR changes across different cases, ROC remains the same.

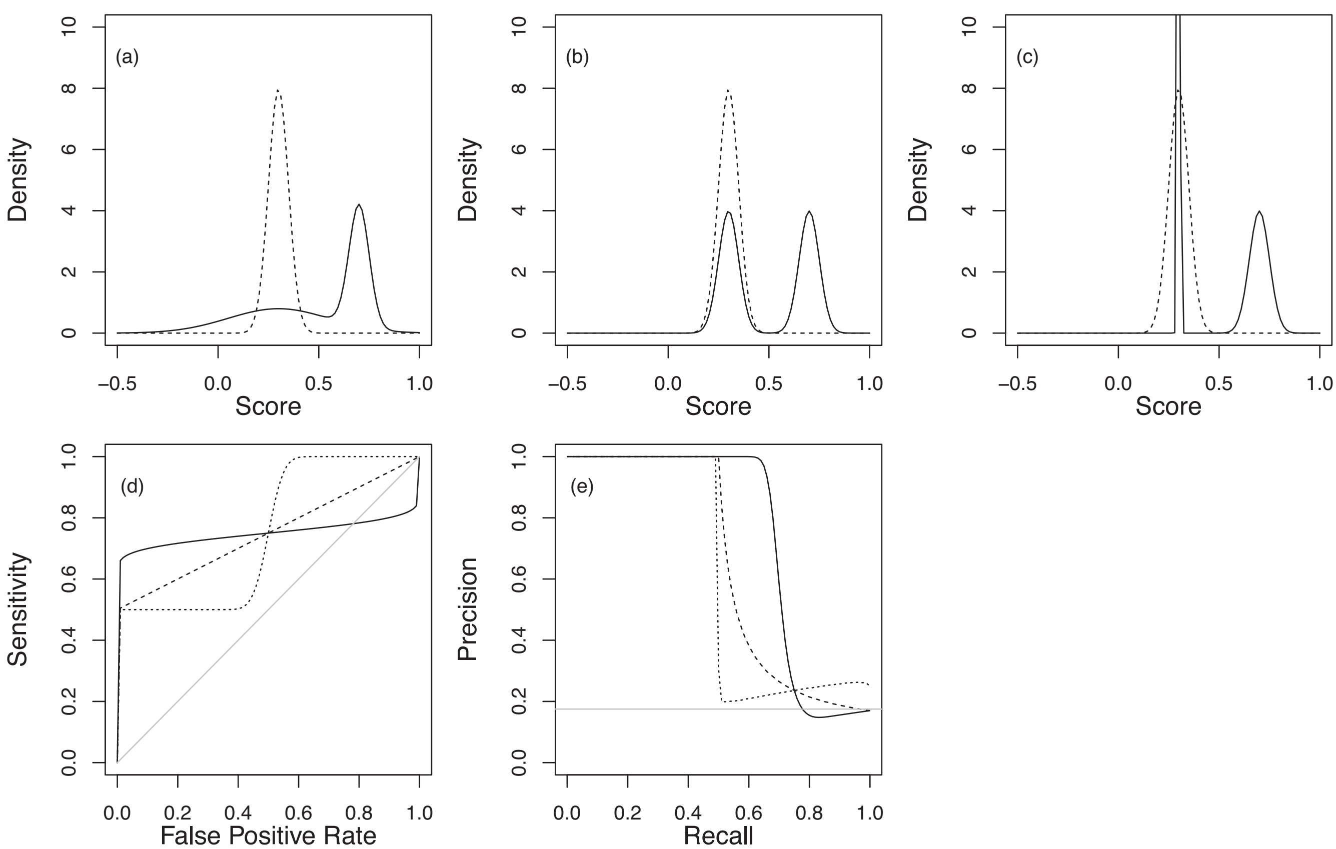

Figure 4. Densities, ROC curves, and PR curves for three cases of population-level algorithms. (a), (b), (c) display the densities corresponding to negative objects as dashed curves, and densities for positive objects as solid curves; (a) is Case 1, (b) is Case 2, and (c) is Case 3 as described in Table 1. (d) displays ROC curves for all three cases: Case 1 is solid, Case 2 is dashed, Case 3 is dotted, and the grey curve depicts random guessing. (e) displays PR curves for all three cases: Case 1 is solid, Case 2 is dashed, Case 3 is dotted, and the grey curve depicts random guessing. $ROC = 0.75$ for all three cases, but PR correctly assesses Case 1 as best able to rank objects ($PR = 0.749$), and Case 3 as worst ($PR = 0.616$).

$F_+(\cdot)$, which is always a two-component mixture of normals where the component far from the negative distribution $F_-(\cdot)$ always has mean 0.7 and standard deviation 0.05. The other component of $F_+(\cdot)$ has the same mean as $F_-(\cdot)$, namely 0.3, but its standard deviation varies from a high of 0.25, to 0.05, to 0.005. Densities for all cases are shown in Figures 4(a), 4(b), and 4(c).

The mixing proportion of 0.5 for distribution F_+ implies that all three algorithms will quickly identify 50 percent of the positives, namely those coming from the rightmost component of F_+. The ROC curves and PR curves of Figures 4(d) and 4(e) show this as all curves overlap for recall (sensitivity) values between zero and 0.5. The curves separate for other values of recall. Case 1, with the largest standard deviation for the leftmost component of F_+, perfectly identifies an additional 12 percent of positives before it struggles significantly, even dropping worse than random; it has the largest PR. Case 3, with the smallest standard deviation for the leftmost component of F_+, is exquisitely poor after identifying the first 50 percent of positives. It creates over 40 percent of false positives before restarting to find true positives at a uniform rate until all are found. Case 3 has the smallest PR. Case 2 finds the second half of positives at a uniform rate after finding the first half without errors. In other words, Case 1 does a better job of finding the actives early, Case 3 finds the actives late, and Case 2 finds the actives uniformly. [19] argue that algorithms designed for the "early recognition" (aka ranking) problem should be able to properly distinguish these cases. PR does an excellent job of assessing these algorithms. ROC, on the other hand, remains exactly the same (0.75) for all three cases. ROC is not able to make a distinction. The ROC curves clearly differ, and if focus is limited to early in the curves, it is clear that the proper ordering of algorithms is obtained. But this information is lost when area under the entire curve is calculated as the primary measure of performance, as might become necessary when comparing multiple algorithms.

Investigating Uncertainty

When data is used to obtain the PR and ROC curves and their associated PR and ROC values, effort must be expended to understand the uncertainty or variability associated with these estimates. A large body of literature exists on statistical inference for ROC; see, for example, [14], [19], [13]. The literature is scant regarding inference on PR, but findings generally suggest that bootstrap sampling is the low-risk method of choice ([2], [3]). I will use bootstrap sampling to explore both PR and ROC.

Consider the scenario presented in the MAO example, where there are two competing algorithms. "Which is better?" may well be answered by the difference between their areas under the curves, namely $PR_2 - PR_1$ or $ROC_2 - ROC_1$. Algorithm 1 is better if $PR_2 - PR_1 < 0$, so it makes sense to obtain a confidence interval for $PR_2 - PR_1$, then the degree to which the interval contains negative values determines our answer. By now, skepticism should be high in your mind regarding the utility of $ROC_2 - ROC_1$ for assessing ranking performances, so we do not expect the confidence interval to the useful.

		Case for Algorithm 2		
		1	2	3
Case for Algorithm 1	1	A	B	C
	2		D	E
	3			F

Table 2. Six scenarios, A–F, for algorithmic comparison. Each scenario describes two algorithms, with the eventual goal of determining which algorithm is better. For example, Scenario B has algorithm 1 defined according to Case 1 as given in Table 1, and algorithm 2 defined according to Case 2. In Scenarios A, D, and F, the two algorithms have the exact same probabilistic settings, so no differences are expected.

Six scenarios are studied, as outlined in Table 2. All scenarios apply distributional settings for both algorithms from among Cases 1, 2, and 3 of Table 1. Scenarios A, D, and F are **null** in that both algorithms have exactly the same probabilistic settings, so there should be no differences in values of PR; the confidence intervals are expected to include zero. Scenarios B, C, and E are **non-null** and differences are expected. The differences for C are expected to be largest (approximately $0.616 - 0.749 = -0.133$), followed by B (approximately $0.650 - 0.749 = -0.099$), then E (approximately $0.616 - 0.650 = -0.034$).

Bootstrap sampling reduces the need for assumptions that are often tenuous at best. New pseudo-samples are created from the original sample by sampling with replacement. My sampling was stratified to maintain the level of imbalance. For each simulation replicate within each Scenario A to F, 1000 bootstrap samples are obtained and these samples are used to determine a percentile confidence interval. R package `boot` [5] was used for bootstrapping. Figure 5(a) displays 95 percent confidence intervals for $PR_2 - PR_1$ from 50 simulation replicates for all of Scenarios A–F. Consistent with expectations, the confidence intervals for Scenario C are most negative, followed by Scenario B then E. It was surprising to me that the confidence intervals for Scenarios A, D, and F do not overwhelmingly include zero. They are close to zero, so may be treated as not practically different from zero, but they bounce around, sometimes above other times below zero. Increasing the

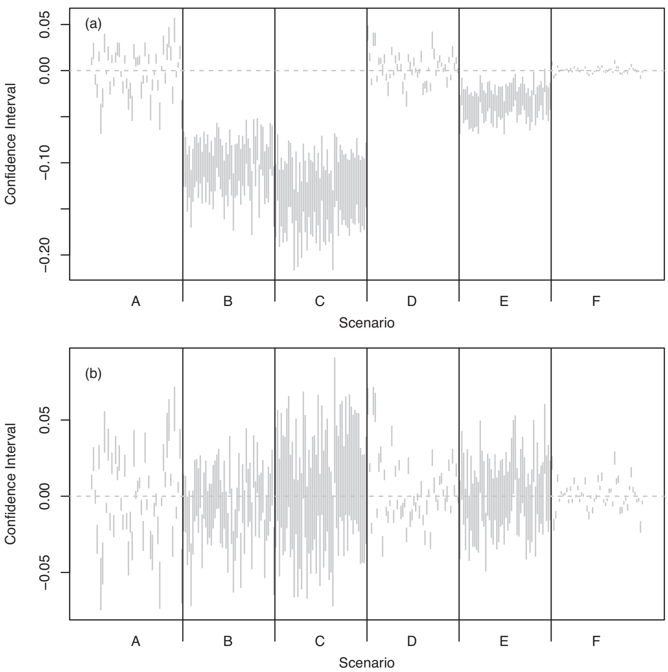

Figure 5. Ninety-five percent confidence intervals for $PR_2 - PR_1$ (plot (a)) and $ROC_2 - ROC_1$ (plot (b)) from the six scenarios of Table 2. For each scenario and each independently obtained sample, a bootstrap percentile confidence interval is shown where upper and lower endpoints are connected by grey lines. For scenarios where both algorithms are probabilistically equivalent (i.e., A, D, and F), one expects approximately 95 percent of the intervals to include zero. Results are shown based on 1000 bootstrap samples for each of 50 independent samples generated from each scenario.

number of bootstrap samples had no impact. Another observation is that the uncertainty level (wide intervals mean greater uncertainty) of the estimated $PR_2 - PR_1$ decreases as the estimate itself gets closer to zero, and more rapidly so if the individual components (PR_2, PR_1) are small.

Figure 5(b) displays 95 percent confidence intervals for $ROC_2 - ROC_1$ from 50 simulation replicates for Scenarios A–F. Recall that ROC for Cases 1–3 are all 0.75, so no differences are expected. Confidence intervals for Scenarios B, C, and E now mostly cross zero, unlike in Figure 5(a). It is surprising that Scenarios A, D, and F behave similar to how they are in Figure 5(a), with confidence intervals hardly ever crossing zero but bouncing above or below zero.

Summary

The PR and ROC curves, and their corresponding area under the curve measures, were compared as assessment criteria for effective ranking by prediction algorithms for the two-class problem. ROC is shown to be particularly deficient while PR is quite effective. A real dataset revealed a bimodal score distribution for the positive class, and this resulted in problems with discrimination between scores for the different classes of objects. A simulation study revealed unusual patterns in confidence intervals for differences $PR_2 - PR_1$ and $ROC_2 - ROC_1$ when algorithms 1 and 2 are probabilistically equivalent. Further studies are needed to understand these uncertainties.

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Jacqueline M. Hughes-Oliver

Credits

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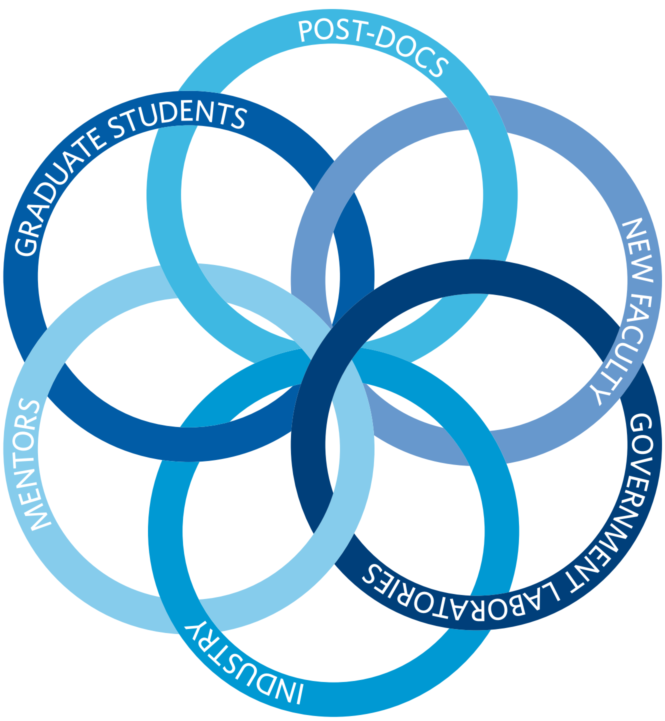
The Early Career Section is a new community project, featured here in the *Notices*. This column will provide information and suggestions for graduate students, job seekers, junior academics of all types, and those who mentor them. Angela Gibney serves as the editor of this section.
This month's theme is Research. Next month's theme will be Writing.

On Choosing a Thesis Advisor

Robert Lipshitz



No external factor has as much influence on your graduate career as your PhD advisor. The goal of this article is to help you find an advisor you can work with successfully. I have no particular expertise, beyond being happy with my choice of thesis advisor, and these thoughts are drawn mainly from watching students handle this decision over the years, and some discussions with other mathematicians.

I will focus implicitly on the US academic system, where thesis advisors are usually chosen in the second or third year of graduate school, after students finish some general course work and qualifying exams. At most universities, students take reading courses (formally or informally) with several potential advisors before choosing one, though some departments have other systems. Attending seminars, reading groups, and other events (e.g., seminar dinners) in fields of interest, starting in your first year of graduate school, is another good way to get to know the faculty.

A thesis advisor's role is to help guide students' research, help students navigate the job application process and, to a limited extent, help with other professional issues that may arise, such as questions related to teaching or professional conduct. Many thesis advisors continue to help, as mentors, well after the PhD. The key word throughout is "help": You are most responsible for your success.

Below are some questions to consider about potential advisors *X*. No advisor will be a perfect match, so you will have to balance which questions are most important to you. Once you have chosen an advisor, plan how you will handle areas of concern: again, this is your responsibility, not your advisor's. Some of the questions you should ask potential advisors, some you might get more useful answers from current (or recent) PhD students, and some you will need to figure out on your own.

Are you interested in *X*'s research area? Almost no one is competent to advise a student in a different field, so you will spend about 30 hours per week for at least the next three years thinking about mathematics in your advisor's area. (Over a longer research career, your interests will

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probably evolve.) This is one reason to take a wide range of courses early on: if you have not taken a graduate course in the field, you cannot tell if you like it. When choosing an advisor, ask what kinds of projects you might work on.

Different fields in mathematics also have different cultures, and you might find some more comfortable than others. Writing and speaking style, and level of rigor also vary by field; you can get a feel for this by reading some research papers before choosing an advisor.

Are you comfortable discussing mathematics with *X*? You must be willing to ask your advisor naive questions and be able to benefit from the answers. Some advisors give detailed answers, while others more often tell students to "go away and think about it"; both can be valuable. You should get used to asking questions, and learn how different faculty address them, by attending office hours starting in your first years. Ask yourself if you feel energized or discouraged after talking to *X*, and whether talking to *X* leads you to work your hardest.

What preparation do you need to work in *X*'s area? All mathematics is challenging, but the amount of background required varies enormously. Ask potential advisors what you would need to master to get started on research and how long it should take.

Is *X* research-active? The cutting edge of mathematics progresses rapidly, so faculty who are not research-active may not know problems of current interest and new techniques. Look for recent papers on MathSciNet® or the arXiv. Some faculty stay active mainly through their students' research, so consider recent students' work as well. The Mathematics Genealogy Project can be useful for finding past students, though your department may have more complete data.

Has *X* advised other students recently? Advising is a different skill from research or classroom teaching. If possible, ask some of *X*'s recent students about their experiences (positive, negative, and neutral) working with *X*.

Past students are also important to be able to assess:

Do some of *X*'s other students' career paths match your goals? Keep in mind that this can say more about how selective a thesis advisor is in choosing students to work with than anything else. Most faculty advise students who go on to a range of career paths.

Does your university have a community of researchers (faculty/postdocs/students) in *X*'s area? Seminars, reading seminars, topics courses, and informal conversations help with learning material and staying focused. More senior graduate students are especially valuable.

Does *X* have tenure? Many junior faculty are focused on getting their own research programs off the ground, and may not be ready to advise others.

Does *X* expect to be around for the rest of your PhD? Mathematicians move for both personal and professional reasons. Even if they make arrangements for students to move with them or to advise students from afar, moves are disruptive. Faculty also take sabbaticals and other research leaves, which can also be difficult if you are just starting on a problem. An advisor cannot commit to staying, but you should ask their current plans.

Is *X* active in the department and mathematical community? This may affect what relevant opportunities *X* is aware of, as well as *X*'s ability to advocate for you.

How often does *X* meet with each PhD student? This varies from several times a week to a couple of times per year, with an hour per week being roughly average. What is optimal for you depends on how independent you are, but less than every-other-week may not be ideal.

How does *X* expect students to find research problems? Most advisors give their students their first research problems, but some run seminars discussing open questions or expect their students to find even their first problems in other ways.

What teaching load and research support can you expect if you work with *X*? In some departments, all students have roughly the same teaching load. In others, it varies widely from field to field or advisor to advisor, depending on what grants faculty have or how actively they seek internal funding for students.

Would you be comfortable discussing other professional concerns with *X*? A secondary role of a thesis advisor is to give professional advice—for instance, about writing a research statement when applying for jobs, responding to a referee's report, or, perhaps, coping with instances of implicit or explicit bias. By contrast, personal issues are outside your advisor's purview and, likely, expertise; if personal issues are affecting your work, seek advice first from appropriate staff members or your Director of Graduate Studies.

Other Questions and Remarks

Should I have more than one thesis advisor?

Probably not. Get advice from, and discuss mathematics with, many people, but have one designated thesis advisor who is keeping track of your progress.

Can I change thesis advisors?

Yes, though this will likely set back your research program. In my experience, a handful of students switch advisors for some reason, and another handful have difficulty working with their advisors but do not switch. If your advisor-student relationship is not working, get advice from your Director of Graduate Studies, your Department Chair, or other faculty who know you.

Can I have an advisor at a nearby university?

Yes, especially if no one at your university works in your field of interest. Usually, you also need an advisor at your university monitoring your progress. Consult with your Director of Graduate Studies about how to proceed.

What if *X* does not want to work with me?

Faculty can decline to advise students, and if *X* does not want to advise you, it is his/her responsibility to say so.

Beyond that, typically it is better not to second guess how X feels: while all faculty are busy, teaching and advising graduate students is an important, and potentially rewarding, part of a mathematician's job. Be bold about talking to faculty, and let them worry about managing their time.

Concluding Remarks

To reiterate, these questions are not requirements for a successful student-advisor relationship, and there are notable exceptions to all of the ideals proposed above. You, not your advisor, are responsible for your academic success. Your advisor will be strong in some areas but not in others. That's okay: there should be other people—your unofficial and unrecognized Board of Advisors—to whom you can also turn. Think about which of the above criteria are most important to you in finding a thesis advisor with whom you will work well. Find other people and other ways to get the remaining help you need to succeed.

Moving Ahead in Your Research

E. E. Eischen

Doing research can be exhilarating, but finding problems and figuring out what to do when you get stuck can feel daunting. To develop a research program and overcome associated challenges, it is important to identify approaches to research that work for you. Here are some of my favorites.

1. **Journal.** Keeping a math journal has been my most helpful professional habit, from shortly before I began working on my dissertation through being awarded tenure.[1] You can use a journal to log goals and your progress toward them, record details of important mathematical conversations, track hours spent on different research tasks, or determine how new approaches—such as those suggested below—impact your research.
2. **Keep a problem list.** During the past twelve years, I have generated a list of over 150 potential research problems. Even if ninety percent are flawed, that leaves fifteen good ones. Keeping such a list can insulate you from the anxiety of having nothing to work on. It also provides a low-stakes exercise in finding problems. The skills we developed as students can be useful for finding new problems. For example, we were all trained to ask what happens when we change hypotheses in a theorem, investigate how much further a new technique allows us to extend a result, and make conjectures based on examples we have computed. If you do not self-edit, you will likely find new problems along the way. Keep several of your problems in mind, and continually refine them, also tying them to more central problems and techniques. You might need to reformulate a problem several times to make it feasible and interesting. As ideas strike, jot details on your list. For help identifying ones worth pursuing, discuss them with more experienced members of the field. Like anything, finding problems gets easier with practice.
3. **Embrace "mistakes" and "failure."** Many items on my problem list landed there as a byproduct of my attempt to overcome an obstacle in my research. In his seminal talk "You and Your Research," Richard Hamming observed, "[O]ften the great scientists, by turning the problem around a bit, changed a defect to an asset. For example, many scientists when they found they couldn't do a problem finally began to study why not. They then turned it around the other way and said, 'But of course, this is what it is' and got an important result." If your goal is simply to explore what is true and why, unexpected outcomes become guideposts rather than stop signs.
4. **Continually learn and relearn.** To develop mathematical intuition, it helps to learn not just the newest results but also the history of a field. I have found papers from several decades ago particularly helpful for developing intuition in my field, which has, in turn, helped me learn newer material. Often, you can find important older papers by tracing references in newer papers back repeatedly. Terry Tao's article "Learn and re-learn your field" (part of the wealth of helpful career advice on his blog) offers specific advice for engaging with material while learning it, while Ravi Vakil's "Three Things" exercise (among other useful advice on his webpage) can help you engage with research talks.

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[1] *Thanks to Julianna Tymoczko for encouraging me to start journaling about my research twelve years ago (on August 2, 2006, my journal shows).*

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5. **Commit specific times to research.** As with any skill, you get better at research through practice. In "How to get a PhD in Mathematics in a Timely Fashion," Sara Billey recommends twenty highly focused hours per week for research. Balancing this with other responsibilities that also require peak focus can be challenging, so find a target that is achievable for you. You can maximize your chances of finding time for research by scheduling it on your calendar and adhering to the schedule.
6. **Protect the time required for tasks needing different kinds of energy.** To determine how much time you need for different aspects of research, track your time. To the extent possible, schedule research tasks for times you can focus sufficiently. (You can look up references in the fifteen minutes before class, but carefully patching a hole in a proof is likely to require a bigger chunk of uninterrupted focus.) If you are frequently interrupted, consider joining a weekly writing circle to help protect some of your time.
7. **Communicate in different ways to see both the forest and the trees.** Communicating about your research in a variety of ways with different audiences can help deepen your own understanding. Writing a paper often forces you to see the trees, while a good talk typically emphasizes the forest. Either, done carefully, is likely to lead you to new insights and questions.
8. **Work supportively with others, as well as alone.** Working alone can help you develop individual strengths valuable not only in independent research but also for collaborations. Collaborating gives you the opportunity to learn from others and pool different areas of expertise for more powerful results. Different members may support a collaboration in different ways, for example via technical knowledge, novel ideas, or clear writing. While most collaborations begin informally, by one researcher mentioning a problem to another (for example, from your problem list), there are also structured programs to promote collaborations, such as Math Research Communities, Research Collaboration Conferences for Women, and the NSF/NIH Innovation Lab.
9. **Bite off one piece at a time.** If developing a new skill or project feels overwhelming, break it into manageable pieces. For example, an individual who finds it difficult to ask experts in their field questions could make a list of all the experts with whom they might want to communicate and then ask one of them a question each week.
10. **Eat, sleep, exercise, breathe.** Theorem-proving devices don't actually run solely on coffee, at least not indefinitely. It is important to take care of yourself. Otherwise, all the tips here soon become irrelevant.


Ellen Eischen

Credits

Author photo is by Miles Truesdell.

Finding New Problems to Work On

Chris Woodward

Often mathematicians know what problems we would like to solve. If your current idea is working out, congratulations! You can stop reading and go back to writing. If you're still reading, it may be because you are stuck, are out of ideas, or perhaps have just finished a project and are worried that the next one you have in mind is a minor extension of something you already understand.

If you feel stuck, it may help to keep in mind that the reality is that we spend most of our research time failing to solve the problems that interest us the most. It's easy to get frustrated, and often it's not clear what to do next. André Weil [Wei74] famously wrote, "In 1947, in Chicago, I felt bored and depressed and, not knowing what to do, I started reading Gauss' memoirs…" By learning something new at least you can preserve your sanity until progress comes along.

When looking for new directions and inspiration, you should take at least some time to read up a bit and see what seems exciting. Of course, it can feel that it's hard enough to keep up with one's own specialty, let alone read papers in another, and reading the arXiv can seem like drinking from a firehose.

If you decide on spending at least part of your time on a new direction, how important is it to look into those that seem fashionable? Popular areas may seem heavily populated by established mathematicians and their graduate students, and may not always feel welcoming. Try not to be intimidated. Learning about the active researchers and making sure to give them proper credit can go a long way towards becoming part of the community. On the other hand, pursuing something less fashionable takes real guts, particularly at an early stage in one's career. In this case you have the advantage of having more of the field to yourself.

What if you feel you are out of ideas? Fortunately there are plenty of excellent but under-explored ideas of others waiting to be rediscovered in the literature. I always found Raoul Bott's attitude inspirational. Bott didn't seem to view his own results on the topology of Lie groups the result of his own creativity as much as his ability to appreciate Morse's theory, which he called his "secret weapon" [Bes08]. His passion for promoting the ideas of other mathematicians was, I thought, one of the reasons he was so successful.

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Writing for *Mathematical Reviews*®

Kelly Jabbusch

Since 1940 *Mathematical Reviews*® has provided mathematicians with a comprehensive view of published mathematical articles and books. Beginning as a monthly print publication, it now delivers its content daily on MathSciNet®, informing the mathematical community of new and interesting publications. More than 100,000 new items are added to the database each year, and as of July 2018 there are more than 3.5 million publications and 908,800 authors indexed. One key component of *Mathematical Reviews*, which distinguishes it from other bibliographic databases, is the individual reviews of each item written by other mathematicians. Lars Ahlfors, Richard Courant, Paul Erdős, Einar Hille, Alston S. Householder, Saunders Mac Lane, John von Neumann, and Oskar Zariski are just a few mathematicians who wrote reviews for Volume 1.[1] Currently there are more than 22,000 mathematicians who actively contribute, writing short reviews that provide a valuable service to readers.

Reading *Mathematical Reviews* can be enlightening, as they often provide more detail than can be found in the abstract or summary. I have on numerous occasions found myself on MathSciNet, looking up a paper, reading its review, and then following a chain of citations and their reviews. A review often guides me towards more background information, if needed, and places the significance of the result in a broader context. A review may also include citations not found in the original article, but that refer the reader to more recent related developments. In short, a review is a recommendation from an expert.

Since finishing graduate school I've contributed to *Mathematical Reviews* as a reviewer, and have found the work stimulating and rewarding. Reviewers may select fields of interest (by Mathematics Subject Classification) and the maximum number of items assigned at a time. Articles are then sent to reviewers throughout the year, with the hope of a six-week turnaround time. Since these articles are already published, writing a review is less demanding that writing a referee report. As a referee my job is to determine if the result is correct, interesting, and appropriate for the given journal; as a reviewer my task is to provide enough information for a fellow mathematician to decide if they want to read the original article. To that end I try to do a little more than summarize the main results. I may describe a result, example, or technique that could be of independent interest, or mention if the article contains a summary of background material that makes it accessible to a larger audience.

Sometimes I am familiar with the paper sent to me, having perhaps read a preliminary pre-print version. More often, though, the papers I review contain results that are new to me. Taking the time to carefully read the article and summarize the main results and methods enriches my work as a mathematician. I learn of new results, techniques, and questions, which I might not have encountered had I not been sent the article to review. It is not uncommon for reviewers to expand on results from articles they review, opening a sort of dialogue with the paper. In this way, reviewing an article is extremely useful for me as a mathematics researcher.

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[1] *See* `www.ams.org/publications/math-reviews/reviewersvolume1` *for a complete list.*

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As mathematicians we're lucky to have MathSciNet and *Mathematical Reviews* providing us with such an invaluable resource. The large community of dedicated reviewers has consistently been a key component in the work of *Mathematical Reviews*. Participating in this community puts you in great company, and can have impact on your future work.

Kelly Jabbusch

Credits

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Research With Undergraduates

Amanda Folsom and Sam Payne

TO ADVISE OR NOT TO ADVISE

Advising undergraduate research is not for everyone, but it can be rewarding, both professionally and personally. The stakes for mentoring undergraduate research are high; young talent is precious, and students may be turned off by a bad experience, such as a dull or impossibly difficult project. Working on research with undergraduates is only one of many ways to give back to the community. If you are not confident that this particular activity is a good fit for you at this point in your life and career, or may take too much time away from other professional pursuits and obligations, you could consider smaller-scale alternatives that still allow for meaningful mathematical interactions with students outside of the classroom. If you don't want to include undergraduates in your research, that's perfectly okay. On the other hand, with the right motivation and forethought, there can be benefits for everyone.

SOME POTENTIAL BENEFITS...

...for students. There are undergraduates with talent and drive who can, with suitable guidance, do excellent and worthwhile research. Undergraduate research can also be an opportunity for motivated but underprepared students to break into real mathematics. What doing mathematics research really means can feel mysterious until you do it. A research opportunity as an undergraduate can shed light on many aspects of mathematics that a student won't typically encounter in the classroom, and can be a pivotal experience.

...for advisors. Students sometimes make substantial and meaningful contributions to important projects. It is also often an advisor's market—there are generally many qualified, interested students. There is great satisfaction in seeing talented young mathematicians experience a positive and realistic first taste of research, including the sweet thrill of solving an open problem. It is absolutely possible to learn new things while working with students.

...for junior faculty. Postdocs mentoring undergrads often find that this experience helps them stand out in the pool of tenure track job applicants, and it is a common topic brought up in on-campus interviews. Young faculty applying for grants may also find that including carefully designed projects suitable for collaboration with undergraduates is generally viewed favorably by review panels, especially when the PI has an established track record of successful mentoring.

GUIDING PRINCIPLES AND PITFALLS TO AVOID

Choosing a project. Choose your project with care. Find a topic of genuine mathematical interest that is also accessible. Many different kinds of projects can work, but it's important to be organized and prepared in order to offer meaningful guidance. In some situations, the ideal research may be open-ended and motivated by readily computable examples. Others may call for more specific and narrowly tailored assignments. In any case, students will sense whether or not you are genuinely interested, and also whether or not you already know how to solve the problem. Proceed wisely.

Organization. Plan the structure and schedule that you want to implement, and communicate expectations clearly at the beginning of your program. An effective daily and weekly timetable may be fixed or flexible; consider in advance what will work best for you and your students. Junior faculty especially may need to protect some time for themselves.

Timing. Research projects typically take longer than expected. Do you need to wrap up your project within a certain time frame, and have you set reasonable goals in order for this to happen? Adaptability can be important

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for ensuring that you and your students have a positive experience, even when the unexpected happens.

Struggling students. Intensive research can be immensely frustrating, and sometimes leads to loneliness or isolation. A student working on a solo project over the summer when campus has cleared out and the department is empty is perfect fodder. Group work is not an antidote, but it often helps. Plugging into a program that includes multiple groups with planned activities (colloquia, opportunities to present work, attend conferences, and social events) may increase the chances of a positive overall experience. If different groups have projects that are related enough to be able to have meaningful conversations (and distinct enough that each group feels genuine ownership of their own problem), that's even better. We may joke about machines for turning coffee into theorems, but students are human beings. Stay tuned in to what's going on, and reach out to help when struggles arise. Consider including having a good time as part of your definition of success.

Group dynamics. Even if mathematics is your top priority, human foibles will be part of the experience. Anticipate common hurdles such as impostor syndrome and issues around coauthorship. Interpersonal conflicts quickly steal energy away from the mathematics. Consider partnering with expert professional educators (e.g., from a campus teaching center) who can help with orientation activities at the start of the program, and with communicating effectively about goals and expectations. You may be surprised by what you learn, and how your program benefits, especially when working with larger groups.

Presentation skills. Giving a formal lecture with beamer slides or at the blackboard, making a presentation to the working group, and chatting math over tea are new experiences for most undergraduate researchers. These skills are learnable, and students progress quickly with honest feedback and practical tips from a relaxed, patient, and encouraging mentor. Make sure your students get plenty of practice before they need to present at a conference. And remember that they will look to you as a role model.

Presentation and communication skills are equally valuable for students who decide not to continue in mathematics, or even in academia. Indeed, the ability to communicate quantitative and technical information clearly, confidently, and effectively is needed, valued, and in short supply both in other STEM fields, and also in industry.

Success. A successful or positive undergraduate research program does not necessarily equate to publishing. Having a broad enough definition of success is important so that everyone can walk away with their head held high even if the hoped for results do not pan out. Goals can include having a positive experience and getting a real taste of research, which includes frustration, getting stuck, and some promising directions simply not working out. Students should take away transferrable skills such as the presentation and communication skills mentioned above, as well as reading the research literature, searching the arXiv and MathSciNet®, using LaTeX, and writing for a mathematical audience, regardless of whether any paper gets published.

Not all students who participate in a math research program will continue to study math or stay in academia. This is perfectly OK, and does not mean your program was unsuccessful. Someone having a positive experience with the project and reaching an informed decision that they do not want to go to graduate school in mathematics is a positive outcome. It may be much better for a student to reach this same decision as an undergraduate than after a first research experience in the third or fourth year of a PhD program.

Resources. You may find it helpful to seek and use existing resources, for both your students (e.g., presentation or software guides/templates, including LaTeX, Beamer, Mathematica, etc.) and yourself (e.g., see [1, 2, 3]). Your colleagues might also serve as resources. Students can benefit from the opportunity to meet and discuss with other mathematicians, and will ideally develop a sense of comfort and confidence in doing so.

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Amanda Folsom


Sam Payne

Amanda Folsom and Sam Payne have personally mentored dozens of undergraduate researchers over the past decade, and founded the Summer Undergraduate Research in Mathematics in Yale (SUMRY) program, which continues to thrive. Both authors also benefited from and enjoyed participating in research programs in mathematics as undergraduates.

Credits

Photo of Amanda Folsom is © Amherst College.

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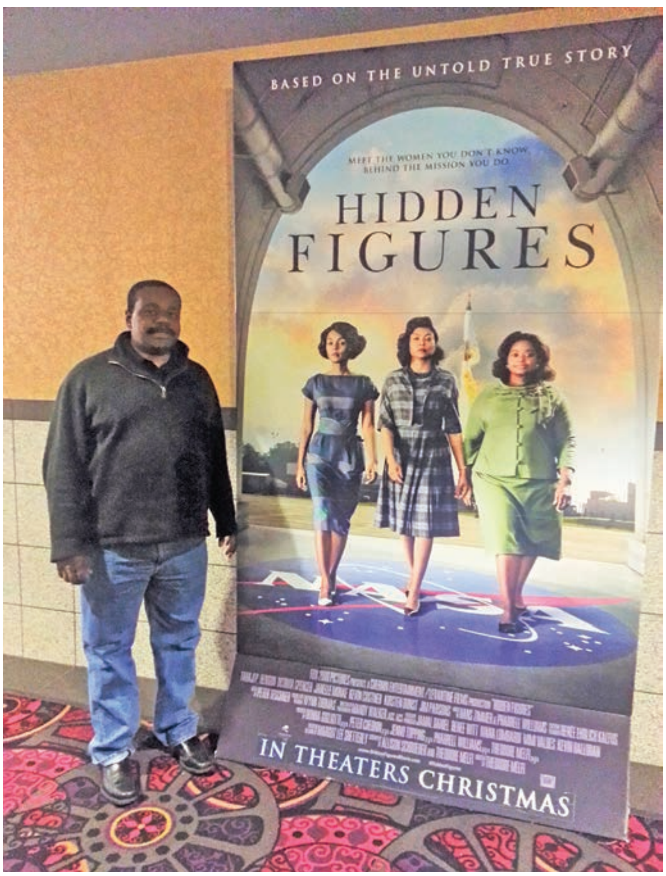
Rudy Lee Horne: The Hidden Figure of *Hidden Figures*

1968–2017



Rudy Lee Horne, mathematics consultant for *Hidden Figures*.

Della Dumbaugh

Introduction

Rudy Lee Horne was a man with an easy, memorable laugh and a gigantic personality. He devoted his life to advancing applied mathematics and the next generation of mathematicians, especially mathematicians from underrepresented groups. He was also a mathematician with the rare distinction of a Hollywood movie credit. Horne's expert and affable work as the mathematics consultant on the Oscar-nominated film *Hidden Figures* helped make the possibilities and power of mathematics available to millions of people across the globe. *Hidden Figures* recounts the previously unknown story of African American women and their critical contributions to the space program at NASA.

Horne's work on *Hidden Figures* earned him the Reverend Jesse Jackson's Rainbow PUSH (People United to Serve Humanity) Coalition "Excellence in Stem Education" Award in July 2017 and the National Association of Mathematicians Lifetime Achievement Award, conferred posthumously in January 2018. He is one of a handful of mathematicians with a listing on the Internet Movie Database. His movie work did not outdistance the importance of his day job at Morehouse College, however. As Talitha Washington, an Associate Professor of Mathematics at Howard University and a program officer at the National Science Foundation described it, "Rudy Lee Horne was a direct role model for African-American male students because they could see

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[1] *Maureen O'Donnell, "Rudy L. Horne dies at 49; Chicago native checked the math in 'Hidden Figures,'" Chicago Sun Times, January 21, 2018,* `https://chicago.suntimes.com/news/rudy-l-horne-dies-at-49-chicago-native-checked-the-math-in-hidden-figures-2`.

themselves as hardcore applied mathematicians and have fun while doing it."[1]


Young Rudy Lee Horne.

Early Years

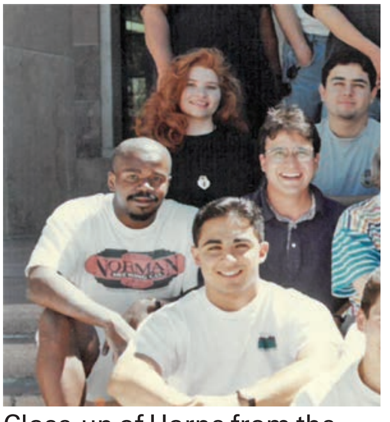
Rudy Lee Horne was born on October 18, 1968 in Englewood, Illinois, a Cook County neighborhood on the southwest side of Chicago. His father, Rudy Horne, Sr., worked at the Sherwin-Williams plant and his mother, Carolyn Horne, helped run a day care center. His entrepreneurial parents owned a 2-story duplex near 79th and Hermitage. The Hornes lived on the top floor and rented out the first floor. Rudy was the oldest of three children. Rudy's younger brother William described their young life as "normal brothers doing normal things."[2] They enjoyed sports and playing outside with kids, albeit with very competitive spirits. If the first floor of their home was unoccupied, the children played and slept there. But Rudy took his older brother role seriously and watched out for William and their sister Frances. William described Rudy as a "brainiac…like a Doogie Howser."[3] Rudy loved comic books as a boy, especially Marvel, DC (Batman, Superman, etc.), Star Wars, and Star Trek. He maintained this interest throughout his life. Rudy's family later moved to Country Club Hills and he graduated from Crete-Monee High School in 1986. It was in high school that Horne realized he liked calculus. Rudy's close-knit family included his Aunt Fran and Uncle Pete (Houston) who were both teachers. They emphasized school and college in particular. "Anything the aunts and uncles said, Rudy soaked it up," William observed. "For me, it was in one ear and out the other." With his interest in calculus and the encouragement of his family, he set off for the University of Oklahoma to pursue a college degree.

Undergraduate and Graduate School: University of Oklahoma and University of Colorado at Boulder

Horne attended the University of Oklahoma for his undergraduate studies and graduated in 1991 with a double major in Mathematics and Physics. In the summer after his sophomore year at Oklahoma, Rudy participated in the inaugural year of the SMART program at the University of Colorado in Boulder in 1989. Designed for undergraduates interested in pursuing graduate degrees in STEM fields, SMART offered a 10-week undergraduate research experience for students from underserved populations from across the country. Rudy met physics professor Paul Beale through this summer experience. "By luck," Beale described it, "someone put his file in front of me." Beale went on to serve as his advisor that summer, introducing Rudy to nonlinear wave equations.[4] This introduction to the University of Colorado had lasting implications for Rudy.


University of Colorado Summer Bridge, 1995.

After graduating from Oklahoma, Rudy spent the next decade at the University of Colorado where he earned a master's in physics in 1994, a master's in applied mathematics in 1996, and a doctorate in applied mathematics in 2001. He was the first African-American to earn a PhD in applied mathematics at Colorado. Professor of applied mathematics Mark Ablowitz served as his dissertation advisor. Ablowitz pointed out that "Horne's very strong background in physics and classical mathematics made it natural for him to consider questions in applied mathematics, especially physical applied mathematics."[5] "In those days," Ablowitz explained, "we were


Close-up of Horne from the Summer Bridge group photo.

[2]Horne, William. Phone interview. 28 June, 2018.

[3]O'Donnell, "Rudy L. Horne."

[4]Beale, Paul. Phone Interview. 29 June, 2018.

[5]Ablowitz, Mark. Phone Interview. 10 July, 2018.

interested in long distance transmission of signals via fiber optics." By the late 1990s, one of the key questions focused on how to send multiple signals in the same fiber and, in particular, how to address the instabilities caused by these multiple signals interacting with each other. Horne's task was to consider how these fibers interacted with dispersion management and without dispersion management. His work hinged on mathematical approximation techniques that could be implemented using nothing more than a laptop. Ablowitz noted that Horne was especially skillful at this more mathematical approach to an applied question. While Ablowitz oversaw Horne's thesis work, Beale continued his support of Horne in his role as the external examiner on his PhD dissertation committee. He and Horne "became close and life-long friends during those years."

Horne at CAARMS 1995.

While at Colorado, Rudy continued to work with the SMART program as a graduate student mentor. In this role, he provided SMART students with feedback on their projects and presentations, enjoyed meals with them in the dining hall, and coordinated weekend activities. In general, as Barbara Kraus, Program Manager of the Colorado Diversity Initiative described it, Rudy and other graduate student mentors helped the SMART students "adjust, academically and socially, to life on a top tier, predominantly white research campus."[6] As a natural extension of the SMART design, a cohort of graduate student mentors often emerged as a support and mentoring network for the graduate students themselves. This type of network remained important to Horne throughout his career, most notably in his active participation in the Conference for African American Researchers in Mathematical Sciences, CAARMS. In addition to his work with SMART students, Rudy also taught in the 5-week intensive summer bridge program for rising first-year engineering students at Colorado. Drawing from the work of Uri Treisman, the Multicultural Engineering Program Summer Bridge hinged on dynamic study groups [Treisman]. Rudy's large, affable personality combined with his sheer joy for mathematics to make him a natural for this study group design. In this position, as David Aragon, Assistant Vice Chancellor in the Office of Diversity, Equity and Community Engagement at the University of Colorado put it, Rudy "served in a very influential role helping scores (maybe hundreds) of underrepresented minority engineering students succeed in their gateway calculus and physics courses during his time as a graduate student."[7] Rudy's work with the Colorado SMART and Bridge programs ultimately served as a beginning for what would become a lifelong commitment to advancing underrepresented students in mathematics. His decade at Colorado gave him the opportunity to do good work and make a difference in people's lives, as well as earn a degree that would provide him with the credentials to do that throughout his career.

Postdoc: University of North Carolina at Chapel Hill

After a year at the California State University, Hayward (now known as California State University, East Bay), Horne secured a postdoctoral position with Professor Chris Jones, the Bill Guthridge Distinguished Professor at the University of North Carolina, from 2002 to 2005. At the time, Jones' research focused on non-linear optics. Jones welcomed Horne's expertise in four-wave mixing and his collegial ability to bring this new approach to the group. He especially welcomed Horne's enthusiasm. "He was the kind of guy who lifted your spirit, he was enthusiastic, he had a love of life. He kept us all buoyant."[8]

Written in the form of a personal letter, Jones captured the richness of Horne's contributions to his lab in his personal tribute to him for SIAM. "As a postdoc with me," Jones wrote, "you brought all your ideas about wave interactions into the group. This led to thinking about random effects and their influence on such interactions. You showed us how to think about this and guided us to a deeper understanding. Looking back, this is not something I would have foreseen working on, let alone figuring out, but without you we would not have done. This happened again just a few years ago when you showed me the world of PT-symmetry and the work you had done. We were just beginning to see the possibilities in this area."[9]

Jones' comments hint at one of Horne's greatest strengths: his collaborative nature. Horne found success by doing and sharing mathematics. Horne "would pull his own weight mathematically" and advance the group and himself in the process. Along the way, Horne benefited from other experienced colleagues in the mathematical community supporting, encouraging, and advising him. Horne and Jones continued their work together after he left UNC and Horne, Jones, and Jones' family remained steadfast friends throughout Rudy's life. In July 2018, Jones

[6] *Kraus, Barbara to Della Dumbaugh, July 6, 2018.*

[7] *Aragon, David to Della Dumbaugh, July 5, 2018.*

[8] *Jones, Chris. Phone Interview. 14 July, 2018.*

[9] *Chris Jones, "In Memoriam: Rudy L. Horne,"* The Dynamical Systems Web, https://dsweb.siam.org/The-Magazine/Article/in-memoriam-rudy-l-horne.

gave the inaugural Rudy Lee Horne Applied Mathematics Lecture at CAARMS.

Jones also hinted at the reality of Horne's experiences as an African American mathematician when he said, "I cannot fully understand the obstacles you had to overcome to get where you did. The times you were not supported, the rejections you had to face and all the doubts you must have had whether you could make it. But you persisted," Jones celebrated, "and make it you did." Jones extended this assessment when he commented that Horne was like the 1970s Weeble toy, "the proverbial thing that bounces back, no matter how many times you try to knock it down." Years later, Horne might have recognized this exact same spirit in his student, Zerotti Woods, when he met him at Morehouse (see below).

After his postdoc at UNC, Horne served on the faculty at Florida State University before he accepted a tenure track position at Morehouse College, a historically black college and university (HBCU), in 2010.

Morehouse College: "You can, you will, you should."

At Morehouse, Horne seemed to have found his life's work—in more ways than one. He achieved the traditional standards in academia, earning tenure and promotion to Associate Professor in 2016. His colleague, Ulrica Wilson, captured the essence of his contributions to Morehouse. She described Horne as "active and engaged with students. He was a young black man that was a role model for African American students pursuing math—whether those students were non-majors getting through their required coursework or majors pursuing careers in math."[10] Wilson underscored the challenges facing young black men as they live in the world today. "When people see a black man, people often see all kinds of images—images informed by their perception," Wilson explained. "Black men are trying to excel and live their lives in a world that does not always see their promise. Sometimes, in that body, it's hard to live and pursue an identity that does not fit others' expectation. Who we are at the park, who we are at the grocery store—for some people this is not stifling, for us and for black men, in particular, it can be." When young black men come to Morehouse they are surrounded, often for the first time, by other black men students and role models that allow them to focus on and develop new possibilities for their lives.

As current Morehouse College student Kevin Womack expressed it, Rudy Lee Horne "had a billion degrees in some intense subjects. He understood the importance of math and how it can be applied to do some powerful things." Dr. Horne encouraged Womack to major in math and computer science. Womack just finished his third summer as a Google intern thanks, in part, to this combination of majors.[11]

Womack took Calculus III and Ordinary Differential Equations with Dr. Horne. He described each day in Calc III as something "like an episode of a TV show. Dr. Horne would spin a story. We would ask him questions. Our questions told him what we could do. And then he'd say, 'wait until you see what we are doing tomorrow.' We were excited to be in the class, he was excited to have us. Everyone in the room was a black male in an advanced math class. I can't think of that happening anywhere else other than an HBCU."

Zerotti Woods, Jasekani Furbert-Wade, Mohlomi Taoana, Horne, Ryan Hynd, Umaru Waizoba, Jordan Clark, Duane Cooper.

Morehouse College student Zerotti Woods met Horne soon after he joined the faculty. Woods took Complex Variables and Numerical Analysis with him. "It was always funny how much of a nerd he was in class," Woods recalled, "he loved what he did and it showed. He had enthusiasm. It never seemed like he was working." He described Horne as a "really big mentor" for him. In particular, Horne showed him "the importance of teaching, of grooming the next generation of mathematicians who look like us. He never made it seem like it was weird to want to be a mathematician."[12]

Before Woods graduated from Morehouse in 2014 and left to pursue a PhD at the University of Georgia, Horne "made it clear" that he would struggle with things in graduate school, that he would not be accepted straight away. Woods attributes his success at Georgia to what he learned at Morehouse, especially the confidence the faculty instilled in him, and to his upbringing in inner city Atlanta. "Every time someone tries to build a narrative that they made it in

[10] *Wilson, Ulrica. Phone Interview. 15 June, 2018.*

[11] *Womack, Kevin. Phone Interview. 26 June, 2018.*

[12] *Woods, Zerotti. Phone Interview. 28 June, 2018.*

spite of coming from the inner city of Atlanta, I point out that I made it *because* of coming from inner city Atlanta. I've never seen anything work the first time for any of the people I love. I've always seen an iterative process. They were okay with it, so I was too." He did not mind failing and getting back up, staying up until the wee hours of the night if that's what it took to do the work. Woods' success testifies to the power of Morehouse and other HBCUs in preparing students for graduate work. From 2010 to 2014 (the latest national data available), 38.1% of black doctorate recipients in mathematics held baccalaureate degrees from HBCUs.[13]

"One of the things I miss about Morehouse," Woods observed, "is that I could identify as a mathematician. There wasn't anything else I had to identify with first. Now, at Georgia, I have to identify first as a black man." The movie *Hidden Figures* helped him understand the importance of his Morehouse experiences even more. When he saw the buildings from Morehouse in the film, "I wanted to stand up in the theater and shout, 'hey, that is my school.'" He was surprised and delighted to recognize Dr. Horne's handwriting on the board (in the form of the young Katherine Goble Johnson's factorization of a quadratic). But he was not surprised at all about Euler's method appearing in the movie. Woods fondly recalled that Horne "loved Euler's method and the bisection method. He droned on and on about them in numerical analysis."

Dr. Horne worked with *Hidden Figures* in the summer of 2016, just between Kevin Womack's classes in Calculus III and ODE. Womack described the excitement of the Morehouse students when they learned over the summer that Dr. Horne was working with Taraji Henson. Horne emerged as something of a celebrity on the campus. "He played it so cool. He made it seem so casual," Womack remembered. "I can't imagine explaining equations to Taraji Henson," he reflected. Horne's job included ensuring the actors and actresses represented the mathematics accurately. Taraji Henson, for example, learned mathematics like she would learn lines.[14] Womack had to laugh when he learned Taraji Henson felt challenged by learning some of the mathematics. "I would expect it would be hard for Taraji. I don't remember any of Dr. Horne's classes being easy. She got a fair Dr. Horne experience."

Wilson similarly recalled that Horne did not "gloat" about the movie work but when he talked about it, "he would just shine." Even after his work with the movie was over, the movie came out in theaters and "2017 was the best year of his life."

Overall, Horne supported Womack and other students at Morehouse. "He tried everything he could to make Morehouse a safe place," Womack reflected, "his message was not about the people who would block us or serve as obstacles. He wanted to be our champion. His message was: You can, you will, you should." Morehouse College provided the perfect home for Horne to continue to make a difference in the lives of the next generation of mathematicians. It also provided the unexpected opportunity for Horne to work with the *Hidden Figures* movie cast and crew and subsequently influence millions of people who would never set foot in his mathematics classroom.

Hidden Figures Work

In March of 2014, a 55-page nonfiction proposal about black women mathematicians at NASA in Hampton, Virginia found its way to the desk of Donna Gigliotti, producer of *Shakespeare in Love* and *The Reader*. "I kind of couldn't get over the fact that this was a true story and I didn't know anything about it," Gigliotti confessed. "I thought well, this is a movie." Those 55-pages grew into the 368-page book we now know as *Hidden Figures: The American Dream and the Untold Story of the Black Women Mathematicians Who Helped Win the Space Race* and the associated movie that debuted on Christmas Day, 2016 with Taraji P. Henson, Octavia Spencer, and Janelle Monae with the simpler title of *Hidden Figures*.[15] In part, the idea appealed to Gigliotti's natural affinity for strong female characters. She partnered with director Theodore Melfi who won Gigliotti over when he took himself out of consideration for the next Spider Man film so he could make what we have come to know as *Hidden Figures*.

Gigliotti and Melfi made a point to capture the realities of a segregated workplace and to portray realistic home lives of middle-class African Americans in the 1960s. They were also committed to the accuracy of on-screen calculations. For this part of their work, they relied on Horne "to tutor the cast and crew on set." Initially, Horne turned down the offer to work as the mathematical consultant on the film fearing that it would take too much time away from his teaching and research. His colleagues encouraged him to accept the position, however, and he changed his mind. He would never regret it.

Over the 43 days of filming in Atlanta, Horne visited the set about a dozen times to work with the cast and crew. As Horne described it, "[a]ny math that she [Taraji P. Henson]

[13] *"Baccalaureate origins of black doctorate recipients," accessed July, 2018,* `https://www.nsf.gov/statistics/2017/nsf17310/digest/enrollment/hbcus.cfm`.

[14] *Joe Ebentraut, "This 'Hidden Figures' Mathematician Inspired Generations of Black Students,"* dose, `https://dose.com/articles/this-hidden-figures-mathematician-inspired-generations-of-black-students`.

[15] *Justine Neubarth, "Do the Math-An Amazing True Story Plus A Dedicated Team Adds Up to 'Hidden Figures,'"* Producers Guild of America, *December 13, 2016,* `https://www.producersguild.org/blogpost/1537650/264534/DO-THE-MATH--An-Amazing-True-Story-Plus-A-Dedicated-Team-Adds-Up-To-Hidden-Figures`.

wrote on the board, I was responsible for training her to write said math on the board. My other task was primarily to check that the mathematics on the blackboards in the background scenes and in note books was consistent with the things that NASA did at the time." These "background scenes" included Horne's own handwriting. In one of the early scenes of the movie, a young Katherine Johnson [Lidya Jewett] factors a quadratic on a chalkboard. Horne actually did this calculation on the board.

When working with Henson, Horne encouraged her to think of the mathematics she had to write and speak in the movie as lines in a script. The stakes were high for Henson. The mathematical aspect of Johnson's life made it difficult for Henson to play her character. She admitted that "[t]hinking about doing the calculations on the board, I broke out in hives last night." Her fear arose because she wanted "to do right by Ms. Katherine Johnson, who's still alive, and her family, and I want to do right by her legacy."[16] Horne was so successful at his work that Gigliotti later suggested that Henson "should be nominated for an Academy Award just because what she is doing on the chalkboard. Everything that she is doing, she is doing accurately."

Horne also influenced the script of the movie. He offered Melfi the suggestion of Euler's method as a way to solve a problem related to the calculation of the trajectory to bring John Glenn back to earth. Melfi "really liked that concept," Horne later commented. "I didn't expect him to, but he actually put that into the script."[17] None of Horne's numerical analysis students were surprised by Horne's suggestion.

Professor William Massey, the Edwin S. Wilsey Professor of Operations Research and Financial Engineering at Princeton University, noted lightheartedly that the mathematics community owes Horne a great deal for his efforts to ensure that Henson pronounced Euler as "Oiler" in the movie rather than "Youler."[18]

As with his more traditional students, Horne's affable personality and sheer love of mathematics endeared him to the actresses he worked with closely. After his death, Henson acknowledged "the passing of Rudy Lee Horne, who was part of our team that brought the story of Katherine Johnson and the 'Hidden Figures' in the space program to the screen and let the world know of their accomplishments." She said *Hidden Figures* was "a blessing to all of us and shed light on a story hidden for far too long."[19] After Horne's death Jewett tweeted "I just got an A- on my first Algebra 1 test. You told me from the math room on set I could do it. And now I really do it! RIP Mr. Rudy." Jewett's tweet calls attention to the "math room" on the set of an Oscar nominated film. That combination seems like an oxymoron. If there was anyone who could bridge the worlds of Hollywood and mathematics, it was Rudy Lee Horne. "It made sense that they asked [Horne] to do it," his former PhD advisor Mark Ablowitz explained, "he was good at

Horne delivering the Blackwell–Wilkins Keynote Address at CAARMS 2017.


Horne at CAARMS 2017.

[16] *Sarah Lewin, "On 'Hidden Figures' Set, NASA's Early Years Take Center Stage,"* Space.com, *September 2, 2016,* https://www.space.com/33886-hidden-figures-movie-set-visit.html.

[17] *Jacob McCleland, "OU Graduate Makes Sure 'Hidden Figures' Math Adds Up," February 15, 2017,* kgou.org/post/ou-graduate-makes-sure-hidden-figures-math-adds.

[18] *Massey, William. Phone Interview. 27 July, 2018.*

[19] *O'Donnell, "Rudy L. Horne."*

doing math, good at learning math, and good at teaching math. He was a high-quality mathematician, high-quality person, and a high-quality teacher. He did it all."

Impact of Horne's contributions to *Hidden Figures*

After his work on the film, Horne was quoted in more than two dozen stories about *Hidden Figures*. He was a guest on National Public Radio's *Closer Look* and was interviewed in insidescience.org and science.com. He received invitations to give talks at various venues, including the 23rd annual CAARMS at the University of Michigan in July 2017. At CAARMS, Horne delivered the Blackwell-Wilkins Keynote Address with a talk on "Hidden Figures: Bringing Math, Physics, History & Race to Hollywood."

William Massey had invited Horne to deliver this address. "If I had hesitated [to invite him to speak]," Massey recently reflected, "I would have regretted that the rest of my life." Massey had originally met Horne at the first CAARMS conference in 1995, now called CAARMS1. At the time, Massey was a Member of the Technical Staff at the Mathematical Sciences Research Center at Bell Laboratories. Massey had observed his African American science colleagues at Bell Labs participate in annual meetings with their fellow Chemists or Physicists, for example. He wondered why African American research mathematicians did not have a dedicated annual meeting. Consequently, Massey helped organize CAARMS1 and every conference since that time. With Horne's talk on *Hidden Figures*, he became the first CAARMS participant to begin as a student member of the group and later give an invited address. Horne's involvement with CAARMS had, literally, spanned his entire career. He attended the first CAARMS at MSRI as a graduate student and remained active and involved at every stage of his career.[20]

Massey noted two important lessons from the movie for the broader public, both focusing on change. "For white America," as Massey described it, "the message is that you have to learn to accept other people as equal. For black people, the message is that you have to learn how to program in Fortran. Dorothy Vaughan was not standing around waiting to be someone's victim. She does not conveniently fit into the noble victim narrative. She did something about it." For Horne, Massey noted that "he lived long enough to see, to feel something was coming back to him, that more people became aware of what he did. Any researcher would give anything to have 1/100 of the number of people that saw the movie to know about their work."

Perhaps the greatest satisfaction for Horne came from the students the movie inspired. "I had a student here at Morehouse who came up to me and mentioned that after he had seen the film it really inspired him to want to do mathematics," Horne said. "If the film does that, if it helps get more women in science, it gets more African Americans, or for that matter more people—whether an African American, white, Hispanic, whatever. If it gets more people involved in math and science and STEM fields, that's a great thing."[21]

Taraji Henson expressed a similar viewpoint, only from the Red Carpet at the Screen Actors Guild (SAG) Awards on January 29, 2017, a place where few mathematicians or proponents of mathematics are represented. "The movie is important, and I don't want another young girl thinking that math and science is not for her," Henson said.[22]

Horne also hoped to use his *Hidden Figures* opportunity to convey the excitement of learning mathematics. "When you've banged your head against the wall of a problem for hours or days or weeks or months," Horne said, "then you finally get a notion of or know how to solve said problem. Sometimes, you wander around for a while, lost, and then you figure out what you need to do. That sense of accomplishment is sometimes hard to describe to people who aren't doing that."[23]

Left to right: Professor Rudy Lee Horne, Professor Monica Jackson (American University), Dr. Gelonia Dent (Brown University / Medgar Evers College), Dr. Idris Stovall (Mathematical Sciences Institute), and Professor William A. Massey (Princeton University).

[20]*For more on CAARMS, see the website at* www.caarms.net *and W. A. Massey, "Mathematics is Four Dimensional," Council for African American Researchers in the Mathematical Sciences: Volume III.* Contemporary Mathematics, *editors Alfred Noel, Earl Barnes and Sonya A. F. Stephens. Volume 275, 2001, pp. 147–158. For more on the creation of CAARMS, see Raymond Johnson, "Conference for African American Researchers in the Mathematical Sciences at MSRI,"* Notices of the American Mathematical Society, *December 1995, pp. 1496–1500.*

[21]*McCleland, "OU Graduate."*

[22]*Marissa G. Muller, "The* Hidden Figures *Effect is Real: How It's Inspiring Young Women to Seek Careers in Science and Technology,"* Glamour, *January 30, 2017,* https://www.glamour.com/story/hidden-figures-inspiring-young-women-science-and-technology.

[23]*Lewin, "On 'Hidden Figures' Set."*

Epilogue

At the end of the fall 2017 semester, Horne had to miss a few days of classes for a health issue related to his heart. "He came back one day and told us everything was okay now," his student Kevin Womack remembered. "But that was the last time I saw him." Horne died at the age of 49 on December 12, 2017, less than one year after the debut of *Hidden Figures*.

It's impossible to write a Memorial Tribute to Rudy Lee Horne as either the *individual* mathematician or the man. Horne didn't exist as a single person; he lived life as part of a wide community. He was at his best when surrounded by family, colleagues, students, and friends. His very presence made the people around him better. As Professor Chris Jones described it, Horne's "impact on our lives will persist…[he] changed us all for the better and that will be inherited by people who never even met [him]."

Horne's career underscores the power of a mathematical community that stretches over a lifetime and across institutions and organizations. His success points to the importance of programs like SMART, of organizations that dedicate resources to encouraging underrepresented groups and, above all, of colleagues that intentionally support and guide future generations of mathematicians. His life affirms the practical and lasting outcomes of these types of institutional and personal initiatives that transform a life through mathematics. Horne's student, Kevin Womack, for example, is currently applying to PhD programs in computational and applied mathematics, a certain indication of his former professor's lasting impression on him.

Aside from his meaningful impact on the people around him, Horne's legacy will live on through the CAARMS annual Rudy Lee Horne Applied Mathematics Lecture, the Dr. Rudy L. Horne Scholarship at Morehouse College, and the Rudy Lee Horne Memorial Scholarship at the University of Colorado Boulder.

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Della Dumbaugh

Credits

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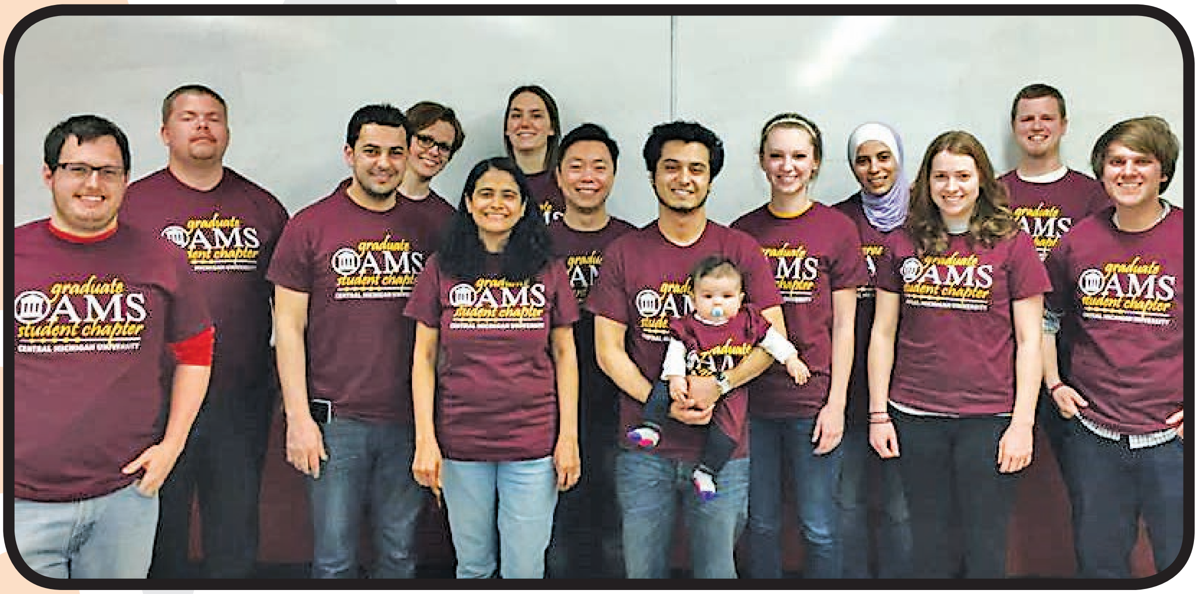

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Doctoral Programs in Mathematics Education: A status report of size, origin of program leadership, and recommended institutions

Robert Reys, Barbara Reys, Jeff Shih, and Farshid Safi

This paper provides a status report on the production of doctorates in mathematics education at institutions across the country. It also reports on where faculty members in active doctoral programs in mathematics education received their doctorates, and it provides a peer-based identification of high quality doctoral programs in mathematics education.

This study uses data gathered from the Survey of Earned Doctorates (SED). The SED began gathering data in 1920 and added mathematics education as a discipline in 1962. The SED is conducted annually and gathers data from every institution in the USA that awards earned doctorates. While the SED has some limitations [1] it provides the most comprehensive data available on the production of doctorates in mathematics education.

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Production of doctorates in mathematics education over the past three decades

There were 205 different institutions of higher education that graduated at least one doctorate in mathematics education between 1987 and 2016. The results of the SED survey for the 50 institutions producing the most doctoral graduates in mathematics education are shown in ten-year intervals in Tables 1, 2, and 3. The total number of doctoral graduates in mathematics education during these ten-year periods grew from 734 (1987–1996), 928 (1997–2006), and 1428 (2007–2016). These numbers reflect dramatic increases over these periods. One reason for the increases is an effort to address the shortage of doctorates in mathematics education that was reported nearly 20 years ago [2 & 3]. Since many job opportunities exist for mathematics educators, institutions have tried to address this need. Additionally, from 2000 to 2003 the National Science Foundation funded several Centers for Learning and Teaching (CLT) that were designed to prepare more doctorates in mathematics (and science) education and that effort prompted an additional bump in the number of graduates.

The data show that the number of doctoral graduates in mathematics education, and the minimum number of doctoral graduates per institution, have increased during each of the ten-year periods. This overall growth of the

Table 1*. Institutions producing the largest number of doctoral graduates in mathematics education from 1987–1996

Rank	Institution	# of graduates
1	**Teachers College, Columbia University**	70
2	**University of Georgia**	51
3	**University of Texas, Austin**	41
4	**Ohio State University**	34
5	**Georgia State University**	32
6	University of Iowa	20
7	**SUNY, Buffalo**	18
	Rutgers University	18
	University of Pittsburgh	18
10	New York University	17
	American University	17
	University of Maryland	17
13	**Florida State University**	16
	Temple University	16
15	**Indiana University**	14
	University of Oklahoma	14
17	Boston University	13
18	**University of Wisconsin**	12
19	**North Carolina State University**	11
	University of Massachusetts Amherst	11
	Vanderbilt University	11
22	**Syracuse University**	10
23	**University of Illinois, Urbana–Champaign**	9
	Oregon State University	9
	University of South Carolina	9
	University of Tennessee	9
27	**University of Minnesota**	8
	Cornell University	8
	University of California, Berkeley	8
30	University of Southern Illinois	7
	University of South Florida	7
32	Purdue University	6
	Illinois State University	6
	University of Massachusetts–Lowell	6
	University of Missouri, Columbia	6
	University of Northern Colorado	6
	University of Virginia	6
38	Auburn University	5
	Texas A & M University	5
	University of Michigan	5
	Michigan State University	5
	University of Florida	5
	Oklahoma State University	5
	University of Alabama	5
45	Washington State University	4
	Montana State University	4
	University of Houston	4
48	Eleven** institutions graduated 3 doctorates	3

*Institutions in **bold** appear in Tables 1, 2, and 3.*

***University of Chicago, Stanford University,* **Penn State University, University of Delaware,** *University of Mississippi,* **Ohio University,** *University of Toledo, University of Denver, University of North Carolina, Greensboro, Claremont Graduate School, Kent State University*

Table 2. Institutions producing the largest number of doctoral graduates in mathematics education from 1997–2006

Rank	Institution	# of graduates
1	Teachers College, Columbia University	74
2	University of Georgia	49
3	University of Texas, Austin	40
4	Illinois State University	38
5	North Carolina State University	29
6	Florida State University	27
7	Ohio State University	25
8	Rutgers University	23
9	Georgia State University	22
	University of Oklahoma	22
11	American University	21
12	University of Maryland	20
13	Oregon State University	18
14	University of Northern Colorado	17
15	Temple University	15
16	Indiana University	14
	University of Missouri, Columbia	14
18	University of Minnesota	13
	University of Wisconsin	13
20	Ohio University	12
	SUNY, Buffalo	12
	Vanderbilt University	12
23	Syracuse University	11
	University of South Florida	11
	University of Virginia	11
26	Auburn University	10
	Michigan State University	10
	University of California, Berkeley	10
29	University of Illinois, Urbana–Champaign	9
	University of Iowa	9
	University of Massachusetts, Lowell	9
	University of Pittsburgh	9
33	Boston University	8
	University of Michigan	8
	University of South Carolina	8
	University of Tennessee	8
37	Penn State University	7
	University of Arizona	7
	University of New Hampshire	7
	Western Michigan University	7
41	Montana State University	6
	Montclair State University	6
	New York University	6
	University of Central Florida	6
	University of Houston	6
	University of Kansas	6
	University of Massachusetts, Amherst	6
48	Eight** institutions graduated 5 doctorates	5

***Claremont Graduate University, San Diego State University–University of California, San Diego (SDSU-UCSD), University of Alabama, University of Arizona, University of California–Davis, University of Chicago, University of Delaware, University of Southern Mississippi*

Table 3. Institutions producing the largest number of doctoral graduates in mathematics education from 2007–2016		
Rank	Institution	# of graduates
1	Teachers College, Columbia University	112
2	University of Georgia	81
3	Rutgers University	46
4	Georgia State University	39
5	North Carolina State University	33
6	Michigan State University	30
7	Illinois State University	29
8	University of Texas, Austin	28
9	University of California, Berkeley	26
10	University of Missouri, Columbia	25
11	Ohio State University	24
	University of Michigan	24
13	Florida State University	23
14	Arizona State University	22
15	SUNY, Buffalo	19
	Texas State University	19
	San Diego State University/ University of California–San Diego (Joint)	19
	University of Northern Colorado	19
	University of Tennessee	19
20	University of Delaware	18
	University of Virginia	18
22	Indiana University	17
	Texas A & M University	17
24	Ohio University	15
	Oregon State University	15
26	Stanford University	15
	Temple University	15
	University of Central Florida	15
	University of Maryland	15
30	George Mason University	14
	Montana State University	14
	Portland State University	14
	University of Minnesota	14
34	Penn State University	13
35	University of Arizona	12
	University of Wisconsin	12
37	Syracuse University	11
	University of Louisville	11
39	Purdue University	10
	University of Florida	10
	University of Kentucky	10
	University of Massachusetts Lowell	10
	University of Nebraska	10
	University of New Hampshire	10
	University of North Carolina	10
	University of Western Michigan	10
47	Nine institutions graduated 9 doctorates	9

**Illinois Institute of Technology, Kansas State University, Middle Tennessee State University, University of California–Davis, University of Illinois, Urbana–Champaign, University of Oklahoma, University of South Florida, University of Southern Mississippi, Utah State University

number of doctoral graduates in mathematics education was reported earlier in some detail [4]. In addition to the increasing number of graduates, an examination of Tables 1, 2, and 3 shows a number of patterns. The Teachers College and the University of Georgia graduated by far the most doctorates in mathematics education during each time period. Georgia State University, Rutgers University, and the University of Texas were the only other institutions to be included in the top ten producers of doctoral graduates during each ten-year period. While there is some shuffling in the order of the largest producers during these three time periods, 32 institutions appear in each of the three tables, and these institutions are shown in bold in Table 1.

Most doctoral programs in mathematics education in institutions shown in Tables 1, 2, and 3 are offered in the college/school of education. However, some institutions house the doctoral program in mathematics education in the mathematics department, including Arizona State University, Illinois State University, Montclair State University, Portland State University, University of New Hampshire, University of Northern Colorado, Texas State University, and Western Michigan University. Other institutions, such as Michigan State University, Oklahoma State University, Oregon State University, Syracuse University, University of Nebraska, University of South Carolina, and the University of Arizona, offer a doctorate in mathematics education in both the mathematics department and in the college/school of education.

Tables 2 and 3 show that several institutions, such as Michigan State University and University of California-Berkeley more than doubled their production in PhDs in mathematics education. More than one-half the institutions in Table 2 showed a growth in the number of doctoral graduates compared to Table 3. Two institutions, Arizona State University and Texas State University, appeared for the first time in Table 3. Notably, these two programs, together with the joint program at SDSU/UCSD, did not appear among the top producers in the earlier period covered by Table 1. Two institutions that appear in Table 1 and 2, the University of Chicago and American University, closed their doctoral programs and so do not appear in Table 3. It is worth noting that the doctoral program in mathematics education at University of Chicago was over 100 years old. It was a highly respected program and was the second oldest doctoral program in mathematics education in the United States, second only to Teachers College, Columbia University [5].

Where faculty producing future mathematics educators earned their doctorate

This is an important factor because the academic preparation of faculty members involved in doctoral programs impacts the curriculum of the program; it may influence the research experiences, as well as admission procedures, advisement, comprehensive exams, dissertations, and follow-up of doctoral graduates. Furthermore, institutions hiring new faculty may want to hire people with doctorates from certain institutions to strengthen their programs. They may be seeking new faculty members with certain kinds of knowledge or research expertise, or who have studied with specific people who are nationally recognized, or who have graduated from highly respected institutions. Information about where people earned their doctorate as well as the faculty who implemented the programs can be useful. To obtain these data, we developed and facilitated a survey.

Survey participants were faculty members in doctoral programs in mathematics education. The targeted institutions had graduated at least ten doctorates in mathematics education during the last 20 years and had produced at least one graduate in the last five years. These parameters generated a list of 64 different institutions. These 64 institutions have graduated 2249 doctorates in mathematics education during the 30-year period (1987–2016), or about 73% of all of the people receiving a doctorate in mathematics education during that time period. Consequently, these 64 institutions represent a major pipeline of producers of doctorates in mathematics education. A faculty member at each institution was contacted and asked to identify all of the faculty members (such as mathematics educators and mathematicians) that were actively involved in their doctoral program in mathematics education. Part-time adjunct appointments and retired faculty were not included. A list of 297 faculty members were identified, and 266 (almost 90%) responded to the survey. One of them did not hold an earned doctoral degree and nine of them earned a doctorate from an international institution. Therefore, data from 256 faculty members served as the basis for analysis.

Table 4. The top 20 institutions producing faculty members in doctoral programs in mathematics education in institutions graduating the most doctorates in mathematics education

Rank	Institution	# of doctoral graduates from these institutions contributing to doctoral programs
1	University of Georgia	21
2	Michigan State University	16
3	University of California, Berkeley	12
4	University of Wisconsin	11
5	San Diego State University & University of California, San Diego (Joint SDSU-UCSD)	10
6	University of Texas	9
7	Illinois State University	9
8	Indiana University	8
	Ohio State University	8
	Stanford University	8
11	University of Maryland, College Park	7
	University of Michigan	7
13	Harvard University	6
	Penn State University	6
	Rutgers University	6
16	Northwestern University	5
	Teachers College, Columbia University	5
	University of Central Florida	5
	University of Minnesota	5
	University of Missouri	5
	Vanderbilt University	5
	TOTAL:	174

Surveyed faculty were asked to identify the institution where they earned their doctorate. Table 4 identifies the 21 institutions that produced the largest number of faculty members currently working in doctoral programs in mathematics education. Table 4 shows that 174 of the 256 faculty members surveyed (69%) earned their doctorate at one of these 21 institutions.

An examination of Table 4 shows that the University of Georgia is the institution graduating the most people currently serving as faculty in doctoral programs in mathematics education. Although Teachers College, Columbia University appears in Table 4, given the large number of graduates Teachers College has produced, it is surprising that their graduates were not often found among the faculty of doctoral programs in these institutions.

Institutions recommended by faculty to potential doctoral students in mathematics education

Specific information about doctoral programs in mathematics education is helpful to potential doctoral students seeking a program. Recommendations from faculty can also be beneficial in shortening a list of possible institutions to consider. Each faculty member in the survey was presented this question:

> Suppose you were asked by a potential doctoral student in mathematics education to identify "particularly strong doctoral programs" (regardless of location or areas of interest) to consider attending. Which institutions would you recommend?

Surveyed faculty were allowed to name up to 8 institutions, and told it was not necessary to order the institutions. They were also asked not to name the institution where they were currently employed. Some faculty members identified eight institutions, some only a couple.

More than 260 of the faculty members responded, but a few of them did not name any institution. In such cases, they said their recommendation of an institution would depend on the interest of the doctoral student and the specialty or niches found at specific institutions. One faculty member indicated he/she would not recommend an institution, but rather seek to match the research interest of the potential doctoral student with a particular scholar independent of an institution.

A total of 94 different institutions were recommended, including 6 institutions in other countries. No institution was recommended by every faculty member, but three institutions (University of Georgia, Michigan State University, and the University of Michigan) were named by more than one-half of the faculty members. A list of all the institutions recommended by more than ten faculty members is shown in Table 5.

Table 5. Doctoral programs in mathematics education recommended by more than ten different surveyed faculty (order in an earlier 2007 survey shown in parenthesis).

Rank	Institution	# of times institution was recommended
1	Michigan State University (2)	162
2	University of Georgia (1)	154
3	University of Michigan (3)	147
4	San Diego State University/University of California (Joint program) (7)	83
5	University of Missouri–Columbia (4)	78
6	University of Wisconsin–Madison (5)	51
	Vanderbilt University (11)	51
	University of Delaware (12)	51
9	University of California–Berkeley (8)	49
10	Stanford University (12)	39
11	Arizona State University (14)	34
12	North Carolina State University (19)	28
	Penn State University (8)	28
14	UCLA (19)	20
	University of Maryland (6)	20
16	Portland State University (23)	19
17	Rutgers University (NA)	17
18	Virginia PolyTechnic Institute & State U. (NA)	16
19	University of Texas–Austin (19)	15
20	Indiana University (10)	14
	University of Washington (NA)	14
22	Illinois State University (15)	13
23	Teachers College, Columbia University (23)	12
	University of Arizona (NA)	12
25	Ohio State University (16)	11

NA—Not Appearing in 2007 survey

All of the institutions in Table 5, except for the University of Washington and Virginia Tech, are among the programs graduating the most doctorates in mathematics education. Their appearances prompted us to examine their history of producing doctoral graduates. According to the SED data, the University of Washington had a total of three doctorates in mathematics education from 1987 to 2006, yet graduated eight from 2007–2016, so it just fell short of making Table 3. Large growth was also shown by Virginia Tech, changing from 3 to 6 doctoral graduates in mathematics education during the same time periods.

About ten years ago, a list of 'particularly strong doctoral programs in mathematics education' was reported [6]. A comparison of that list with Table 5 shows that seven institutions appear in the top ten of each list, and the first three institutions are the same, except that the University of Georgia and Michigan State University have interchanged. Several institutions not included in the 2007 survey have surfaced in Table 5, including Rutgers University, University of Arizona, University of Washington, and Virginia Tech.

There are some common traits of the institutions shown in Table 5. They each have a core of established and nationally recognized mathematics education faculty members who are engaged in research and scholarly writing as well as active leadership in professional organizations. They have a critical mass of doctoral students and regularly graduate doctorates in mathematics education. Additionally, the faculty at most of these institutions have a record of success in gaining external funds that supports some of their doctoral students.

Summary

While the list of institutions producing the largest number of doctorates in mathematics education is constantly changing, over one-half of the largest producers during 1987–1996 continued to be among the largest producers in the past two decades. There was a significant growth (nearly 54%) in the number of new doctoral graduates in mathematics education from 2007–2016 over the previous ten years.

This paper identifies where the faculty members involved in major doctoral programs in mathematics education received their doctorate. This information is important because their experiences as doctoral students will likely shape the direction of future doctoral preparation.

Finally, since there is no accreditation of doctoral programs in mathematics education, it is helpful to potential doctoral students to learn of institutions with doctoral programs in mathematics education that faculty in other doctoral institutions recommend. This peer recognition of doctoral programs in mathematics education can be useful to potential doctoral students in establishing a list of institutions to consider.

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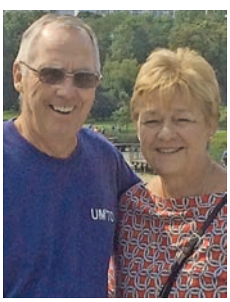

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Recognizing Black and Latinx Mathematical Excellence: The Blackwell–Tapia Prize

Ron Buckmire

Despite the many contributions mathematicians have made to the world, there are too few accolades to recognize excellence in mathematics. For example, there is no Nobel Prize in mathematics. The most prestigious prize for research in mathematics is widely considered to be the Fields Medal; it is presented to mathematicians under 40 years old at the quadrennial International Congress of Mathematicians. There have been 60 people who have won the Fields Medal and only two have been members of underepresented groups (Maryam Mirzakhani and Artur Avila). For mathematicians who are active in mathematical research as well as interested in broadening the participation of underrepresented minorities in mathematics, the most prestigious award is the Blackwell–Tapia Prize. The Prize is awarded at the biennial Blackwell–Tapia Conference hosted by the National Science Foundation Mathematical Sciences Institutes. The conference and prize are named after David Blackwell and Richard Tapia.

Blackwell is one of the most celebrated African-American mathematicians of all time. He is the seventh Black person to receive a PhD in mathematics, which he did at the age of 22 in 1941. He was the first African American to be inducted into the National Academy of Sciences. Blackwell was a full professor and head of the Mathematics Department at Howard University, which is a Historically Black College and University (HBCU). In 1954 he joined the faculty at the University of California at Berkeley (UCB) and became their first Black tenured professor. In 2018, UCB opened its newest residence hall, the David Blackwell Hall. Before Blackwell died in 2010, he made fundamental contributions to mathematical statistics, game theory, and information theory through his research, which led to notable results like the Rao–Blackwell theorem, Black-

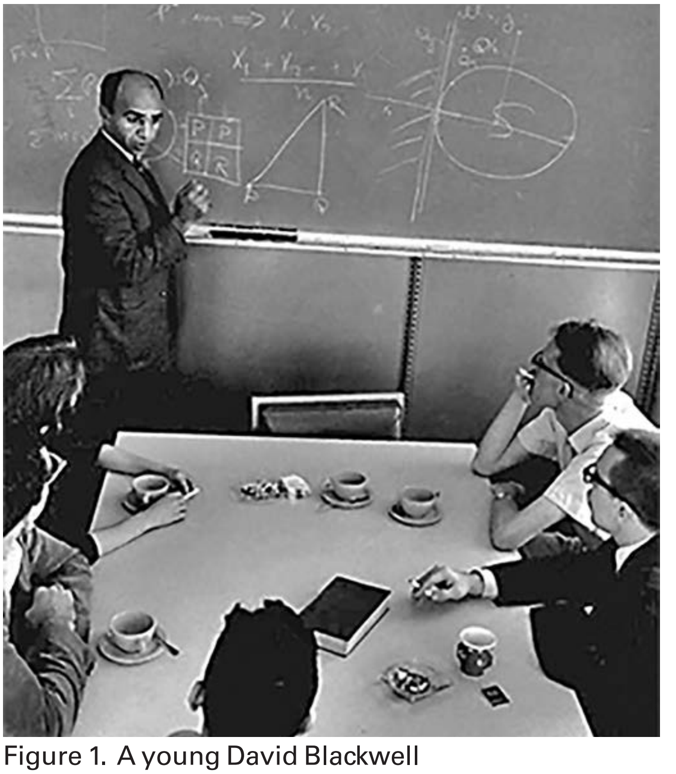

Figure 1. A young David Blackwell

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well's Approachability theorem, and Blackwell channels. Additionally, he wrote and published one of the very first textbooks on Bayesian statistics in 1969.

Richard Tapia is one of the most celebrated Latinx mathematicians in America. He is a longtime professor at Rice University in Houston, where among his many laurels he is a University Professor (one of six people ever designated with this title in the history of the institution). Tapia is also Maxfield-Oshman Professor of Engineering, and director of multiple initiatives to broaden the participation of underrepresented minorities in mathematics. He was awarded the National Medal of Science in 2011 by President Obama and in 1996 was appointed to the National Science Board by President Clinton. Tapia's mathematical research areas include scientific computing, specifically iterative methods for numerical solution of nonlinear problems, and various topics in mathematical optimization. Many know him for his work on integrating mathematics with drag racing. He is also the namesake for the Richard Tapia Celebration of Diversity in Computing conference.

Figure 2. Richard Tapia

The Blackwell–Tapia Prize is awarded every two years to a mathematician who has a significant research portfolio and demonstrated interest and success in broadening the participation of underrepresented minority groups in mathematics. The prize has only been given nine times since 2002. The prize winners (in chronological order) are Arlie Petters (Duke University, 2002), Rodrigo Bañuelos (Purdue University, 2004), William S. Massey (Princeton University, 2006), Juan Meza (University of California, Merced and National Science Foundation, 2008), Trachette Jackson (University of Michigan, 2010), Ricardo Cortez (Tulane University, 2012), Jacqueline Hughes-Oliver (North Carolina State University, 2014), Mariel Vázquez (University of California, Davis, 2016) and Ronald E. Mickens (Clark Atlanta University, 2018).

Figure 3. Five previous Blackwell–Tapia Prize winners: (From left to right) William Massey (2006), Richard Tapia, Ricardo Cortez (2012), Trachette Jackson (2010), Jill Pipher (founding director of ICERM), Juan Meza (2008), Rodrigo Banuelos (2004), and Carlos Castillo Chavez.

Mickens was announced to be the winner of the 2018 prize by the Blackwell–Tapia committee in April. The award was presented at the Tenth Blackwell–Tapia Conference held at the Institute for Computational and Experimental Research in Mathematics (ICERM) in November 2018. Mickens is the first person from an HBCU to win the Blackwell–Tapia Prize.

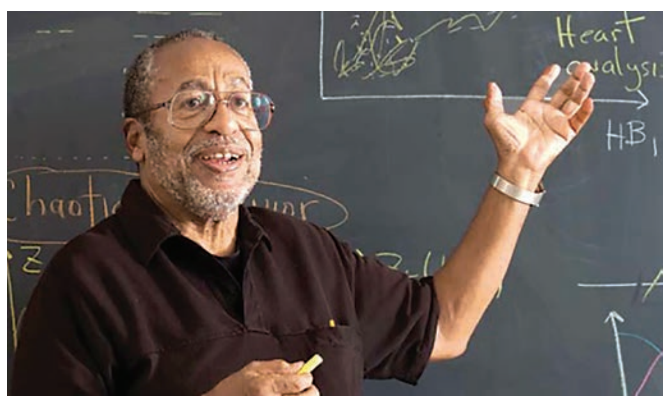

Figure 4. 2018 Blackwell–Tapia Prize winner Ronald E. Mickens

Mickens is renowned for his voluminous and significant contributions to several areas of applied mathematics. Primarily he is known for his contributions to the analysis, approximation, and solution of differential equations, nonlinear oscillations, and difference equations. In particular, he developed and shaped the field of non-standard finite differences (NSFD) through his astonishing research productivity in this area, which is documented in eight books, more than 300 peer-reviewed publications, and countless talks and presentations.

In addition to his prodigious mathematical contributions, Mickens has a longstanding interest in the underrepresentation of people of African and Hispanic descent in mathematics and the other sciences. In 2002, he published the book *Edward Bouchet: the First African American Doctorate* and has regularly worked to excavate and publicize the accomplishments of many Black scientists. In addition to being a Ford Foundation Postdoctoral Fellow himself in 1980, Mickens has long mentored subsequent Ford Fellows, in many disciplines. As a faculty member at an HBCU, Mickens has served as a role model to thousands of students who have been in his classes and as an unofficial advisor to dozens of professionals in the mathematical sciences who are from various groups that are underrepresented based on race, gender, ethnicity, and sexual orientation.

Mickens holds a PhD in Theoretical Physics from Vanderbilt University in 1968 and a Bachelor of Science in Physics from Fisk University in 1964. He is currently the Distinguished Fuller E. Callaway Professor in the Department of Physics at Clark Atlanta University, where he has been on the faculty since 1990. Before that, Mickens taught in the physics departments at Atlanta University (1982-1990) and Fisk University (1970-1982).

This author was one of several people who nominated Mickens for the Blackwell–Tapia Prize. The careers, the outlook, and the ambitions of people like myself and many others have been positively impacted by the example of excellence in mathematics that Dr. Ronald Mickens portrays to us all.

Ron Buckmire

Credits

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A Centennial Year (2019) Reflection on the Life and Contributions of Mathematician David H. Blackwell (1919–2010)

Johnny L. Houston

Born on April 24, 1919 in Centralia, Illinois, a town of about 12,000, David Harold Blackwell spent ten years attending public schools there (in elementary school he was twice promoted a year beyond his next grade level). In 1935, at the age of sixteen, he entered the University of Illinois in Champaign-Urbana, earning his AB degree in 1938, his AM in 1939, and his PhD in 1941, all in mathematics. At the age of 22 he had earned a PhD in mathematics and was awarded a prestigious Rosenwald post-doctoral fellowship at the Institute for Advanced Study in Princeton, New Jersey. Yes, David H. Blackwell was viewed as an exceptionally talented person at that time, being only the seventh African American to earn a PhD in mathematics. However, no one at the time envisioned that this was only the beginning of his more than fifty professional years where the annals of history would come to recognize him as a world-class mathematician-statistician and a gifted teacher.

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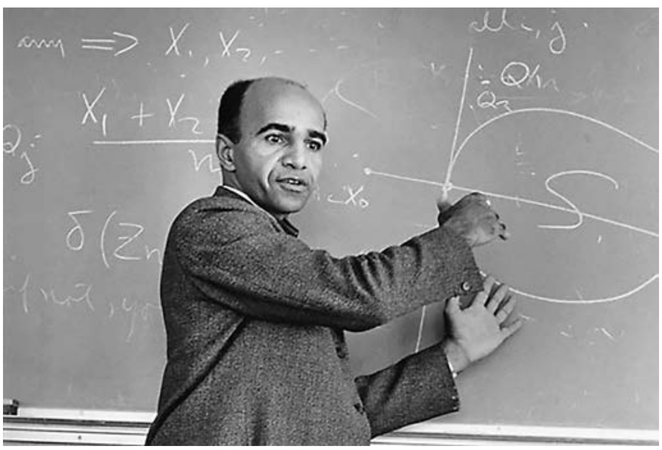
David was raised in a family that expected and supported working hard to improve the quality of life for members of the family. David's parents were Grover Blackwell and Mabel Johnson Blackwell, and David was the oldest of their four children. Grover Blackwell worked for the Illinois Central Railroad looking after the locomotives, while Mabel looked after the family, nurturing David, his two brothers John Wesley (Skeet) and Joe, and their sister Elizabeth. As a young child, David taught himself how to read even before entering school.

A fair amount of racism existed in Centralia when David was growing up. There were two racially segregated schools, one that only Blacks could attend and one that only Whites could attend, and there was one integrated school. As an African American, David might well have attended the all Black school; however, he lived in a mixed neighborhood and was permitted to attend the integrated school. Blackwell said, "I was fortunate to have attended the integrated school and I had no sense of being discriminated against. My parents protected us from whatever racism existed in general and I didn't encounter enough of it in the schools to notice it [3]."

David was always adept at mathematics, but in high school he did not like algebra or trigonometry. "I could do them and I could see that they were useful, but they were not exciting." However, in high school, geometry was the subject that really caught his attention. His high school geometry teacher, Ms. Caroline Luther, made the subject beautiful for him. "Geometry connected with activity and motion [1,3]," David said, and he loved the idea of the "helping line," a line that could be drawn to clarify a problem that had previously seemed unsolvable. The advisor to the high school mathematics club, Mr. Huk, encouraged David's interest in math. He would look for problems in the journal of the School Science and Mathematics Association, and bring them in for the club to solve. David's name appeared in the journal three times with others, and once alone when he solved a problem and published the solution with Mr. Huk's assistance.

At the University of Illinois in Champaign-Urbana his interest in mathematics continued to grow. He recalled, "The most interesting thing I remember from calculus was Newton's method for solving equations. That was the only thing in calculus I really liked. The rest of it looked like stuff that was useful for engineers in finding moments of inertia and volumes and such." However, during his junior year,



A young David Blackwell teaching.

he took a course in real analysis, based on Hardy's *Pure Mathematics*. David said, "It was in this class that I realized for the first time that serious mathematics was for me. It became clear that it was not simply a few things that I liked. The whole subject was just beautiful [3]."

His undergraduate years were not easy for David, but he figured out how to make wise use of his time and resources. When he realized that his father was having to borrow money to finance his studies, he took on available jobs, such as "being a waiter," "washing dishes," and working in an entomology lab to help earn money. At the same time he took courses over the summers and was able to graduate with an AB in 1938 after only three years of study. He financed his own education after his freshman year with no further support from his parents. While at the university, he joined the Alpha Phi Alpha fraternity and lived in the Alpha House during five of his six years in residency at the University of Illinois [1]. Initially, David's goal was to become an elementary school teacher. However, after completing his AB, David continued to study math at the university. He was awarded his AM degree in 1939 and completed his PhD in 1941 with a thesis on Markov chains, supervised by Professor Joseph Doob.

After completing his PhD, he became a Rosenwald Postdoctoral Fellow at the IAS in Princeton. This, however, led Blackwell to encounter blatant racism. The standard practice at IAS was that fellows of the Institute would be honorary faculty members of Princeton University, but in Blackwell's case this practice was challenged. Blackwell was the first African American to be a fellow of the IAS. At that time, Princeton University had never had a Black undergraduate student and certainly not a Black faculty member. Blackwell learned later that Princeton University had in fact rejected Paul Roberson's application to be a student because he was Black. The president of Princeton University wrote to the director of the Institute for Advanced Study saying that the Institute was abusing the hospitality of Princeton University by providing a Black American with such an appointment. In addition, Blackwell was denied the right to attend faculty lectures on the Princeton University campus [5].

Although Blackwell was aware of the problems that his appointment was causing, he was excited to be at the Institute where he met John von Neumann, who was interested in talking to him about his thesis. At IAS, he interacted with other influential mathematicians such as Jimmy Savage, Paul Halmos, Shizno Kakutani, Sam Wilks, and Dorothy Meharam. Some colleagues at the IAS wanted to extend Blackwell's post-doc beyond the one-year appointment. However, the president of Princeton University organized such opposition that the IAS would not do this.

At the end of his post-doc at the IAS, Blackwell suspected that no predominantly White institution of higher learning would hire him. Blackwell applied to all 105 Black institutions of higher learning in the country to see if any position was available. Out of the 105 applications, Blackwell received three job offers. He chose Southern University in Baton Rouge, LA, over West Virginia State College and Clark

From left to right: J. Ernest Wilkins, Johnny Houston, David Blackwell, and Lee Lorch.

College (now Clark Atlanta University) in Atlanta, GA. "It was the first offer I got," Blackwell said [2].

In spite of his doubt that a predominantly White institution would hire him, Blackwell did apply for a faculty position at the University of California at Berkeley (UCB) and was interviewed by Jerzy Neyman. Neyman strongly supported his appointment, but others in the department had far too strong prejudices to allow Blackwell to be hired. Blackwell and Neyman remained professional colleagues and friends, and they eventually worked together at UCB.

Blackwell was an instructor at Southern University during 1942-43, followed by a year at Clark College. In 1944, Blackwell joined the faculty of Howard University as an instructor. At the time, Howard University "was the ambition of every black scholar." In three years, Blackwell went from instructor to the rank of full professor and became the math department chair at Howard. In spite of heavy teaching and administrative duties and limited research support, Blackwell published a substantial amount of research while he was at Howard.

Blackwell credits a 1945 American Statistical Association lecture by Meyer Abe Girshick with turning his interests toward statistics. Girshick gave a talk on sequential analysis that revolutionized the way Blackwell thought about sampling in two ways. First, where some scientists would limit their sampling before they even began, Blackwell remembers Girshick recommending sampling "until you've seen enough." Blackwell was intrigued by how this approach might be used to more effectively address real world problems. Second, Girshick announced a theorem that Blackwell thought was false. Blackwell worked up a counterexample and sent it to Girshick. It was wrong, but sending it was right. "Instead of dismissing it as the work of a crank," Blackwell said, "he called and invited me to lunch. [2]" Thus began a collaboration that lasted about a dozen years.

By 1954, Blackwell had spent several summers working on game theory at the RAND Corporation. One of the "games" he worked on was that of two duelists who approached each other each with loaded pistols. If one fires early his chance of hitting the target decreases, but if he waits too long he increases his chance of getting hit. The question is, what is the optimal moment for the duelist to shoot? In a variation of the game the guns are silent and so a player does not know if his opponent has fired unless he is hit. The Cold War did much to promote interest in this type of game, and Blackwell soon became a leading expert [13].

Except for spending the summers of 1948-1950 at the RAND Corporation, and spending 1950-1951 at Stanford University as a visiting professor of statistics, Blackwell was at Howard University for the ten-year period 1944-1954. While at Howard, he distinguished himself as an excellent teacher, an able leader (department chair, 1947-1954), and a very productive scholar, publishing more than twenty papers during his tenure there.

The same year that Blackwell joined the faculty at Howard University, he married Ann Madison, whom he had met at Clark College. The Blackwells had eight children; four survived his death: son Hugo Blackwell and daughters Ann Blackwell, Vera Gleason, and Sarah Hunt Dahlquist.

In 1954, Blackwell was invited to give an address in probability at the International Congress of Mathematicians (ICM) in Amsterdam. Shortly after his presentation, Blackwell was offered a visiting position in the mathematics and statistics department at the University of California, Berkeley (UCB). Blackwell accepted the offer. The following year, he was elected president of the Institute of Mathematical Statistics and granted a full professorship in the newly formed department of statistics at UCB. Blackwell was the first African American tenured full professor at UCB. After serving as assistant dean of the College of Arts and Sciences, Blackwell served as chair of the statistics department from 1957 to 1961. In the mid-1970s, Blackwell served abroad as director of the University of California Study Center for the United Kingdom and Ireland. With this international appointment came the presidency of the International Association for Statistics in the Physical Sciences. Also, Blackwell was appointed to be the W. W. Rouse Ball Lecturer at Cambridge, and he was made an Honorary Fellow of the Royal Statistical Society.

David Blackwell.

At Berkeley, Blackwell continued the stellar record of research he had started at Howard, working with Elbert Cox, Dudley Woodard, and William W. S. Claytor, the first, second, and third African Americans to earn PhDs in mathematics. He published an additional 50 scholarly papers prior to his retirement in 1989 (a total of 80 altogether), plus he wrote two books: *The Theory of Games and Statistical Decisions*, with M. A. Girshick in 1954, and *Basic Statistics* in 1969.

At UC Berkeley, his reputation as a gifted teacher and great mentor was also well known. Many mathematically talented students went to UC Berkeley with the hope of being in some of his classes or fortunate enough to have him as a thesis advisor. His mathematical genealogy reflects this. Blackwell was highly sought as a visiting scholar and guest lecturer, both nationally and internationally. Students and scholars alike enjoyed attending his presentations. The writer of this document had the privilege of being in his audience on more than one occasion. Each was a scholarly delight. His prolific and impactful productivity as a scholar gained him international fame and high regards; this has not been forgotten.

I. SOME AREAS OF BLACKWELL's SCHOLARLY LEGACY AND HIS BOOKS

A. Rao–Blackwell Theorem or Rao-Blackwellization (sometimes referred to as the Rao–Blackwell–Kolmogorov theorem)

B. Blackwell channel

C. Blackwell's approachability theory

D. Arbitrarily varying channel

E. Infinite Games and Analytic Sets

F. Game Theory

G. Games of imperfect information

H. Dirichlet distribution

I. Bayesian statistics

J. Dynamic Programming

K. Operation Research

L. Mathematical economics

M. Recursive economics

N. Sequential analysis

O. *Basic Statistics*, 1969

P. *Theory of Games and Statistical Decisions*, with M.A. Girshick, 1954

II. BLACKWELL MATHEMATICAL DESCENDANTS

The Mathematical Genealogy Project indicates that Prof. Blackwell was the PhD advisor of 65 students and he has 389 descendants; such a genealogy is highly exceptional for any mathematician. His students reflected diversity: by country of origin, by gender, and by ethnicity. Moreover, Prof. Blackwell has inspired, encouraged, and mentored hundreds of people (some vicariously) who have pursued mathematics as a goal or a career.

III. SOME OF BLACKWELL'S PROFESSIONAL POSITIONS

A. The International Association for Statistics in Physical Sciences, President
B. The Institute of Mathematical Statistics, President
C. The Bernoulli Society, President
D. International Statistical Institute, Vice President
E. American Statistical Association, Vice President
F. American Mathematical Society, Vice President

IV. BLACKWELL, SOME HONORS, RECOGNITIONS, AND FIRSTS

A. The International Congress of Mathematicians, 1954, Invited Speaker (first African American mathematician invited to give an ICM major address)

B. The National Academy of Sciences, 1965 (Elected)(first and only African American mathematician elected to this Academy)

C. The American Academy of Arts and Sciences, 1968 (Elected)

D. Institute for Advanced Study, Princeton, NJ, 1941 (first African American)

E. Awarded the John von Neumann Prize, 1979 (Operation Research Society)

F. The R. A. Fisher Award, 1986 (by the Presidents of Statistical Societies)

G. Appointed Director of UC Study Center, United Kingdom/Ireland (mid 70s)

H. Appointed W. W. Rouse Ball Lecturer; Cambridge, England

I. Appointed Fellow of the Royal Statistical Society

J. The Annual NAM-MAA Blackwell Lecture named in his honor, 1994

K. The Biennial David Blackwell-Richard Tapia Conference/Prize, began 2000

L. Selected by the American Mathematical Society to be featured in a film: *Guessing at Random*

M. A Medley of Tributes (30 + pages) to David Blackwell, (After His Death) by the *Notices of AMS* (928, Volume 58), George G. Roussas, Coord. Editor

N. Awarded the National Medal of Science in Mathematics, 2014 (posthumously)

O. The David Blackwell Building (UC Berkeley Campus), Opened Aug. 2018

P. Awarded Twelve Honorary Doctor of Science Degrees, by:

1. Harvard 2. Yale 3. University of Illinois
4. Howard University 5. Carnegie-Mellon
6. University of Southern California
7. Michigan State 8. Syracuse
9. Southern Illinois 10. University of Warwick

11. Nat'l Univ. of Lesotho 12. Amherst College

V. SOME PERSPECTIVES FOR KNOWING BLACKWELL BETTER

A. Blackwell said that the work that gave him the most satisfaction was "Infinite games and analytic sets," which he published in the *Proceedings of the National Academy of Sciences* in 1967. He had found a game theory proof of the Kuratowski Reduction Theorem, connecting the areas of game theory and topology. He said: "This gave me real joy, connecting these two fields that had not been previously connected [3]."

B. Blackwell, who became a diehard Bayesian, said "Jimmy Savage convinced me that the Bayes approach is absolutely the right way to do statistical inference." Savage had been at the Institute for Advanced Study with him, and they worked together again at RAND. Blackwell said, "I think I was looking for Bayes all along," and Savage was the one who brought him to it [1].

C. Blackwell's extensive teaching career has yielded many exciting moments, but one of the ones that stands out is a time when he was working on a problem involving choosing experiments, and he and several of his peers couldn't seem to find the solution. He mentioned it to a graduate student, and the student was tempted to try the problem. Blackwell said he was initially inclined to discourage the student, who was not among the top students in the program, but he was insistent. The student found the solution, and Blackwell chuckled about it often, so pleased that one of his students solved the problem that it outweighs not solving it himself. This student's focus was a kind that Blackwell could appreciate.

D. When students asked his advice about the profession, he told them what he would tell them about any other profession: keep trying different things and try to find something you like.

E. In 1957, Blackwell became the chair of the Berkeley's department of statistics, but the administrative role was less fulfilling. For about a year after stepping down, he said, "my first thought was, 'I am no longer chairman,' and it made my day."

F. "As a scholar, he went from one area to another, and he would write a fundamental paper in each," Thomas Ferguson, an emeritus professor of statistics at UCLA said. "He would come into a field that had been well studied and find something really new, that was remarkable. That was his forte."

G. "He had this great talent for making things appear simple," Peter Bickel, a statistics professor at Berkeley, once said. "He liked elegance and simplicity. That is the ultimate best thing in mathematics, if you have an insight that something seemingly complicated is really simple, but simple after the fact."

H. In an interview Blackwell was asked the question: "Of the areas in which you have worked, which do you think are the most significant?" He replied, "I've worked in so many areas; I'm sort of a dilettante. Basically, I'm not interested in doing research and I never have been ... I'm interested in understanding, which is quite a different thing. And often to understand something, you have to work it out for yourself because no one else has done it [5]."

VI. LA FIN

The annals of history have recorded that David Harold Blackwell understood much and he shared much.

Blackwell is recognized in the mathematical and statistical communities as being a world-class mathematician-statistician. He is one of the more influential and productive mathematicians-statisticians of recent eras, as well as a very gifted teacher. Many persons consider him the most well-known and famous African American mathematician-statistician, worldwide.

His life and his contributions (before his transition on July 8, 2010) left much for us to reflect during his centennial year, 2019.

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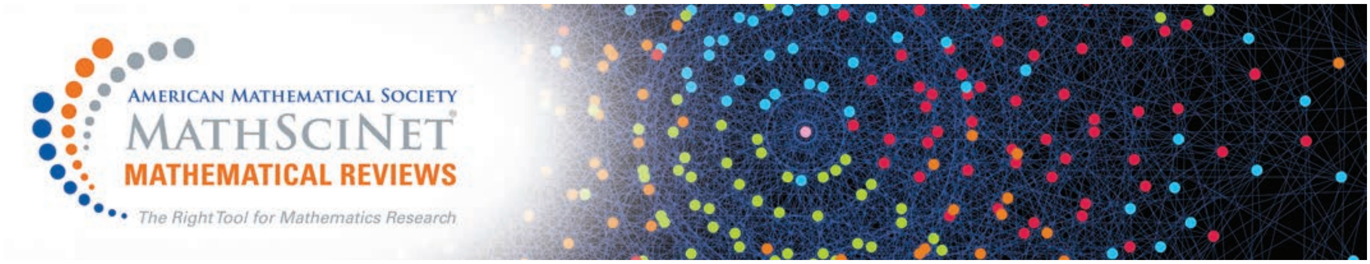
Johnny L. Houston

Credits

Photos of Blackwell are courtesy of UC Berkeley/Bancroft Library.
Author photo is courtesy of Johnny L. Houston.



Credit: Marjin Heule

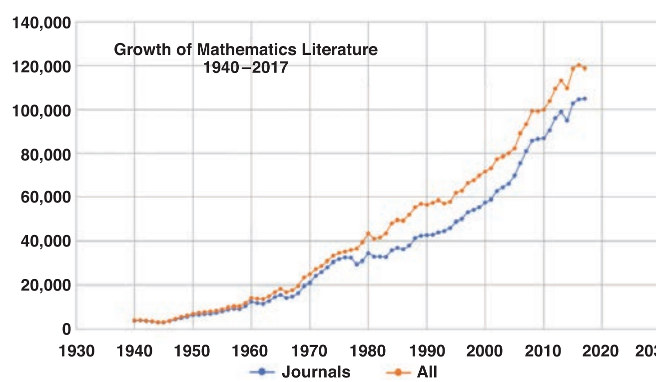
Looking at the Mathematics Literature

Edward Dunne

If you think your job is getting harder, you are correct. The mathematics literature is growing relentlessly, and becoming harder to figure out along the way. The vehicles for publishing are more varied than ever. Meanwhile, bibliometrics are tempting to administrators for their apparent objectivity, which then tempts researchers to respond accordingly. This is a look at these three issues, using information contained in the Mathematical Reviews Database (MRDB), which is what powers MathSciNet®. Mathematical Reviews has been indexing and reviewing the research literature in mathematics since 1940. We have collected a considerable amount of information about this corpus over the years. As of this writing, the database contains roughly 3.6 million items and profiles for over 900,000 authors.

1. Growth of the Literature

Counting the number of items indexed by Mathematical Reviews per year from 1985 to 2017, the number of new articles per year is well modelled by exponential growth at a rate of about 3% percent per year. Counting just journal articles, the rate is about 3.6%. That rate has a doubling time of just over 19 years. So far, we have counted 104,953 journal articles published in 2017. When I finished my PhD, in 1984, just over 35,000 mathematical articles in journals were published that year. For graduate students finishing their PhDs last year, that number had essentially tripled. Moreover, this model says they should expect more than 400,000 journal articles to be published per year by the time they are thinking about retirement.



This trend is not unique to mathematics, of course. Looking at the data in Web of Science from 1985 to today, the scientific literature overall is growing at about 3.9% per year.

The data from the arXiv are harder to compare with publication data, as there are two phenomena occurring simultaneously. The arXiv started in 1991, with almost all the submissions being in certain areas of physics, such as high-energy physics. One way that the arXiv has grown is by attracting papers in different subjects and by the increasing participation of researchers in those subjects. As a result, the increasing number of uploads per year to the arXiv reflects

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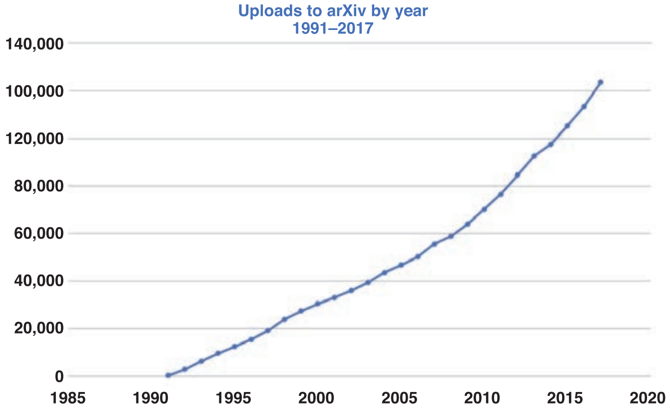
both a growth in the number of papers being written and a growth in the number of researchers choosing to post their papers on the arXiv. A rough analysis of arXiv data indicates that it is growing according to a power law that is in between linear and quadratic.



A note about the arXiv: There is a belief that most of what is published is available on the arXiv. However, that is not so true for the items in MathSciNet. In particular, for all items in the Math Reviews Database with publication dates in the five-year period from 2013 to 2017 (inclusive), just 23% were also in the arXiv. This corresponds reasonably well with the findings of Larivière, et al., who reported that 21% of all mathematics papers in journals covered in Web of Science were also on the arXiv, using data with publication year 2010. Mathematics had the highest proportion of published papers on the arXiv. Physics was second, at 20%, and the global percentage for all disciplines was 3.6%. Going the other direction, Larivière, et al. found that in their data set, 64% of all arXiv papers were published in a journal indexed by Web of Science. The percentage was highest in condensed-matter physics (80%), and was about 45% for mathematics papers. Of course, there are some interesting examples of papers posted on the arXiv that were never published in a journal, such as Perelman's papers on the Poincaré Conjecture.

2. New Models

Publishing models for journals run from traditional journals that are a hundred years old to new overlay journals. Journals offer several versions of open access: green, gold, diamond. Many journals are now hybrid journals, where you can pay an APC to make your article open access, but some of the other articles in the journal will be behind a paywall. When Mathematical Reviews started in the 1940s, it was difficult to start a journal.[1] At a minimum, you needed resources to print and to distribute the journal. You also needed to establish a subscription base. Now, those obstacles have fallen away, making it much easier to start a journal. The effect has been to broaden the types of journals that are started.

There have been some interesting new journals: The *Cambridge Journal of Mathematics* is a traditional journal published by International Press that started publishing in 2013 and quickly attracted good authors who submitted good papers. The now-hybrid journal *Research in Mathematical Sciences* from Springer, which began as an open access journal in 2014, also quickly attracted good papers from recognized researchers. The overlay journal *Discrete Analysis*, started in 2016, hosts the papers on the arXiv, rather than developing their own infrastructure. There has also been a surge in journals of questionable value. Mathematical Reviews tries to identify those that meet the standards in our Editorial Statement `www.ams.org/publications/math-reviews/mr-edit`, in particular that are publishing refereed research in the mathematical sciences.

How do you know if a journal is "good"? For a few journals, this is easy to answer. They have been around for some years and have consistently published good papers—papers that you know have had an impact in the field. However, for most journals, this is very difficult to answer. Ideally, the measure would be the quality of the papers published in the journal. But is this "quality" the depth of the ideas, the correctness of the paper, its influence, or something else? Since most of these characteristics are hard to quantify, we end up adopting what we can count.

3. The Mismeasure of Math

Bibliometrics are designed to provide a statistical analysis of published research. There are two basic quantities that are used: citations and publication counts. Citations are meant to be a proxy for "influence," with the assumption being that highly cited papers are influential, hence important and deep. Counts are meant to be a measure of productivity. A researcher who writes many papers is presumed to be more productive than a researcher who writes few.

Counting citations is harder than it appears. People often ask why their citation counts are lower in MathSciNet than in Google Scholar. It is because they count different things. Google does not give a precise definition for their source of citations, but it is clearly broad. Mathematical Reviews provides a list on MathSciNet of the sources used in our citation counts. In looking at a particular paper from number theory from 2006, Google Scholar finds 48 citations, while Mathematical Reviews finds 19. The Google Scholar list includes: duplicates—a preprint on the arXiv and the published version of the paper; papers posted on a web site (not the arXiv) but not published; papers from conference proceedings; a research plan by one of the authors of the paper; the syllabus to a course at MIT posted on MIT OpenCourseWare plus a separate set of lecture notes for that course. These non-journal references indicate the reach of the work but are different from what Mathematical

[1] *Duke University Press has posted a nice history of the founding of the* Duke Mathematical Journal, *which is well worth a read.*

Reviews citations are trying to indicate: adoption in the research literature.

Matching citations can be tricky. References are matched to the original works algorithmically. It is hard for the algorithms to predict all the ways a citation might vary. An author contacted Mathematical Reviews to say that some citations were missing from their book. They provided specifics. In looking at the papers that were citing the book, we found that the citations were careless. The title was missing or was wrong. In at least one case, the year of publication given was actually the volume number. Meanwhile, the formats for references in many physics journals omit the title, give just initials for the authors' names, and sometimes give only the starting page number instead of the full page range. While these formats may be traditional, they provide fewer hooks for finding a match. Perhaps someday physicists will adopt a less telegraphic format for their references. If they do, they may find that their citation counts go up immediately.

Even if the matches are perfect, the question of whether they mean anything remains. There is a story that people like to tell when discussing consultants:

> A mathematical prodigy named Jedediah Buxton was taken to see David Garrick perform in Shakespeare's *Richard III* at the Drury Lane theatre. When asked whether he had enjoyed the play, his reply was that it contained 12,445 words. His analysis was correct, but did seem to miss some significant aspects.

The Jedediah Buxton tale is often a prelude to a discussion of the maxim: You get what you measure. That is to say, the system responds to the measuring, which leads to the problem of people gaming the system. For impact factors of journals, gaming includes excessive self-citations and citation stacking. The first is self-explanatory. The latter, also known as a citation cartel, is an arrangement whereby a group of journals have a policy of inflating citations to other journals in the group.

Individual researchers can try to game the system, aiming to maximize publication counts. The best-known tool is "salami slicing." The goal is to find the least publishable unit (LPU) or publon, which is the minimum amount of content that can be used to get a paper accepted in a journal. We have seen more than one series of papers where a result was first proved for second-order equations, followed by another paper proving the result for third-order equations, possibly even a paper on fourth-order equations.

Prolific authors and award-winning mathematicians

In mathematics, having a significant impact need not be correlated to having published lots of papers. The following is a list of prolific authors combined with a list of some award-winning authors. The lists of award-winning mathematicians and prolific authors have overlaps, but the symmetric difference is interesting.

Author	Total pubs in MRDB	# Citations in MRDB	# of Citing Authors in MRDB	h-index in MRDB
Ravi Agarwal	1,549	13,807	5,211	51
Paul Erdős	1,445	17,097	9,563	58
Donal O'Regan	1,295	10,220	3,716	42
Israel Gel'fand	486	9,075	7,171	45
Terence Tao	305	13,186	6,199	56
Stanley Osher	272	14,802	8,210	54
Jean-Pierre Serre	270	14,650	7,701	62
Michael Atiyah	262	9,284	5,471	47
George Lusztig	244	8,277	2,011	47
Masaki Kashiwara	240	6,044	2,110	42
Peter Lax	211	6,649	5,612	37
Sylvia Serfaty	84	1,524	582	22
Cathleen Morawetz	82	1,045	603	19
Andrew Wiles	28	1,824	789	15
Maryam Mirzakhani	20	324	267	9
Peter Scholze	19	233	143	8

Data current as of October 3, 2018.

4. The Human Factor

At Mathematical Reviews, we rely on computers and data, but we rely even more on experts. We have two departments that function like departments in a research library: maintaining relations with publishers, keeping track of what we have received and what we are still waiting for, and carefully cataloging material—especially author identification. Many of them have graduate degrees in library science or information science. They are valuable internal resources on publishing standards.

Our database tools help our staff do their work more quickly and more accurately. For instance, the program that helps with author identification tries to match the author of a newly received paper to an author already in our database, using subject area, coauthors, institution, and more. Many times, there is an obvious match. Plenty of other times, there are multiple possible matches. At this point, the catalogers need to rely on their training, experience, and guile to find the correct match.

We also have 18 PhD mathematicians who make editorial decisions every day. All of them have research programs, which help them identify what journals, proceedings, and books meet our editorial standards. They also rely on their years of experience in working with the mathematical literature, which gives them a powerful perspective.

Finally, we have 22,000 research mathematicians who serve as reviewers. We rely on their training and expertise in specific areas of mathematics to comment on the published literature. The reviewers' familiarity with subfields can be particularly helpful in pointing out overlaps between papers or duplicate publications.

The mathematics literature is complex. It is useful to count it and to measure it in various different ways. But it is also subtle, and we only truly understand it by reading it and engaging with it.

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Edward Dunne

Credits

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AMERICAN MATHEMATICAL SOCIETY

2018 Election Results

In the elections of 2018, the Society elected a vice president, a trustee, five members at large of the Council, three members of the Nominating Committee, and two members of the Editorial Boards Committee.

Vice President


Photo courtesy of Abigail Thompson

Abigail Thompson
University of California, Davis

Term is three years (*February 1, 2019–January 31, 2022*).

Trustee


Photo courtesy of Matthew Ando

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Term is five years (*February 1, 2019–January 31, 2024*).

Members at Large of the Council

Terms are three years (*February 1, 2019–January 31, 2022*).


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Terms are three years (*January 1, 2019–December 31, 2021*).

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Terms are three years (*February 1, 2019–January 31, 2022*).

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AMERICAN MATHEMATICAL SOCIETY

2019 Election

Nominations by Petition

Vice President or Member at Large

One position of vice president and member of the Council *ex officio* for a term of three years is to be filled in the election of 2019. The Council intends to nominate at least two candidates, among whom may be candidates nominated by petition as described in the rules and procedures.

Five positions of member at large of the Council for a term of three years are to be filled in the same election. The Council intends to nominate at least ten candidates, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

Petitions are presented to the Council, which, according to Section 2 of Article VII of the bylaws, makes the nominations.

Prior to presentation to the Council, petitions in support of a candidate for the position of vice president or of member at large of the Council must have at least fifty valid signatures and must conform to several rules and procedures, which are described below.

Editorial Boards Committee

Two places on the Editorial Boards Committee will be filled by election. There will be four continuing members of the Editorial Boards Committee.

The President will name at least four candidates for these two places, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

The candidate's assent and petitions bearing at least 100 valid signatures are required for a name to be placed on the ballot. In addition, several other rules and procedures, described below, should be followed.

Nominating Committee

Three places on the Nominating Committee will be filled by election. There will be six continuing members of the Nominating Committee.

The President will name at least six candidates for these three places, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

The candidate's assent and petitions bearing at least 100 valid signatures are required for a name to be placed on the ballot. In addition, several other rules and procedures, described below, should be followed.

Rules and Procedures

Use separate copies of the form for each candidate for vice president, member at large, member of the Nominating or Editorial Boards Committees.

1. To be considered, petitions must be addressed to Carla D. Savage, Secretary, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA, and must arrive by 24 February 2019.
2. The name of the candidate must be given as it appears in the American Mathematical Society's membership records and must be accompanied by the member code. If the member code is not known by the candidate, it may be obtained by the candidate contacting the AMS headquarters in Providence (`amsmem@ams.org`).
3. The petition for a single candidate may consist of several sheets each bearing the statement of the petition, including the name of the position, and signatures. The name of the candidate must be exactly the same on all sheets.
4. On the next page is a sample form for petitions. Petitioners may make and use photocopies or reasonable facsimiles.
5. A signature is valid when it is clearly that of the member whose name and address is given in the left-hand column.
6. When a petition meeting these various requirements appears, the secretary will ask the candidate to indicate willingness to be included on the ballot. Petitioners can facilitate the procedure by accompanying the petitions with a signed statement from the candidate giving consent.

AMERICAN MATHEMATICAL SOCIETY

Nomination Petition for 2019 Election

The undersigned members of the American Mathematical Society propose the name of

____________________________________ as a candidate for the position of (check one):

- ☐ **Vice President** (*term beginning 02/01/2020*)
- ☐ **Member at Large of the Council** (*term beginning 02/01/2020*)
- ☐ **Member of the Nominating Committee** (*term beginning 01/01/2020*)
- ☐ **Member of the Editorial Boards Committee** (*term beginning 02/01/2020*)

of the American Mathematical Society.

Return petitions by February 24, 2019 to:
Secretary, AMS, 201 Charles Street, Providence, RI 02904-2213 USA

Name and address (printed or typed)

Name and address (printed or typed)	
	Signature
	Signature
	Signature
	Signature
	Signature
	Signature

COMMUNICATION

Plans for ICM 2022

Andrei Okounkov

The next International Congress of Mathematicians (ICM) will take place in Saint Petersburg, Russia, in July of 2022. ICMs are the largest and perhaps the most significant meetings in our profession. As we congratulate the organizers of ICM 2018 in Rio on the very successful realization of their plans, it is already time to start the preparations for the next congress.

Saint Petersburg occupies a very important place on the mathematical map of the world. This is the city in which Leonhard Euler spent 31 incredibly productive years, where Perelman proved Thurston's geometrization conjecture, where Chebyshev invented his polynomials, Markov—his processes, Lyapunov—his exponents, where analysis and mathematical physics were taken to new heights by Sobolev, Ladyzhenskaya, Faddeev, V. Smirnov, and their schools. This list can certainly be continued, but perhaps St. Petersburg's, and indeed Russia's, main asset is not the past accomplishments, but the students who are learning mathematics now, and who are curious, talented, and inspired to go further than their teachers and grandteachers. In connection with the ICM, 2022 will be declared the Year of Mathematics in the Russian Federation. One of its main objectives is to reach this talented youth, as well as to reinforce Russia's traditional broad public interest in mathematics.

Saint Petersburg, the second largest city in Russia, may not be quite the whirlwind of human activity that Moscow is, but it is second to none when it comes to architectural

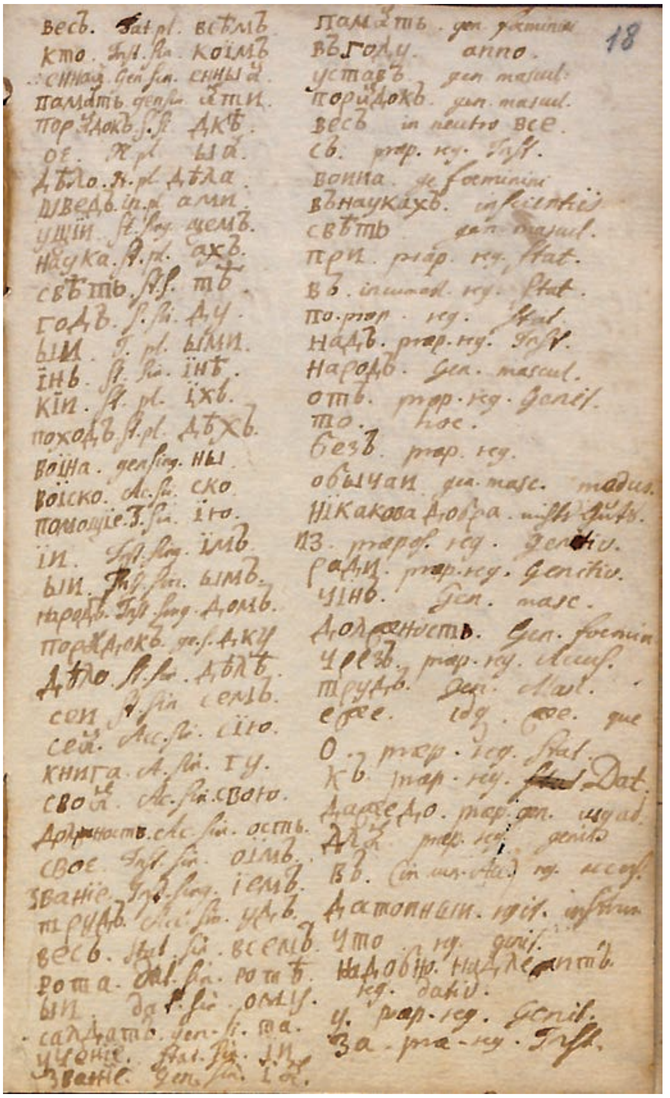

Figure 1. Euler's notebooks from the time he was learning Russian. The great mathematician had an excellent command of spoken and written Russian.

Andrei Okounkov teaches mathematics at Columbia University, and also at Skoltech and Higher School of Economics in Moscow. He is a member of the ICM 2022 Organizing Committee. His email is `okounkov@math.columbia.edu`.

The author was a member of the team that prepared the St. Petersburg ICM bid and presented it to the General Assembly of the IMU in São Paulo. This paper outlines the central points of this document and of the organizers' plans.

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and natural beauty, the treasures of its museums, the expression and perfection of its musical performances, and so many other things that attract millions of visitors every year. Many visitors come to take part in international congresses and conventions—St. Petersburg is a leading destination for international gatherings, hosting such high profile events as its annual international economic (SFIEF) and legal forums. The city is used to extending its warm hospitality on a really large scale: it hosted several games of the soccer World Cup in 2018 and will be welcoming soccer fans again for UEFA Euro 2020.

Figure 2. Views of the Expoforum convention and exhibition centre.

The venue for ICM 2022 will be the newly constructed, state-of-the-art Expoforum convention and exhibition centre. Its three pillar-free exhibition pavilions can be freely partitioned with mobile walls and can be transformed into comfortable congress halls of any size up to 8,600 participants. They come equipped with modern concert-quality sound, light, and projection systems. Energy spent on mathematics may be replenished at the two on-site hotels and a number of restaurants. As to the proverbial free lunches, the organizers are budgeting for free or significantly subsidized lunches for all registered participants, and are also planning a variety of options for meals and snacks. The Expoforum's catering service is experienced in welcoming guests with diverse dietary restrictions and needs.

Frequent special buses will bring ICM participants to the subway (15 min) and to the city center (about 30 min). The trip to the city center goes via one of the city's main arteries—Moskovsky Prospect. Along it, many more hotels are situated, making for convenient pickup and dropoff. The connection from the Expoforum to the Pulkovo International Airport is particularly easy and takes only a few minutes. Pulkovo has recently completed a large-scale upgrade and was recognized in 2015 as Europe's best airport. There will be people there to help ICM participants and no lines. Other convenient arrival options include high-speed trains and cruise ships.

Figure 3. View of Saint Petersburg: the Peter and Paul fortress, the Neva River, the Winter palace (the Hermitage museum), and the Saint Isaac's cathedral in the background.

What should make getting to and around the ICM especially easy is the system that has been already proven effective during the World Cup and other large-scale international events. The ICM registration tag will work like the soccer Fan ID—it will eliminate the need for a visa, public transit tickets, etc. Registered participants will be able to print their tags before arriving in Russia, or at the airport. Its magic visa properties will extend well beyond the ICM itself, so that people can go to satellite events or tour Russia.

Meeting conveniences are, of course, only auxiliary to the quality of the ICM events themselves. The local organizers are confident that the IMU Program Committee will put together a superb collection of prize, plenary, and sectional talks and are preparing to complement them with top quality public lectures, industry events, exhibitions, and other activities. A new feature of ICM 2022 will be a crucial involvement of the newly created IMU Structure Committee in shaping the scientific program. The overall structure of ICM is certainly a topic that needs the thoughtful and careful attention of our whole community, and the people who represent it through IMU. Concerning ICM events that are within the influence of local organizers, we strongly encourage people to send their ideas and suggestions, either to us directly (see, e.g. my email address in the p 236 footnote.) or to one of the members of our distinguished International Advisory Committee.

Such input will be especially valuable because the organizers have in mind a large variety of events that go beyond the boundaries of mathematics as they may be narrowly defined. We are discussing plans for a festival of science films, a convention of science journalists, large-scale mathematics outreach events, and so on. We are discussing the possibility of having pretalks on the eve of plenary talks that will provide useful background information at a more comfortable pace. ICM outreach events will be naturally crosslinked with the program of the Year of Mathematics in Russia mentioned above. Of course, we are also planning social events, like receptions, city and museum tours, special concerts and performances for ICM participants, etc. In fact, the dates of the ICM have been moved to July to make sure the participants catch more of the enchanting endless northern summer light and the various festivals inspired by it.

Another direction in which local organizers are inviting international participation is the organization of satellite events. St. Petersburg was created as Russia's window on the Baltic and Europe and is within easy reach of many centers of mathematics and conference venues of great historical significance. In our budget, we should have means to cosponsor many satellite events. Of course, we hope that people who come to Russia for the ICM will also take time to tour some of the so many exciting destinations within Russia. An excellent opportunity for this is to team up with some local people to run a satellite at one of the many potential locations. The strengthening of ties between the Russian and the international mathematics community is certainly among the major goals of the organizers.

An important feature of our budget is a very extensive financial and logistic help for many categories of ICM participants. Following Russia's longstanding tradition of reaching out to our colleagues in developing countries, 1,000 of them will receive full financial support. The recipients will be chosen by the IMU and it is very important to get the word out early and widely. Up to 200 invited speakers will also be fully supported. We are planning to support an additional 1,300 young mathematicians from all over the world in partnership with their local societies and science foundations. A possible arrangement could be: the local organizers pay local expenses while the participant's travel expenses are covered at home. We are hoping the AMS, and others societies and funding agencies worldwide, will find this cooperation proposal attractive.

Even people paying the registration fee in full, which we estimate to be less than US$200, will probably find it very reasonable given what we are planning to provide in return: a tablet with a congress app, free mobile access for the ICM duration, ICM proceedings, medical and accident insurance for the duration of the event, the aforementioned free lunches, free conference dinner, free public transport, and at least one free social and one cultural event. This all in addition to the intangibles of being in the middle of the most exciting mathematics there is.

The organizers believe they are working for an excellencause and are determined to make the 2022 ICM a successful and impactful event. We are reaching out to the readers of the *Notices* and to the whole mathematics community for their input in planning the event; we hope to see everybody in St. Petersburg in 2022!

Andrei Okounkov

Credits

Figure 1 is courtesy of the St. Petersburg Branch of the Archive of the Russian Academy of Sciences.

Figure 2 is the author's collage of images by Expoforum International Ltd.

Figure 3 is courtesy of the city of St. Petersburg.

Thinking Positive: Arithmetic Geometry in Characteristic p

Renee Bell, Julia Hartmann, Valentijn Karemaker, Padmavathi Srinivasan, and Isabel Vogt

The authors of this piece are organizers of the AMS 2019 Mathematics Research Communities summer conference on **Explicit Methods in Arithmetic Geometry in Characteristic p**, one of three topical research conferences offered this year that are focused on collaborative research and professional development for early career mathematicians.

Additional information can be found at http://www.ams.org/programs/research-communities/2019MRC-Explicit.
Applications are open until **February 15, 2019.**

Arithmetic geometry arose as a beautiful and powerful theory unifying geometry and number theory, formalizing striking analogies between them in a way that allowed tools, results, and intuition of each to be transported to the other—a notable example of this is the proof of the centuries-old problem, Fermat's Last Theorem. This theory provides a geometric viewpoint of objects over fields of prime characteristic p, like finite fields. Decades after these ideas were formalized, characteristic p arithmetic geometry is rapidly expanding to include work in the vibrant young fields of arithmetic dynamics and derived algebraic geometry.

Fields of positive characteristic p have a fundamentally distinct flavor from the classical setting of fields of characteristic 0. In addition, having to let go of the archimedean framework requires a radically different geometric intuition. Working in characteristic p therefore comes with additional challenges, but there are also additional tools and structure that can be exploited, which have led to remarkable results. In some instances these results even carry over to solve open problems in characteristic 0.

Historical overview. While number theorists had been studying the rational numbers and other number fields using the Riemann and Dedekind zeta functions, E. Artin in his 1921 thesis [1] first proposed an analogous theory of zeta functions for curves over finite fields. Hasse extended this to prove the Riemann hypothesis for elliptic curves over finite fields. Their work further sparked the development of the theory of function fields, as well as the theory of algebraic varieties, by several mathematicians, including Weil. In the 1940's, Weil published a book [11] on the foundations of algebraic geometry in characteristic p, and subsequently proved the Riemann hypothesis for function fields over finite fields [12]. Moreover, in 1949, Weil proposed conjectures on the behavior of zeta functions of varieties over finite fields—including the Riemann hypothesis—which came to be known as the Weil conjectures [13]. The breakthrough which enabled Grothendieck,

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M. Artin, and Verdier to prove all but one of the Weil conjectures in 1964 [5] was their development of ℓ-adic cohomology. Building on this, the Riemann hypothesis was eventually proven in two different ways by Deligne ([3, 1974], [4, 1980]). For more historical details, see e.g. [8].

Weil conjectures and counting points. The zeta function $Z_X(T)$ of an algebraic variety X over $\mathbb{F}_q$ encompasses information about point counts over all finite extensions of $\mathbb{F}_q$:

$$Z_X(T) = \exp\left(\sum_{n\geq 0} \#X(\mathbb{F}_{q^n})\frac{T^n}{n}\right).$$

The Weil conjectures predict that the power series $Z_X(T)$ is a rational function, whose zeros and poles can be described in terms of natural group actions on the associated étale cohomology groups. For example, when $X = \mathbb{P}^1_{\mathbb{F}_q}$, direct calculation gives $\#\mathbb{P}^1(\mathbb{F}_{q^n}) = q^n + 1$, and

$$Z_{\mathbb{P}^1}(T) = \frac{1}{(1-qT)(1-T)}.$$

In 1954 Lang and Weil [7] proved a weaker version of the Weil Conjectures: if X has d irreducible components of maximal dimension r, then

$$\#X(\mathbb{F}_{q^n}) \sim d(q^n)^r + O(q^{r-1/2}),$$

as n goes to infinity. These Lang–Weil estimates are useful in both directions: information either about point counts or about the parameters r and d can be traded for the other. This technique is particularly striking in combination with the technique of spreading out from characteristic 0 to characteristic p: counting points on a specialization to characteristic p can give information about dimension and irreducibility in characteristic 0!

This idea was used in recent work of Browning–Vishe [2], building on an idea of Ellenberg–Venkatesh, to show that certain spaces parameterizing rational curves on hypersurfaces over $\mathbb{C}$ are irreducible of the expected dimension. This is a beautiful illustration of the synergy between characteristic 0 and characteristic p algebraic geometry.

Characteristic p phenomena.

Example 1: Elliptic curves of unbounded rank. Another notable application of these modern tools is an explicit example, due to Ulmer [10], of a family of elliptic curves of unbounded rank defined over a function field over a finite field. The construction makes clever use of the known cases of the Tate conjecture [9] over finite fields. The existence of such a family of elliptic curves over number fields is a topic of heated debate amongst number theorists today! Numerous heuristics have been developed about the behavior of ranks and other fundamental invariants of elliptic curves.

Example 2: Automorphisms. A broad theme in mathematics is that one should study the symmetries (i.e. automorphisms) of an object alongside the object itself. In the simplest case of an algebraic curve over $\mathbb{C}$, the automorphism group varies by the topological type: it is infinite for curves of genus $g = 0$ or 1, but once $g \geq 2$, Hurwitz famously proved that its order is at most $84(g-1)$. In characteristic p, an algebraic curve could have additional symmetries! For example, the projective plane curve $x^{p+1} + yz^p + zy^p$ over $\mathbb{F}_{p^2}$ has genus $p(p-1)/2$, but $O(p^8)$ symmetries. These symmetries arise in analogy with the fact that the unitary group $\mathrm{PGU}_3(\mathbb{C})$ preserves the Hermitian form $Q(x,y,z) = x\bar{x} + y\bar{z} + z\bar{y}$, and so acts on $Q = 0$. Replacing complex conjugation with the Frobenius involution $x \mapsto \bar{x} = x^p$ (special to characteristic p) of $\mathbb{F}_{p^2}$ over $\mathbb{F}_p$ gives rise to an action of $\mathrm{PGU}_3(\mathbb{F}_{p^2})$ on this curve.

Example 3: Fundamental groups. An important objective of mathematics is to classify spaces; a natural approach to this is introducing and comparing algebraic invariants associated to a space. One important example of such an invariant is the fundamental group, which captures information about the geometry of a space and its maps to other spaces. For a variety X over $\mathbb{C}$, the topological fundamental group $\pi_1(X)$ has a description in terms of loops on X. In particular, the line $\mathbb{A}^1_{\mathbb{C}}$ has trivial π_1, since it's contractible. Using an equivalent description of π_1, this says that $\mathbb{A}^1_{\mathbb{C}}$ has no nontrivial unramified covers. The same is true when we replace $\mathbb{C}$ with any other characteristic 0 field. In characteristic p, the theory of étale covers gives a direct analog $\pi_1^{\text{ét}}$ of the topological fundamental group. Grothendieck proved that the prime-to-p part of $\pi_1^{\text{ét}}$ is the same as that of π_1 of an analogous curve over $\mathbb{C}$ [6]. However, the p-part of $\pi_1^{\text{ét}}$ detects that the theory of covers is much richer in this setting. For example, over a characteristic p ground field, $\pi_1^{\text{ét}}(\mathbb{A}^1)$ is far from trivial; in fact, its cardinality is huge and depends on the ground field. It is even conjectured that $\pi_1^{\text{ét}}(\mathbb{A}^1_k)$ determines the ground field when k is algebraically closed.

Mathematics research communities. Arithmetic geometry in characteristic p lends itself to a rich and varied collection of accessible problems, in topics such as isogeny classes of abelian varieties over finite fields, Galois covers of curves and lifting problems, and arithmetic dynamics. Many of these problems are existential, and can be attacked by an explicit or computational approach. Significant progress in this setting has been made in the last year alone, due to both recent theoretical technical innovations and to recent computational advances that have aided experimentation. Relatedly, there are new approaches to explicitly constructing examples exhibiting certain phenomena.

We invite early-career mathematicians from a wide range of backgrounds to continue this story at our upcoming MRC, "Explicit Methods in Arithmetic Geometry in Characteristic p."

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Renee Bell

Julia Hartmann

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Credits

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Representation Theory and the Elliptic Frontier

Daniel Berwick Evans, Emily Cliff, Nora Ganter, Arnav Tripathy, and Josh Wen

The authors of this piece are organizers of the AMS 2019 Mathematics Research Communities summer conference on **Geometric Representation Theory and Equivariant Elliptic Cohomology,** one of three topical research conferences offered this year that are focused on collaborative research and professional development for early career mathematicians.

Additional information can be found at `http://www.ams.org/programs/research-communities/2019MRC-Geometry`. *Applications are open until* **February 15, 2019.**

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Equivariant elliptic cohomology is a topological invariant born out of deep work in algebraic topology and mathematical physics. Intriguingly, it both organizes previous work and points towards new terrain in the established but mysterious subject of elliptic representation theory. The central examples bridge a variety of subjects, including manifold invariants, integrable systems, algebraic combinatorics, and enumerative geometry. This article is meant to serve as a relaxed hike through this broad landscape.[1]

Functoriality in algebraic topology. The goal of algebraic topology is to associate algebraic invariants to topological spaces. In practice the most useful invariants are *functorial*: in addition to assigning an invariant to a space, continuous maps between spaces beget maps between invariants. A first example is cohomology, which assigns a graded abelian group $\mathrm{H}^\bullet(X)$ to a space X and a linear map $f^*\colon \mathrm{H}^\bullet(Y) \to \mathrm{H}^\bullet(X)$ (the *pullback*) to a continuous map $f\colon X \to Y$ between spaces. For nice maps $\pi\colon X \to B$ (e.g., with compact, oriented manifold fibers), we obtain a linear map $\pi_!\colon \mathrm{H}^\bullet(X) \to \mathrm{H}^{\bullet-d}(B)$ (the *pushforward*), where d is the fiber dimension of π. A more sophisticated invariant is K-theory, $\mathrm{K}(X)$, an abelian group generated by complex vector bundles on X. Once again, there are pullbacks $f^*\colon \mathrm{K}(Y) \to \mathrm{K}(X)$ associated to maps $f\colon X \to Y$ and pushforwards $\pi_!\colon \mathrm{K}(X) \to \mathrm{K}(B)$ for sufficiently nice maps $\pi\colon X \to B$ (e.g., when the fibers of π are compact complex manifolds).

Equivariant algebraic topology seeks to associate algebraic invariants to G-spaces, i.e., spaces endowed with the action of a Lie group G. For a G-space X, equivariant cohomology is $\mathrm{H}^\bullet_G(X) = \mathrm{H}^\bullet((X \times EG)/G)$ where EG is a

[1] *For simplicity, we make some technical elisions at various points, such as conflating homological and cohomological constructions or a compact Lie group and its complexification.*

$$\begin{tikzcd} & X \arrow[dl, "\pi_1"'] \arrow[dr, "\pi_2"] & \\ Y & & B \end{tikzcd} \quad \rightsquigarrow \quad \begin{tikzcd} & E(X) \arrow[dr, "(\pi_2)_!"] & \\ E(Y) \arrow[ur, "\pi_1^*"] & & E(B) \end{tikzcd}$$

contractible space on which G acts freely, while equivariant K-theory, $\mathrm{K}_G(X)$ is generated by G-equivariant vector bundles on X. Both of these define algebraic invariants of G-spaces that are (contravariantly) functorial with respect to equivariant maps, and both theories have pushforwards for appropriate maps $\pi\colon X \to B$.

Push-pull constructions in geometric representation theory. Applying functorial invariants to spaces built out of algebraic groups often results in rich representation-theoretic structures. [2] These can be loosely organized as *push-pull* constructions. Given a functorial invariant E and a pair of maps between manifolds on the left, we obtain the *push-pull map* $(\pi_2)_! \circ \pi_1^*\colon E(Y) \to E(B)$ as the composition on the right.

Example 1 (Convolution). Let X_1, X_2, X_3 be compact oriented smooth manifolds with dimension $\dim(X_i) = d_i$ and let $\pi_{ij}\colon X_1 \times X_2 \times X_3 \to X_i \times X_j$ be the projection. The *convolution product* is the map

$$- * - \colon \mathrm{H}^k(X_1 \times X_2) \otimes H^l(X_2 \times X_3) \to H^{k+l-d_2}(X_1 \times X_3)$$

determined by the formula $\omega_{12} * \omega_{23} := (\pi_{13})_!(\pi_{12}^*\omega_{12} \smile \pi_{23}^*\omega_{23})$, where $\smile$ is the cup product. When X_1, X_2, X_3 are 0-dimensional (so just *finite sets*) $\mathrm{H}^\bullet(X_i \times X_j)$ can be identified with the vector space of $d_i \times d_j$ matrices, and the convolution product is the matrix product.

Example 2 (Springer theory). Consider the *flag variety*, G/B, where $G = \mathrm{GL}(n, \mathbb{C})$, B is the subgroup of upper triangular matrices and $\tilde{\mathcal{N}} = T^*(G/B)$ its cotangent bundle. The cotangent fiber over each flag is the space of nilpotent $n \times n$ matrices preserving that flag. Defining the *nilpotent cone* $\mathcal{N}$ as all nilpotent $n \times n$ matrices, there is a projection map $\tilde{\mathcal{N}} \to \mathcal{N}$ called the *Springer resolution*. The *Steinberg variety* is the fibered product $\tilde{\mathcal{N}} \times_{\mathcal{N}} \tilde{\mathcal{N}}$. Then the push-pull construction associated to $\tilde{\mathcal{N}} \xleftarrow{\pi_1} \tilde{\mathcal{N}} \times_{\mathcal{N}} \tilde{\mathcal{N}} \xrightarrow{\pi_2} \tilde{\mathcal{N}}$ defines a product on (compactly supported) cohomology $\mathrm{H}_c^\bullet(\tilde{\mathcal{N}})$

$$w * v = (\pi_2)_!(\pi_1^* w \smile \pi_2^*(v)), \quad v, w \in \mathrm{H}_c^\bullet(\tilde{\mathcal{N}}).$$

With this algebra structure, $\mathrm{H}_c^{\mathrm{top}}(\tilde{\mathcal{N}} \times_{\mathcal{N}} \tilde{\mathcal{N}})$ may be identified with $\mathbb{Z}[W]$, the group algebra of the Weyl group. In this case this is simply S_n, the symmetric group on n letters. By considering similar constructions for the fibers $\tilde{\mathcal{N}}_x$ of the Springer resolution, one may endow their cohomologies with actions of $\mathbb{Z}[W]$ and derive geometrically, for example, the standard classification of irreducible representations of S_n by partitions.

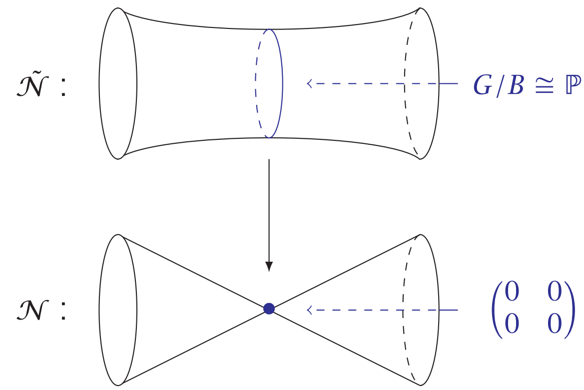

Figure 1. The Springer resolution for $G = GL(2, \mathbb{C})$.

Example 3 (Affine Hecke algebra). Continuing the previous example, let $T < G$ be the n-dimensional torus of diagonal matrices. The Springer resolution is naturally T-equivariant. Furthermore $\tilde{\mathcal{N}}$ has an additional $\mathbb{C}^\times$-action from scaling the cotangent fibers. These actions carry over to the Steinberg variety $\tilde{\mathcal{N}} \times_{\mathcal{N}} \tilde{\mathcal{N}}$, and so we can form the equivariant K-group $K_{T \times \mathbb{C}^\times}(\tilde{\mathcal{N}} \times_{\mathcal{N}} \tilde{\mathcal{N}})$. The analogous push-pull construction identifies this K-group with the affine Hecke algebra $\mathbb{H}^{\mathrm{aff}}$, a deformation of the group algebra of $\Sigma_n \ltimes \mathbb{Z}^n$. This generalizes the Iwahori–Hecke algebra of p-adic representation theory, which controls (spherical) representations of the p-adic Lie group $G(\mathbb{Q}_p)$. As such, this yields a geometric construction of the Iwahori–Hecke algebra and a geometric classification of irreducible representations of $\mathbb{H}^{\mathrm{aff}}$, first proved by Kazhdan and Lusztig.

Example 4 (Nakajima quiver varieties [6]). Consider the cotangent bundle $T^*G(k,n)$ of the Grassmannian $G(k,n)$ of k-planes in $\mathbb{C}^n$. A cotangent vector over a point $V_k \subset \mathbb{C}^n$ is an element $E \in \mathrm{Hom}(V_k, \mathbb{C}^n/V_k)$. The *Hecke correspondence* $Z_{k,k+1} \subset T^*G(k,n) \times T^*G(k+1,n)$ is the subspace of pairs (V_k, E_1) and (V_{k+1}, E_2) where $V_k \subset V_{k+1}$, (so there is a projection $p\colon \mathbb{C}^n/V_k \twoheadrightarrow \mathbb{C}^n/V_{k+1}$) and $E_1 = E_2 \circ p$. For fixed n, we consider the disjoint union over k of $T^*G(k,n)$ and the Hecke correspondences between them. Applying equivariant cohomology yields a representation of the *Yangian* $Y(\mathfrak{sl}_2)$, while applying equivariant K-theory yields a representation of the *quantum affine algebra* $U_q(\widehat{\mathfrak{sl}}_2)$. Both of these quantum algebras contain commutative subalgebras important to quantum integrable systems, and in both cases, the subalgebra is diagonalized by the fixed point basis. This diagonalization paradigm carries over to other well-known examples of quiver varieties, such as the Hilbert of scheme of points on the plane. Here, fixed point classes in equivariant K-theory have been associated to Macdonald polynomials in Haiman's celebrated proof of Macdonald positivity. On the other hand, Macdonald polynomials appear in integrable systems as eigenfunctions for the Macdonald operators, and the action of these operators can be realized geometrically through this kind of K-theoretic construction of a quantum group.

The elliptic frontier. The above examples prompt an obvious question: what happens when we apply ever more complicated functorial invariants to the spaces built out of algebraic groups? In algebraic topology, the chromatic filtration organizes generalized cohomology theories according to their *height*, which is a rough measure of complexity: ordinary cohomology with complex coefficients has height 0 while K-theory has height 1. The most interesting example of a height 2 theory is elliptic cohomology. *Equivariant* elliptic cohomology is still very much under development. The visionary insights of Grojnowski [3] defined the theory over $\mathbb{C}$ while Lurie's more recent work [5] gives indications of what one can expect from the full theory over $\mathbb{Z}$.

One mystery to be solved is the basic representation theoretic content of equivariant elliptic cohomology. To start with an analogy, the coefficients for equivariant K-theory are $K_G(\mathrm{pt}) = \mathrm{Rep}(G)$, the representation ring of G. The complexification of $\mathrm{Rep}(G)$ is the ring of class functions, where characters of representation take their values. Grojnowski's elliptic cohomology over $\mathbb{C}$ has as its coefficients a ring of θ-functions. One might therefore speculate that the coefficients for equivariant elliptic cohomology is some ring of *elliptic representations* whose characters are θ-functions. Positive energy loop group representations have θ-functions as characters, and so are expected to play a central role. However, this appears to only be one part of the story.

A second focal point comes from elliptic generalizations of constructions in geometric representation theory. The examples above point the way to the deep theories of elliptic Hecke algebras, elliptic quantum groups, and elliptic Macdonald polynomials. The recent work of Aganagic–Okounkov [1] suggests a presentation for elliptic cohomology of flag varieties in a precursor to a long-anticipated but unrealized elliptic Schubert calculus. Prominent work in related directions has been pursued by Etingof, Felder, Grojnowski, Rains, Schiffmann, Varchenko, Vasserot, Yang, Zhao, and Zhong among many others.

A third body of research aims to clarify the long-sought but still-mysterious connection between elliptic cohomology and mathematical physics. This was born out of Witten's string theory interpretation [8] of manifold invariants called *elliptic genera*. Conjectured properties of these invariants (e.g., rigidity) were proved using equivariant elliptic cohomology. Segal suggested [7] that these connections between string theory and topology could be deepened by constructing a cocycle model for elliptic cohomology in terms of a suitable space of 2-dimensional field theories. Although the full picture remains unrealized, many of the underlying ideas drove great advances in homotopy theory. Led by Hopkins and collaborators [4], a high point of these results is the string orientation of elliptic cohomology which gives a families version of elliptic genera. One of the more recent fruits of this labor has been the intriguing role of *categorical groups* in equivariant elliptic cohomology. In short, whereas groups are automorphisms of sets, categorical groups are automorphisms of categories. The *string group* is a particular 2-group that has a preferred role in elliptic cohomology and also appears to have an origin in physics as the automorphisms of a particular quantum field theory. Indeed, there are yet vast realms of representation theory to be explored along this elliptic frontier.

Want to learn more? Supported by the AMS, the authors are organizing a Mathematical Research Community on geometric representation theory and equivariant elliptic cohomology for PhD students and early career researchers. The event will be held June 2–8, 2019 in Rhode Island, during which time participants will work in small groups on research problems. Prior to the workshop, there will be an opportunity to learn relevant background material through a guided reading course. The AMS also provides support after the workshop for participants to continue to collaborate and to attend a special session at the JMM in January 2020.

To find out more, visit `www.ams.org/programs/research-communities/2019MRC-Geometry`, or email any of the organizers. Applications are due on February 15, 2019. We look forward to welcoming a diverse group of participants from different mathematical backgrounds—expertise in all areas is not at all expected!

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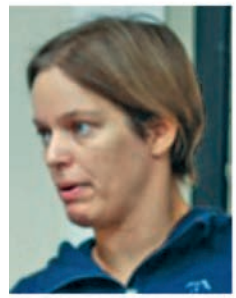
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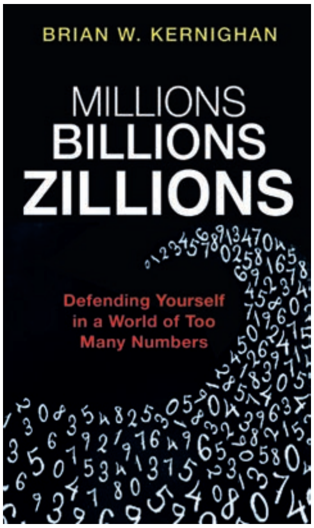

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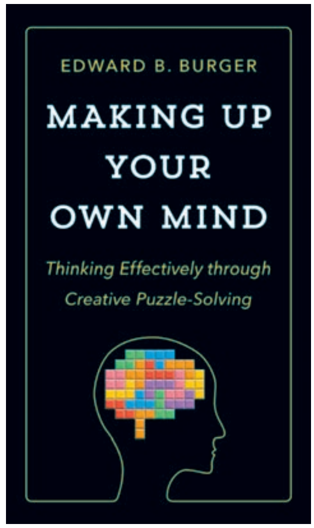

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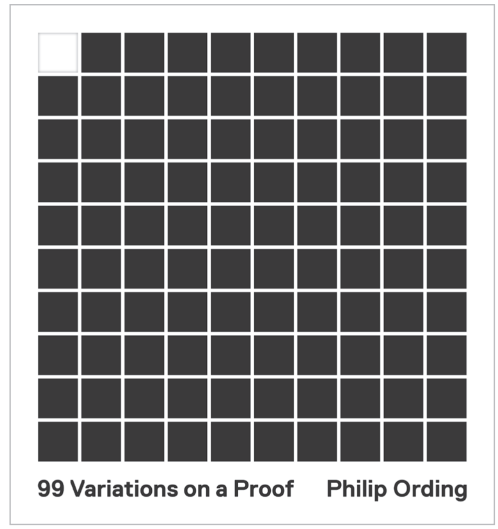

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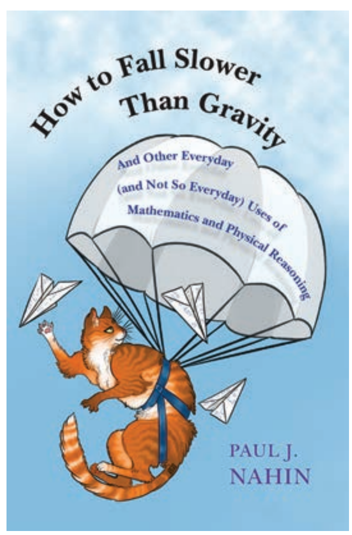

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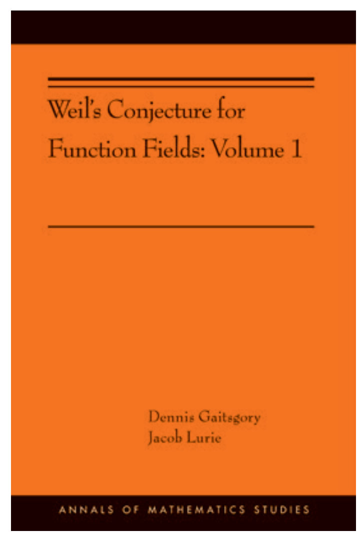

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Annals of Mathematics Studies

Making Dance, Making Math: Parallels

Sophia D. Merow

Mathematicians have no monopoly on problem solving. Anesthesiologists do it, as do sound designers and air traffic controllers. Dancers, too. In a piece called *necessary and sufficient: a dance of mathematics* performed in Seattle last spring, math educator and dance improviser Katherine Cook explored the parallels between how humans puzzle their way out of tight spots, be it in math or in dance improvisation.

Cook has intimate, expert knowledge of both disciplines. She holds a masters degree in mathematics from the University of Washington and is creative director of Seattle-based Math For Love, an organization committed to transforming how math is taught and learned worldwide. She develops curricula and games, writes about math and math education. When not helping learners of all ages develop a meaningful relationship to math, Cook teaches and performs across the country as a dance improviser and conducts dance improvisation research with collaborators at the Institute for the Study of Somatic Communication and the Seattle CI Lab. (CI stands for Contact Improvisation, a dance form nurtured in the post-modern experiments of the 1970s.)

Katherine Cook conceived of and performed in *necessary and sufficient: a dance of mathematics.*

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Dancers think through their bodies, says Cook—and mathematicians do, too.

Part of Cook's motivation for staging *necessary and sufficient* was to give the audience a glimpse of the sparsely populated niche she occupies.

"I feel like I live all of the time in the overlap between dance and mathematics," she explains, "and I really sort of selfishly just wanted to invite other people in."

Mathematically inclined viewers stand to gain, Cook believes, from recognizing and mulling over the commonalities between dance improvisation and the core work of mathematics.

"I think understanding how one field solves its creative problems is of great use to another field," she says. Someone who spends lots of time stuck on math problems might be curious, then, how someone facing a dance problem intuits her way to a path forward.

If your only experience of dance is stumbling through waltz basics or bopping to the beat (more or less) in a club, the idea of a "dance problem" may mystify you. It's doesn't refer—in Cook's dance improvisation context, anyway—to having two left feet or no sense of rhythm.

A dance improvisation performance is created in real time, with the choreography or *score* of the piece not dictating the dancers' movements but instead directing their attention, guiding their thought. In one segment of *necessary and sufficient*, for example, the score stipulates that the four dancers "not not mimic"—no, that double negative isn't a typo—one another. It's up to them to figure out what that means and how to do it, and their actions naturally differ from one performance to the next.

"The score is like a set of axioms," wrote Cook in the *necessary and sufficient* program, "giving a shape, a skeleton, which the dancers fill out with creations and discoveries."

A dance improvisation problem is like a mathematical one, then, in that it involves determining what is possible under a given set of constraints. A dancer is constrained by the score's instructions and the performance space, by her body and the physics of our world.

"Exploring the intuitive and creative act of finding our way through systems that begin with constraints is what we do in both mathematics and in dance [improvisation]," Cook says. And dancers and mathematicians alike can be surprised by what a particular set of constraints can produce: a beguilingly dynamic interaction between a quartet of performers, for instance, or hyperbolic geometry.

Some math-inspired works of dance respond to surface features. They may meet math at a purely notational level by, for example, translating the symbols delineating a sheet of calculus problems into a sequence of body configurations. In conceiving *necessary and sufficient*, Cook wanted to delve deeper, to bypass the mathematical shallows, avoid getting bogged down in the discipline's manifold technicalities (which she personally appreciates, but knows can be rough going for a lay audience), and instead access "some really meaty ideas in the schema of mathematics." She settled on *equivalence and morphism*.

Cook's notion of *morphism* derives from category theory's generalization of the structure-preserving maps found throughout mathematics: functions between sets; homomorphisms between groups, rings, or fields; linear transformations between vector spaces; and so on. In the *necessary and sufficient program*, Cook called morphisms "the mathematician's metaphors." "They are arrows connecting disparate objects and structures," she wrote, "highlighting similarities that might be missed elsewhere." In category theory, objects are completely defined by the morphisms between them, making the morphisms, in a way, the stars of the show. In *necessary and sufficient*, dancers play the role of morphisms.

"Everything the dancer does [in the piece] is a morphism," Cook explains, "a transformation from perceived phenomena to generated phenomena. The dancers are trying to preserve structure in these transformations and the result is an exploration of equivalence and sameness."

Through a category theoretic lens, equivalences can be considered morphisms with special properties, but Cook found it impracticable to convey this in her chosen medium. Though she toyed with trying to incorporate into the piece the conditions for an equivalence relation, she couldn't reconcile the time independence of mathematics with the ephemerality of dance. While a mathematical structure persists stably as long as one cares to study it, a dance structure dissipates as soon as it arises. Try establishing even something as straightforward as reflexivity under such conditions! Cook ended up "playing a little fast and loose with the language of *equivalence*" and interpreted the word largely in its colloquial sense.

A site-specific work, *necessary and sufficient* played out in an art gallery—first Seattle's Center on Contemporary Art, then Schack Art Center in Everett, WA—during the run of the exhibit Art ∩ Math, curated by Cook and Math For Love founder Dan Finkel. The piece opens with the four performers filing into the gallery and standing abreast before one of the show's installations. Here the exploration of morphism begins, as each dancer responds bodily to the work in view, becomes a locus of transformation from one

necessary and sufficient asked dancers (left to right: Brandin Steffensen, Cook, Corrie Befort, Aaron Swartzman) and onlookers alike to ponder flexible notions of sameness.

In parts of the piece, Cook and her fellow performers respond to the visual art on display, become morphisms taking art as input and yielding movement as output.

kind of stimulus to another, from a piece of visual art to a spontaneously generated movement phrase.

"The dancers are working like functions, essentially, from the set of visual stimuli to kind of movement and sound phenomena," says Cook.

Soon the dancers-as-morphisms are chaining themselves together into compositions (as do category theory's morphisms, according to composition laws). One dancer looks at a sculpture, painting, or print, and transforms it into movement; another dancer observes that movement and transforms it into another movement.

Responding to one another rather than to the art is what the performers do in the aforementioned not-not-mimic segment of the piece, which probes notions of equivalence. Cook shied away from writing a straight-up mimic instruction into the score because she is interested in how two things can be deemed the same when in some way they clearly are not. While there are obvious visual similarities between what the performers are doing at the outset of the not-not-mimic portion of the program, it invariably morphs into a deeper exploration of a more sophisticated, nuanced sense of sameness. Akin, perhaps, to the one that equates coffee cups and doughnuts.

Cook sees elegance in the view that subfields of mathematics are defined by their definitions of equivalence and suggests that there's a sense in which much of math comes down to determining when things are the same. "It's almost like different domains are hard at work sorting the objects in their universe according to those ideas of equivalence," she says. The not-not-mimic directive burdens the dancers with trying to find natures of equivalence; it asks the performers to do in their discipline what mathematicians do so tirelessly in theirs.

Press coverage of *necessary and sufficient* (see https://bit.ly/2y35wsy) played on the work's challenge to the perception of math and dance as enterprises without overlap. For Cook, though, the commonality between her two passions is not incidental, but fundamental: they derive from the same source.

necessary and sufficient is a celebration of both math and dance, "and of their shared provenance which, after all, is centered in humanity," Cook wrote in the program. "Our bodies are entangled with our thoughts, and both dance and mathematics spring from this entanglement."

Dancers think through their bodies, Cook observes, and any mathematician who feels she doesn't should remember the location of her brain. "All of our thinking happens in the body that we have," Cook says. "That's just the nature of being a person." Cook believes that embracing and interrogating thought as an embodied phenomenon could enrich and inform humankind's mathematical endeavors.[1]

In one regard, of course, mathematics benefits from a lack of physicality. Mathematics can only maintain its "obscene amount of precision and rigor," Cook concedes, because "the world of [pure] mathematics is imagined, so we don't have to deal with all the messy reality that bodies or anything else suggests."

But if the mathematically inclined could relax their strict standards temporarily, could for a period dispense with the restrictive exactitude of the field, perhaps they could come to appreciate a deeper union between mathematics and other human strivings.

Cook suspects that mathematicians may be loath to let go, even briefly, of the technical machinery of mathematics, even if doing so could afford them insight into what lies beneath. "But I think it's a grave mistake not to do that from time to time," she says, "because there's so much there."

Sophia D. Merow

Credits

Photo of Katherine Cook is by Amber Lee.
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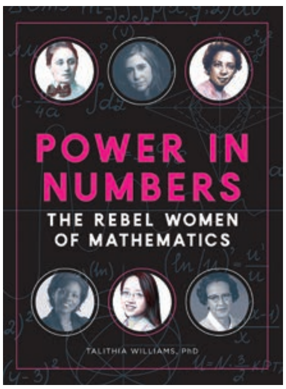
[1] *Pressed to elaborate, Cook notes that children formalize mathematical understanding more successfully when they use their bodies, when they manipulate blocks, count on their fingers, move in space. She also references a 2014 study (see* https://bit.ly/2D4k8Nm*) that found that moving in accordance with a math problem—going up (e.g., in an elevator) or walking rightward when adding, for instance—can facilitate its solution. Watching dancers problem-solve their way through a score can help observers recognize and appreciate the role of the body in cognition.*

BOOK REVIEW

Power in Numbers

Reviewed by Emille Davie Lawrence



Power in Numbers: The Rebel Women of Mathematics
By Talithia Williams
Race Point Publishing, 2018
Hardcover, 224 pages
ISBN: 978-1631064852

The year 2018 has been heralded by some as "The Year of the Woman," with record numbers of women candidates in midterm congressional elections and gubernatorial races, as well as an unprecedented outcry against harassment in the workplace and unequal pay for equal work. This women's movement has without a doubt been accelerated by our current political climate and an intense desire to fight back against a "boys will be boys" school of thought. We are all aware of the public perception that, for the most part, men hold the keys to mathematical understanding. Thus, Williams' compilation *Power in Numbers: The Rebel Women of Mathematics* seems especially timely. Even her choice of the word "rebel" in its title is indicative of a certain pushback that is currently reverberating throughout our society.

Upon starting the book, the first thing I noticed was the sheer beauty in each page. Williams clearly sets out not only to educate the reader, but to do so in a visually tantalizing way. In addition to vibrant portraits of each woman she features, every page has one or two mathematical diagrams, photographs, or historical paintings that absolutely make the book a joy to look at (see Figure 1). There are also pages that have random equations, expressions, and diagrams written in the background, which is sure to amuse a general audience and mathematicians, alike.

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Figure 1. Portrait of mathematician, computer scientist, and rocket scientist Annie Easley.

Williams gives a candid look inside her own mathematical trajectory in the introduction (including a math-inspired pick-up line her husband used to win over her heart) and makes it clear that the women she chose to spotlight in the book are all women who overcame certain stereotypes and persisted in the face of adversity. Although every woman mathematician has overcome a stereotype, I love the women that Williams chose to feature.

The book is organized into three parts: The Pioneers, From Code Breaking to Rocket Science, and Modern Math Mavens, with each section highlighting different women through short biographies. There is also one woman who is a "Spotlight" in each section, though Williams does not make it apparent why the subject of this Spotlight should necessarily be set apart from all the rest. Part 1 contains only six featured women, and among these there are some usual suspects, such as Sofia Kovalevskaya and Emmy Noether. But I was also glad to see some women who are likely lesser known to the mathematical community, such

Figure 2. This 1910 photograph shows first graders at the Minor Normal School brushing their teeth. Lofton-Haynes attended the same school and graduated with distinction just a year earlier in 1909.

as mathematician, astronomer, and poet Wang Zhenyi from the Qing Dynasty and Euphemia Haynes, the first female African American mathematics PhD. Williams emphasizes how many of these women had to use pseudonyms or submit work anonymously in order to advance in the field—a point that is not surprising, but also not widely known. Two of my favorite photographs appear in The Pioneers: a photograph of an all-woman archery team from Bryn Mawr College (one of the places Emmy Noether taught) and an adorable photograph of first graders at the Minor Normal School (see Figure 2), a school that Euphemia Haynes attended. Neither of these photos have a direct connection to mathematics, but they both add to the historical tone I believe Williams is striving for.

Part 2 moves forward to the mid-twentieth century when women were joining the workforce in greater numbers and making significant contributions to computing and space exploration. One of my favorite anecdotes from the book came in the piece on Grace Hopper. Williams describes how Hopper and her colleagues discovered that a large moth caught in the Mark II computer was causing it to malfunction. From that day forward, they began calling any problem with the computer a "bug" and fixing the problem "debugging." Bonus points to Hopper for contributing to our scientific lexicon! Williams also highlights Katherine Johnson and Dorothy Vaughan, two women who have already been prominently featured in the book and feature film *Hidden Figures*, but thankfully Williams manages to put forth fresh information about both. We also get to know in the book Mary Golda Ross, a Cherokee-American woman who went from teaching mathematics in rural Oklahoma public schools to becoming an engineer who worked on defense missiles at Lockheed.

Part 3 is the longest and features sixteen Modern Math Mavens. All of the Modern Mavens, with the unfortunate exception of Maryam Mirzakhani (see Figure 3), whose profile does not appear until almost the end of the book, are still making meaningful contributions to their respective fields. Some of the most fascinating stories, I found, were those of Tatiana Toro and Carla Cotwright-Williams. Toro through her own persistence found her way onto Colombia's IMO team at the age of 17 and went on to earn a PhD and numerous prestigious academic appointments and accolades. Cotwright-Williams has a unique story of growing up in a tough neighborhood in South Central Los Angeles but earned a PhD from the University of Mississippi. After a short career in academia, she made a successful career change to working for the US government and is currently a data scientist for the Department of Defense. Both of these stories really underscore the book's theme of finding one's way in the face of opposition. I also especially enjoyed reading about how Ami Radunskaya spent much of her life as a professional cellist and composer but went on to become a professor at Pomona College and is currently president of the AWM. My most favorite quote of the book comes from Chelsea Walton, associate professor at the University of Illinois at Urbana-Champaign: "The only person you should compare yourself with is you yesterday." I just love that.

Overall, the thirty women that Williams chose for this book are from a diverse array of backgrounds—mathematical and otherwise. Williams has unquestionably done a considerable amount of research on each of these women. This comes through in the level of detail she puts into each biography. I found myself marveling at the personal anecdotes and quotes throughout the book. Furthermore, for those who want an even deeper dive, she gives a list of resources for further reading on each woman at the end of the book.

Power in Numbers is unmistakably a book written for the general public. The explanations of the mathematics to which each of these women contributed are light-handed

Figure 3. The front pages of several Iranian newspapers celebrate Mirzakhani a day after her death on July 14, 2017. Some of them broke cultural taboo, picturing the mathematician with her hair uncovered.

and intended for the casual reader. However, one particularly nice touch is that in each piece there is an inset that gives a bit more detail about the mathematics or history of a certain topic for the more curious reader. For example, there is a short explanation of Fermat's Last Theorem along with a diagram to demonstrate the Pythagorean theorem. There are also diagrams that define the Cartesian coordinate system and give a visual representation of some of the Fibonacci numbers, to name a few. A general reader will undoubtedly be amused by recalling some mathematics from their past and also learning about mathematics they've never seen before. This is not at all to say that the average mathematician will not enjoy reading this book because that is certainly not the case. *Power in Numbers* could be used as a coffee table conversation starter or an additional teaching resource in a History of Math course. It remarkably delivers on the author's self-proclaimed objective of bringing awareness to woman contributors to the mathematical sciences and with hope, will inspire more such women.

This book should not be put on a shelf to gather dust. It is a vital storytelling of revolutionary women that should be shared with your students and amongst your colleagues and loved ones. I, myself, look forward to the day when I can enjoy this book with both my daughter and my son. Williams has with this book given girls and women a bounty of rebel math warriors to admire, emulate, or even surpass.

Emille Davie
Lawrence

Credits

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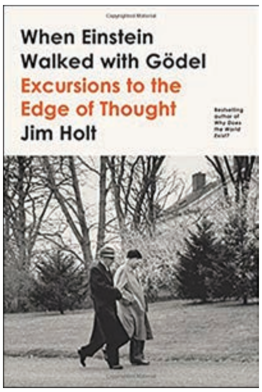
New and Noteworthy Titles on our Bookshelf
February 2019

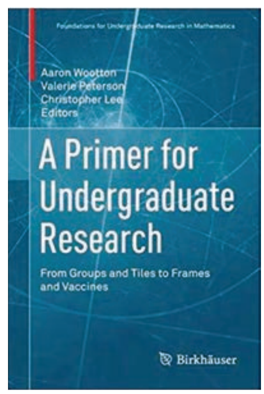


When Einstein Walked with Gödel: Excursions to the Edge of Thought
by Jim Holt (Farrar, Straus and Giroux, 2018, 384 pages).

This book is a collection of 24 essays on mathematics, science, and philosophy, each around 10 pages, and 15 short essays, each a page or two in length. The author will be familiar to many readers, since he has written many math- and science-related pieces for a number of popular outlets such as the *New York Times* and *The New Yorker*.

The essays were written over the course of the past 20 years and are spread out over nine chapters. Four of the first five focus on mathematics and proceed in a largely non-technical and humanistic fashion. The eponymous essay concerns the conversations, debates, and discussions between two giants: Einstein and Gödel. The titles of the individual essays convey the general approach; for example, "Sir Francis Galton, the Father of Statistics…and Eugenics," "Worshipping Infinity: Why the Russians Do and the French Don't." Even those readers who are familiar with the mathematics will learn a thing or two about the characters and stories behind the theorems. Later sections concern the foundations of computer science, cosmology, and philosophy.

It must be said that Holt has strong opinions and not every reader will agree with everything that he says. Platonists and the deeply religious will grimace at the lines "I think that prime numbers will lose their transcendental reputation when we come to understand them more completely. Then we will see that like the rest of mathematics (or like religion, for that matter) they are man-made, a terrestrial artifact" (p.48). He also takes a dim view of Ada Lovelace's abilities and praises a biographer who "deftly contrasted the grandiosity of Ada's aspirations with the modesty of her gifts and the slimness of her output" (p.178). However, whether one agrees with Holt or not, it must be admitted that he writes with lucidity and style.



A Primer for Undergraduate Research: From Groups and Tiles to Frames and Vaccines
edited by Aaron Wootton, Valerie Peterson, Christopher Lee (Birkhauser, 2017, 313 pages).

This volume consists of 11 self-contained introductory articles on various topics in pure and applied mathematics, although the content leans somewhat more toward the pure side. For example, there are articles on Coxeter groups, graph coloring, matroids, and Lie theory, along with introductions to vaccination strategies and mathematical decision-making.

Each article begins with a list of suggested prerequisites. As befits the book's intent, these prerequisites are minimal, with group theory being among the most advanced topics required. Suggested research problems and warm-up exercises are provided for each topic, and there are many useful figures, including some in color. The pieces themselves are inviting and compete for the reader's attention. Professors seeking jumping-off points for student research projects will find many good ideas here.

This primer is the first title in Birkhauser's new "Foundations for Undergraduate Research in Mathematics" book series. At least two more titles are in the pipeline. One concerns undergraduate research in computational and mathematical biology. The second is a project-based guide about starting and sustaining accessible undergraduate research.

The Bookshelf is prepared monthly by Notices *Associate Editor Stephan Ramon Garcia.*

Appearance of a book in the Notices *Bookshelf does not represent an endorsement by the* Notices *or by the AMS.*

Suggestions for the Bookshelf can be sent to `notices-booklist@ams.org`.

AMS BOOKSHELF

The AMS Book Program serves the mathematical community by publishing books that further mathematical research, awareness, education, and the profession while generating resources that support other Society programs and activities. As a professional society of mathematicians and one of the world's leading publishers of mathematical literature, we publish books that meet the highest standards for their content and production. Visit **bookstore.ams.org** to explore the entire collection of AMS titles.

Exercises in Mathematical Style
(NML/50, 2017)
by John McCleary

Raymond Queneau, founder of Oulipo, was inspired in his literary experimentation by the forms and patterns of mathematics. In a nice bit of reciprocity, John McCleary's *Exercises in (Mathematical) Style* was inspired by Queneau's *Exercises in Style*. Queneau tells 99 versions of the same—meaningless, uninteresting—story of an encounter on a bus in 99 stylistically distinct ways. McCleary tells 99 stories of the binomial coefficients in a variety of voices, styles, and forms. McCleary's stories have the advantage that they are not uninteresting and are not all the same; it's not just style, it's substance, too.

In the voice of Newton, McCleary explains the discovery of the binomial theorem via interpolation in the rows of Pascal's triangle. In Gauss's voice, he sums the series:

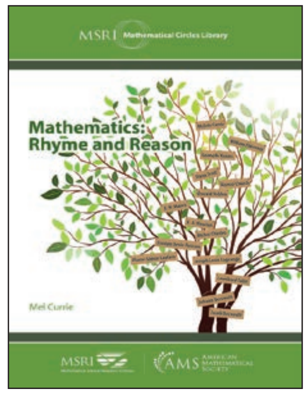
$$\sum_{k=0}^{N} \cos(2\pi k^2/N)+i\ \sin(2\pi k^2/N).$$

In Bourbaki's voice—the piece starts with two completely unmotivated definitions and a proposition—he counts the number of surjections from an n-element set to a k-element set. We overhear a Newton/Abel afterlife conversation, and listen in on a call-in radio show with two Bernoulli brothers standing in for Tom and Ray Magliozzi. A variety of literary forms are represented: a limerick, a tanaka, a lipogram, and a series of tweets! All explain something interesting about the binomial coefficients.

My absolute favorite chapter has to be "On The Bus," McCleary, in a nice homage, retells the banal story of Queneau's Exercises while incorporating a proof of the, weird-looking, identity:

$$\sum_{k} \binom{n}{k}^2 * \binom{3n+k}{2n} = \binom{3n}{n}^2.$$

The AMS Bookshelf is prepared bimonthly by AMS Acquisitions Specialist for MAA Press titles Stephen Kennedy. His email address is `kennedy.maapress@ams.org`.



Mathematics: Rhyme and Reason
(MCL/22, 2018)
by Melvin J. Currie

Mel Currie's *Mathematics: Rhyme and Reason* is not exactly a memoir, it contains too much expository mathematics to be just that, but it's not merely exposition either. In the Preface he says that he tried to produce a book that would have appealed to, and informed, his eighteen-year-old self. He wanted to show that eighteen-year-old some beautiful mathematics, clearly and elegantly explained, but also something of the culture of mathematics, and some inkling of what it might be like to have a life in mathematics, and how one might go about getting there.

Currie calls his stories "nursery rhymes of mathematics," in part to reflect their mathematical level, but even more to indicate his purpose: to introduce his reader to the culture of mathematics. One of the most compelling features of the book is the way Currie intertwines the personal and mathematical. Every nursery rhyme is about a topic in mathematics, but the topics chosen are all ones which featured in Currie's life and development as a mathematician, topics with which he has a personal relationship. In the chapter on primes, for example, he presents Euclid's proof that there are infinitely many, describes Fermat's Last theorem and the Twin Prime conjecture. At the same time he also tells the story of his introduction to these topics as a high school senior, tells the story of Yitang Zhang and his prime gap work and its reception, and he relates the charming story of watching Zhang listen to Andrew Granville explain Zhang's work at the Joint Mathematics Meetings.

Mixed in with the mathematics and the personal history are a fair amount of gossip and tomfoolery. There is a great story about what it's like to have Erdős as a houseguest (and waking to find him standing over your bed in the middle of the night). And one about what happens when you introduce the topic of anagrams into a crowd of mathematicians (Currie describes it as introducing "a virus in a vulnerable population.") The whole thing is lively, compelling and, ultimately, beguiling.

See over 130 Mathematical Moments, **hear** people talk about how they use math, and **read translations** in 13 languages.

NEW!

Winning the Race

The book and film *Hidden Figures* tell the true story of African-American women who worked as "computers" for NASA in the early days of the space program. Among the film's many memorable moments is when future Presidential Medal of Freedom winner Katherine Johnson (pictured bottom right) figures out how to solve the differential equations that describe a space capsule's orbit and re-entry so that it will splash down close to its intended target. Using her work, the machines that we now call computers then calculated the precise trajectory. And in today's world, that would have been the end of it. But the astronaut John Glenn, whose life depended on the calculations being correct, insisted that the human computer, Katherine Johnson, verify the machine's work. Only then was he confident that he could rocket into space and return home.

Christine Darden
Retired from NASA

Hidden Figures is a triumphant tale but an unfinished one. Even today, the proportions of women and African Americans who work in mathematics are smaller than those proportions in the general population, which is ironic for a profession that puts so much emphasis on logical thinking. Many in the mathematics community are working to create programs that encourage broader participation, so that future Katherine Johnsons are an integral part of the profession … and not hidden.

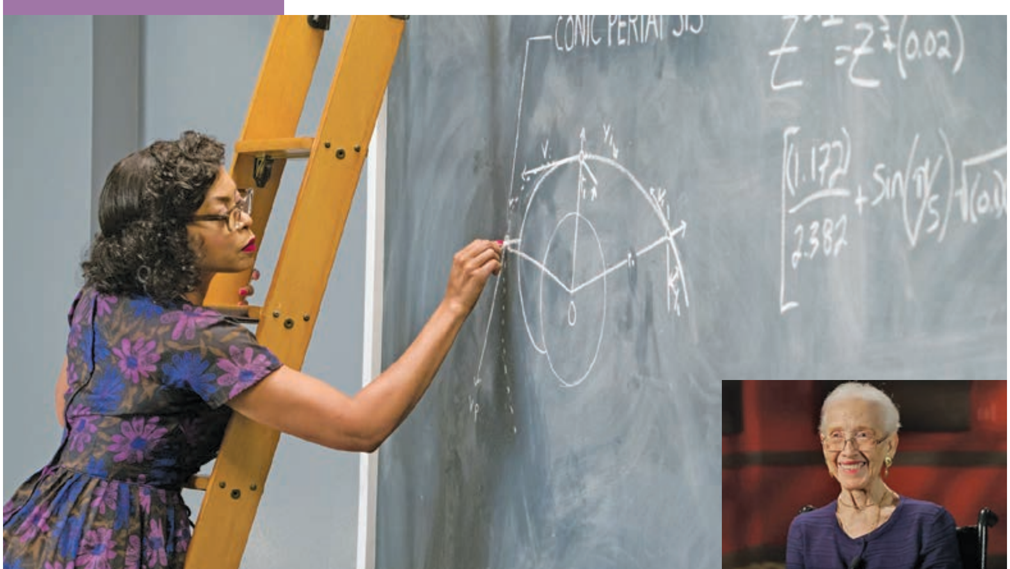


Image: ©20th Century Fox, inset: NASA Langley Research Center.

For More Information:
Hidden Figures, Margot Lee Shetterly, 2016.

Listen Up!

MM/137.s

AMS
American Mathematical Society

*The **Mathematical Moments** program promotes appreciation and understanding of the role mathematics plays in science, nature, technology, and human culture.*

www.ams.org/mathmoments

Call for Nominations

LEROY P. STEELE PRIZES

The selection committee for these prizes requests nominations for consideration for the 2020 awards..

Three Leroy P. Steele Prizes are awarded each year in the following categories: (1) the Steele Prize for Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through PhD students; (2) the Steele Prize for Mathematical Exposition: for a book or substantial survey or expository-research paper; and (3) the Steele Prize for Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research. In 2019 the prize for Seminal Contribution to Research will be awarded for a paper in Analysis/Probability.

Further information and instructions for submitting a nomination can be found at the Leroy P. Steele Prizes website: www.ams.org/steele-prize.

Nominations for the Steele Prizes for Lifetime Achievement and for Mathematical Exposition will remain active and receive consideration for three consecutive years.

For questions contact the AMS Secretary at secretary@ams.org.

The nomination period is February 1, 2019 through March 31, 2020.

AMS
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Advancing research. Creating connections.

Community Updates

Building a Permanent AMS Resource for Early Career Mathematicians

A special campaign is underway in the AMS community. The Campaign for The Next Generation is building an endowment designated to supporting early career mathematicians. Campaign volunteers and AMS staff are working to raise $1.5 million to meet a generous benefactor's matching challenge gift.

This philanthropist has long supported rising mathematicians at the AMS, and began The Next Generation Fund to create a permanent resource dedicated to their success: "It is my hope that this fund will impact many individuals who are limited not by their abilities or their goals but simply by financial concerns. I believe this fund can help provide them with the kinds of opportunities to which they may not otherwise have access."

Early career mathematicians work intensely to establish their careers. Doctoral students strive to finish dissertations, and PhD recipients compete for employment while working to deepen their research. Financial support can play a critical role, yet traditional university and federal funding is increasingly difficult to obtain.

The Next Generation Fund will be a permanent resource for many doctoral and post-doctoral mathematicians each year. It will support proven AMS programs that up until now have existed on temporary funding, such as AMS Graduate Student Travel Grants, Mathematics Research Communities, Graduate Student Chapters, and others.

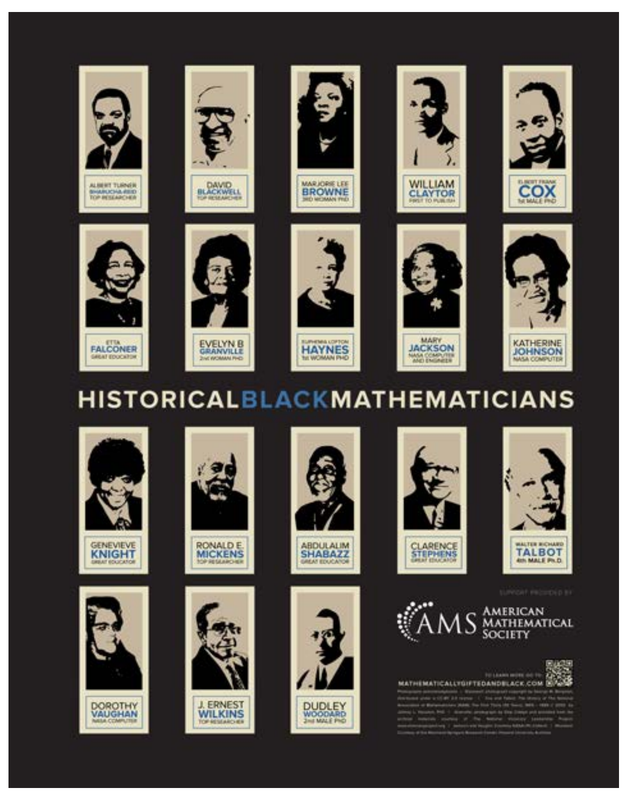
To explore further, visit: `https://www.ams.org/nextgen` or contact the Development office at (401) 455-4111 or `development@ams.org`.

Tokieda Gives Arnold Ross Lecture

Tadashi Tokieda of Stanford University delivered the 2018 Arnold Ross Lecture, titled "A World from a Sheet of Paper," in St. Louis, Missouri, on October 24, 2018. For more details and photos, see `https://www.ams.org/programs/students/wwtbam/arl2018`.

From the AMS Public Awareness Office

Historical Black Mathematicians poster. Celebrate Black History Month by ordering a copy of this free poster to display in your office, department or library. See `www.ams.org/posters`.



AMS Email Support for Frequently Asked Questions

A number of email addresses have been established for contacting the AMS staff regarding frequently asked questions.

The following is a list of those addresses together with a description of the types of inquiries that should be made through each address:

abs-coord@ams.org for questions regarding a particular abstract or abstracts questions in general.

acquisitions@ams.org to contact the AMS Acquisitions Department.

ams@ams.org to contact AMS Headquarters in Providence, Rhode Island.

amsdc@ams.org to reach the AMS Washington Office about government relations and advocacy programs.

amsfellows@ams.org to inquire about the Fellows of the AMS.

amsmem@ams.org to request information about membership in the AMS and about dues payments or to ask any general membership questions; may also be used to submit address changes.

ams-mrc@ams.org for questions about the AMS Mathematics Research Communities.

ams-simons@ams.org for information about the AMS Simons Travel Grants Program.

ams-survey@ams.org for information or questions about the Annual Survey of the Mathematical Sciences or to request reprints of survey reports.

bookstore@ams.org for inquiries related to the online AMS Bookstore.

captions@ams.org to submit entries for the Back Page Caption Contest in the *Notices*.

classads@ams.org to submit classified advertising for the *Notices*.

cust-serv@ams.org for general information about AMS products (including electronic products), to send address changes, or to conduct any general correspondence with the Society's Sales and Member Services Department.

development@ams.org for information about charitable giving to the AMS.

eims-info@ams.org to request information about Employment Information in the Mathematical Sciences (EIMS). For ad rates and to submit ads go to `eims.ams.org`.

emp-info@ams.org for information regarding AMS employment and career services.

eprod-support@ams.org for technical questions regarding AMS electronic products and services.

exdir@ams.org to contact the AMS Executive Director.

gradprg-ad@ams.org to inquire about a listing or ad in the Find Graduate Programs online service.

mathjobs@ams.org for questions about the online job application service `Mathjobs.org`.

mathcal@ams.org for questions regarding posting on the Mathematics Calendar.

mathprograms@ams.org for questions about the online program application service `Mathprograms.org`.

mathrev@ams.org to send correspondence to *Mathematical Reviews* related to reviews or other editorial questions.

meet@ams.org to request general information about Society meetings and conferences.

mmsb@ams.org for information or questions about registration, housing, and exhibits for any Society meetings or conferences (Mathematics Meetings Service Bureau).

mr-exec@ams.org to contact the Executive Editor of the Mathematical Reviews Division regarding editorial and related questions.

mr-librarian@ams.org to contact the Mathematical Reviews Division regarding the acquisition and cataloging of the mathematical literature indexed in MathSciNet®.

msn-support@ams.org for technical questions regarding MathSciNet®.

notices@ams.org to send correspondence to the managing editor of the *Notices*.

notices-ads@ams.org to submit electronically paid display ads for the *Notices*.

notices-booklist@ams.org to submit suggestions for books to be included in the "Bookshelf" section of the *Notices*.

notices-letters@ams.org to submit letters and opinion pieces to the *Notices*.

notices-whatis@ams.org to comment on or send suggestions for topics for the "WHAT IS…?" column in the *Notices*.

noti-backpage@ams.org to submit original short amusing stories, math-related humor, cartoons, and ideas to the Back Page of the *Notices.*

noti-gradsec@ams.org to submit ideas to the Graduate Student Section of the *Notices.*

opportunities@ams.org for questions about submitting to the Opportunities webpage.

paoffice@ams.org to contact the AMS Public Awareness Office.

president@ams.org to contact the AMS president.

prof-serv@ams.org to send correspondence about AMS professional programs and services.

promorequests@ams.org to request AMS giveaway materials such as posters, brochures, and catalogs for use at mathematical conferences, exhibits, and workshops.

publications@ams.org to send correspondence to the AMS Publication Division.

publisher@ams.org to contact the AMS Publisher.

pub-submit@ams.org to submit accepted electronic manuscripts to AMS publications (other than Abstracts). See www.ams.org/submit-book-journal to electronically submit accepted manuscripts to the AMS book and journal programs.

reprint-permission@ams.org to request permission to reprint material from Society publications.

reviewcopies@ams.org to request a book for review.

royalties@ams.org for AMS authors to direct questions about royalty payments.

sales@ams.org to inquire about reselling or distributing AMS publications or to send correspondence to the AMS Sales and Member Services Department.

secretary@ams.org to contact the AMS Secretary.

student-serv@ams.org for questions about AMS programs and services for students.

tech-support@ams.org to contact the Society's typesetting Technical Support Group.

textbooks@ams.org to request examination/desk copies or to inquire about using AMS publications as course texts.

webmaster@ams.org for general information or for assistance in accessing and using the AMS website.

Deaths of AMS Members

John D. Blanton, of Pittsford, New York, died on December 18, 2016. Born on January 23, 1927, he was a member of the Society for 57 years.

Lincoln Ellsworth Bragg, of Alexandria, Virginia, died on May 17, 2013. Born on January 25, 1936, he was a member of the Society for 43 years.

Paromita Chowla, of State College, Pennsylvania, died in December, 2016. Born on February 1, 1967, she was a member of the Society for 49 years.

James J. Cross, of Australia, died on January 6, 2017. Born on March 24, 1937, he was a member of the Society for 45 years.

Philip C. Curtis Jr., of Los Angeles, California, died on December 19, 2016. Born on March 6, 1928, he was a member of the Society for 63 years.

James M. D'Archangelo, of Annapolis, Maryland, died on February 2, 2017. Born on April 11, 1947, he was a member of the Society for 44 years.

Charles G. Fain, of Largo, Florida, died on November 4, 2014. Born on September 13, 1932, he was a member of the Society for 51 years.

James M. G. Fell, of Gladwyne, Pennsylvania, died on December 16, 2016. Born on December 4, 1923, he was a member of the Society for 67 years.

Pal Fischer, professor, University of Guelph, died on November 16, 2016. Born on June 11, 1940, he was a member of the Society for 30 years.

Richard P. Goblirsch, of Minneapolis, Minnesota, died on January 6, 2017. Born on January 1, 1930, he was a member of the Society for 65 years.

Elaine M. Hubbard, of Woodstock, Georgia, died on November 18, 2016. Born on March 19, 1950, she was a member of the Society for 41 years.

Arnold A. Johanson, of Largo, Florida, died on November 18, 2016. Born on March 15, 1924, he was a member of the Society for 61 years.

Amy C. King, of Lexington, Kentucky, died on June 7, 2014. Born on December 30, 1928, she was a member of the Society for 46 years.

Hitoshi Kitada, of Tokyo, Japan, died on November 10, 2016. Born on February 15, 1948, he was a member of the Society for 33 years.

Helmut Klingen, professor, University of Freiburg, died on January 23, 2017. Born on December 16, 1927, he was a member of the Society for 55 years.

Bertram Kostant, of Newton, Massachusetts, died on February 2, 2017. Born on May 24, 1928, he was a member of the Society for 65 years.

Helen F. Kriegsman, of Pittsburg, Kansas, died on November 25, 2016. Born on February 27, 1924, she was a member of the Society for 45 years.

Mathematics People


Alice Guionnet

Guionnet Awarded Pascal Medal

Alice Guionnet of ENS Lyon is the recipient of the 2018 Blaise Pascal Medal for her work in random matrices. According to the prize citation, she is "an inspiring leader in the field of probability and random matrices. She has established surprising links with various other fields of mathematics as spectral theory, operator algebra, free probability, which led her to several outstanding results. Her 'single ring theorem' is a real masterpiece of analysis." She founded the theory of "matrix models." Her honors include the Rollo Davidson Prize (2003), the Loève Prize (2009), and selection as a Simons Investigator (2012). The medal recognizes "an outstanding and demonstrated personal contribution to science and technology and the promotion of excellence in research and education."

—From a European Academy of Sciences announcement

Seward and Wright Awarded 2018 Brin Prize


Brandon Seward

Brandon Seward of the Courant Institute of Mathematical Sciences and Alex Wright of the University of Michigan have been awarded the 2018 Michael Brin Dynamical Systems Prize for Young Mathematicians. Seward was recognized "for his groundbreaking work on entropy theory for actions of countable groups, extending to this general setting fundamental results of the classical theory such as Sinai's factor theorem and Krieger's finite generators theorem." Wright was recognized for his fundamental work on the Teichmüller geometry and dynamics of the moduli space of Riemann surfaces, especially on the classification of $SL_2(\mathbb{R})$-invariant orbifolds and the discovery of unexpected examples.


Alex Wright

Seward received his PhD in 2015 from the University of Michigan under the supervision of Ralf Spatzier. He spent a year at the Hebrew University of Jerusalem before joining the Courant Institute in 2016. Wright received his PhD in 2014 from the University of Chicago under the supervision of Alex Eskin. He then spent four years at Stanford University, where he worked under the supervision of and collaborated with Maryam Mirzakhani. He joined the University of Michigan in the fall of 2018. He received a Clay Research Fellowship in 2014.

The prize is funded by an endowment from Michael Brin and is awarded for specific contributions to the field of dynamical systems made by researchers within four years of their PhD degrees. It carries a cash award of US$4,000.

—Giovanni Forni, Chair, Prize Selection Committee

Heineman Prize Awarded


Bill Sutherland

Bill Sutherland of the University of Utah, Francesco Calogero of the University of Rome "La Sapienza," and Michel Gaudin of Commissariat de l' Energie Atomique (CEA) have been awarded the 2019 Dannie Heineman Prize for Mathematical Physics by the American Institute of Physics (AIP) and the American Physical Society (APS). According to the prize citation, they were honored "for profound contributions to the field of exactly solvable models in statistical mechanics and many body physics, in particular the construction of the widely studied Gaudin magnet and the Calogero–Sutherland, Shastry-Sutherland, and Calogero–Moser models." Sutherland tells the *Notices:* "I grew up for the most part in the small farming community of Marshall, Missouri; I would agree that I have many Missouri character traits, for better or for worse. My local teachers were very good, especially 'Granny' Crutcher who taught me math, and asked me to evaluate new math textbooks the salesmen had given her." Sutherland was the first in his family to go to college; a summer program in physics at the Missouri School of Mines led to his forgoing his senior year in high school to go directly to college. In later years he and his wife, Veronica, taught in an open classroom that their children attended in Salt Lake City, where they enjoyed cross-country skiiing. Calogero is an arms control and disarmament activist who served as secretary general of the Pugwash Conference on Science and World Affairs from 1989 to 1997; he served as chair of the Pugwash Council from 1997 to 2002 and is still an ex officio member.

The Heineman Prize recognizes outstanding publications in the field of mathematical physics. The prize carries a cash award of US$10,000.

—From an AIP-APS announcement

Rossman Awarded Aisenstadt Prize

Benjamin Rossman

Benjamin Rossman of the University of Toronto has been awarded the 2018 André Aisenstadt Prize of the Centre de Recherches Mathématiques (CRM).

The prize citation reads: "Ben works in computational complexity theory, a branch of theoretical computer science that classifies problems according to their relative difficulty. His research seeks to quantify the minimum resources required to solve basic problems in combinatorial models such as Boolean circuits. Through creative techniques based in logic and the probabilistic method, Ben has derived groundbreaking lower bounds on the complexity of detecting cliques and determining connectivity in random graphs. His other notable results include size and depth hierarchy theorems for bounded-depth circuits, answering long-standing questions. This work has contributed to a reemergence of interest in circuit complexity, a concrete approach to *P* vs *NP*, which had seen little progress since breakthroughs of the 1980s." He delivered the Aisenstadt Prize lecture, "The complexity of detecting cliques and cycles in random graphs," at the University of Montreal in November 2018.

Rossman received his PhD in 2010 from the Massachusetts Institute of Technology under the direction of Madhu Sudan. He held postdoctoral positions at the Tokyo Institute of Technology, the Simons Institute for the Theory of Computing at Berkeley, and the National Institute of Informatics in Tokyo before joining the University of Toronto in 2016. He was a Sloan Fellow (2017) and an invited speaker at the International Congress of Mathematicians in Rio de Janeiro (2018). In 2018 he co-organized a special semester in Lower Bounds in Computational Complexity at the Simons Institute. Rossman tells the it *Notices*: "These days I spend nearly all my free time playing with my one-year-old son."

—From a CRM announcement

Zhitnitsky Receives CAP-CRM Prize

Ariel Zhitnitsky

Ariel Zhitnitsky of the University of British Columbia has been awarded the CAP-CRM Prize in Theoretical and Mathematical Physics "for his ground-breaking contributions to theoretical high energy physics, in particular for his development of the 'invisible axion' model, and for his work on the vacuum structure of non-Abelian gauge theories." In what the prize citation calls one of his "most influential ideas," he proposed that the Strong CP problem in the standard model could be resolved by a nearly invisible axion; presently several experimental groups are searching for such axions. According to the citation, "This paper has over 1000 citations and has influenced experimental searches, and the proposed axions are a candidate for cosmological cold dark matter. Another influential work was accomplished with V. Chernyak, providing a set of wave-functions that allow computation of exclusive amplitudes at high energies, such as form-factors or two-particle decays of heavy mesons. A series of papers with D. Son analyzed anomalous topological nondissipating currents in dense matter using an effective Lagrangian approach. Zhitnitsky later investigated the roles of these topological currents in neutron stars as a model for kicks and superconductivity. With D. Kharzeev, he further used these results to explain the CP-odd asymmetries observed at the relativistic heavy ion collider, and proposed that the bulk of dark matter is anti-baryonic so that the universe as a whole could be baryon-symmetric. Zhitnitsky has made key contributions to our understanding of the QCD phase transition, hadron physics, dark matter, QCD axions and neutron stars."

The prize is awarded by the Canadian Association of Physicists (CAP) and the Centre de Recherches Mathématiques (CRM) and recognizes exceptional achievements in theoretical and mathematical physics.

—From a CAP-CRM announcement

Anantharaman Awarded Infosys Prize

Nalini Anantharaman of the University of Strasbourg, France, has been awarded the 2018 Infosys Prize for Mathematical Sciences for her work related to quantum chaos, "specifically for the effective use of entropy in the study of semiclassical limits of eigenstates in quantum analogs of chaotic dynamical systems and for her work on the delocalization of eigenfunctions on large regular graphs." The prize citation states: "The quantum world is one of the deepest secrets of the universe and mathematics is the language that helps us understand this world. Mathematicians and physicists have been trying for decades to unravel the mysteries of this subatomic world. . . . Anantharaman's work impressively explores the deep relationship between classical and quantum systems and the unexpected use of entropy to prove some of the hard results."

Anantharaman was born in Paris, France, and received her PhD from Université Pierre et Marie Curie in 2000 under François Ledrappier. She has held positions at Ecole Normale Supérieure de Lyon, Ecole Polytechnique (Palaiseau), University of California Berkeley, Université Paris-Sud, Orsay, and the Institute for Advanced Study, Princeton. Her honors include the Grand Prix Jacques Herbrand (2011), the Salem Prize (2011), and the Henri Poincaré Prize for Mathematical Physics in 2012 (with Freeman Dyson, Barry Simon, and Sylvia Serfaty).

The Infosys Prizes recognize outstanding researchers and scientists in the fields of mathematical sciences, engineering and computer science, humanities, life sciences, physical sciences, and social sciences.

—From an Infosys announcement

Packard Fellowships Awarded

Keenan Crane

Two researchers whose work involves the mathematical sciences have been awarded 2018 Packard Fellowships by the David and Lucile Packard Foundation. Keenan Crane of Carnegie Mellon University received a Fellowship in computer and information sciences. Crane describes his work as focusing on "mathematical, computational, and mechanical foundations for designing complex 3D structures that can be built from shape-shifting 2D materials. New kinds of dynamic, shape-shifting matter such as programmable metamaterials, self-folding robotics, and active hydrogels driven by changes in heat, light, or humidity open up an abundance of new possibilities for applications. But with each new material comes new questions about geometry: which shapes can we make, and how?" Crane's work "aims to dramatically increase the geometric complexity of flexible, shape-shifting objects, by developing new computational tools for shape-shifting design. This work builds on the emerging field of discrete differential geometry, which provides faithful computational analogues for fundamental objects from geometry and physics." He tells the *Notices:* "I never feel like I truly understand something until I can draw a picture of it—there is something about turning mathematics into code into imagery that forces a serious reality check on your understanding! Somehow this disposition led me to the field of discrete differential geometry, which has further deepened my appreciation for the beautiful things that can happen when mathematicians and computer scientists work together."

Mahdi Soltanolkotabi of the University of Southern California received a Fellowship in computer and information sciences. He describes his work as follows: "Nonconvex learning algorithms are enabling transformative societal changes by revolutionizing how we process data. Despite wide empirical success, a satisfactory understanding of the behavior of these algorithms is still lacking. In particular, as systems and processes become increasingly automated with algorithms aiding or replacing human judgment, the importance of more reliable learning methodologies coupled with a thorough understanding of their behavior intensifies.

"The overarching goal of my research is to develop guiding theory, scalable algorithms and tools that make the modern practice of nonconvex learning more principled, reliable, and effective. I intend to contribute significantly to the transformation of modern data analysis from a collection of effective yet mysterious heuristics to a principled and completely reliable scientific discipline. Such a development will not only demystify existing practices but will facilitate new algorithms and system designs that better utilize computational and data resources."

Packard Fellows receive US$875,000 over five years to pursue their research. The Fellowships are designed to allow maximum flexibility in how the funding is used.

—From Packard Foundation announcements

Efron Awarded International Prize in Statistics

Bradley Efron of Stanford University has been awarded the International Prize in Statistics in recognition of the "bootstrap," a method he developed in 1977 for assessing the uncertainty of scientific results that has had extraordinary impact across many scientific fields. According to the prize citation, "With the bootstrap, scientists are able to learn from limited data in a simple way that enables them to assess the uncertainty of their findings. In essence, it is possible to simulate a potentially infinite number of data sets from an original data set and—in looking at the differences—measure the uncertainty of the result from the original data analysis." The prize is awarded every other year by a foundation consisting of the American Statistical Association, the Institute of Mathematical Statistics, the International Biometric Society, the International Statistical Institute, and the Royal Statistical Society.

—International Prize in Statistics announcement

Bhatnagar Prizes Awarded

The Shanti Swarup Bhatnagar (SSB) Prizes for Science and Technology have been awarded in the mathematical sciences. Amit Kumar of IIT Delhi was honored for his work in combinatorial optimization and graph theoretic algorithms. Nitin Saxena of IIT Kanpur was recognized for work in algebraic complexity. The prize recognizes excellence in scientific research in India.

—From a Bhatnagar Prize announcement

2018 NSF CAREER Awards

The National Science Foundation (NSF) has named a number of recipients of 2018 Faculty Early Career Development (CAREER) Awards. The awards support early-career faculty who have the potential to serve as academic role models in research and education and to lead advances in the mission of their departments or organizations. Following are the names, institutions, and proposal titles of the awardees selected by the NSF Division of Mathematical Sciences (DMS).

- Christine Breiner, Fordham University: Existence and regularity of solutions to variational problems in geometric analysis
- Aaron Brown, University of Chicago: Rigidity of group actions on manifolds
- Wei-Kuo Chen, University of Minnesota–Twin Cities: Mean field spin glasses and related applications
- Xiaohui Chen, University of Illinois at Urbana–Champaign: Computer-intensive statistical inference on high-dimensional and massive data: From theoretical foundations to practical computations
- Eric Chi, North Carolina State University: Stable and scalable estimation of the intrinsic geometry of multiway data
- Tudor Dan Dimofte, University of California–Davis: The algebraic structures of three-dimensional gauge theory
- Semyon Dyatlov, Massachusetts Institute of Technology: Classical and quantum chaos
- Ellen Eischen, University of Oregon, Eugene: Structure and interpolation in number theory and beyond
- Brittany Froese Hamfeldt, New Jersey Institute of Technology: Generated Jacobian equations in geometric optics and optimal transport
- Kristen Hendricks, Michigan State University: Equivariant Floer theory and low-dimensional topology
- Leah Johnson, Virginia Polytechnic Institute and State University: Quantifying heterogeneity and uncertainty in the transmission of vector borne diseases with a Bayesian trait-based framework
- Eric Katz, Ohio State University: Tropical and diophantine geometry
- Rongjie Lai, Rensselaer Polytechnic Institute: Geometry and learning for manifold-structured data in 3D and beyond
- Qin Li, University of Wisconsin–Madison: Applicable kinetic computation with boundaries and rough media
- Galyna Livshyts, Georgia Institute of Technology: High-dimensional geometry and its applications
- Po-Ling Loh, University of Wisconsin–Madison: Something old, something new: Robust statistics in the 21st century
- Li Ma, Duke University: Advances in multi-scale Bayesian inference and learning on massive data
- Daniel McDonald, Indiana University: Calibrating regularization for enhanced statistical inference
- Hoi H. Nguyen, Ohio State University: Littlewood–Offord theory and universality in random structures
- Stefan Patrikis, University of Utah: Galois representations: Deformation theory and motivic origins
- Braxton Osting, University of Utah: Variational and geometric methods for data analysis

• Julius Ross, University of Illinois at Chicago: Stability, Kahler geometry, and the Hele–Shaw flow

• Lars Ruthotto, Emory University: A flexible optimal control framework for efficient training of deep neural networks

• Christopher Rycroft, Harvard University: Adapting the fluid projection method to model elasto-plastic materials

• Steven Sam, University of California–San Diego: Categorical and classical symmetries in commutative algebra and algebraic geometry

• Hayden Schaeffer, Carnegie-Mellon University: Sparse model selection for nonlinear evolution equations

• Laura Schaposnik, University of Illinois at Chicago: Branes in the moduli space of Higgs bundles

• Benjamin Shaby, Pennsylvania State University: Hierarchical models for spatial extremes

• Anne Shiu, Texas A&M University: Biochemical reaction systems: From structure to dynamics

• Nike Sun, University of California–Berkeley: Phase transitions in randomized combinatorial search and optimization problems

• Thomas Trogdon, University of California–Irvine: Numerical linear algebra, random matrix theory and applications

• Paula Vasquez, University of South Carolina at Columbia: Multi-scale modeling of biological gels by coupling Langevin equations and fractional viscoelastic constitutive models

• Chunmei Wang, Texas Tech University: Primal-dual weak Galerkin finite element methods

• Ying Wang, University of Oklahoma Norman: Mathematical analysis and numerical methods for the underground oil recovery models

• Zhiren Wang, Pennsylvania State University: Dynamical rigidity related to group actions and arithmetics

• Liang Xiao, University of Connecticut: Slopes of p-adic modular forms

• Yue Yu, Lehigh University : A local–nonlocal coupling framework for tissue damage in fluid-structure interaction

• Wenliang Zhang, University of Illinois at Chicago: Local cohomology, de Rham cohomology and D-modules

—From NSF announcements

Credits

Photos of Brandon Seward and Alex Wright are by Dan Lesher, Huck Institutes of the Life Sciences, Penn State University.
Photo of Bill Sutherland is courtesy of Veronica Sutherland.
Photo of Benjamin Rossman is courtesy of the Faculty of Arts & Science, University of Toronto.
Photo of Ariel Zhitnitsky is courtesy of Ariel Zhitnitsky.
Photo of Keenan Crane is courtesy of Keenan Crane.
Photo of Alice Guionnet is courtesy of Frederic Bellay.

Mathematics Opportunities

Listings for upcoming mathematics opportunities to appear in Notices *may be submitted to* notices@ams.org.

Early Career Opportunity

Launch the NExT Stage of your Career!

MAA Project NExT (New Experiences in Teaching) is a year-long professional development program of the Mathematical Association of America (MAA) for new or recent PhDs in the mathematical sciences. The program is designed to connect new faculty with master teachers and leaders in the mathematics community and address the three main aspects of an academic career: teaching, research, and service.

MAA Project NExT welcomes and encourages applications from new and recent PhDs in postdoctoral, tenure-track, and visiting positions. We particularly encourage applicants from under-represented groups (including women and minorities). Applications for the 2019 cohort of MAA Project NExT Fellows are due on **April 15, 2019**, and can be found at projectnext.maa.org.

—Alisa Crans, MAA

The 2018 Cohort of the MAA Project NExT Fellows.

Early Career Opportunity

Call for Nominations for CRM Aisenstadt Prize

The Centre de Recherches Mathématiques (CRM) solicits nominations for the André Aisenstadt Mathematics Prize, awarded to recognize talented young Canadian mathematicians. The deadline for nominations is **March 1, 2019**. See www.crm.umontreal.ca/prix/prixAndreAisenstadt/prix_attributionAA_an.shtml.

—From a CRM announcement

Early Career Opportunity

NRC Research Associateship Programs

The National Research Council (NRC) Research Associateship Programs promote excellence in scientific and technological research conducted by the US government through the administration of programs offering graduate-, postdoctoral-, and senior-level research opportunities at sponsoring federal laboratories and affiliated institutions. Application deadlines are **February 1, May 1, August 1**, and **November 1, 2019**. For details, see sites.nationalacademies.org/pga/rap.

—From an NRC announcement

Early Career Opportunity

Call for Applications for the Seventh Heidelberg Laureate Forum

The Heidelberg Laureate Forum Foundation (HLFF) will hold its seventh forum September 22–27, 2019, bringing together winners of the Abel Prize and the Fields Medal, both in mathematics, as well as the Turing Award and the Nevanlinna Prize, both in computer science. Young researchers (from undergraduate level to five years' postdoc) interested in attending should submit applications online at `application.heidelberg-laureate-forum.org` by **February 15, 2019**; see `www.heidelberg-laureate-forum.org` for more information.

—From an HLFF announcement

Call for Nominations for Traub Prize

The Joseph F. Traub Prize is given for outstanding achievement in information-based complexity. The deadline for nominations is **March 31, 2019**. Nominations should be sent to `erich.novak@uni-jena.de`. See `https://www.journals.elsevier.com/journal-of-complexity/awards/joseph-f-traub-prize`.

—Traub Prize Award Committee

Classified Advertising

Employment Opportunities

TENNESSEE

University of Memphis
Department of Mathematical Sciences

The Department of Mathematical Sciences at the University of Memphis invites applications for the non-tenure-track Ralph Faudree Assistant Professorship, beginning Fall 2019. Candidates should have research activity in an area of Combinatorics and Graph Theory. Applicants must be on track to receive a PhD in the relevant field by May 2019 and hold a PhD at the time of the appointment. Details are available at www.memphis.edu/msci/news/positions.php. Application should be completed on-line at https://workforum.memphis.edu/postings/. Reviews begin January 2019. Email: pbalistr@memphis.edu for questions. EOE.

05

CHINA

Tianjin University, China
Tenured/Tenure-Track/Postdoctoral Positions at the Center for Applied Mathematics

Dozens of positions at all levels are available at the recently founded Center for Applied Mathematics, Tianjin University, China. We welcome applicants with backgrounds in pure mathematics, applied mathematics, statistics, computer science, bioinformatics, and other related fields. We also welcome applicants who are interested in practical projects with industries. Despite its name attached with an accent of applied mathematics, we also aim to create a strong presence of pure mathematics. Chinese citizenship is not required.

Light or no teaching load, adequate facilities, spacious office environment and strong research support. We are prepared to make quick and competitive offers to self-motivated hard workers, and to potential stars, rising stars, as well as shining stars.

The Center for Applied Mathematics, also known as the Tianjin Center for Applied Mathematics (TCAM), located by a lake in the central campus in a building protected as historical architecture, is jointly sponsored by the Tianjin municipal government and the university. The initiative to establish this center was taken by Professor S. S. Chern. Professor Molin Ge is the Honorary Director, Professor Zhiming Ma is the Director of the Advisory Board. Professor William Y. C. Chen serves as the Director.

TCAM plans to fill in fifty or more permanent faculty positions in the next few years. In addition, there are a number of temporary and visiting positions. We look forward to receiving your application or inquiry at any time. There are no deadlines.

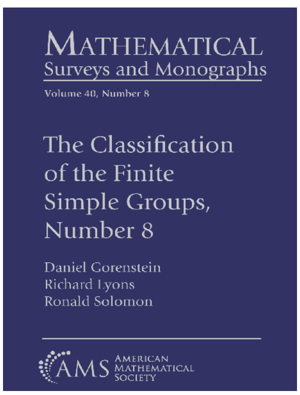
Please send your resume to mathjobs@tju.edu.cn.

For more information, please visit cam.tju.edu.cn or contact Ms. Erica Liu at mathjobs@tju.edu.cn, telephone: 86-22-2740-6039.

01

New Books Offered by the AMS

Algebra and Algebraic Geometry



The Classification of the Finite Simple Groups, Number 8

Part III, Chapters 12–17: The Generic Case, Completed

Daniel Gorenstein, *Richard Lyons, Rutgers University, Piscataway, NJ*, and **Ronald Solomon**, *The Ohio State University, Columbus, OH*

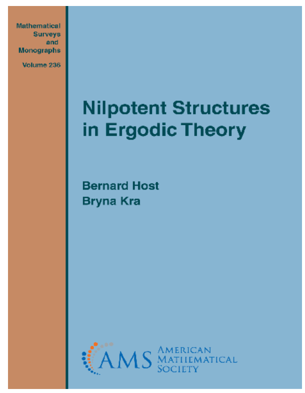
This book completes a trilogy (Numbers 5, 7, and 8) of the series The Classification of the Finite Simple Groups treating the generic case of the classification of the finite simple groups. In conjunction with Numbers 4 and 6, it allows us to reach a major milestone in our series—the completion of the proof of the following theorem:

Theorem O: Let G be a finite simple group of odd type, all of whose proper simple sections are known simple groups. Then either G is an alternating group or G is a finite group of Lie type defined over a field of odd order or G is one of six sporadic simple groups.

Mathematical Surveys and Monographs, Volume 40
January 2019, 488 pages, Hardcover, ISBN: 978-1-4704-4189-0, LC 94-23001, 2010 *Mathematics Subject Classification:* 20D05, 20D06, 20D08; 20E25, 20E32, 20F05, 20G40, List US$122, **AMS members US$97.60**, **MAA members US$109.80**, Order code SURV/40.8

bookstore.ams.org/surv-40-8

Analysis



Nilpotent Structures in Ergodic Theory

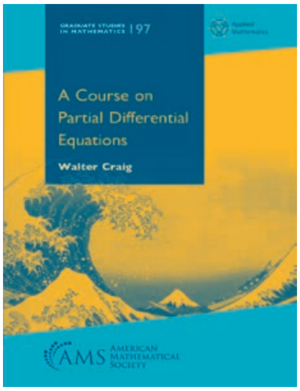
Bernard Host, *Université Paris-Est Marne-la-Vallée, Champs-sur-Marne, France*, and **Bryna Kra**, *Northwestern University, Evanston, IL*

Nilsystems play a key role in the structure theory of measure preserving systems, arising as the natural objects that describe the behavior of multiple ergodic averages. This book is a comprehensive treatment of their role in ergodic theory, covering development of the abstract theory leading to the structural statements, applications of these results, and connections to other fields.

Mathematical Surveys and Monographs, Volume 236
January 2019, 427 pages, Hardcover, ISBN: 978-1-4704-4780-9, LC 2018043934, 2010 *Mathematics Subject Classification:* 37A05, 37A30, 37A45, 37A25, 37B05, 37B20, 11B25, 11B30, 28D05, 47A35, List US$122, **AMS members US$97.60**, **MAA members US$109.80**, Order code SURV/236

bookstore.ams.org/surv-236

Differential Equations



A Course on Partial Differential Equations

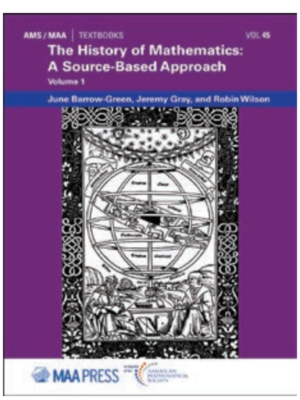
Walter Craig, *McMaster University, Hamilton, ON, Canada, and Fields Institute, Toronto, ON, Canada*

This book puts together the three main aspects of partial differential equations, namely theory, phenomenology, and applications, from a contemporary point of view. In addition to the three principal examples of the wave equation, the heat equation, and Laplace's equation, the book has chapters on dispersion and the Schrödinger equation, nonlinear hyperbolic conservation laws, and shock waves..

Graduate Studies in Mathematics, Volume 197
January 2019, 205 pages, Hardcover, ISBN: 978-1-4704-4292-7, LC 2018035511, 2010 *Mathematics Subject Classification:* 35–XX, 42–XX, List US$73, **AMS members US$58.40**, **MAA members US$65.70**, Order code GSM/197

bookstore.ams.org/gsm-197

General Interest



The History of Mathematics: A Source-Based Approach Volume 1

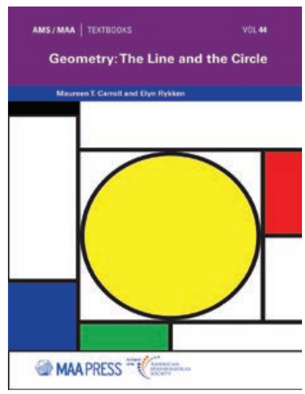
June Barrow-Green, *The Open University, Milton Keynes, United Kingdom*, **Jeremy Gray**, *The Open University, Milton Keynes, United Kingdom*, and **Robin Wilson**, *The Open University, Milton Keynes, United Kingdom*

The History of Mathematics: A Source-Based Approach is a comprehensive history of the development of mathematics. This, the first volume of the two-volume set, takes readers from the beginning of counting in prehistory to 1600 and the threshold of the discovery of calculus. It is, in addition to being an innovative and insightful textbook, an invaluable resource for students and scholars of the history of mathematics.

AMS/MAA Textbooks, Volume 45
March 2019, approximately 496 pages, Hardcover, ISBN: 978-1-4704-4352-8, LC 2018034323, 2010 *Mathematics Subject Classification:* 01–01, 01A05; 01A15, 01A16, 01A17, 01A20, 01A25, 01A32, 01A35, 01A45, List US$89, **AMS Individual member US$66.75**, AMS Institutional member US$71.20, **MAA members US$66.75**, Order code TEXT/45

bookstore.ams.org/text-45

Geometry and Topology



Geometry: The Line and the Circle

Maureen T. Carroll, *University of Scranton, PA*, and **Elyn Rykken**, *Muhlenberg College, Allentown, PA*

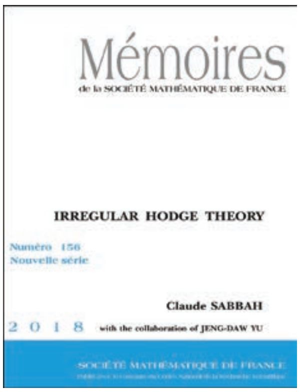
Starting with Euclid's Elements, this book connects topics in Euclidean and non-Euclidean geometry in an intentional and meaningful way, with historical context. It is an undergraduate text with a strong narrative that is written at the appropriate level of rigor for an upper-level survey or axiomatic course in geometry.

AMS/MAA Textbooks, Volume 44
January 2019, 480 pages, Hardcover, ISBN: 978-1-4704-4843-1, LC 2018034790, 2010 *Mathematics Subject Classification:* 51–01, List US$109, **AMS Individual member US$81.75**, AMS Institutional member US$87.20, **MAA members US$81.75**, Order code TEXT/44

bookstore.ams.org/text-44

New AMS-Distributed Publications

Algebra and Algebraic Geometry



Irregular Hodge Theory

Claude Sabbah, *Ecole Polytechnique, Palaiseau, France*
with the collaboration of Jeng-Daw Yu

The author introduces the category of irregular mixed Hodge modules consisting of possibly irregular holonomic D-modules which can be endowed in a canonical way with a filtration known as the irregular Hodge filtration. Mixed Hodge modules with their Hodge filtration naturally belong to this category, as well as their twist by the exponential of any meromorphic function. This category is stable by various standard functors, which produce many more filtered objects.

The irregular Hodge filtration satisfies the E_1-degeneration property with respect to any projective morphism. This generalizes some results previously obtained by H. Esnault, J.-D.Yu, and the author. The author also shows that, modulo a condition on eigenvalues of monodromies, any rigid irreducible holonomic D-module on the complex projective line underlies an irregular pure Hodge module. In a chapter written jointly with Jeng-Daw Yu, the author makes explicit the case of irregular mixed Hodge structures for which the author proves a Thom-Sebastiani formula.

This item will also be of interest to those working in analysis.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the US, Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Mémoires de la Société Mathématique de France, Number 156
September 2018, 126 pages, Softcover, ISBN: 978-2-85629-887-9, 2010 *Mathematics Subject Classification:* 14F40, 32S35, 32S40, List US$60, **AMS members US$48**, Order code SMFMEM/156

bookstore.ams.org/smfmem-156

A New Approach to Kazhdan-Lusztig Theory of Type *B* via Quantum Symmetric Pairs

Huanchen Bao, *University of Maryland, College Park, MD*, and **Weiqiang Wang**, *University of Virginia, Charlottesville, VA*

The authors show that Hecke algebra of type B and a coideal subalgebra of the type A quantum group satisfy a double centralizer property, generalizing the Schur-Jimbo duality in type A. The quantum group of type A and its coideal subalgebra form a quantum symmetric pair.

A new theory of canonical bases arising from quantum symmetric pairs is initiated. It is then applied to formulate and establish for the first time a Kazhdan-Lusztig theory for the BGG category O of the ortho-symplectic Lie superalgebras $\mathfrak{osp}(2m+1|2n)$. In particular, the authors' approach provides a new formulation of the Kazhdan-Lusztig theory for Lie algebras of type B/C.

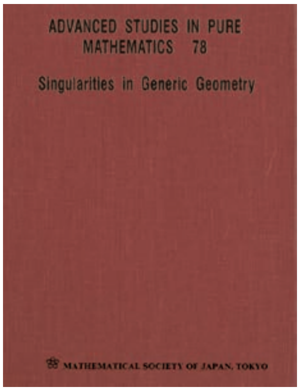
A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the US, Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Astérisque, Number 402
October 2018, 134 pages, Softcover, ISBN: 978-2-85629-889-3, 2010 *Mathematics Subject Classification:* 17B10, 17B20, 17B37, List US$52, **AMS members US$41.60**, Order code AST/402

bookstore.ams.org/ast-402

NEW BOOKS

Analysis



Singularities in Generic Geometry

Shyuichi Izumiya, *Hokkaido University, Japan*, **Goo Ishikawa**, *Hokkaido University, Japan*, **Minoru Yamamoto**, *Hirosaki University, Japan*, **Kentaro Saji**, *Kobe University, Japan*, **Takahiro Yamamoto**, *Tokyo Gakugei University, Japan*, and **Masatomo Takahashi**, *Muroran Institute of Technology, Japan*, Editors

This volume contains the proceedings of the workshop Singularities in Generic Geometry and Applications—Kobe–Kyoto 2015 (Valencia IV), which was held at Kobe University from June 3–6, and the Research Institute for Mathematical Sciences (RIMS), Kyoto University, from June 8–10, 2015. The workshop was the fourth in a series of biennial workshops. The first workshop was in Valencia, Spain, in 2009.

The volume consists of fifteen original research articles and three survey articles by specialists in singularity theory and its applications to differential topology and differential geometry. It is highly recommended for researchers and graduate students who are interested in these areas.

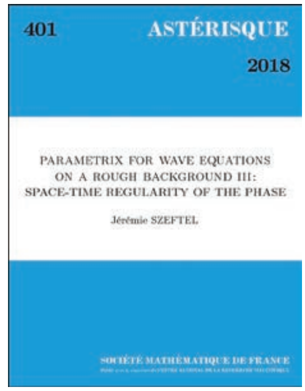
This item will also be of interest to those working in discrete mathematics and combinatorics

Published for the Mathematical Society of Japan by Kinokuniya, Tokyo, and distributed worldwide, except in Japan, by the AMS.

Advanced Studies in Pure Mathematics, Volume 78
September 2018, 486 pages, Hardcover, ISBN: 978-4-86497-055-6, 2010 *Mathematics Subject Classification:* 58K99; 53A99, List US$97, **AMS members US$77.60**, Order code ASPM/78

bookstore.ams.org/aspm-78

Mathematical Physics



Parametrix for Wave Equations on a Rough Background III: Space-Time Regularity of the Phase

Jérémie Szeftel, *Université Pierre et Marie Curie, France*

This book is dedicated to the construction and the control of a parametrix to the homogeneous wave equation $\Box_{\mathbf{g}}\phi=0$, where $\mathbf{g}$ is a rough metric satisfying the Einstein vacuum equations. Controlling such a parametrix as well as its error term when one only assumes L^2 bounds on the curvature tensor $\mathbf{R}$ of $\mathbf{g}$ is a major step of the proof of the bounded L^2 curvature conjecture proposed by Sergiu Klainerman and solved by Sergiu Klainerman, Igor Rodnianski, and the author.

On a more general level, this book deals with the control of the eikonal equation on a rough background and with the derivation of L^2 bounds for Fourier integral operators on manifolds with rough phases and symbols, and as such is also of independent interest.

This item will also be of interest to those working in differential equations

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the US, Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Astérisque, Number 401
September 2018, 321 pages, Softcover, ISBN: 978-2-85629-882-4, 2010 *Mathematics Subject Classification:* 83C05; 35Q75, 58J45, List US$90, **AMS members US$72**, Order code AST/401

bookstore.ams.org/ast-401

Meetings & Conferences of the AMS
February Table of Contents

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited on this page for more detailed information on each event.

Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated.

The most up-to-date meeting and conference information can be found online at: `www.ams.org/meetings`.

Important Information About AMS Meetings: Potential organizers, speakers, and hosts should refer to page 127 in the January 2019 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts: Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit `www.ams.org/cgi-bin/abstracts/abstract.pl`. Questions about abstracts may be sent to `abs-info@ams.org`. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Associate Secretaries of the AMS

Central Section: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; email: `benkart@math.wisc.edu`; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18015-3174; email: `steve.weintraub@lehigh.edu`; telephone: 610-758-3717.

Southeastern Section: Brian D. Boe, Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403, email: `brian@math.uga.edu`; telephone: 706-542-2547.

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; email: `lapidus@math.ucr.edu`; telephone: 951-827-5910.

Meetings in this Issue

See `https://www.ams.org/meetings` *for the most up-to-date information on the meetings and conferences that we offer.*

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See www.ams.org/meetings/.

Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL .

Baltimore, Maryland

Baltimore Convention Center, Hilton Baltimore, and Baltimore Marriott Inner Harbor Hotel

January 16–19, 2019

Wednesday – Saturday

Meeting #1145

Joint Mathematics Meetings, including the 125th Annual Meeting of the AMS, 102nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM)and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate Secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2018
Program first available on AMS website: November 1, 2018
Issue of *Abstracts*: Volume 40, Issue 1

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/national.html.

Joint Invited Addresses

Sarah Koch, University of Michigan, *What is the shape of a rational map?* (AMS-MAA Invited Address).
Bryna Kra, Northwestern University, *Dynamics of systems with low complexity* (AWM-AMS Noether Lecture).
Cathy O'Neil, ORCAA, *Big data, inequality, and democracy* (MAA-AMS-SIAM Gerald and Judith Porter Public Lecture).
Daniel A Spielman, Yale University, *Miracles of Algebraic Graph Theory* (AMS-MAA Invited Address).

AMS Invited Addresses

Jesús A. De Loera, University of California, Davis, *Algebraic, Geometric, and Topological Methods in Optimization*.
Benedict H. Gross, University of California San Diego, *Complex multiplication: past, present, future* (AMS Colloquium Lectures: Lecture I).
Benedict H. Gross, University of California San Diego, *Complex multiplication: past, present, future* (AMS Colloquium Lectures: Lecture II).
Benedict H. Gross, University of California San Diego, *Complex multiplication: past, present, future* (AMS Colloquium Lectures: Lecture III).
Peter Ozsvath, Princeton University, *From knots to symplectic geometry and algebra*.

Lior Pachter, California Institute of Technology, *A mathematical introduction to the molecular biology of the cell*.
Karen Hunger Parshall, University of Virginia, *The roaring twenties in American mathematics*.
Alan S. Perelson, Los Alamos National Laboratory, *Immunology for mathematicians* (AMS Josiah Willard Gibbs Lecture).
Lillian B. Pierce, Duke University, *On torsion subgroups in class groups of number fields*.

AMS Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at jointmathematicsmeetings.org/meetings/abstracts/abstract.pl?type=jmm.

Some sessions are cosponsored with other organizations. These are noted within the parenthesis at the end of each listing, where applicable.

25 years of Conferences for African-American Researchers in the Mathematical Sciences (CAARMS times 25), **William A. Massey**, Princeton University.

A Showcase of Number Theory at Undergraduate Institutions, **Adriana Salerno**, Bates College, and **Lola Thompson**, Oberlin College.

Advances and Applications in Integral and Differential Equations, **Jeffrey T. Neugebauer**, Eastern Kentucky University, and **Min Wang**, Kennesaw State University.

Advances by Early Career Women in Discrete Mathematics, **Jessalyn Bolkema**, State University of New York at Oswego, and **Jessica De Silva**, California State University, Stanislaus.

Advances in Operator Theory, Operator Algebras, and Operator Semigroups, **Joseph Ball**, Virginia Tech, **Marat Markin**, California State University, Fresno, **Igor Nikolaev**, St. John's University, and **Ilya Spitkovsky**, New York University, Abu Dhabi.

Advances in Quantum Walks, Quantum Simulations, and Related Quantum Theory, **Radhakrishnan Balu**, US Army Research Lab, **Chaobin Liu**, Bowie State University, and **Takuya Machida**, Nihon University, Japan.

Agent-based Modeling in Biological and Social Systems (a Mathematics Research Communities Session), **Maryann Hohn**, University of California Santa Barbara, **Angelika Manhart**, Imperial College, London, **Christopher Miles**, Courant Institute, New York University, and **Cole Zmurchok**, Vanderbilt University.

Algebraic Structures Motivated by Knot Theory, **Mikhail Khovanov**, Columbia University, and **Jozef H. Przytycki** and **Alexander Shumakovitch**, George Washington University.

Algebraic and Geometric Methods in Discrete Optimization, **Amitabh Basu**, Johns Hopkins University, and **Jesus De Loera**, University of California, Davis.

Algebraic, Discrete, Topological and Stochastic Approaches to Modeling in Mathematical Biology, **Olcay Akman**, Illinois State University, **Timothy D. Comar**, Benedictine University, **Daniel Hrozencik**, Chicago State University, and **Raina Robeva**, Sweet Briar College.

Algorithmic Dimensions and Fractal Geometry, **Jack H. Lutz**, Iowa State University, and **Elvira Mayordomo**, University of Zaragoza, Spain (AMS-ASL).

Analysis and Geometry of Nonlinear Evolution Equations, **Marius Beceanu**, University at Albany, State University of New York, and **Dan-Andrei Geba**, University of Rochester.

Analysis of Fractional, Stochastic, and Hybrid Dynamic Systems with Applications, **John R. Graef**, University of Tennessee at Chattanooga, **G. S. Ladde**, University of South Florida, and **A. S. Vatsala**, University of Louisiana at Lafayette.

Analytic Number Theory, **Thomas A. Hulse**, Boston College, **Angel V. Kumchev** and **Nathan McNew**, Towson University, and **John Miller**, The Johns Hopkins University.

Arithmetic Statistics, **Michael Chou** and **Robert Lemke Oliver**, Tufts University, and **Ari Shnidman**, Center for Communications Research—Princeton.

Bifurcations of Difference Equations and Discrete Dynamical Systems with Applications, **Arzu Bilgin**, Recep Tayyip Erdogan University, Turkey, and **Toufik Khyat**, Trinity College.

Commutative Ring Theory: Research for Undergraduate and Early Graduate Students, **Nicholas Baeth**, Franklin and Marshall College, and **Branden Stone**, Hamilton College.

Continued Fractions, **Geremías Polanco Encarnación**, Hampshire College, **James McLaughlin**, West Chester University, **Barry Smith**, Lebanon Valley College, and **Nancy J. Wyshinski**, Trinity College.

Counting Methods in Number Theory, **Lillian Pierce**, Duke University, **Arindam Roy**, Rice University, and **Jiuya Wang**, University of Wisconsin.

Definability and Decidability Problems in Number Theory, **Kirsten Eisenträger**, Pennsylvania State University, **Deidre Haskell**, McMaster University, Ontario, Canada, **Jennifer Park**, University of Michigan, and **Alexandra Shlapentokh**, East Carolina University (AMS-ASL).

Differential Equations on Fractals, **Patricia Alonso-Ruiz**, University of Connecticut, **Joe Chen**, Colgate University, **Luke Rogers**, University of Connecticut, **Robert Strichartz**, Cornell University, and **Alexander Teplyaev**, University of Connecticut.

Enumerative Combinatorics, **Miklos Bona**, University of Florida, and **Cheyne Homberger**, University of Maryland, Baltimore County.

Financial Mathematics, **Maxim Bichuch**, Johns Hopkins University, **Anja Richter**, Baruch College, City University of New York, and **Stephan Sturm**, Worcester Polytechnic Institute.

Geometric and Topological Combinatorics, **Anastasia Chavez** and **Jamie Haddock**, University of California, Davis, and **Annie Raymond**, University of Massachusetts, Amherst.

Geometric and Topological Generalization of Groups, **Amrita Acharyya**, University of Toledo, and **Bikash C. Das**, University of North Georgia.

Geometry Labs United: Research, Visualization, and Outreach, **Marianne Korten**, Kansas State University, and **Sean Lawton** and **Anton Lukyanenko**, George Mason University.

Geometry and Dynamics of Continued Fractions, **Anton Lukyanenko**, George Mason University, and **Joseph Vandehey**, Ohio State University.

Geometry of Representation Spaces, **Sean Lawton**, George Mason University, **Chris Manon**, University of Kentucky, and **Daniel Ramras**, Indiana University-Purdue University Indianapolis.

Group Representation Theory and Character Theory, **Mohammad Reza Darafsheh**, University of Tehran, Iran, and **Manouchehr Misaghian**, Prairie View A&M University.

Harmonic Analysis, Partial Differential Equations, and Applications, **Russell Brown**, University of Kentucky, and **Irina Mitrea**, Temple University.

Harmonic Analysis: Recent Developments on Oscillatory Integrals (a Mathematics Research Communities Session), **Xiumin Du**, University of Maryland, **Taryn C. Flock**, University of Massachusetts Amherst, and **Yakun Xi**, University of Rochester.

History of Mathematics, **Sloan Despeaux**, Western Carolina University, **Jemma Lorenat**, Pitzer College, **Daniel E. Otero**, Xavier University, and **Adrian Rice**, Randolph-Macon College (AMS-MAA-ICHM).

Hopf Algebras and Tensor Categories, **Siu-Hung Ng**, Louisiana State University, **Julia Plavnik**, Texas A&M University, and **Henry Tucker**, University of California, San Diego.

How to Guard an Art Gallery and Other Discrete Mathematical Adventures (In Memory of T. S. Michael, 1960 to 2016), **Joseph Bonin**, The George Washington University, **Carolyn Chun**, US Naval Academy, and **Nancy Neudauer**, Pacific University.

If You Build It They Will Come: Presentations by Scholars in the National Alliance for Doctoral Studies in the Mathematical Sciences, **David Goldberg**, Purdue University, and **Phil Kutzko**, University of Iowa.

Latinx in Math, **Alexander Diaz-Lopez**, Villanova University, **Laura Escobar**, University of Illinois, and **Juanita Pinzón-Caicedo**, North Carolina State University.

Lattice Path Combinatorics and Applications, **Christian Krattenthaler**, University of Vienna, Austria, and **Alan Krinik** and **Randall J. Swift**, California State Polytechnic University.

Localization and Delocalization for Disordered Quantum Systems, **Peter D. Hislop**, University of Kentucky, **Christoph A. Marx**, Oberlin College, and **Jeffery Schenker**, Michigan State University.

Low Complexity Models in Data Analysis and Machine Learning, **Emily J. King**, University of Bremen, Germany, **Nate Strawn**, Georgetown University, and **Soledad Villar**, New York University.

Mappings on Metric and Banach Spaces with Applications to Fixed Point Theory, **Torrey M. Gallagher**, Bucknell University, and **Christopher J. Lennard**, University of Pittsburgh.

Mathematical Analysis in Fluid Dynamics, **Yanqiu Guo**, Florida International University, **Jinkai Li**, South China Normal University, **Jing Tian**, Towson University, and **Yuncheng You**, University of South Florida.

Mathematical Investigations of Spatial Ecology and Epidemiology, **Leah Shaw** and **Junping Shi**, College of William and Mary, and **Zhisheng Shuai**, University of Central Florida.

Mathematical Models in Ecology, Epidemiology, and Medicine, **Richard Schugart**, Western Kentucky University, and **Najat Ziyadi**, Morgan State University.

Mathematicians at Sea (in the Sky, or on Land): Defense Applications of Mathematics, **Tegan Emerson**, **Timothy Doster**, and **George Stantchev**, Naval Research Laboratory.

Mathematics in the Realm of Cyber Research, **Daniel Bennett**, Army Cyber Institute, **Paul Goethals**, United States Military Academy, and **Natalie Scala**, Towson University.

Mathematics of Coding Theory and Applications, **Hiram Lopez-Valdez** and **Felice Manganiello**, Clemson University, and **Gretchen L. Matthews**, Virginia Tech.

Multiscale Problems in the Calculus of Variations, **Elisa Davoli**, University of Vienna, Austria, and **Rita Ferreira**, King Abdullah University of Science and Technology, Saudi Arabia.

Natural Resources Modeling, **Julie Blackwood**, Williams College, and **Shandelle M. Henson**, Andrews University.

Network Science, **David Burstein**, Swarthmore College, **Franklin Kenter**, United States Naval Academy, and **Feng 'Bill' Shi**, University of North Carolina.

New Directions in the Theory of Complex Multiplication, **Henri Darmon**, McGill University, **Samit Dasgupta**, Duke University, and **Benedict Gross**, Harvard University.

Nonlinear Evolution Equations and Their Applications, **Mingchao Cai**, Morgan State University, **Gisele Mophou Loudjom**, University of French West Indies, Guadeloupe, France, and **Gaston N'Guerekata**, **Alexander Pankov**, **Xuming Xie**, and **Guoping Zhang**, Morgan State University.

Not KNerds: A Community for Knot Theory, **Moshe Cohen**, Vassar College, **Elizabeth Denne**, Washington and Lee University, and **Adam Lowrance**, Vassar College.

Number Theoretic Methods in Hyperbolic Geometry (a Mathematics Research Communities Session), **Samantha Fairchild**, University of Washington, **Junxian Li**, Universität Göttingen, and **Richard Vradenburgh**, University of Virginia.

Number Theory, Arithmetic Geometry, and Computation, **Brendan Hassett**, Brown University, **Drew Sutherland**, Massachusetts Institute of Technology, and **John Voight**, Dartmouth College.

Numerical Methods for PDEs and Applications, **Wenrui Hao**, **Qingguo Hong**, and **Jinchao Xu**, Pennsylvania State University.

Optimal Methods in Applicable Analysis: Variational Inequalities, Low Rank Matrix Approximations, Systems Engineering, Cyber Security, **Aritra Dutta**, King Abdullah University of Science and Technology, Saudi Arabia, **Ram Mohapatra**, University of Central Florida, **Gayatri Pany**, Singapore University of Technology and Design, Singapore, and **Nabin Kumar Sahu**, Dhirubhai Ambani Institute of Information and Communication Technology, India.

Orthogonal Polynomials, Quantum Probability, Harmonic and Stochastic Analysis, **Nobuhiro Asai**, Aichi University of Education, Kariya, Japan, **Rodica Costin**, The Ohio State University, **Aurel I. Stan**, The Ohio State University at Marion, and **Hiroaki Yoshida**, Ochanomizu University, Tokyo, Japan.

Partition Theory and Related Topics, **Dennis Eichhorn**, University of California, Irvine, **Tim Huber**, University of Texas, Rio Grande Valley, and **Amita Malik**, Rutgers University.

Problems in Partial Differential Equations, **Alex Himonas**, University of Notre Dame, and **Curtis Holliman**, The Catholic University of America.

Quantum Symmetries: Subfactors and Fusion Categories (a Mathematics Research Communities Session), **Cain Edie-Michell** and **Lauren Ruth**, Vanderbilt University, and **Yilong Wang**, Louisiana State University.

Quaternions, **Terrence Blackman**, Medgar Evers College, City University of New York, and **Johannes Familton** and **Chris McCarthy**, Borough of Manhattan Community College, City University of New York.

Recent Advancements in Mathematical Modeling of Cancer, **Kamila Larripa**, Humboldt State University, and **Hwayeon Ryu**, University of Hartford.

Recent Advances and Trends in Computable Structure Theory (in honor of J. Remmel), **Jennifer Chubb**, University of San Francisco, and **Tim McNicholl**, Iowa State University.

Recent Advances in Biological Modeling and Related Dynamical Analysis, **Joshi Raj Hem**, Xavier University, and **Yanyu Xiao**, University of Cincinnati.

Recent Advances in Homological and Commutative Algebra, **Neil Epstein**, George Mason University, **Claudiu Raicu**, Notre Dame University, and **Alexandra Seceleanu**, University of Nebraska.

Recent Advances in Inverse Problems and Imaging, **Kui Ren**, University of Texas at Austin, and **Yang Yang**, Michigan State University.

Recent Advances in Regularity Lemmas, **Gabriel Conant**, University of Notre Dame, **Rehana Patel**, and **Julia Wolf**, University of Bristol, UK.

Recent Progress in Multivariable Operator Theory, **Dmitry Kaliuzhnyi-Verbovetsky** and **Hugo Woerdeman**, Drexel University.

Research in Mathematics by Early Career Graduate Students, **Marat Markin**, **Morgan Rodgers**, **Khang Tran**, and **Oscar Vega**, California State University, Fresno.

Research in Mathematics by Undergraduates and Students in Post-Baccalaureate Programs, **Darren A. Narayan**, Rochester Institute of Technology, **Khang Tran**, California State University, Fresno, **Mark David Ward**, Purdue University, and **John Wierman**, The Johns Hopkins University (AMS-MAA-SIAM).

Riordan Arrays, **Alexander Burstein** and **Dennis Davenport**, Howard University, **Asamoah Nkwanta**, Morgan State University, **Lou Shapiro**, Howard University, and **Leon Woodson**, Morgan State University.

Statistical, Variational, and Learning Techniques in Image Analysis and their Applications to Biomedical, Hyperspectral, and Other Imaging, **Justin Marks**, Gonzaga University, **Laramie Paxton**, Washington State University, and **Viktoria Taroudaki**, Eastern Washington University.

Stochastic Analysis and Applications in Finance, Actuarial Science and Related Fields, **Julius N. Esunge**, University of Mary Washington, **See Keong Lee**, University of the Sciences, Malaysia, and **Aurel I. Stan**, The Ohio State University at Marion.

Stochastic Differential Equations and Applications, **Carey Caginalp**, University of Pittsburgh.

Symbolic Dynamics, **Van Cyr**, Bucknell University, and **Bryna Kra**, Northwestern University.

The Mathematics of Gravity and Light (a Mathematics Research Communities Session), **Sougata Dhar**, University of Maine, **Chad R. Mangum**, Niagara University, and **Nadine Stritzelberger**, University of Waterloo.

The Mathematics of Historically Black Colleges and Universities (HBCUs) in the Mid-Atlantic, **Edray Goins**, Purdue University, **Janis Oldham**, North Carolina A&T, **Talithia Washington**, Howard University, and **Scott Williams**, University at Buffalo, State University of New York.

Topological Data Analysis: Theory and Applications, **Justin Curry**, University at Albany, State University of New York, **Mikael Vejdemo-Johansson**, College of Staten Island, City University of New York, and **Sara Kalisnik Verovsek**, Wesleyan University.

Topology, Structure and Symmetry in Graph Theory, **Lowell Abrams**, George Washington University, and **Mark Ellingham**, Vanderbilt University.

Using Modeling to Motivate the Study of Differential Equations, **Robert Kennedy**, Centennial High School, Ellicott City MD, **Audrey Malagon**, Virginia Wesleyan University, **Brian Winkel**, SIMIODE, Cornwall NY, and **Dina Yagodich**, Frederick Community College.

Women in Topology, **Jocelyn Bell**, Hobart and William Smith Colleges, **Rosemary Guzman**, University of Chicago, **Candice Price**, University of San Diego, and **Arunima Ray**, Max Planck Institute for Mathematics, Germany.

Auburn, Alabama

Auburn University

March 15–17, 2019

Friday – Sunday

Meeting #1146

Southeastern Section
Associate Secretary: Brian D. Boe

Announcement issue of *Notices*: January 2019
Program first available on AMS website: January 31, 2019
Issue of *Abstracts*: Volume 40, Issue 2

Deadlines

For organizers: Expired
For abstracts: January 29, 2019

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Grigoriy Blekherman, Georgia Institute of Technology, *Do sums of squares dream of free resolutions?*
Carina Curto, Pennsylvania State University, *Graphs, network motifs, and threshold-linear algebra in the brain.*
Ming Liao, Auburn University, *Invariant Markov processes under actions of Lie groups.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Algebraic and Discrete Methods in Mathematical Biology (Code: SS 21A), **Carina Curto**, The Pennsylvania State University, **Katherine Morrison**, University of Northern Colorado, and **Nora Youngs**, Colby College.

Applications of Algebraic Geometry (Code: SS 25A), **Greg Blekherman**, Georgia Institute of Technology, **Michael Burr**, Clemson University, and **Tianran Chen**, Auburn University at Montgomery.

Clustering Methods and Applications (Code: SS 23A), **Benjamin McLaughlin**, Naval Surface Warfare Center Panama City Division (NSWCPCD), and **Sung Ha Kang**, Georgia Institute of Technology.

Combinatorial Matrix Theory (Code: SS 2A), **Zhongshan Li**, Georgia State University, and **Xavier Martínez-Rivera**, Auburn University.

Commutative and Combinatorial Algebra (Code: SS 3A), **Selvi Kara Beyarslan**, University of South Alabama, and **Alessandra Costantini**, Purdue University.

Developments in Commutative Algebra (Code: SS 1A), **Eloísa Grifo**, University of Michigan, and **Patricia Klein**, University of Kentucky.

Differential Equations in Mathematical Biology (Code: SS 7A), **Guihong Fan**, Columbus State University, **Zhongwei Shen**, University of Alberta, and **Xiaoxia Xie**, Idaho State University.

Discrete and Convex Geometry (Code: SS 17A), **Andras Bezdek**, Auburn University, **Ferenc Fodor**, University of Szeged, and **Wlodzimierz Kuperberg**, Auburn University.

Evolution Equations and Applications (Code: SS 9A), **Dmitry Glotov**, **Wenxian Shen**, and **Paul G. Schmidt**, Auburn University.

Experimental Mathematics in Number Theory, Analysis, and Combinatorics (Code: SS 6A), **Amita Malik**, Rutgers University, and **Armin Straub**, University of South Alabama.

Geometric Flows and Minimal Surfaces (Code: SS 20A), **Theodora Bourni**, University of Tennessee, and **Giuseppe Tinaglia**, King's College London and University of Tennessee.

Geometric Methods in Representation Theory (Code: SS 15A), **Jiuzu Hong** and **Shrawan Kumar**, University of North Carolina, Chapel Hill, and **Yiqiang Li**, University at Buffalo, the State University of New York.

Geometric and Combinatorial Aspects of Representation Theory (Code: SS 19A), **Mark Colarusso**, University of South Alabama, and **Jonas Hartwig**, Iowa State University.

Geometry and Topology of Low Dimensional Manifolds, and Their Invariants (Code: SS 13A), **John Etnyre**, Georgia Institute of Technology, **Bulent Tosun**, University of Alabama, and **Shea Vela-Vick**, Louisiana State University.

Graph Theory in Honor of Robert E. Jamison's 70th Birthday (Code: SS 4A), **Robert A Beeler**, East Tennessee State University, **Gretchen Matthews**, Virginia Tech, and **Beth Novick**, Clemson University.

Hopf Algebras and Their Applications (Code: SS 10A), **Robert Underwood**, Auburn University at Montgomery, and **Alan Koch**, Agnes Scott College.

Mapping Class Groups (Code: SS 27A), **Joan Birman**, Columbia University, and **Kevin Kordek** and **Dan Margalit**, Georgia Institute of Technology.

Mathematical Analysis and Control Theory of Coupled Partial Differential Equation Models (Code: SS 11A), **George Avalos** and **Pelin Gu˝ven Geredeli**, University of Nebraska-Lincoln, and **László Kindrat**, University of New Hampshire.

Nonlinear Reaction-Diffusion Equations and Their Applications (Code: SS 18A), **Jerome Goddard,II**, Auburn University at Montgomery, **Nsoki Mavinga**, Swarthmore College, **Quinn Morris**, Appalachian State University, and **R. Shivaji**, University of North Carolina at Greensboro.

Probability and Stochastic Processes (Code: SS 5A), **Ming Liao**, **Erkan Nane**, and **Jerzy Szulga**, Auburn University.

Random Discrete Structures (Code: SS 24A), **Lutz P Warnke**, Georgia Institute of Technology, and **Xavier Pérez-Giménez**, University of Nebraska–Lincoln.

Recent Advances in Coarse Geometry (Code: SS 12A), **Jerzy Dydak**, University of Tennessee.

Recent Advances in Numerical Methods for PDEs and PDE-constrained Optimization (Code: SS 26A), **Yanzhao Cao**, **Thi-Thao-Phuong Hoang**, and **Junshan Lin**, Auburn University.

Recent Developments in Graph Theory (Code: SS 16A), **Xiaofeng Gu**, **Jeong-Hyun Kang**, **David Leach**, and **Rui Xu**, University of West Georgia.

Representations of Lie Algebras, Algebraic Groups, and Quantum Groups (Code: SS 8A), **Joerg Feldvoss**, University of South Alabama, **Lauren Grimley**, Spring Hill College, and **Cornelius Pillen**, University of South Alabama.

The Modeling and Analysis of Spatially Extended Structures (Code: SS 22A), **Shibin Dai**, University of Alabama, **Keith Promislow**, Michigan State University, and **Qiliang Wu**, Ohio University.

Topological Data Analysis, Statistics and Applications (Code: SS 14A), **Yu-Min Chung**, University of North Carolina at Greensboro, and **Vasileios Maroulas**, University of Tennessee.

Honolulu, Hawaiʻi

University of Hawaiʻi at Mānoa

March 22–24, 2019

Friday – Sunday

Meeting #1147

Central Section
Associate Secretaries: Georgia Benkart and Michel L. Lapidus
Announcement issue of *Notices*: January 2019
Program first available on AMS website: February 7, 2019
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 40, Issue 2

Deadlines

For organizers: Expired
For abstracts: January 29, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Barry Mazur, Harvard University, *On the arithmetic of curves* (Einstein Public Lecture in Mathematics).
Aaron Naber, Northwestern University, *Analysis of geometric nonlinear partial differential equations*.
Deanna Needell, University of California, Los Angeles, *Simple approaches to complicated data analysis*.
Katherine Stange, University of Colorado, Boulder, *Title to be announced*.
Andrew Suk, University of California, San Diego, *On the Erdos-Szekeres convex polygon problem*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Advances in Iwasawa Theory (Code: SS 12A), **Frauke Bleher**, University of Iowa, **Ted Chinburg**, University of Pennsylvania, and **Robert Harron**, University of Hawaiʻi at Mānoa.

Advances in Mathematical Fluid Mechanics (Code: SS 32A), **Kazuo Yamazaki**, University of Rochester, and **Adam Larios**, University of Nebraska–Lincoln.

Algebraic Groups, Galois Cohomology, and Local-Global Principles (Code: SS 3A), **Raman Parimala**, Emory University, **Andrei Rapinchuk**, University of Virginia, and **Igor Rapinchuk**, Michigan State University.

Algebraic Number Theory and Diophantine Equations (Code: SS 20A), **Claude Levesque**, University of Laval.

Algebraic Points (Code: SS 36A), **Barry Mazur** and **Hector Pasten**, Harvard University.

Algebraic and Combinatorial Structures in Knot Theory (Code: SS 9A), **Sam Nelson**, Claremont McKenna College, **Natsumi Oyamaguchi**, Shumei University, and **Kanako Oshiro**, Sophia University.

Algebraic and Geometric Combinatorics (Code: SS 17A), **Andrew Berget**, Western Washington University, and **Steven Klee**, Seattle University.

Analysis of Nonlinear Geometric Equations (Code: SS 23A), **Aaron Naber**, Northwestern University, and **Richard Bamler**, University of California Berkeley.

Analytic and Probabilistic Methods in Convex Geometry (Code: SS 27A), **Alexander Koldobsky**, University of Missouri, **Alexander Litvak**, University of Alberta, **Dmitry Ryabogin**, Kent State University, **Vladyslav Yaskin**, University of Alberta, and **Artem Zvavitch**, Kent State University.

Applications of Ultrafilters and Nonstandard Methods (Code: SS 33A), **Isaac Goldbring**, University of California, Irvine, and **Steven Leth**, University of Northern Colorado.

Arithmetic Dynamics (Code: SS 29A), **Andrew Bridy**, Texas A&M University, **Michelle Manes**, University of Hawaiʻi at Mānoa, and **Bianca Thompson**, Harvey Mudd College.

Arithmetic Geometry and Its Connections (Code: SS 51A), **Laura Capuano**, University of Oxford, and **Amos Turchet**, University of Washington.

Arithmetic and Transcendence of Special Functions and Special Values (Code: SS 56A), **Matthew A. Papanikolas**, Texas A&M University, and **Federico Pellarin**, Université Jean Monnet, St. Étienne.

Coarse Geometry, Index Theory, and Operator Algebras: Around the Mathematics of John Roe (Code: SS 53A), **Erik Guentner**, University of Hawaiʻi at Mānoa, **Nigel Higson**, Penn State University, and **Rufus Willett**, University of Hawaiʻi at Mānoa.

Coding Theory and Information Theory (Code: SS 39A), **Manabu Hagiwara**, Chiba University, and **James B. Nation**, University of Hawai'i.

Combinatorial and Experimental Methods in Mathematical Phylogeny (Code: SS 47A), **Sean Cleary**, City College of New York and the CUNY Graduate Center, and **Katherine St. John**, Hunter College and the American Museum of Natural History.

Commutative Algebra and its Environs (Code: SS 4A), **Olgur Celikbas** and **Ela Celikbas**, West Virginia University, and **Ryo Takahashi**, Nagoya University.

Computability, Complexity, and Learning (Code: SS 45A), **Achilles A. Beros** and **Bjørn Kjos-Hanssen**, University of Hawai'i at Mānoa.

Computational and Data-Enabled Sciences (Code: SS 54A), **Roummel Marcia**, **Boaz Ilan**, and **Suzanne Sindi**, University of California, Merced.

Constructive Aspects of Complex Analysis (Code: SS 7A), **Ilia Binder** and **Michael Yampolsky**, University of Toronto, and **Malik Younsi**, University of Hawai'i at Mānoa.

Differential Geometry (Code: SS 10A), **Vincent B. Bonini**, Cal Poly San Luis Obispo, **Jie Qing**, University of California, Santa Cruz, and **Bogdan D. Suceava**, California State University, Fullerton.

Dynamical Systems and Algebraic Combinatorics (Code: SS 57A), **Maxim Arnold**, University of Texas at Dallas, **Jessica Striker**, North Dakota State University, and **Nathan Williams**, University of Texas at Dallas.

Emerging Connections with Number Theory (Code: SS 43A), **Katherine Stange**, University of Colorado, Boulder, and **Renate Scheidler**, University of Calgary.

Equivariant Homotopy Theory and Trace Methods (Code: SS 58A), **Andrew Blumberg**, University of Texas, **Teena Gerhardt**, Michigan State University, **Michael Hill**, UCLA, and **Michael Mandell**, Indiana University.

Factorization and Arithmetic Properties of Integral Domains and Monoids (Code: SS 31A), **Scott Chapman**, Sam Houston State University, **Jim Coykendall**, Clemson University, and **Christopher O'Neill**, University California, Davis.

Generalizations of Symmetric Spaces (Code: SS 22A), **Aloysius Helminck**, University of Hawai'i at Mānoa, **Vicky Klima**, Appalachian State University, **Jennifer Schaefer**, Dickinson College, and **Carmen Wright**, Jackson State University.

Geometric Approaches to Mechanics and Control (Code: SS 55A), **Monique Chyba**, University of Hawai'i at Mānoa, **Tomoki Ohsawa**, The University of Texas at Dallas, and **Vakhtang Putkaradze**, University of Alberta.

Geometry, Analysis, Dynamics and Mathematical Physics on Fractal Spaces (Code: SS 8A), **Joe P. Chen**, Colgate University, **Lũ(Tim) Hùng**, Hawai'i Pacific University, **Machiel van Frankenhuijsen**, Utah Valley University, and **Robert G. Niemeyer**, University of the Incarnate Word.

Homotopy Theory (Code: SS 48A), **Kyle Ormsby** and **Angélica Osorno**, Reed College.

Interactions between Geometric Measure Theory, PDE, and Harmonic Analysis (Code: SS 41A), **Mark Allen**, Brigham Young University, **Spencer Becker-Kahn**, University of Washington, **Max Engelstein**, Massachusetts Institute of Technology, and **Mariana Smit Vega Garcia**, University of Washington.

Interactions between Noncommutative Algebra and Noncommutative Algebraic Geometry (Code: SS 24A), **Garrett Johnson**, North Carolina Central University, **Bach Nguyen** and **Xingting Wang**, Temple University, and **Daniel Yee**, Bradley University.

Lie Theory in the Representations of Groups and Related Structures—dedicated to the memory of Kay Magaard (Code: SS 14A), **Christopher Drupieski**, DePaul University, and **Julia Pevtsova**, University of Washington.

Mapping Class Groups (Code: SS 35A), **Asaf Hadari**, University of Hawai'i.

Mathematical Analysis of Nonlinear Phenomena (Code: SS 16A), **Mimi Dai**, University of Illinois at Chicago.

Mathematical Methods and Models in Medicine (Code: SS 19A), **Monique Chyba**, University of Hawai'i, and **Jakob Kotas**, University of Hawai'i and University of Portland.

New Trends in Geometric Measure Theory (Code: SS 37A), **Antonio De Rosa**, Courant Institute of Mathematical Sciences, New York University, and **Luca Spolaor**, Massachusetts Institute of Technology.

New Trends on Variational Calculus and Non-Linear Partial Differential Equations (Code: SS 44A), **Craig Cowan**, University of Manitoba, **Michinori Ishiwata**, Osaka University, **Abbas Moameni**, Carleton University, and **Futoshi Takahashi**, Osaka City University.

Nonlinear Wave Equations and Applications (Code: SS 42A), **Boaz Ilan**, University of California, Merced, and **Barbara Prinari**, University of Colorado, Colorado Springs.

Numerical Methods for Partial Differential Equations (Code: SS 50A), **Evan Gawlik**, **Michael Holst**, and **Martin Licht**, University of California, San Diego.

Real and Complex Singularities (Code: SS 34A), **Leslie Charles Wilson**, University of Hawai'i at Mānoa, **Goo Ishikawa**, Hokkaido University, and **David Trotman**, Aix-Marseille University.

Recent Advances and Applications of Modular Forms (Code: SS 1A), **Amanda Folsom**, Amherst College, **Pavel Guerzhoy**, University of Hawai'i at Mānoa, **Masanobu Kaneko**, Kyushu University, and **Ken Ono**, Emory University.

Recent Advances in Lie and Related Algebras and their Representations (Code: SS 28A), **Brian D. Boe**, University of Georgia, and **Jonathan Kujawa**, University of Oklahoma.

Recent Advances in Numerical Methods for PDEs (Code: 2249A), **Hengguang Li**, Wayne State University, and **Sara Pollock**, University of Florida.

Recent Advances in Numerical Methods for PDEs (Code: SS 49A), **Hengguang Li**, Wayne State University, and **Sara Pollock**, University of Florida.

Recent Developments in Automorphic Forms (Code: SS 2A), **Solomon Friedberg**, Boston College, and **Jayce Getz**, Duke University.

Recent Trends in Algebraic Graph Theory (Code: SS 26A), **Sebastian Cioaba**, University of Delaware, and **Shaun Fallat**, University of Regina.

SYZ Mirror Symmetry and Enumerative Geometry (Code: SS 11A), **Siu Cheong Lau**, Boston University, **Naichung Leung**, The Chinese University of Hong Kong, and **Hsian-Hua Tseng**, Ohio State University.

Several Complex Variables (Code: SS 5A), **Peter Ebenfelt**, University of California, San Diego, **John Erik Fornaess**, University of Michigan and Norwegian University of Science and Technology, **Ming Xiao**, University of California, San Diego, and **Yuan Yuan**, Syracuse University.

Spaces of Holomorphic Functions and Their Operators (Code: SS 21A), **Mirjana Jovovic** and **Wayne Smith**, University of Hawai'i.

Sparsity, Randomness, and Optimization (Code: SS 15A), **Deanna Needell** and **Jamie Haddock**, University of California, Los Angeles.

Spectral Geometry: The Length and Laplace Spectra of Riemannian Manifolds (Code: SS 25A), **Benjamin Linowitz**, Oberlin College, and **Jeffrey S. Meyer**, California State University at San Bernardino.

Stability and Singularity in Fluid Dynamics (Code: SS 40A), **Tristan Buckmaster**, Princeton University, **Steve Shkoller**, University of California, Davis, and **Vlad Vicol**, Princeton University.

Structural Graph Theory (Code: SS 30A), **Zixia Song**, University of Central Florida, **Martin Rolek**, College of William and Mary, and **Yue Zhao**, University of Central Florida.

The Mathematics of Cryptography (Code: SS 18A), **Shahed Sharif**, California State University, San Marcos, and **Alice Silverberg**, University of California, Irvine.

Three-dimensional Floer Theory, Contact Geometry, and Foliations (Code: SS 6A), **Joan Licata**, Australian National University, and **Robert Lipshitz**, University of Oregon.

Topics at the Interface of Analysis and Geometry (Code: SS 38A), **Alex Austin** and **Sylvester Eriksson-Bique**, University of California, Los Angeles.

Valuations on Algebraic Function Fields and Their Subrings (Code: SS 46A), **Ron Brown**, University of Hawai'i, **Steven Dale Cutkosky**, University of Missouri, and **Franz-Viktor Kuhlmann**, University of Szczecin.

What is Happening in Mathematical Epidemiology? Current Theory, New Methods, and Open Questions (Code: SS 52A), **Olivia Prosper**, University of Kentucky.

Hartford, Connecticut

University of Connecticut Hartford (Hartford Regional Campus)

April 13–14, 2019

Saturday – Sunday

Meeting #1148

Eastern Section
Associate Secretary: Steven H. Weintraub

Announcement issue of *Notices*: February 2019
Program first available on AMS website: February 21, 2019
Issue of *Abstracts*: Volume 40, Issue 2

Deadlines

For organizers: Expired
For abstracts: February 5, 2019

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Olivier Bernardi, Brandeis University, *Percolation on triangulations and a bijective path to Liouville quantum gravity*.

Brian Hall, Notre Dame University, *Eigenvalues of random matrices in the general linear group in the large-N limit*.

Christina Sormani, Lehman College and CUNY Graduate Center, *Compactness Theorems for Sequences of Riemannian Manifolds*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Algebraic Number Theory (Code: SS 22A), **Harris Daniels**, Amherst College, and **Alvaro Lozano-Robledo** and **Erik Wallace**, University of Connecticut.

Analysis, Geometry, and PDEs in Non-smooth Metric Spaces (Code: SS 1A), **Vyron Vellis**, University of Connecticut, **Xiaodan Zhou**, Worcester Polytechnic Institute, and **Scott Zimmerman**, University of Connecticut.

Banach Space Theory and Metric Embeddings (Code: SS 11A), **Mikhail Ostrovskii**, St. John's University, and **Beata Randrianantoanina**, Miami University.

Chip-firing and Divisor Theory (Code: SS 19A), **Caroline Klivans**, Brown University, and **David Perkinson**, Reed College.

Cluster Algebras and Related Topics (Code: SS 12A), **Emily Gunawan** and **Ralf Schiffler**, University of Connecticut.

Combinatorial Commutative Algebra and Polyhedral Geometry (Code: SS 13A), **Elie Alhajjar**, US Military Academy, and **McCabe Olsen**, Ohio State University.

Computability Theory (Code: SS 2A), **Damir Dzhafarov** and **Reed Solomon**, University of Connecticut, and **Linda Brown Westrick**, Pennsylvania State University.

Convergence of Riemannian Manifolds (Code: SS 17A), **Lan-Hsuan Huang** and **Maree Jaramillo**, University of Connecticut, and **Christina Sormani**, City University of New York Graduate Center and Lehman College.

Discrete Dynamical Systems and Applications (Code: SS 20A), **Elliott J. Betrand**, Sacred Heart University, and **David McArdle**, University of Connecticut.

Invariants of Knots, Links, and Low-dimensional Manifolds (Code: SS 15A), **Patricia Cahn**, Smith College, and **Moshe Cohen** and **Adam Lowrance**, Vassar College.

Knot Theory, the Colored Jones Polynomial, and Khovanov Homology (Code: SS 18A), **Adam Giambrone**, Elmira College, and **Katherine Hall**, University of Connecticut.

Mathematical Cryptology (Code: SS 8A), **Lubjana Beshaj**, United States Military Academy, and **Jaime Gutierrez**, University of Cantabria, Santander, Spain.

Mathematical Finance (Code: SS 14A), **Oleksii Mostovyi**, University of Connecticut, **Gu Wang**, Worcester Polytechnic Institute, and **Bin Zhou**, University of Connecticut.

Modeling and Qualitative Study of PDEs from Materials Science and Geometry. (Code: SS 6A), **Yung-Sze Choi**, **Changfeng Gui**, and **Xiaodong Yan**, University of Connecticut.

Recent Advances in Structured Matrices and Their Applications (Code: SS 16A), **Maxim Derevyagin**, University of Connecticut, **Olga Holz**, University of California, Berkeley, and **Vadim Olshevsky**, University of Connecticut.

Recent Development of Geometric Analysis and Nonlinear PDEs (Code: SS 3A), **Ovidiu Munteanu**, **Lihan Wang**, and **Ling Xiao**, University of Connecticut.

Representation Theory of Quantum Algebras and Related Topics (Code: SS 10A), **Drew Jaramillo**, University of Connecticut, **Garrett Johnson**, North Carolina Central University, and **Margaret Rahmoeller**, Roanoke College.

Special Session on Regularity Theory of PDEs and Calculus of Variations on Domains with Rough Boundaries (Code: SS 5A), **Murat Akman**, University of Connecticut, and **Zihui Zhao**, University of Washington.

Special Values of L-functions and Arithmetic Invariants in Families (Code: SS 21A), **Ellen Eischen**, University of Oregon, **Yifeng Liu**, Yale University, **Liang Xiao**, University of Connecticut, and **Wei Zhang**, Massachusetts Institute of Technology.

Stochastic Analysis and Related Fields (Code: SS 7A), **Fabrice Baudoin**, University of Connecticut, and **Cheng Ouyang**, University of Illinois at Chicago.

Stochastic Processes, Random Walks, and Heat Kernels (Code: SS 4A), **Patricia Alonso Ruiz**, University of Connecticut, and **Phanuel Mariano**, Purdue University.

Sub-Riemannian and CR Geometric Analysis (Code: SS 9A), **Fabrice Baudoin**, University of Connecticut, and **Luca Capogna**, Worcester Polytechnic Institute.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice. Special discounted rates were negotiated with the hotels listed below. Rates quoted do not include an 15% occupancy tax. Participants must state that they are with the **AMS Spring Eastern Sectional Meeting/American Mathematical Society (AMS) Meeting at the University of Connecticut Hartford** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**

Candlewood Suites Hartford Downtown, 70 Market Street, Hartford, CT; (860)724-1074, `https://www.ihg.com/candlewood/hotels/us/en/hartford/hfdcw/hoteldetail`. Rates are US$109 per night for a studio suite with one queen bed or two double beds; this rate is applicable for single or double occupancy. To reserve a room over the phone please contact the hotel directly and identify the block of rooms for the **American Mathematical Society**. This is an all suite property. All rooms include a fully equipped kitchen with appliances, including a full-sized fridge. Amenities include wired or wireless Internet access free to members, fitness center, indoor pool, 24-hour business center, on-site self-laundry services, and convenience store, Candlewood Cupboard. Complimentary parking is available on-site for one car per room; additional cars will carry a parking fee. This property is pet-friendly with a non-refundable fee of US$75 for up to six nights and a US$75 deposit, please contact the hotel directly to make these arrangements. Check-in is at 3:00 pm, check-out is at 11:00 am. This property is located approximately one half mile from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 13, 2019.**

The Goodwin, One Haynes Street, Hartford, CT, 06103; (860)246-1881, `https://www.goodwinhartford.com`. Rates are US$139 per night for a deluxe king bedded room, single or double occupancy. To reserve a room at these rates via email please write to `reservations@goodwinhartford.com` and identify your affiliation with the American Mathematical Society Group. To reserve a room over the phone please contact the hotel directly at (860)246-1881 and identify the block of rooms for the American Mathematical Society Group. Amenities include complimentary wireless Internet, fitness center, in-room mini fridge and Keurig coffee maker, Porron & Pina on-site restaurant serving lunch and dinner, onsite donut shop, and valet parking for US$25 per night. Check-in is at 3:00 pm, check-out is at 11:00 am. This property is located approximately one half mile from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 15, 2019**.

The Hilton Hartford, 315 Trumbull Street, Hartford, CT, 06103; (860)728-5151, `https://www3.hilton.com/en/hotels/connecticut/hilton-hartford-HFDHHHF/index.html`. Rates are US$115 per night for a single king bedded room or a double queen bedded room. To reserve a room by phone please use the Hilton's toll free reservations line at (800)754-7941 and identify the block of rooms for the American Mathematical Society. Amenities include complimentary high-speed internet access, in room mini-fridge, fitness center, indoor swimming pool, M & M Bistro on-site restaurant serving breakfast, lunch, and dinner, Element 315 cocktail lounge and on-site parking for US$19 per night. Check-in is at 3:00 pm, check-out is at noon. This property is located less than one mile from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 19, 2019**.

Marriott Downtown Hartford, 200 Columbus Boulevard, Hartford, CT, 06103; (860)249-8000; `https://www.marriott.com/hotels/travel/bdldt-hartford-marriott-downtown/`.Rates are US$119 per night for a guest room with double occupancy, with two double or one king sized bed. To reserve a room by phone please contact the hotel directly at (860)249-8000 and identify the block of rooms for the **AMS Spring Eastern Sectional Meeting**. Amenities include complimentary wireless internet access, mini-fridge, indoor pool, fitness center, business center, *Starbucks* on property, *Vivo* an on-site Italian and Mediterranean restaurant serving breakfast, lunch, and dinner, *L BAR* cocktail bar open for lunch and dinner, valet parting at a rate of US$23 per night, and on-site self-parking at a rate of US$3.00 per hour/US$19 per night. Check-in is at 4:00 pm, check-out is at 11:00 am. This property is located approximately one half mile from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 22, 2019.**

Red Lion Inn, 50 Morgan Street, S Hartford, CT, 06120; (860)549-2400, `https://www.redlion.com/red-lion-hotel/ct/hartford/red-lion-hotel-hartford`. Rates are US$89 for a guest room, with single or double occupancy. To reserve a room at this rate, please call the hotel at (860)549-2400 or the Red Lion Reservation Desk at (800)733-5466 and ask for the group rate for the **American Mathematical Society Spring Eastern Sectional Meeting**. Guests may also book online at `www.redlion.com/Hartford`. Please enter the arrival/departure dates and the group code: **AMER0412**. Amenities include complimentary wireless internet access, fitness center, business center, on-property *Bistro Z* restaurant serving room service, breakfast, lunch, and dinner, and on-site parking for a fee of US$17 per night. Check-in is at 3:00 pm, check-out is at 11:00 am. This property is located approximately one mile from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations in this block is **March 29, 2019.**

Residence Inn by Marriott Downtown Hartford, 942 Main Street, Hartford, CT, 06103; (860)524-5550, `https://www.marriott.com/hotels/travel/bdlri-residence-inn-hartford-downtown`. Rates are US$149 per night for a king or queen bedded bi-level loft suite, single or double occupancy. To make reservations, by phone, please call the reservations line at (800)331-3131 or call the hotel directly at (860)524-5550. Be sure to identify the group as the **AMS Eastern Spring Sectional Meeting**. Any requests for special room arrangements must be made at the time of this call. The Hotel does not confirm reservations in writing. Reservations may also be booked on line at `www.hartfordresidenceinn.com`. Choose the dates of your stay and then enter the group code, AMSAMSD. This is an all-suite property. Amenities include complimentary wired or wireless internet access, fitness center, pull-out sofa bed, full-sized kitchen, business center, complimentary hot breakfast, sundry/convenience store, onsite restaurant *City Steam Brewery Café* serving lunch and dinner, on-site parking is available for overnight guests identifying themselves as a part of the AMS Spring Eastern Sectional Meeting for US$5 per night (standard rate is US$17 per night). This property is pet-friendly, with prior notice and a US$100 non-refundable fee per pet, per stay. Check-in is at 3:00 pm, check-out is at noon. This property is located less than one half mile from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **March 13, 2019.**

Housing Warning

Please beware of aggressive housing bureaus that target potential attendees of a meeting. They are sometimes called "room poachers" or "room-block pirates" and these companies generally position themselves as a meeting's housing bureau, convincing attendees to unknowingly book outside the official room block. They call people who they think will more likely than not attend a meeting and lure them with room rates that are significantly less than the published group rate—for a limited time only. And people who find this offer tempting may hand over their credit card data, believing they have scored a great rate and their housing is a done deal. Unfortunately, this often turns out to be the start of a long, costly nightmare.

Note that some of these room poachers create fake websites on which they represent themselves as the organizers of the meeting and include links to book rooms, etc. The only official website for this meeting is ams.org and one that has the official AMS logo.

These housing bureaus are not affiliated with the American Mathematical Society or any of its meetings, in any way. The AMS would never call anyone to solicit reservations for a meeting. The only way to book a room at a rate negotiated for an AMS Sectional Meeting is via a listing on AMS Sectional Meetings pages or Notices of the AMS. The AMS cannot be responsible for any damages incurred as a result of hotel bookings made with unofficial housing bureaus.

Food Services

On Campus: There are no outlets for dining on the UConn Hartford Campus.
Off Campus: Local organizers encourage participants to visit the surrounding area of downtown Hartford for options for lunch and dinner during the meeting.
Some dining options nearby to the campus include:

- **Bear's BBQ,** 25 Front Street, (860)785-8772, `https://www.bearsbbq.com`; Saturday and Sunday, 11:00 am–9:00 pm; Local BBQ chain serving smoked-meat entrees & sandwiches, plus comfort sides, in a casual setting (2 min walk).

- **Front Street Bistro**, 9 Front Street, (860)422-7712, frontstreetbistro.com; Saturday 11:30 am–10:30 pm and Sunday 11:30 am–8:30 pm; Mediterranean-influenced cuisine with locally sourced ingredients (3 min walk).
- **The Blind Pig Pizza Co.**, 89 Arch Street, (860)744-4333, https://www.blindpigpizza.com; Saturday 11:00 am–10:00 pm and Sunday 11:00 am–9:00 pm; pizzeria offering inventive pies, including BBQ options, paired with cocktails, beer, & wine. (3 min walk).
- **New York Market and Deli**, 27 Main Street, (860)216-6568, https://newyorkmarketdeli.business.site; Saturday 8:00 am–4:00 pm; Sunday closed; Deli serving breakfast and lunch (4 min walk).
- **Dunkin' Donuts**, 485 Main Street, (860)727-9332, www.dunkindonuts.com; Saturday 6:00 am–2:00 pm; Sunday closed; serving coffee and breakfast items (3 min walk).
- **Republic at the Linden**, 10 Capital Avenue, (860)310-3269, www.republicct.com, Saturday 5:00 pm–11:00 pm, Sunday 11:30 am–9:30 pm; urban gastropub featuring craft beers and artistic culinary creations (5 min walk).
- **Elm Café and Spirits,** 50 Elm Street, (860)206-3322; Saturday 4:00 pm–11:00 pm and Sunday closed; South American and Caribbean inspired small plates (5 min walk).
- **Starbucks**, 20 Front Street, (860)263-2271, https://www.starbucks.com; Saturday 8:00 am-8:00 pm and Sunday 9:00 am-6:00 pm; coffee, beverages, and light fare (3 min walk).
- **Arch Street Tavern**, 85 Arch Street, (860)246-7610, https://www.archstreettavern.com; Saturday 11:30 am–2:00 am and Sunday call for hours; hearty pub fare with live jazz. (3 min walk).
- **Dhaba Wala Indian Kitchen,** 49 Asylum Street, (860)232-1500, dhabawalahartford.com; Saturday 11:30 am-10:00 pm and 5:00 pm–10 pm Sunday; traditional Indian cuisine in a casual atmosphere (8 min walk).
- **Subway,** 493 Main Street, (860)560-2200, www.subway.com; Saturday and Sunday 9:00 am–9:00 pm; build-your-own sandwiches, fast food (3 min walk).

Registration and Meeting Information

Advance Registration: Advance registration for this meeting will open on **January 22, 2019**. Advance registration fees will be US$63 for AMS members, US$95 for nonmembers, and US$10 for students, unemployed mathematicians, and emeritus members. Fees will be payable by cash, check, or credit card. Participants may cancel registrations made in advance by e-mailing mmsb@ams.org. 100% refunds will be issued for any advance registrations canceled by the first day of the meeting. After this date, no refunds will be issued.

On site Information and Registration: The registration desk and coffee service will be located in the atrium, and the AMS book exhibit and AMS membership exhibit will be located on the second floor landing of the Hartford Times Building. Special Sessions and Contributed Paper Sessions will also take place in classrooms in the same building. The Invited Addresses will be held in the Spotlight Theater located nearby on Front Street. For a map and directions to the Spotlight Theater please visit their website here: hartford.spotlighttheatres.com/map.

The registration desk will be open on Saturday, April 13, 7:30 am– 4:00 pm and Sunday, April 14, 8:00 am–12:00 pm. The same fees apply for on-site registration, as for advance registration. Fees are payable on-site via cash, check, or credit card.

Other Activities

Book Sales: Stop by the on site AMS bookstore to review the newest publications and take advantage of exhibit discounts and free shipping on all on site orders! AMS and MAA members receive 40% off list price. Nonmembers receive a 25% discount. Not a member? Ask a representative about the benefits of AMS membership.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you wish to discuss with the AMS, please stop by the book exhibit.

Membership Activities: During the meeting, stop by the AMS Membership Exhibit to learn about the benefits of AMS Membership. Members receive free shipping on purchases all year long and additional discounts on books purchased at meetings, subscriptions to *Notices* and *Bulletin*, discounted registration for world-class meetings and conferences, and more!

Complimentary Refreshments will be served courtesy, in part, by the AMS Membership Department.

Special Needs

It is the goal of the AMS to ensure that its conferences are accessible to all, regardless of disability. The AMS will strive, unless it is not practicable, to choose venues that are fully accessible to the physically handicapped.

If special needs accommodations are necessary in order for you to participate in an AMS Sectional Meeting, please communicate your needs in advance to the AMS Meetings Department by:

- Registering early for the meeting

- Checking the appropriate box on the registration form, and
- Sending an email request to the AMS Meetings Department at mmsb@ams.org or meet@ams.org.

AMS Policy on a Welcoming Environment

The AMS strives to ensure that participants in its activities enjoy a welcoming environment. In all its activities, the AMS seeks to foster an atmosphere that encourages the free expression and exchange of ideas. The AMS supports equality of opportunity and treatment for all participants, regardless of gender, gender identity or expression, race, color, national or ethnic origin, religion or religious belief, age, marital status, sexual orientation, disabilities, or veteran status.

Harassment is a form of misconduct that undermines the integrity of AMS activities and mission.

The AMS will make every effort to maintain an environment that is free of harassment, even though it does not control the behavior of third parties. A commitment to a welcoming environment is expected of all attendees at AMS activities, including mathematicians, students, guests, staff, contractors and exhibitors, and participants in scientific sessions and social events. To this end, the AMS will include a statement concerning its expectations towards maintaining a welcoming environment in registration materials for all its meetings, and has put in place a mechanism for reporting violations. Violations may be reported confidentially and anonymously to (855)282-5703 or at www.mathsociety.ethicspoint.com. The reporting mechanism ensures the respect of privacy while alerting the AMS to the situation. Violations may also be brought to the attention of the coordinator for the meeting (who is usually at the meeting registration desk), and that person can provide advice about how to proceed.

For AMS policy statements concerning discrimination and harassment, see the AMS Anti-Harassment Policy.

Questions about this welcoming environment policy should be directed to the AMS Secretary.

Local Information and Maps

This sectional will take place on the Hartford Campus of the University of Connecticut located at 10 Prospect Street in downtown Hartford.

A campus map can be found at https://hartford.uconn.edu/wp-content/uploads/sites/2222/2017/11/UConn-Hartford-Map-v8-2016.pdf. Information about the University of Connecticut Department of Mathematical Sciences located on the Storrs campus, can be found at https://www.math.uconn.edu.

The meeting will occupy space in the Hartford Times Building and the nearby Spotlight Theater, located at 39 Front Street.

Parking

Parking is available at the Connecticut Convention Center, located at 100 Columbus Boulevard and the Connecticut Science Center/Riverfront Garage located at the corner of Bob Steele Street and Columbus Boulevard, at 250 Columbus Boulevard. The Convention Center Garage is a 24-hour garage; the Science Center/Riverfront Garage is open between 8:00 am and 6:00 pm during the dates of the meeting. At the time of publication, parking rates at the Connecticut Convention Center were US$3 for the first hour and US$2 for each additional hour, up to a daily maximum of US$19; rates at the Convention Center were approximately US$20 for the day (arriving at 8:00 am and departing at 6:00 pm).

Information about parking at the Convention Center can be found here: https://www.ctconventions.com/visitors/parking; information about parking at the Science Center can be found here: https://www.lazparking.com/local/hartford-ct/ct-science-center.

On street metered parking is also available in Hartford. For information on public parking in Hartford please visit https://hartfordparking.com.

Travel

The University of Connecticut Hartford is located in the heart of downtown Hartford at 10 Prospect Street. Bradley International Airport is the closest airport to the University. The most common types of transportation used from the airport are rental cars, taxis, and limo/transportation companies.

By Air:

Bradley International Airport (BDL) is the closest airport to the University of Connecticut Hartford. Bradley International Airport is located approximately 16 miles from the University, in Windsor Locks, CT.

Rental cars are available nearby to BDL. All car rental companies provide complimentary shuttle service to their respective locations. Agencies available at this airport include Alamo, Avis, Budget, Dollar, Enterprise, Hertz, National, and Thrifty.

Taxi, and limousine/ transportation companies are available on the ground level of the terminal. For more information on these options, please visit the BDL website here: www.bradleyairport.com/directions/rental-taxi-limo.

By Train:
The Hartford area is served by Amtrak. Reservations can be made on Amtrak at www.amtrak.com or by calling 1(800) USA-RAIL (1(800)872-7245). All trains utilize Hartford Union Station located at 1 Union Place, Hartford. Information about regional trains on the Hartford Line CT Rail can be found at www.hartfordline.com.

By Bus:
The Hartford area is served by bus lines including Greyhound and Peter Pan. Both bus lines utilize Hartford Union Station located at 1 Union Place, Hartford. For more information on Greyhound service please visit their website at https://www.greyhound.com. For more information on Peter Pan service please visit their website at https://peterpanbus.com.

By Car:
All driving directions will end at the Hartford Times Building located on Prospect Street. Upon arriving at that location, participants can determine a suitable parking option nearby.

From Boston: Take I-93 S to I-90 W, I-495 S, I-90 W and I-84 to Columbus Boulevard in Hartford for 126 miles. Continue on Columbus Boulevard to Prospect Street.

From New York City: Take I-95 N, CT-15 N and I-91 N to Whitehead Highway in Hartford. Take exit 29A from I-91 N. Continue on Whitehead Highway and drive to Prospect Street.

From Albany: Take I-787 N to I-90 E and I-91 S to Whitehead Highway in Hartford. Take exit 29A from I-91 S and Continue on Whitehead Highway and drive to Prospect Street.

Local Transportation

Car Rental: Hertz is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at www.hertz.com, click on the box "I have a discount," and type in our convention number (CV): 04N30008. You can also call Hertz directly at (800)654-2240 (US and Canada) or (405)749-4434 (other countries). At the time of your reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available. Meeting rates include unlimited mileage and are subject to availability. Advance reservations are recommended, blackout dates may apply.

Taxi Service: Licensed, metered taxis are available in Hartford. One option in the Hartford area is Yellow Cab, (860)666-6666.

Both Lyft and Uber also operate in the Hartford area.

Bus Service: CT Transit is the Connecticut Department of Transportation (CTDOT)-owned bus service. CTtransit Hartford operates over 30 local and 13 express bus routes. Many local routes operate 7 days a week. There is a bus stop located on Front Street outside of the Hartford Times Building. For additional information about bus fares and schedules please visit the CT transit website at https://www.cttransit.com.

Weather

The average high temperature for April is approximately 60 degrees Fahrenheit and the average low is approximately 45 degrees Fahrenheit. Visitors should be prepared for inclement weather and check weather forecasts in advance of their arrival.

Social Networking

Attendees and speakers are encouraged to tweet about the meeting using the hashtag #AMSmtg.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the US found at https://travel.state.gov/content/travel/en.html. If you need a preliminary conference invitation in order to secure a visa, please send your request to mac@ams.org.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence

- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the US host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Quy Nhon City, Vietnam

Quy Nhon University

June 10–13, 2019

Monday – Thursday

Meeting #1149

Associate Secretary: Brian D. Boe
Announcement issue of *Notices*: April 2019
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: Expired
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/internmtgs.html`.

Invited Addresses

Henry Cohn, Microsoft Research, *Title to be announced*.
Robert Guralnick, University of Southern California, *Title to be announced*.
Le Tuan Hoa, Hanoi Institute of Mathematics, *Title to be announced*.
Nguyen Dong Yen, Hanoi Institute of Mathematics, *Title to be announced*.
Zhiwei Yun, Massachusetts Institute of Technology, *Title to be announced*.
Nguyen Tien Zung, Toulouse Mathematics Institute, *Title to be announced*.

Madison, Wisconsin

University of Wisconsin–Madison

September 14–15, 2019

Saturday – Sunday

Meeting #1150

Central Section
Associate Secretary: Georgia Benkart
Announcement issue of *Notices*: June 2019
Program first available on AMS website: July 23, 2019
Issue of *Abstracts*: Volume 40, Issue 3

Deadlines

For organizers: February 14, 2019
For abstracts: July 16, 2019

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Nathan Dunfield, University of Illinois, Urbana-Champaign, *Title to be announced*.
Teena Gerhardt, Michigan State University, *Title to be announced*.
Lauren Williams, University of California, Berkeley, *Title to be announced* (Erdős Memorial Lecture).

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic and Geometric Combinatorics (Code: SS 12A), **Benjamin Braun**, University of Kentucky, **Marie Meyer**, Lewis University, and **McCabe Olsen**, Ohio State University.

Analysis and Probability on Metric Spaces and Fractals (Code: SS 10A), **Guy C. David**, Ball State University, and **John Dever**, Bowling Green State University.

Association Schemes and Related Topics – in Celebration of J.D.H. Smith's 70th Birthday (Code: SS 8A), **Kenneth W. Johnson**, Penn State University Abington, and **Sung Y. Song**, Iowa State University.

Computability Theory in honor of Steffen Lempp's 60th birthday (Code: SS 6A), **Joseph S. Miller**, **Noah D. Schweber**, and **Mariya I. Soskova**, University of Wisconsin–Madison.

Extremal Graph Theory (Code: SS 14A), **Józef Balogh**, University of Illinois, and **Bernard Lidický**, Iowa State University.

Geometry and Topology of Singularities (Code: SS 13A), **Laurentiu Maxim**, University of Wisconsin-Madison.

Hodge Theory in Honor of Donu Arapura's 60th Birthday (Code: SS 11A), **Ajneet Dhillon**, University of Western Ontario, **Kenji Matsuki** and **Deepam Patel**, Purdue University, and **Botong Wang**, University of Wisconsin-Madison.

Homological and Characteristic $p > 0$ Methods in Commutative Algebra (Code: SS 1A), **Michael Brown**, University of Wisconsin-Madison, and **Eric Canton**, University of Michigan.

Model Theory (Code: SS 5A), **Uri Andrews** and **Omer Mermelstein**, University of Wisconsin-Madison.

Recent Developments in Harmonic Analysis (Code: SS 3A), **Theresa Anderson**, Purdue University, and **Joris Roos**, University of Wisconsin-Madison.

Recent Work in the Philosophy of Mathematics (Code: SS 4A), **Thomas Drucker**, University of Wisconsin-Whitewater, and **Dan Sloughter**, Furman University.

Several Complex Variables (Code: SS 7A), **Hanlong Fang** and **Xianghong Gong**, University of Wisconsin-Madison.

Special Functions and Orthogonal Polynomials (Code: SS 2A), **Sarah Post**, University of Hawai'i atMānoa, and **Paul Terwilliger**, University of Wisconsin-Madison.

Uncertainty Quantification Strategies for Physics Applications (Code: SS 9A), **Qin Li**, University of Wisconsin-Madison, and **Tulin Kaman**, University of Arkansas.

Binghamton, New York

Binghamton University

October 12–13, 2019

Saturday – Sunday

Meeting #1151

Eastern Section
Associate Secretary: Steven H. Weintraub

Announcement issue of *Notices*: August 2019
Program first available on AMS website: August 29, 2019
Issue of *Abstracts*: Volume 40, Issue 3

Deadlines

For organizers: March 12, 2019
For abstracts: August 20, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Richard Kenyon, Brown University, *Title to be announced*.
Tony Pantev, University of Pennsylvania, *Title to be announced*.
Lai-Sang Young, New York University, *Title to be announced*.

Gainesville, Florida

University of Florida

November 2–3, 2019

Saturday – Sunday

Meeting #1152

Southeastern Section
Associate Secretary: Brian D. Boe
Announcement issue of *Notices*: September 2019
Program first available on AMS website: September 19, 2019
Issue of *Abstracts*: Volume 40, Issue 4

Deadlines

For organizers: April 2, 2019
For abstracts: September 10, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Jonathan Mattingly, Duke University, *Title to be announced*.

Isabella Novik, University of Washington, *Title to be announced*.

Eduardo Teixeira, University of Central Florida, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Fractal Geometry and Dynamical Systems (Code: SS 2A), **Mrinal Kanti Roychowdhury**, University of Texas Rio Grande Valley.

Geometric and Topological Combinatorics (Code: SS 1A), **Bruno Benedetti**, University of Miami, **Steve Klee**, Seattle University, and **Isabella Novik**, University of Washington.

Riverside, California

University of California, Riverside

November 9–10, 2019

Saturday – Sunday

Meeting #1153

Western Section
Associate Secretary: Michel L. Lapidus
Announcement issue of *Notices*: September 2019
Program first available on AMS website: September 12, 2019
Issue of *Abstracts*: Volume 40, Issue 4

Deadlines

For organizers: April 9, 2019
For abstracts: September 3, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Mohsen Aliabadi, University of Illinois at Chicago, Chicago, IL, *A connection between matchings in field extensions and the fundamental theorem of algebra*.

Jonathan Novak, University of California, San Diego, *Title to be announced*.

Anna Skripka, University of New Mexico, Albuquerque, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Inverse Problems (Code: SS 3A), **Hanna Makaruk**, Los Alamos National Laboratory, and **Robert Owczarek**, University of New Mexico, Albuquerque and University of New Mexico, Los Alamos.

Random Matrices and Related Structures (Code: SS 2A), **Jonathan Novak**, University of California, San Diego, and **Karl Liechty**, De Paul University.

Topics in Operator Theory (Code: SS 1A), **Anna Skripka** and **Maxim Zinchenko**, University of New Mexico.

Denver, Colorado

Colorado Convention Center

January 15–18, 2020

Wednesday – Saturday

Meeting #1154

Joint Mathematics Meetings, including the 126th Annual Meeting of the AMS, 103rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM)

Associate Secretary: Michel L. Lapidus
MAA Associate Secretary: Hortensia Soto
Announcement issue of *Notices*: October 2019
Program first available on AMS website: November 1, 2019
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2019
For abstracts: To be announced

Call for Proposals for AMS Special Sessions at the Joint Mathematics Meetings in Denver, CO, January 15–18, 2020

Michel L. Lapidus, Associate Secretary responsible for the AMS program at the 2020 Joint Mathematics Meetings in Denver, solicits proposals for AMS Special Sessions for this meeting (to be held from Wednesday, January 15 through Saturday, January 18, 2020 in Denver, CO). Each proposal must include:

1. the name, affiliation, and e-mail address of each organizer, with one organizer designated as the contact person for all communications about the session;
2. the title and a brief (two or three paragraphs) description of the topic of the proposed special session;
3. a *sample* list of speakers (along with their institutions) whom the organizers plan to invite. (It is not necessary to have received confirmed commitments from these potential speakers.)

Organizers are strongly encouraged to consult the AMS Manual for Special Session Organizers at: `www.ams.org/meetings/specialsessionmanual.html`, in its entirety.

Proposals for AMS Special Sessions should be sent by e-mail to Prof. Lapidus (`lapidus@math.ucr.edu`) and must be received by the deadline for organizers, **April 2, 2019**. Late proposals will **not** be considered [with the possible exception of Special Sessions explicitly associated with (and approved for) one of the invited addresses at the meeting]. No decisions will be made on Special Session proposals until after the submission deadline has passed.

Special Sessions will, in general, be allotted between 5 and 10 hours in which to schedule speakers. To enable maximum movement of participants between sessions, organizers must schedule each speaker for either a) a 20-minute talk, 5-minute discussion and 5-minute break; or b) a 45-minute talk,10-minute discussion and 5-minute break. Any combination of 20-minute and 45-minute talks is permitted, but all talks should begin and end at the scheduled time. (In particular, all the talks should start on the hour or half-hour, except on the first afternoon when special sessions must begin at 2:15pm and hence, talks will start on the quarter or third-quarter hour.)

The number of Special Sessions on the AMS program is limited and not all proposals can be accepted. Furthermore, a large number of high-quality proposals is expected. Therefore, please be sure to submit as informative and convincing a proposal as possible for review by the Program Committee. We aim to notify organizers whether their proposal has been accepted by **May 15, 2019**. Following that deadline, specific additional instructions will be given to the contact persons of the accepted Special Sessions.

Charlottesville, Virginia

University of Virginia

March 13–15, 2020

Friday – Sunday
Southeastern Section
Associate Secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Curves, Jacobians, and Abelian Varieties (Code: SS 1A), **Andrew Obus**, Baruch College (CUNY), **Tony Shaska**, Oakland University, and **Padmavathi Srinivasan**, Georgia Institute of Technology.

Medford, Massachusetts

Tufts University

March 21–22, 2020

Saturday – Sunday
Eastern Section
Associate Secretary: Steven H. Weintraub
Program first available on AMS website: To be announced
Announcement issue of *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Fresno, California

California State University, Fresno

May 2–3, 2020

Saturday – Sunday
Western Section
Associate Secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

El Paso, Texas

University of Texas at El Paso

September 12–13, 2020

Saturday – Sunday
Central Section
Associate Secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

State College, Pennsylvania

Pennsylvania State University, University Park Campus

October 3–4, 2020

Saturday – Sunday
Eastern Section
Associate Secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Salt Lake City, Utah

University of Utah

October 24–25, 2020

Saturday – Sunday
Western Section
Associate Secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Washington, District of Columbia

Walter E. Washington Convention Center

January 6–9, 2021

Wednesday – Saturday
Joint Mathematics Meetings, including the 127th Annual Meeting of the AMS, 104th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate Secretary: Brian D. Boe
Announcement issue of *Notices*: October 2020
Program first available on AMS website: November 1, 2020
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2020
For abstracts: To be announced

Grenoble, France

Université Grenoble Alpes

July 5–9, 2021

Monday – Friday
Associate Secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Buenos Aires, Argentina

Mathematical Congress of the Americas 2021 (MCA2021), the third Mathematical Congress of the Americas (MCA). The University of Buenos Aires

July 19–23, 2021

Monday – Friday
Associate Secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Omaha, Nebraska

Creighton University

October 9–10, 2021

Saturday – Sunday
Central Section
Associate Secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 5–8, 2022

Wednesday – Saturday
Associate Secretary: Georgia Benkart
Announcement issue of *Notices*: October 2021
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Boston, Massachusetts

John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel

January 4–7, 2023

Wednesday – Saturday
Associate Secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2022
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Upcoming Features and Memorial Tributes

In Memory of Marina Ratner
Elon Lindenstrauss, Amie Wilkinson, and Peter Sarnak, eds.

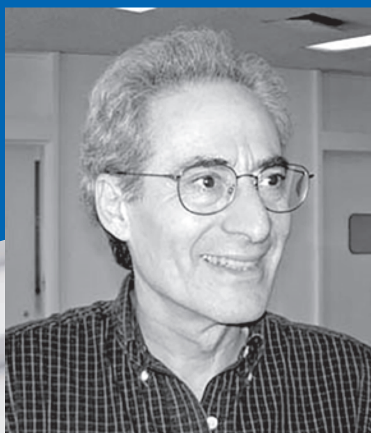
Interview with Abel Laureate Robert P. Langlands
by Bjørn Ian Dundas and Christian Skau

The Mirror's Magic Sights: An Update on Mirror Symmetry
by Timothy Perutz

On Turbulence and Geometry: From Nash to Onsager
by Camillo De Lellis and László Székelyhidi Jr.

The American Mathematical Society Presents

The 2019 AMS Einstein Public Lecture in Mathematics



BARRY MAZUR

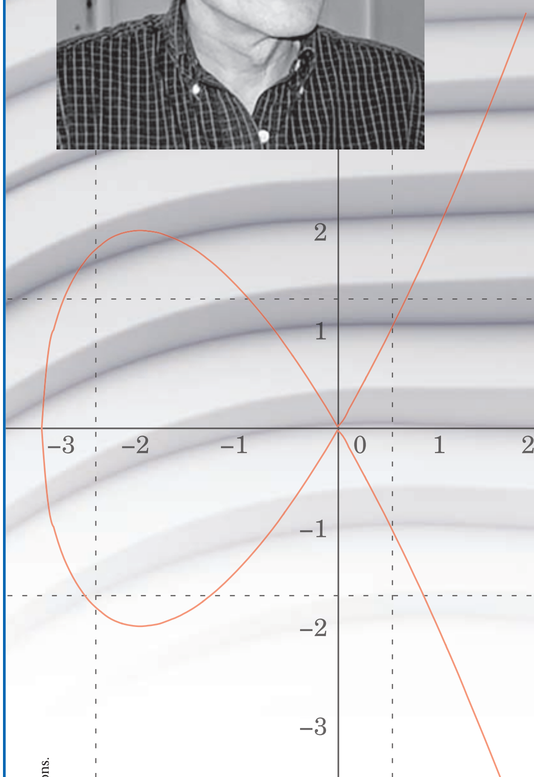
Harvard University

ON THE ARITHMETIC OF CURVES

SATURDAY, MARCH 23 AT 5:30 PM

Kennedy Theatre, University of Hawai'i at Mānoa

Reception to follow.



Barry Mazur is Gerhard Gade University Professor at Harvard University. Winner of many awards and prizes, including the National Medal of Science, Mazur has done outstanding work in many areas of mathematics—his work provided a foundation for the proof of Fermat's Last Theorem—and is known for being able to communicate those results to non-mathematicians and to relish doing so. His former dean, Jeremy Knowles, said:

"Barry is not only a brilliant mathematician, but a wonderful teacher who engages biologists, physicists, economists, and others and seduces them into an understanding of the beauty and use of mathematics."

In this public lecture, Mazur will give a survey of current approaches, results, and conjectures in the vibrant subject of algebraic curves.

Photo of Kennedy Theater by John Wells.
Tschirnhausen cubic: Krishnavedala, Wikimedia Commons.

The Einstein Lecture is part of the 2019 AMS Spring Central and Western Joint Sectional Meeting (March 22–24) at the University of Hawai'i at Mānoa.

Event details: https://www.ams.org/meetings/sectional/2251_events.html
Sectional details: https://www.ams.org/meetings/sectional/2251_program.html

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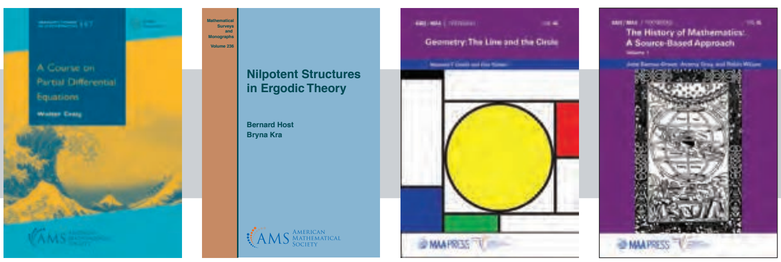
NEW RELEASES
from the AMS



= Textbook
= Applied Mathematics

A Course on Partial Differential Equations

Walter Craig, *McMaster University, Hamilton, ON, Canada, and Fields Institute, Toronto, ON, Canada*

This book puts together the three main aspects of partial differential equations, namely theory, phenomenology, and applications, from a contemporary point of view.

Graduate Studies in Mathematics, Volume 197; 2018; 205 pages; Hardcover; ISBN: 978-1-4704-4292-7; List US$73; AMS members US$58.40; MAA members US$65.70; bookstore.ams.org/gsm-197; Order code GSM/197

Nilpotent Structures in Ergodic Theory

Bernard Host, *Université Paris-Est Marne-la-Vallée, Champs-sur-Marne, France*, and **Bryna Kra**, *Northwestern University, Evanston, IL*

Nilsystems play a key role in the structure theory of measure preserving systems, arising as the natural objects that describe the behavior of multiple ergodic averages. This book is a comprehensive treatment of their role in ergodic theory, covering development of the abstract theory leading to the structural statements, applications of these results, and connections to other fields.

Mathematical Surveys and Monographs, Volume 236; 2018; 427 pages; Hardcover; ISBN: 978-1-4704-4780-9; List US$122; AMS members US$97.60; MAA members US$109.80; bookstore.ams.org/surv-236; Order code SURV/236

AMS / **MAA** Press

Geometry: The Line and the Circle

Maureen T. Carroll, *University of Scranton, PA*, and **Elyn Rykken**, *Muhlenberg College, Allentown, PA*

Starting with Euclid's Elements, this book connects topics in Euclidean and non-Euclidean geometry in an intentional and meaningful way, with historical context.

AMS/MAA Textbooks, Volume 44; 2018; 480 pages; Hardcover; ISBN: 978-1-4704-4843-1; List US$109; AMS members US$81.75; MAA members US$81.75; bookstore.ams.org/text-44; Order code TEXT/44

AMS / **MAA** Press

The History of Mathematics: A Source-Based Approach

Volume 1

June Barrow-Green, *The Open University, Milton Keynes, United Kingdom*, **Jeremy Gray**, *The Open University, Milton Keynes, United Kingdom*, and **Robin Wilson**, *The Open University, Milton Keynes, United Kingdom*

This first volume of The History of Mathematics: A Source-Based Approach takes readers from the beginning of counting in prehistory to 1600 and the threshold of the discovery of calculus.

AMS/MAA Textbooks, Volume 45; 2018; approximately 496 pages; Hardcover; ISBN: 978-1-4704-4352-8; List US$89; AMS members US$66.75; MAA members US$66.75; bookstore.ams.org/text-45; Order code TEXT/45

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