Ultrasound beamforming

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Where innovation starts

Reference:

Comprehensive Biomedical Physics, vol. 2, Chapter 2.13: J.M. Thijssen & M. Mischi, “Ultrasound Imaging Arrays” Elsevier, 2014.

Frequency analysis

Fourier transformation

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

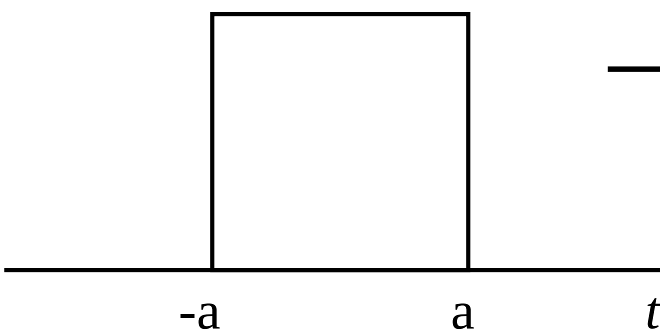
Fourier anti-transformation

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

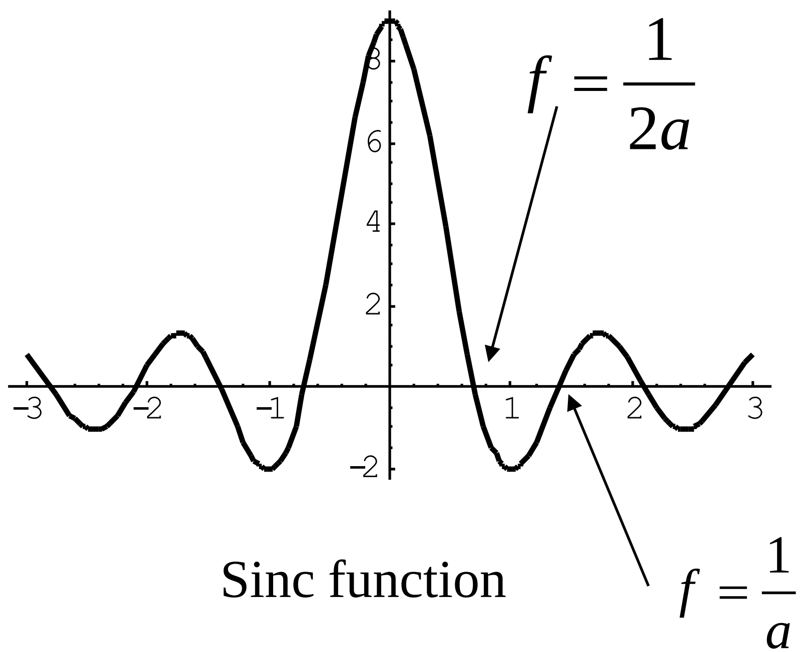
Frequency analysis

The Fourier transform permits the evaluation of the behavior of a system for different frequencies ***f*** **=** $\boldsymbol{\omega/2\pi}$.

$$\Pi_a(t) \xrightarrow{F} 2a \cdot \mathrm{sinc}(2\pi a f)$$



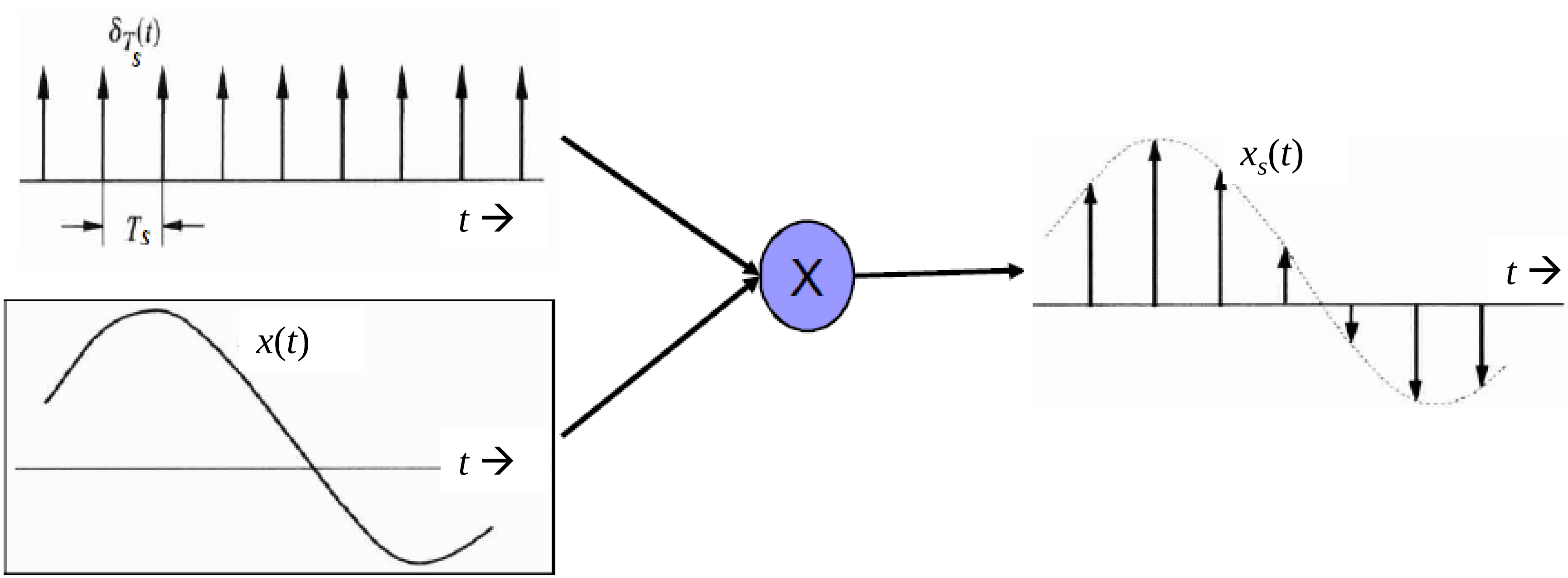
Square wave



Sinc function

Sampling theorem

Sampling $x(t)$ is equivalent to multiply it by a train of impulses…



In formulas…

Time domain

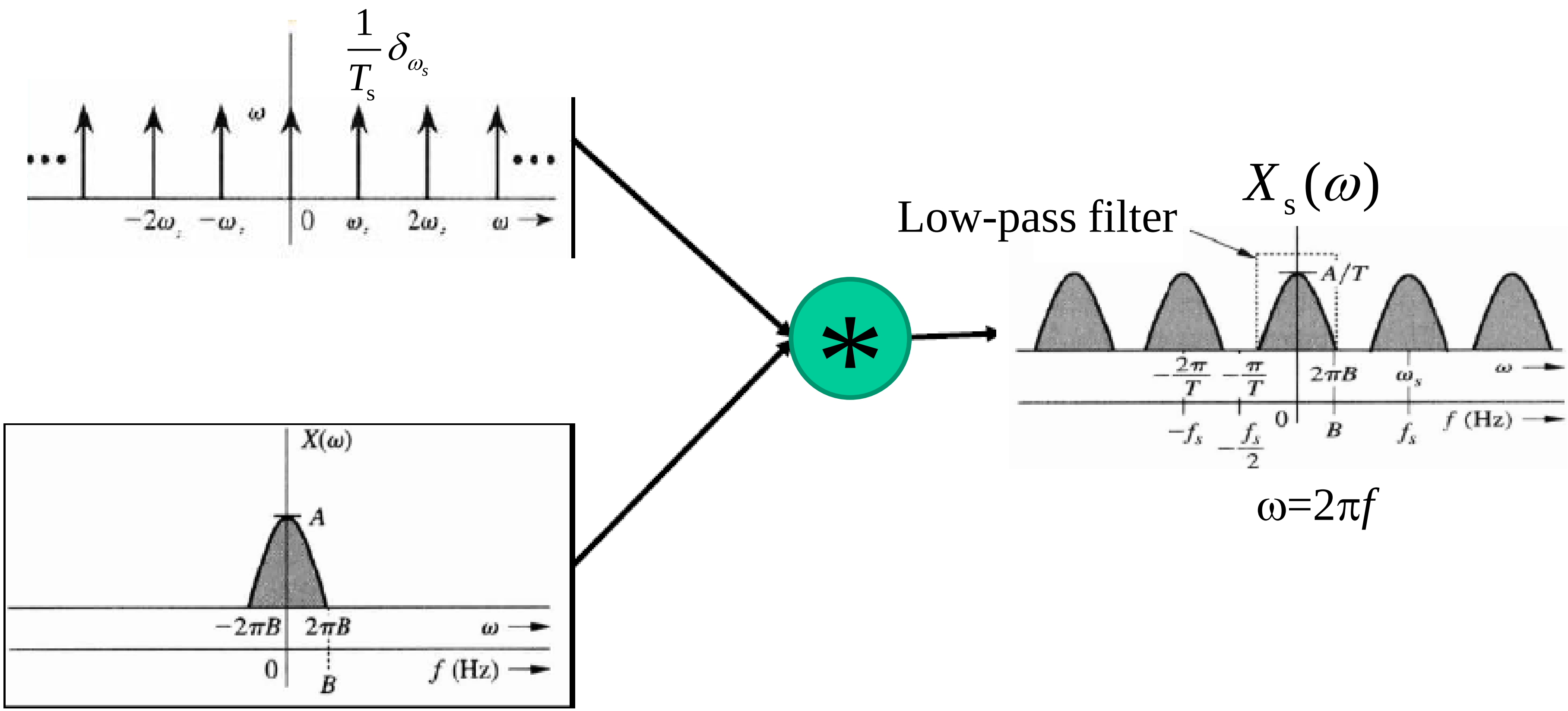
$$x_s(t) = x(t)s(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_s)$$

Frequency domain

$$X_s(\omega) = X(\omega) * \frac{1}{T_s}\sum_{n=-\infty}^{\infty}\delta(\omega - n\omega_s) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty}X(\omega - n\omega_s)$$

Result obtained by using the Fourier expansion of a periodic function (train of pulses).

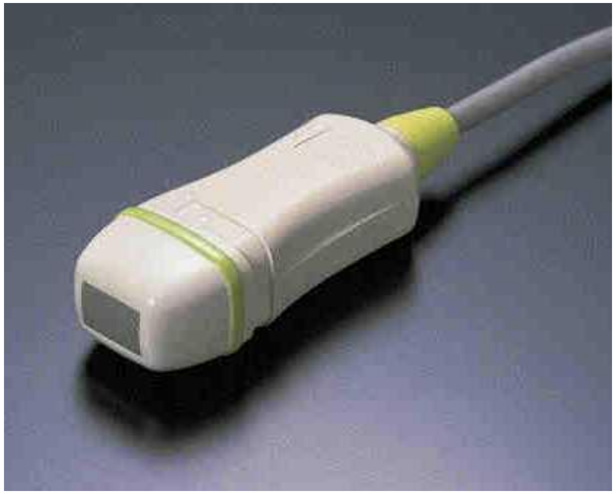
Graphically…



In order to avoid aliasing, the ***Nyquist*** condition must be fulfilled: $\boldsymbol{f_s > 2B}$

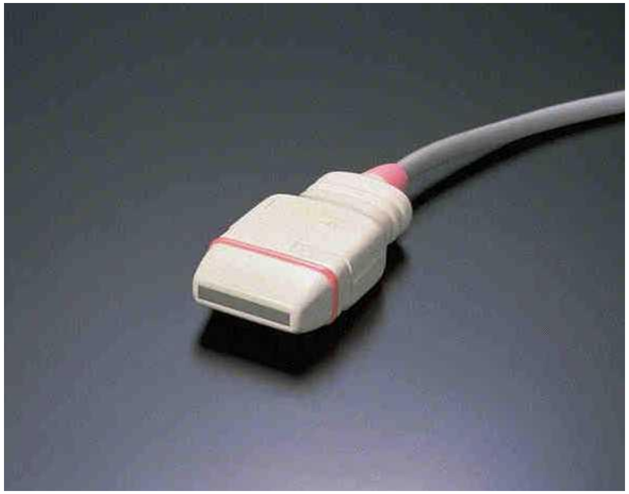
Array transducers

sector

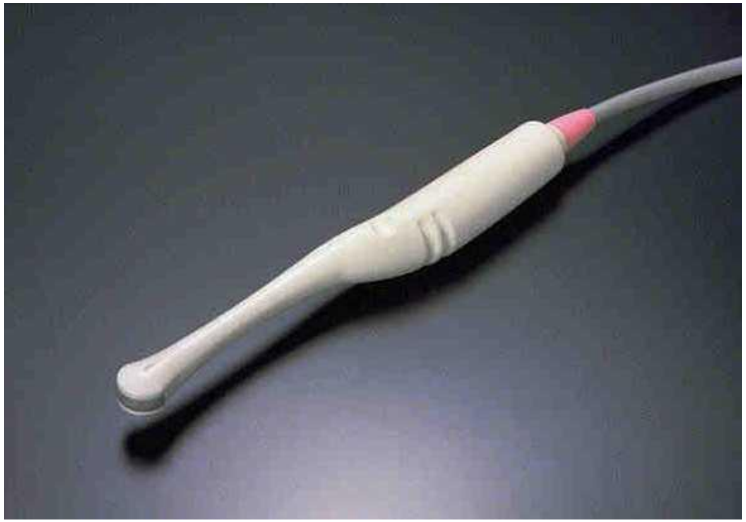


convex

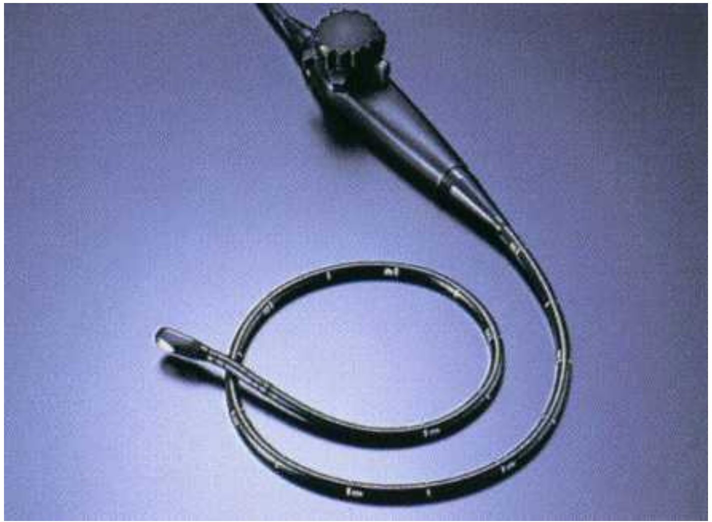
linear



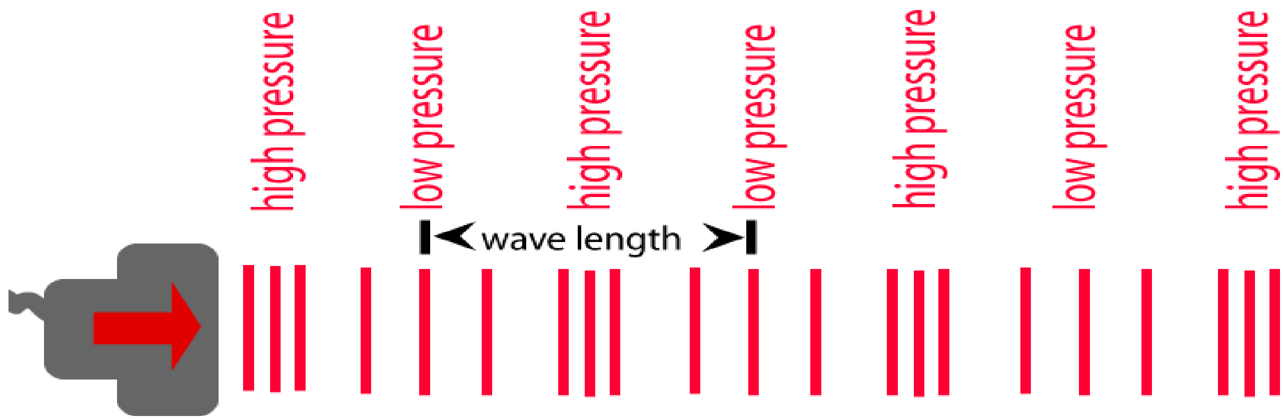
transvaginal/transrectal



transesophageal



Wave equation



$$\frac{\partial^2 A}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2}$$

$$A(t,z) = A_0 e^{ik(ct-z)} \rightarrow \mathrm{Re}[A(t,z)] = A_0 \cos(k(z-ct))$$

$A =$ particle displacement

$c =$ wave propagation velocity

$k =$ wave number $= 2\pi f v^{-1} = 2\pi\lambda^{-1}$

Wave frequency (↑ f) — Wave length (↑ λ)

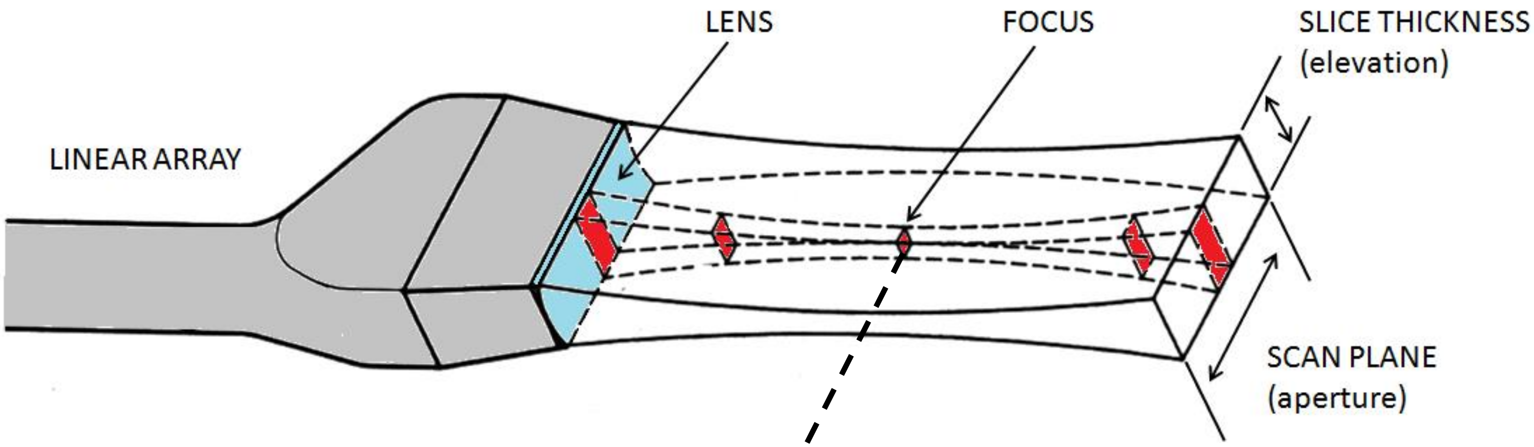
Relation pressure-displacement

$$p = \rho c \frac{\partial A}{\partial t}$$

$p =$ pressure [Pa]

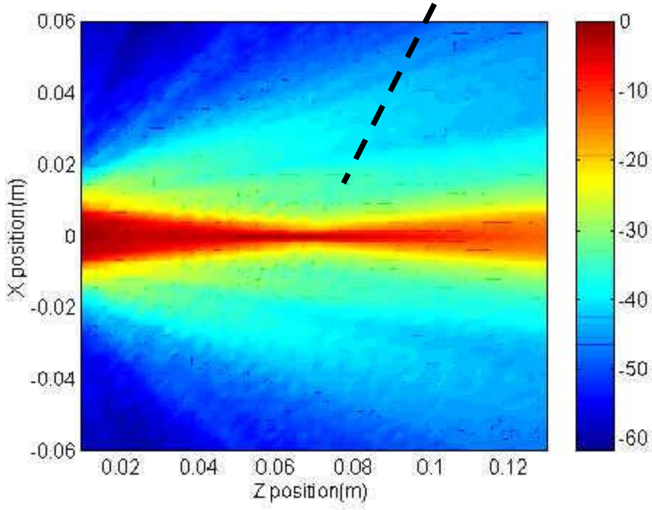
$\rho =$ medium density [kg m^{-3}]

Pressure field



Near field (Fresnel)

Far field (Fraunhofer, distance > D^2/λ)



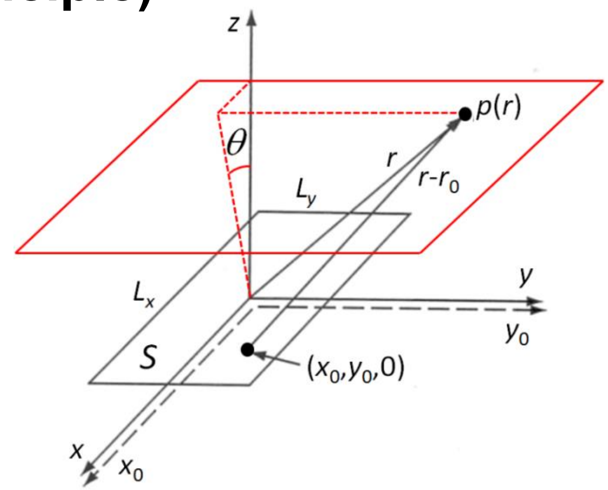
Generated pressure field

Rayleigh-Sommerfeld integral (besed on Huygen’s principle)

$$p(r,t) = \frac{j\rho ck}{2\pi}\iint_S \frac{e^{j[\omega t - k|r-r_0|]}v(r_0)}{|r-r_0|}dS = \frac{j\rho ckv_0}{2\pi}\iint_S \frac{e^{j[\omega t - k|r-r_0|]}a(r_0)}{|r-r_0|}dS$$

with $v(r_0)$ the velocity on the emitting surface, S,

and $a(r_0)$ a weighting function (apodization).



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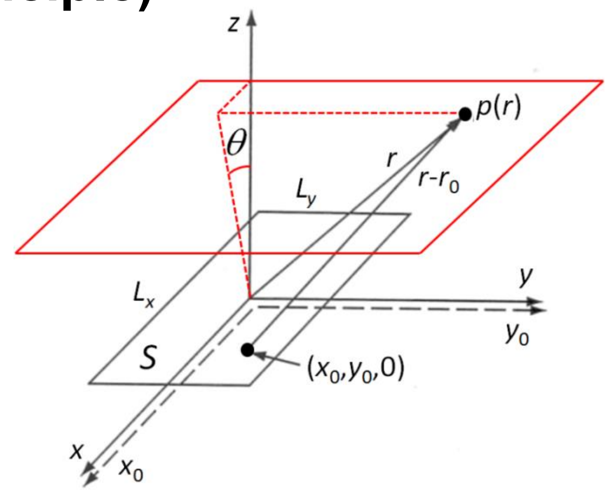


Fresnel approximation (r >> r$_0$)

$$|r-r_0|=\sqrt{z^2+(x-x_0)^2+(y-y_0)^2}=z\sqrt{1+\left(\frac{x-x_0}{z}\right)^2+\left(\frac{y-y_0}{z}\right)^2}\cong z\left[1+\frac{1}{2}\left(\frac{x-x_0}{z}\right)^2+\frac{1}{2}\left(\frac{y-y_0}{z}\right)^2\right]\cong z$$

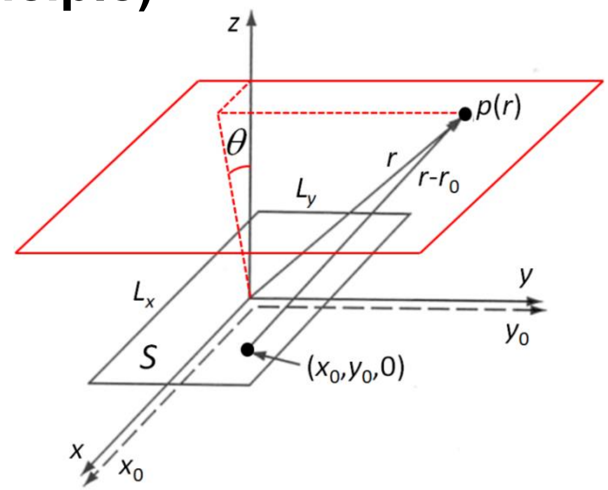
$$p(r,t)=\frac{j\rho ckv_0}{2\pi z\lambda}e^{j(\omega t-kz)}e^{-jk\left(\frac{x^2+y^2}{2z}\right)}\iint_S a(x_0,y_0,0)e^{-jk\left(\frac{x_0^2+y_0^2}{2z}\right)}e^{jk\left(\frac{xx_0+yy_0}{z}\right)}dx_0dy_0$$

Maclaurin expansion

$\sqrt{1+a},\ a\to 0$

Generated pressure field

Rayleigh-Sommerfeld integral (besed on Huygen's principle)



$$p(r,t)=\frac{j\rho ck}{2\pi}\iint_S \frac{e^{j[\omega t-k|r-r_0|]}v(r_0)}{|r-r_0|}dS=\frac{j\rho ckv_0}{2\pi}\iint_S \frac{e^{j[\omega t-k|r-r_0|]}a(r_0)}{|r-r_0|}dS$$

with $v(r_0)$ the velocity on the emitting surface, S,

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Fresnel approximation (r >> r_0)

$$|r-r_0|=\sqrt{z^2+(x-x_0)^2+(y-y_0)^2}=z\sqrt{1+\left(\frac{x-x_0}{z}\right)^2+\left(\frac{y-y_0}{z}\right)^2}\cong z\left[1+\frac{1}{2}\left(\frac{x-x_0}{z}\right)^2+\frac{1}{2}\left(\frac{y-y_0}{z}\right)^2\right]\cong z$$

$$p(r,t)=\frac{j\rho ckv_0}{2\pi z\lambda}e^{j(\omega t-kz)}e^{-jk\left(\frac{x^2+y^2}{2z}\right)}\iint_S a(x_0,y_0,0)e^{-jk\left(\frac{x_0^2+y_0^2}{2z}\right)}e^{jk\left(\frac{xx_0+yy_0}{z}\right)}dx_0dy_0$$

Maclaurin expansion

$\sqrt{1+a},\ a\rightarrow 0$

Rectangular aperture

$$a(x_0,y_0,0)=\prod_{L_x}(x_0)\cdot\prod_{L_y}(y_0),\ \text{with}\ \prod_L(.)\ \text{rectangular function of length}\ L.$$

Generated pressure field

Separable integral: $P(x,y,z) = P(x,z)P(y,z)$.

Neglecting the phase j and oscillating exponent and considering a single integral:

$$p_x(x,z) = \psi_{0x} \mathrm{e}^{-jk\left(\frac{x^2}{2z}\right)} \int_{-\infty}^{\infty} \mathrm{e}^{-jk\left(\frac{x_0^2}{2z}\right)} \prod_{L_x}(x_0) \mathrm{e}^{jk\left(\frac{xx_0}{z}\right)} dx_0,$$

with $\psi_{0x} = \sqrt{\dfrac{\rho c k v_0}{2\pi z}}$

Generated pressure field

Separable integral: $P(x,y,z)=P(x,z)P(y,z)$.

Neglecting the phase j and oscillating exponent and considering a single integral:

$$p_x(x,z)=\psi_{0x}\mathrm{e}^{-jk\left(\frac{x^2}{2z}\right)}\int_{-\infty}^{\infty}\mathrm{e}^{-jk\left(\frac{x_0^2}{2z}\right)}\prod_{L_x}(x_0)\mathrm{e}^{jk\left(\frac{xx_0}{z}\right)}dx_0,$$

with $\psi_{0x}=\sqrt{\dfrac{\rho c k v_0}{2\pi z}}$

For $\lambda z >> x^2$ and $\lambda z >> x_0{}^2$, i.e., far from the transducer surface (*Fraunhofer* approx),
the quadratic phase terms can be neglected:

$$p_x(x,z)=\psi_{0x}\int_{-\infty}^{\infty}\left[\prod_{L_x}[x_0]\right]\mathrm{e}^{j2\pi\left(\frac{x}{\lambda z}\right)x_0}dx_0$$

Pressure field estimation by Fourier Transform

Fourier (anti)transform in space

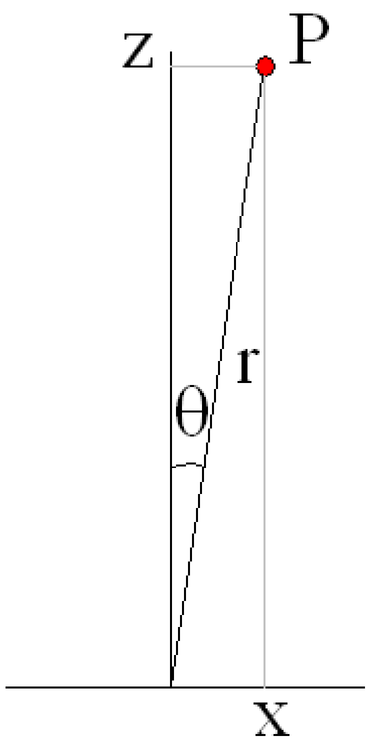
antitransform variable

$$p_x(x,z) = \psi_{0x} \int_{-\infty}^{\infty} \left[\prod_{L_x} [x_0] \right] e^{j2\pi\left(\frac{x}{\lambda z}\right)x_0} dx_0 = L_x \psi_{0x} \mathrm{sinc}\left(\frac{\pi x L_x}{\lambda z} \right)$$

$$p_x(x,z) \cong L_x \psi_{0x} \mathrm{sinc}\left(\frac{\pi L_x}{\lambda} u \right)$$

Based on ***Fraunhofer*** approximation $\lambda z >> x^2$,

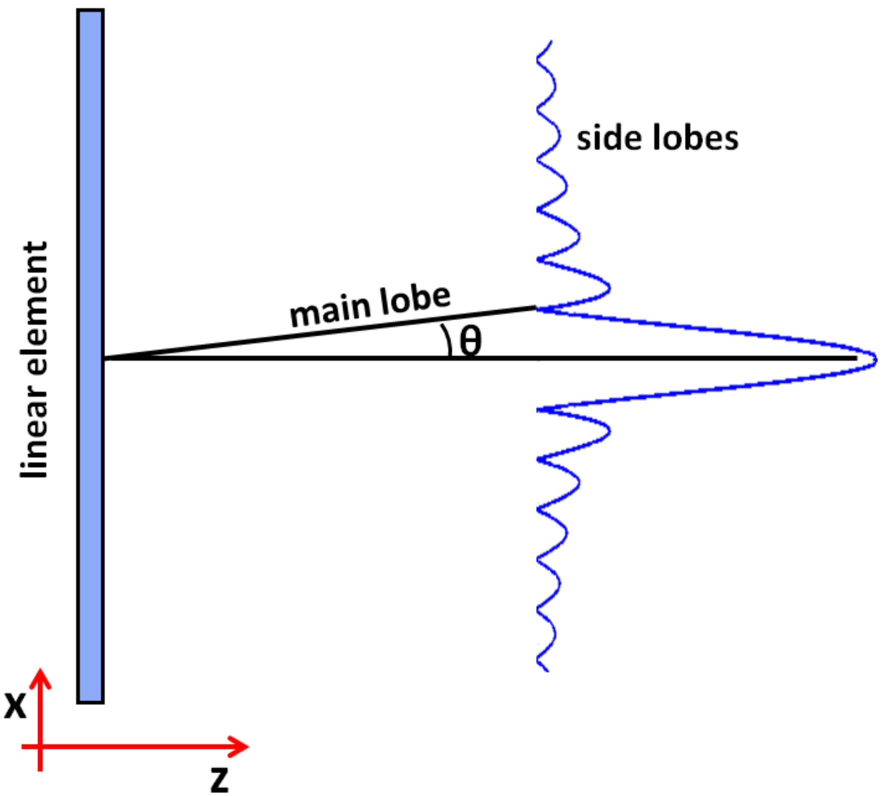
$$x = |r| \sin\theta \cong z \sin\theta \equiv zu$$



Pressure field estimation by Fourier Transform

$$p_x(x,z) \cong L_x \psi_{0x} \operatorname{sinc}\left(\frac{\pi L_x}{\lambda} \sin(\theta) \right)$$

$$\text{For } \sin(\theta) = \frac{\lambda}{L_x} \rightarrow p_x(x,z) = 0$$

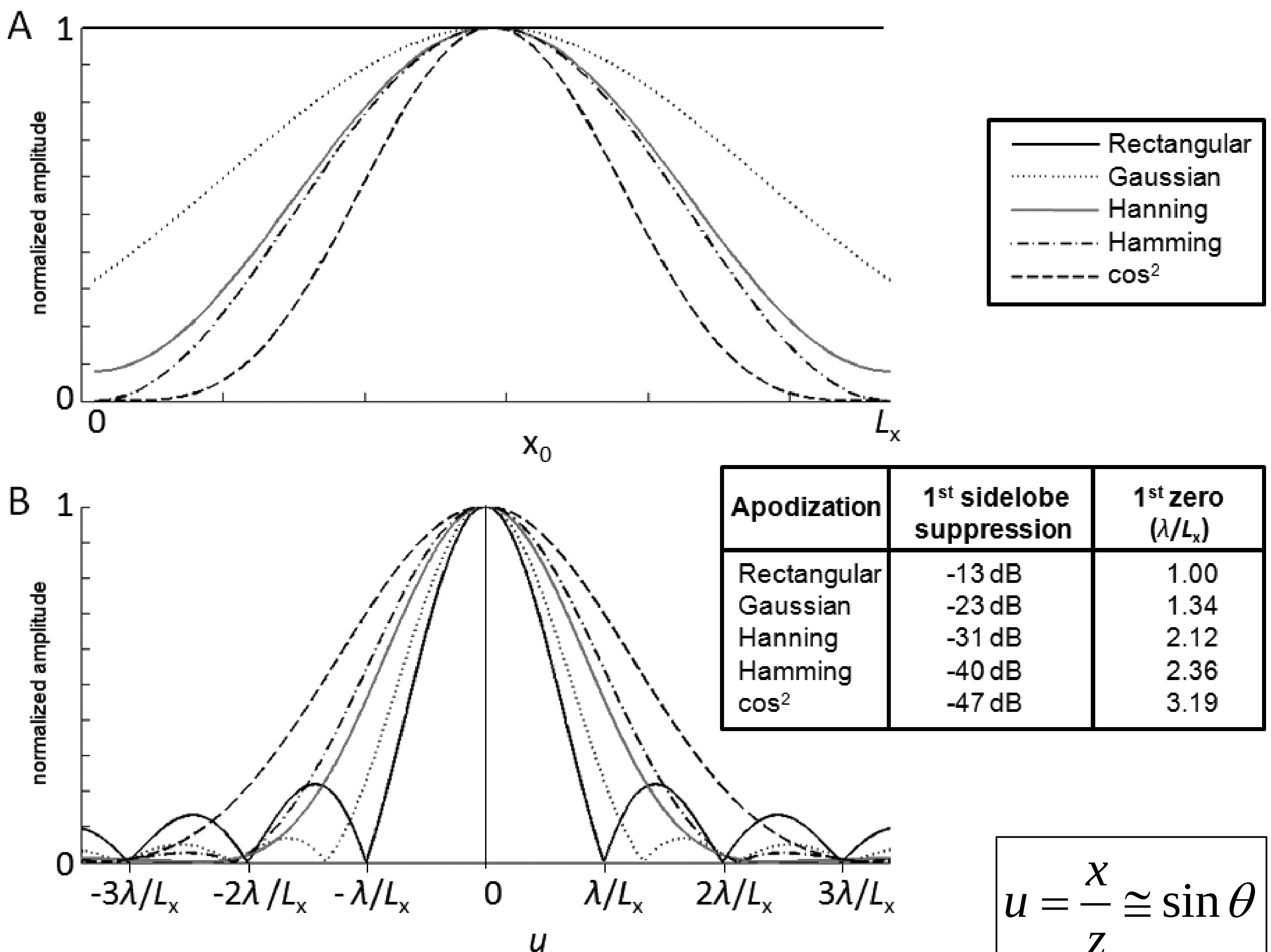


Pressure field estimation by Fourier Transform

Like in signal processing, windowing reduces the side lobes.
Transducer windowing: "*Apodization*".

$$p_x(x,z) = \psi_{0x} \int_{-\infty}^{\infty} \left[\prod_{L_x} [x_0] \mathrm{a}(x_0) \right] \mathrm{e}^{j2\pi\left(\frac{x}{\lambda z}\right)x_0} dx_0 = L_x \psi_{0x} \mathrm{A}\left(\frac{x}{\lambda z}\right) \cong L_x \psi_{0x} \mathrm{A}\left(\frac{u}{\lambda}\right)$$

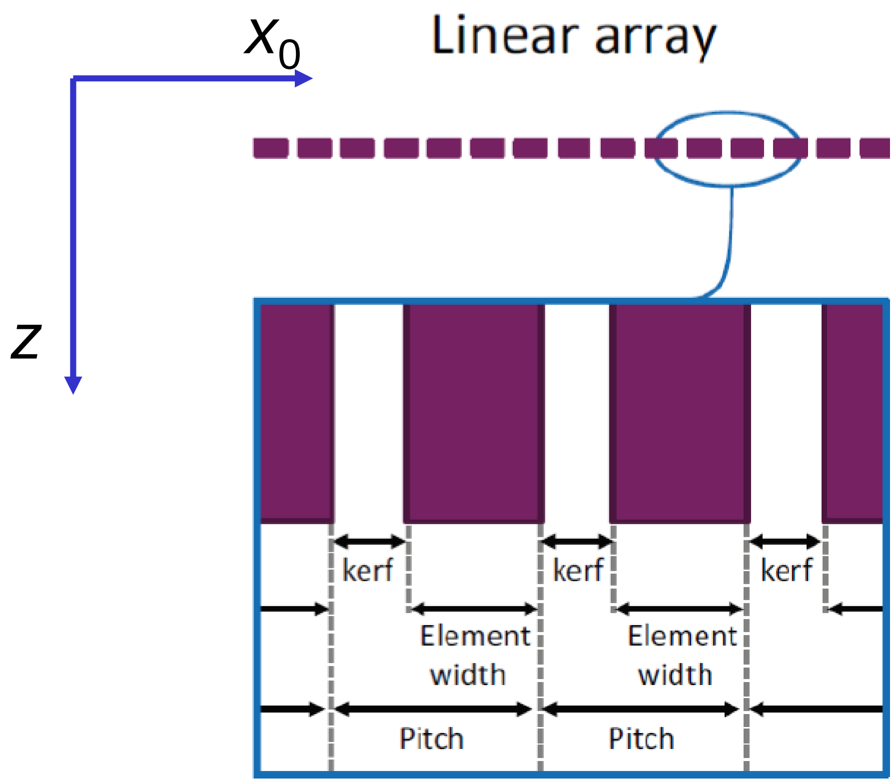
Apodization



Apodization	1st sidelobe suppression	1st zero (λ/L_x)
Rectangular	-13 dB	1.00
Gaussian	-23 dB	1.34
Hanning	-31 dB	2.12
Hamming	-40 dB	2.36
$\cos^2$	-47 dB	3.19

$$u = \frac{x}{z} \cong \sin\theta$$

Multiple elements



Pitch length

$$\prod_{L_x}[x_0] \Rightarrow \prod_{L_x}[x_0]\left(\prod_{w}[x_0] * \sum_{m}\delta(x_0 - md)\right)$$

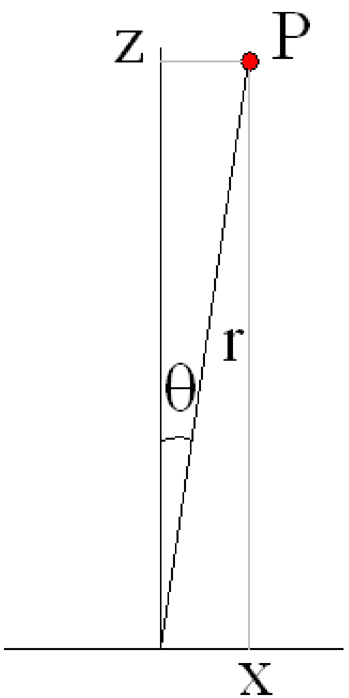
Dirac impulse

Linear-array transducer

$$p_x(x,z)=\psi_{0x}\mathfrak{I}^{-1}\left\{\left(\prod_w[x_0]*\sum_m\delta(x_0-md)\right)\prod_{L_x}[x_0]\right\}$$
$$=\psi_{0x}\mathfrak{I}^{-1}\left\{\prod_w[x_0]\right\}\mathfrak{I}^{-1}\left\{\prod_{L_x}[x_0]\sum_m\delta(x_0-md)\right\}$$

Sampling theorem

$$p_x(x,z)=\psi_{0x}\frac{L_x w}{d}\text{sinc}\left[\frac{\pi w x}{\lambda z}\right]\sum_m \text{sinc}\left[\frac{\pi L_x}{\lambda}\left(\frac{x}{z}-\frac{m\lambda}{d}\right)\right]=$$
$$=\psi_{0x}\frac{L_x w}{d}\text{sinc}\left[\frac{\pi w x}{\lambda z}\right]\sum_m \text{sinc}\left[\frac{\pi L_x}{\lambda}\left(u-\frac{m\lambda}{d}\right)\right]$$



$$x=|r|\sin\theta\cong z\sin\theta\equiv zu$$

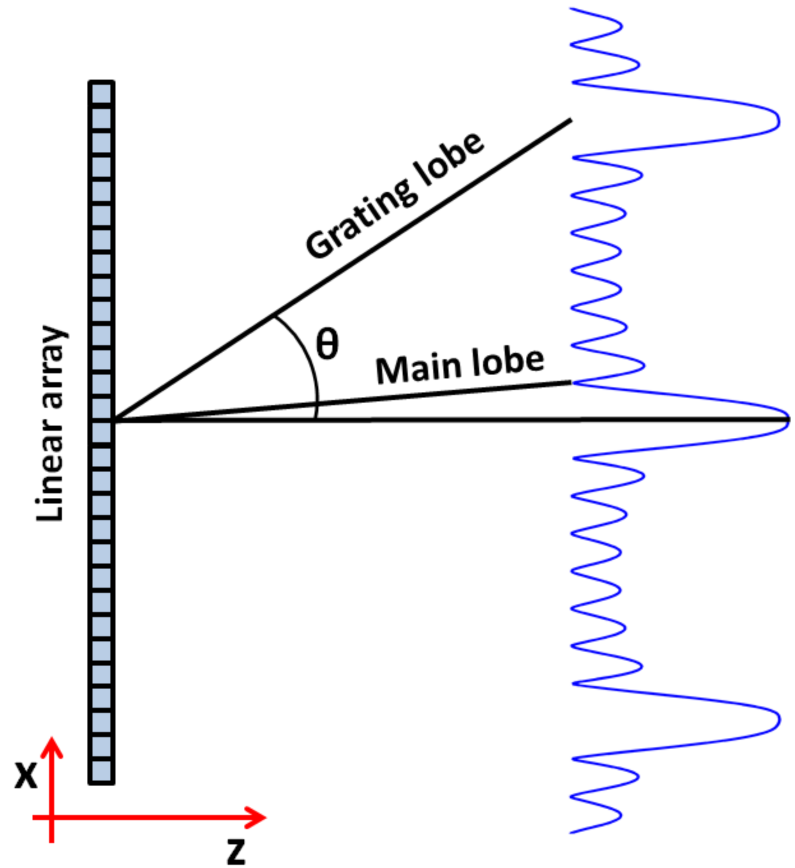
Grating lobes are centered at angles such that the sinc(.) argument equals zero.

$$\theta_g=\pm\arcsin\left(\frac{m\lambda}{d}\right)$$

Grating lobes

$$p_x(x,z) = \psi_{0x}\frac{L_x w}{d}\,\mathrm{sinc}\left[\frac{\pi w x}{\lambda z}\right]\sum_m \mathrm{sinc}\left[\frac{\pi L_x}{\lambda}\left(u - \frac{m\lambda}{d}\right)\right]$$

$$\theta_g = \pm\arcsin\left(\frac{m\lambda}{d}\right)$$



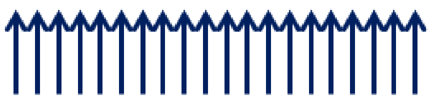
Summary

Aperture L_x

$\prod_{L_x}[x_0]$

$$\text{sinc}\left(\frac{\pi u L_x}{\lambda}\right)$$

Pulse train with period p (pitch)



$\sum_m \delta(x_0 - md)$

$$\sum_m \text{sinc}\left[\frac{\pi L_x}{\lambda}\left(u - \frac{m\lambda}{d}\right)\right]$$

Single element with size w

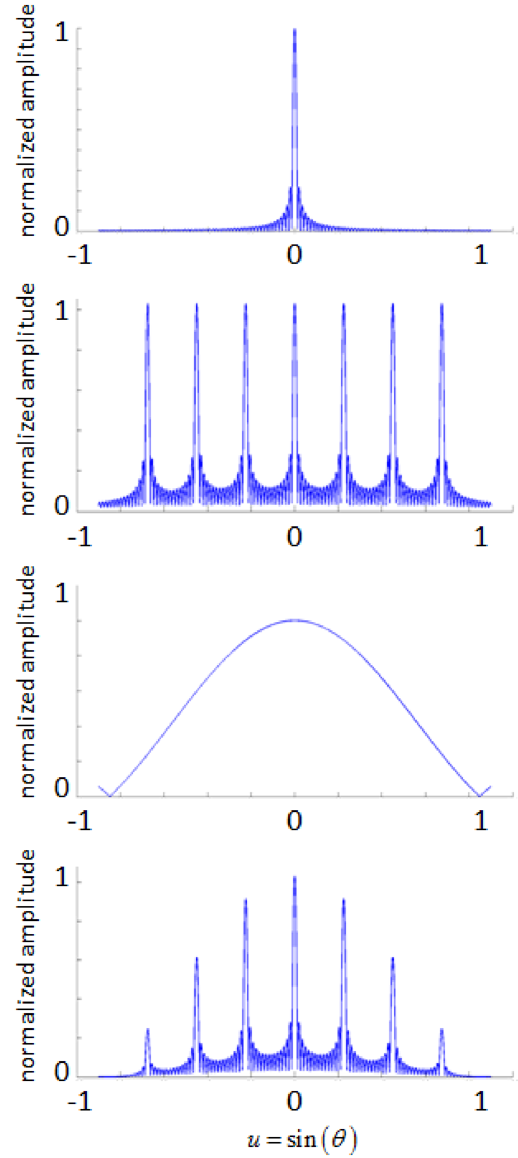
$\prod_{w}[x_0]$

$$\text{sinc}\left[\frac{\pi w u}{\lambda}\right]$$

Full array with aperture L_x

$\prod_{L_x}[x_0]\cdot\left[\prod_{w}[x_0] * \sum_m \delta(x_0 - md)\right]$

$$\text{sinc}\left[\frac{\pi w u}{\lambda}\right]\sum_m \text{sinc}\left[\frac{\pi L_x}{\lambda}\left(u - \frac{m\lambda}{d}\right)\right]$$



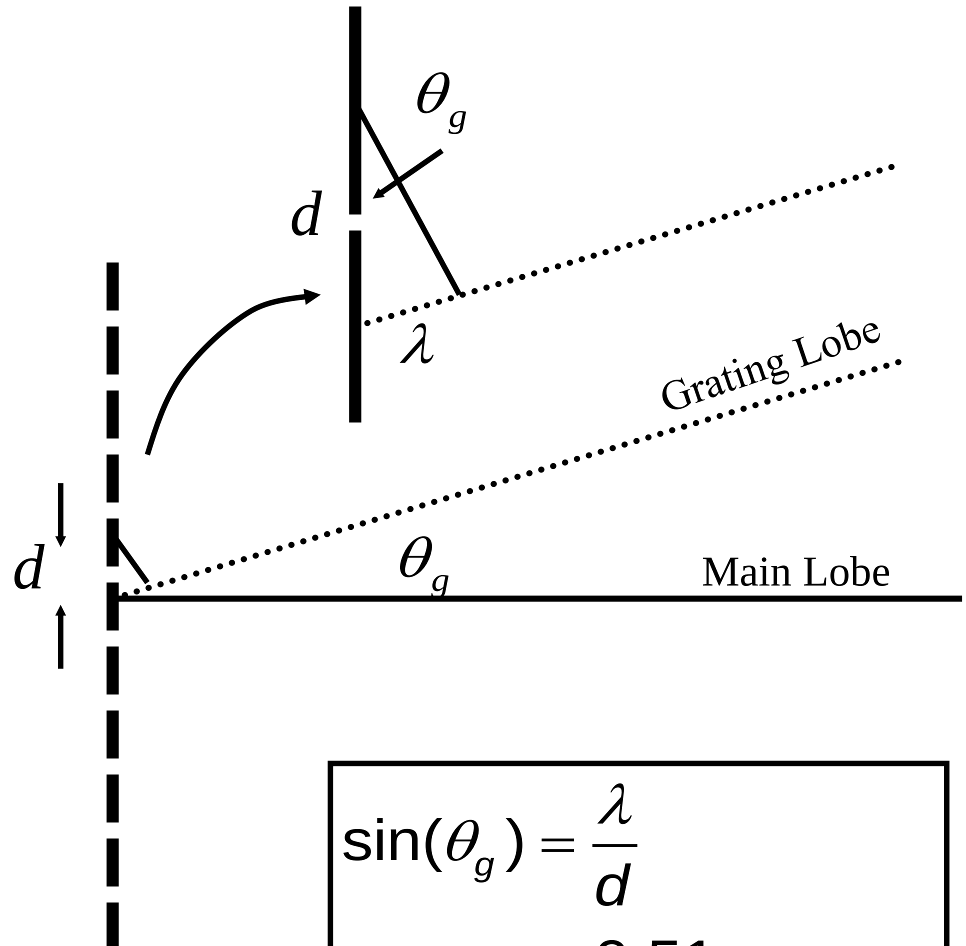
Grating lobe example

How many elements?

What spacing?

Linear array:

- 32 element array
- 3 MHz
- 'pitch' d = 0.4 mm
- λ = 0.51 mm
- $L_x = N\,d$ = 13 mm



$$\sin(\theta_g) = \frac{\lambda}{d}$$

$$\sin(\theta_g) = \frac{0.51}{0.4} = 1.275$$

Apodization example

Same array:

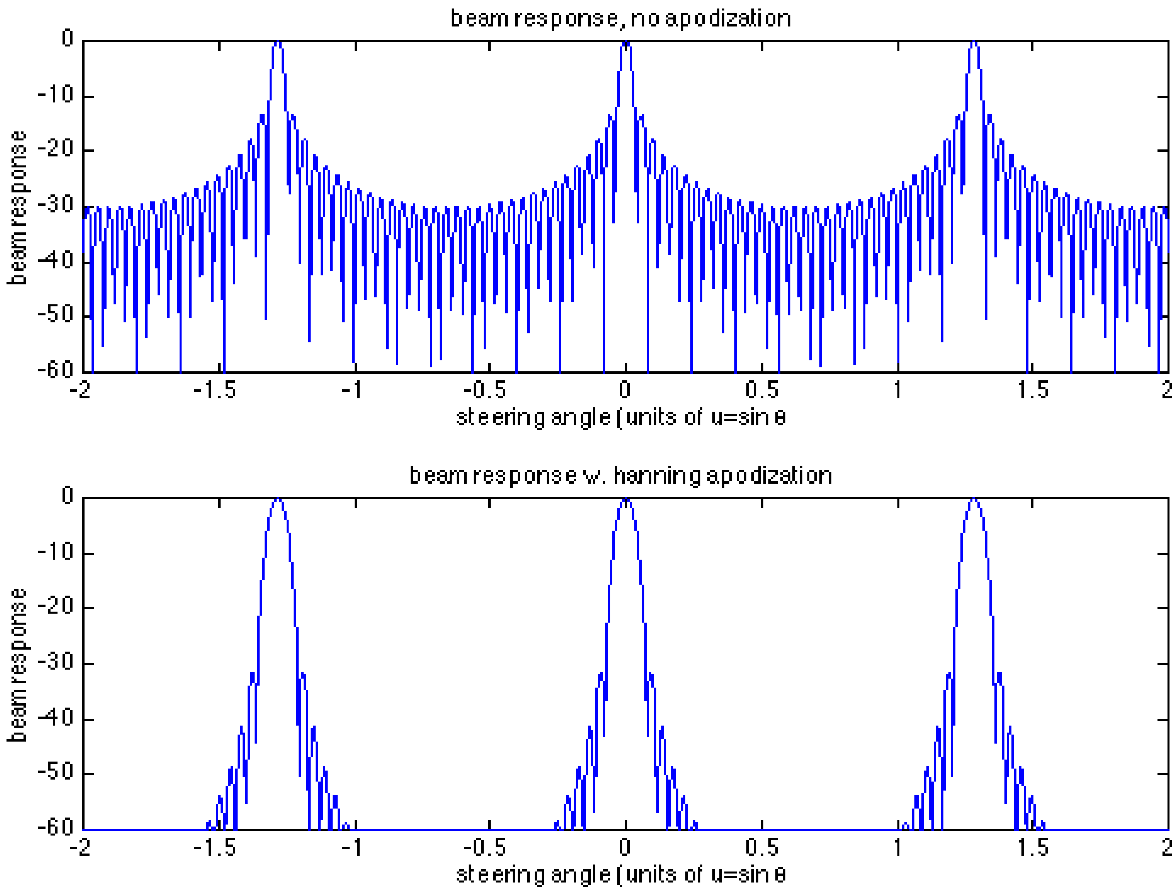
- 32 element array
- 3 MHz
- pitch d = 0.4 mm
- λ = 0.51 mm
- $L_x = N\, d$ = 13 mm.

•With & w/o Hanning windowing.

•Sidelobes way down.

•Mainlobe wider.

•No effect on grating lobes.



Electronic beam steering

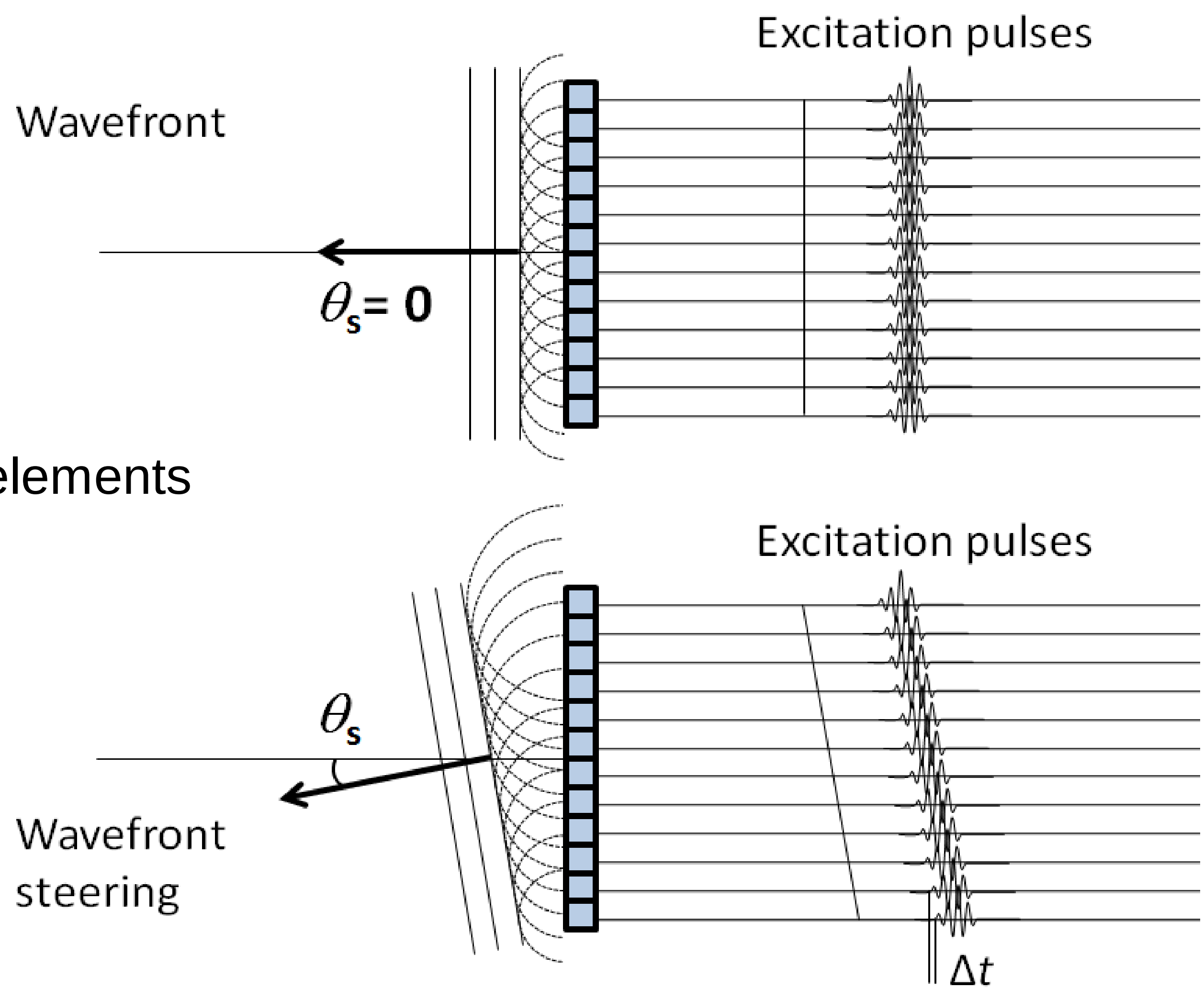
Beam steering by systematic excitation delays.

$$\theta_s = \sin^{-1}\left(\frac{c_0 \Delta t}{d}\right)$$

d = pitch, distance between elements

c_0 = ultrasound velocity

Δt = activation delay



Electronic beam steering

Beam steering by systematic excitation delays.

$$\theta_s = \sin^{-1}\left(\frac{c_0 \Delta t}{d}\right)$$

$$u_s = \sin(\theta_s)$$

$$p_{xs}\left(u, u_s, \lambda\right) = \psi_{0x} \frac{L_x w}{d} \operatorname{sinc}\left[\frac{\pi w x}{\lambda z}\right] \sum_m \operatorname{sinc}\left[\frac{\pi L_x}{\lambda}\left(u - \frac{m\lambda}{d} - u_s\right)\right]$$

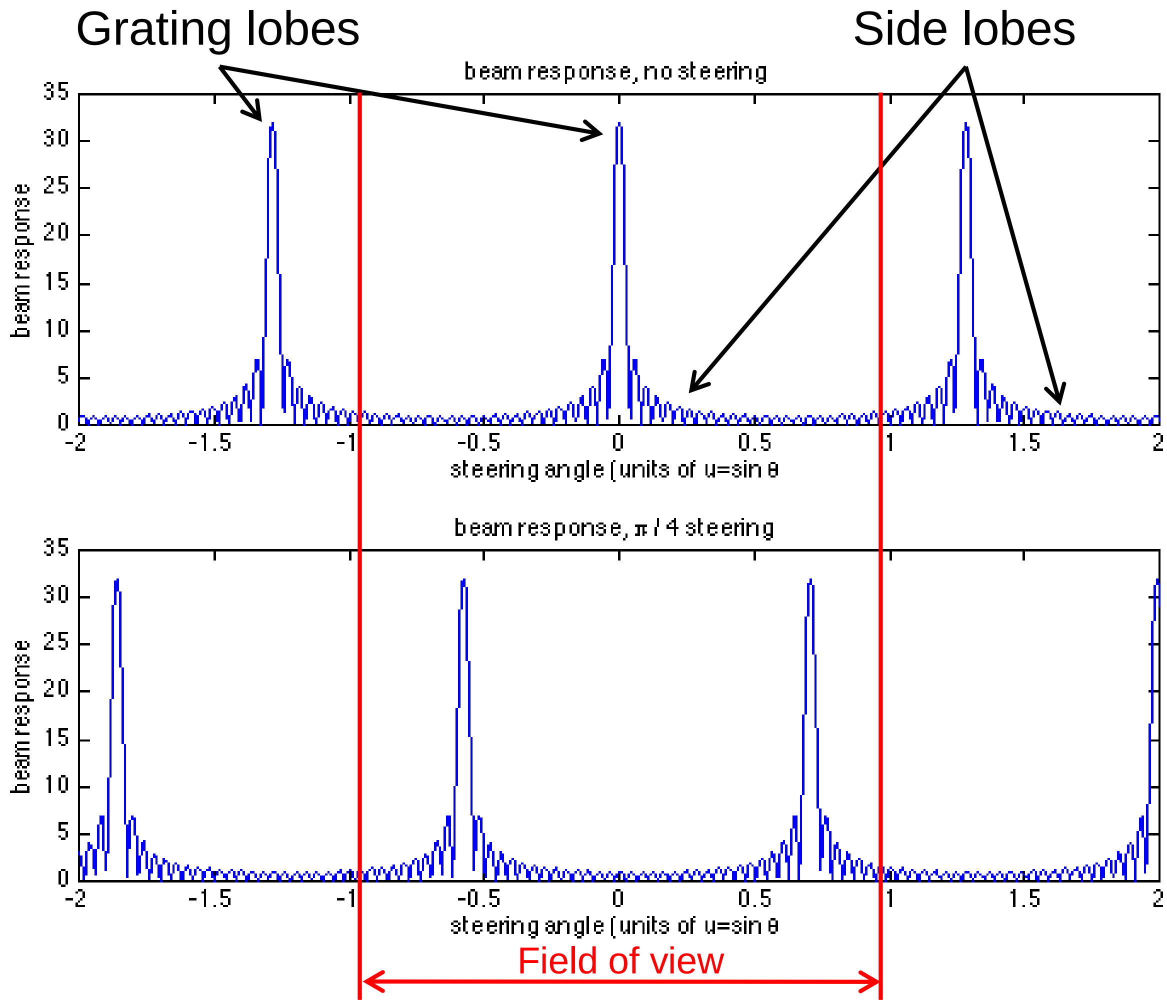
Array design

How many elements?

What spacing?

Linear array:

- 32 element array
- 3 MHz
- 'pitch' d = 0.4 mm
- λ = 0.51 mm
- $L_x = N\,d$ = 13 mm
- θ_s = 45 degrees (π/4)

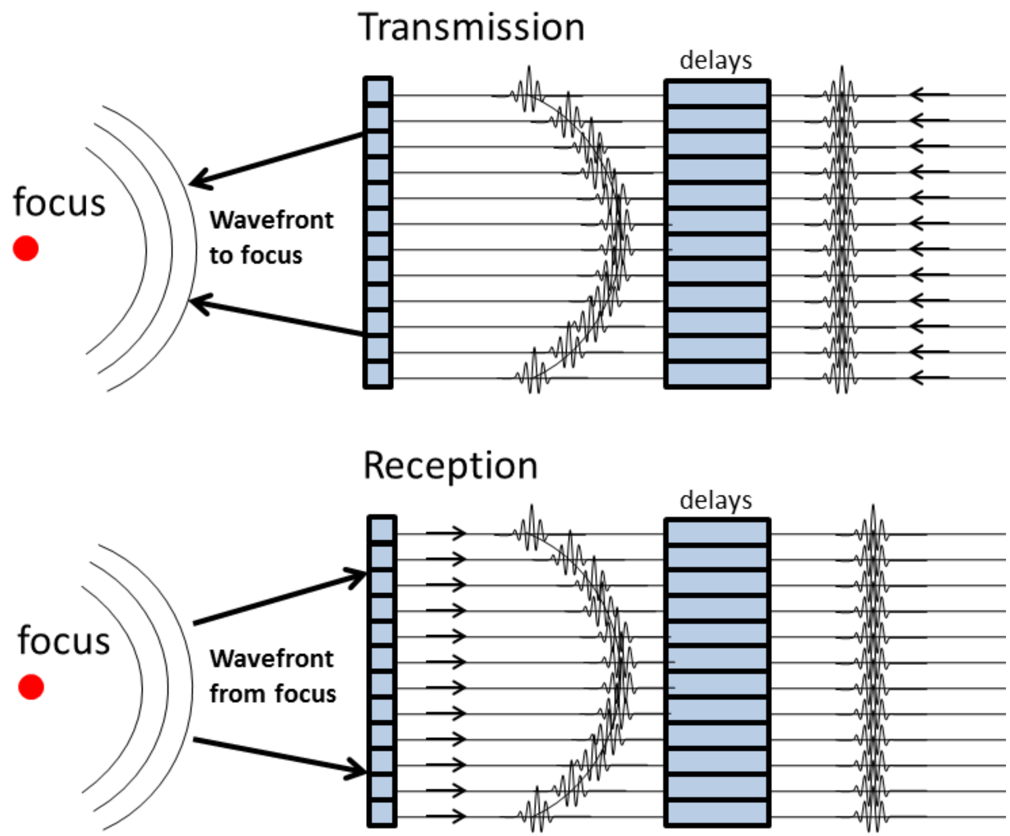


$$d \leq \frac{\lambda}{2}$$

Even steering by 90 degrees (sinθ=1), the first grate lobes remain outside the field of view. Like Nyquist limit for array transducers!

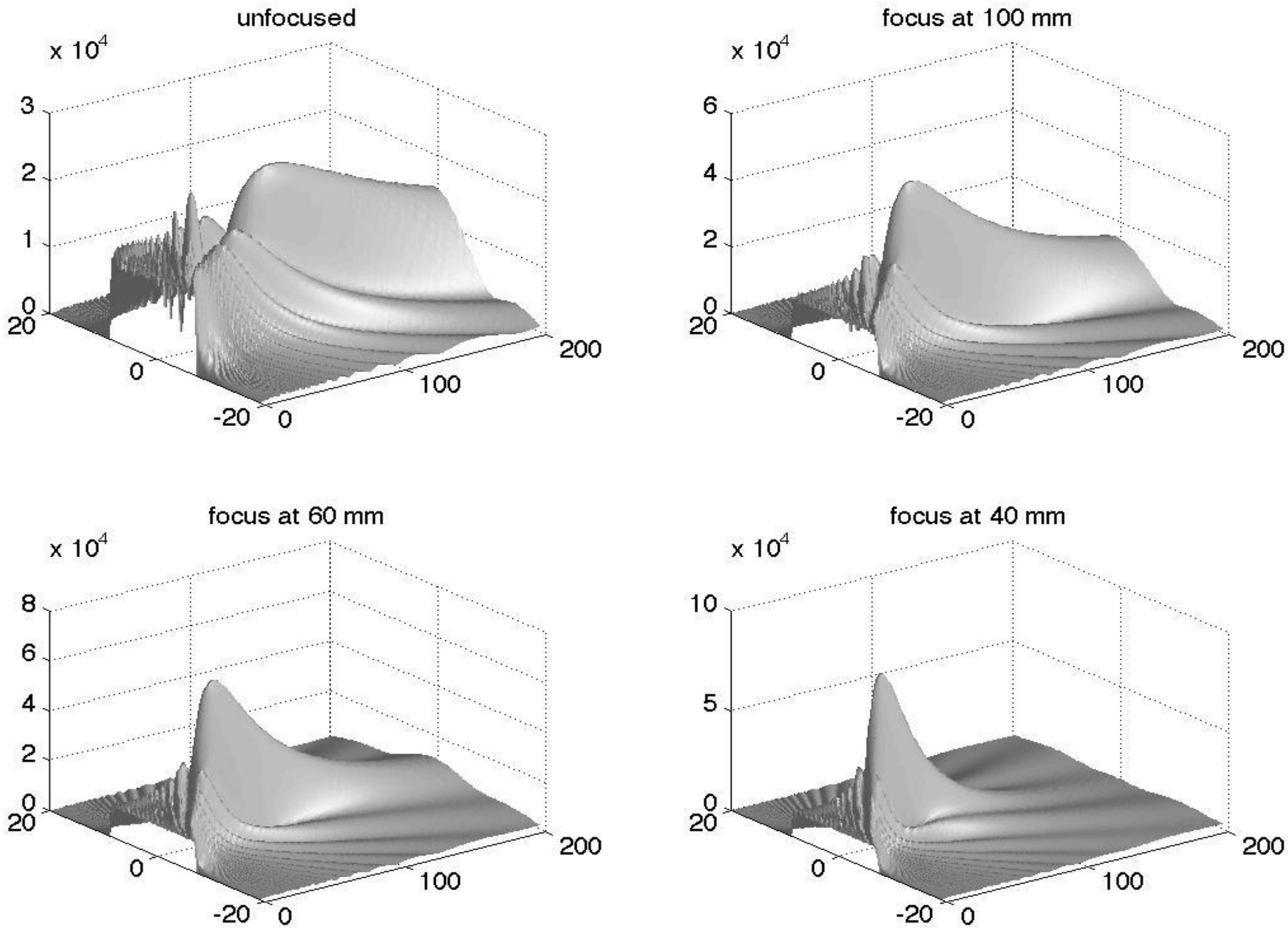
Electronic focusing

Beam forming for dynamic focusing



Focusing effects on the ultrasound beam

Electronic Focusing



Compounding

Improved SNR by averaging different images with same target but different speckle noise.

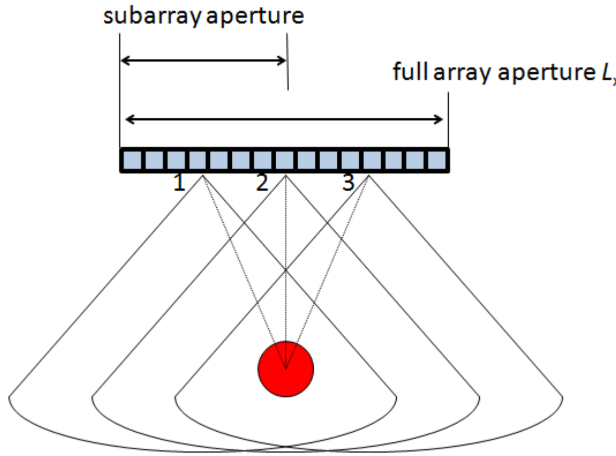
Spatial compounding

- Translational
  (sub-aperture linear stepping)
- Rotational (steering)

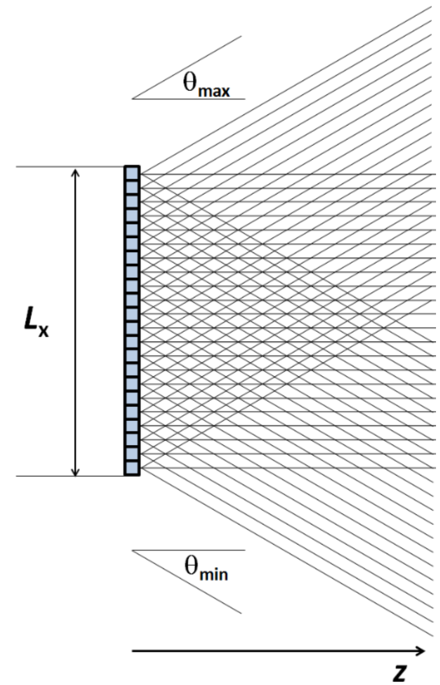
Frequency compounding

- Broadband transducer
- Long pulses (lower axial resolution)

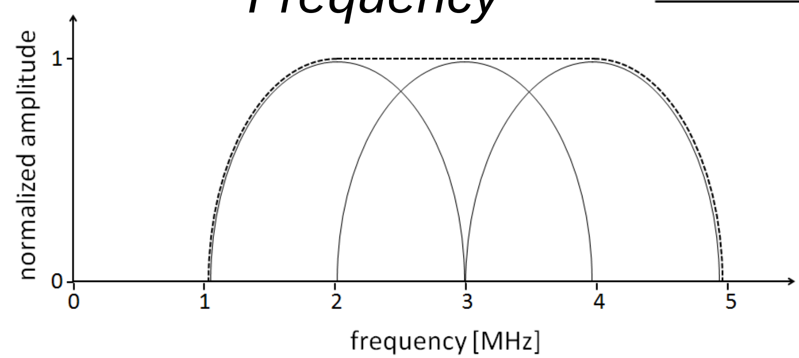
Translational



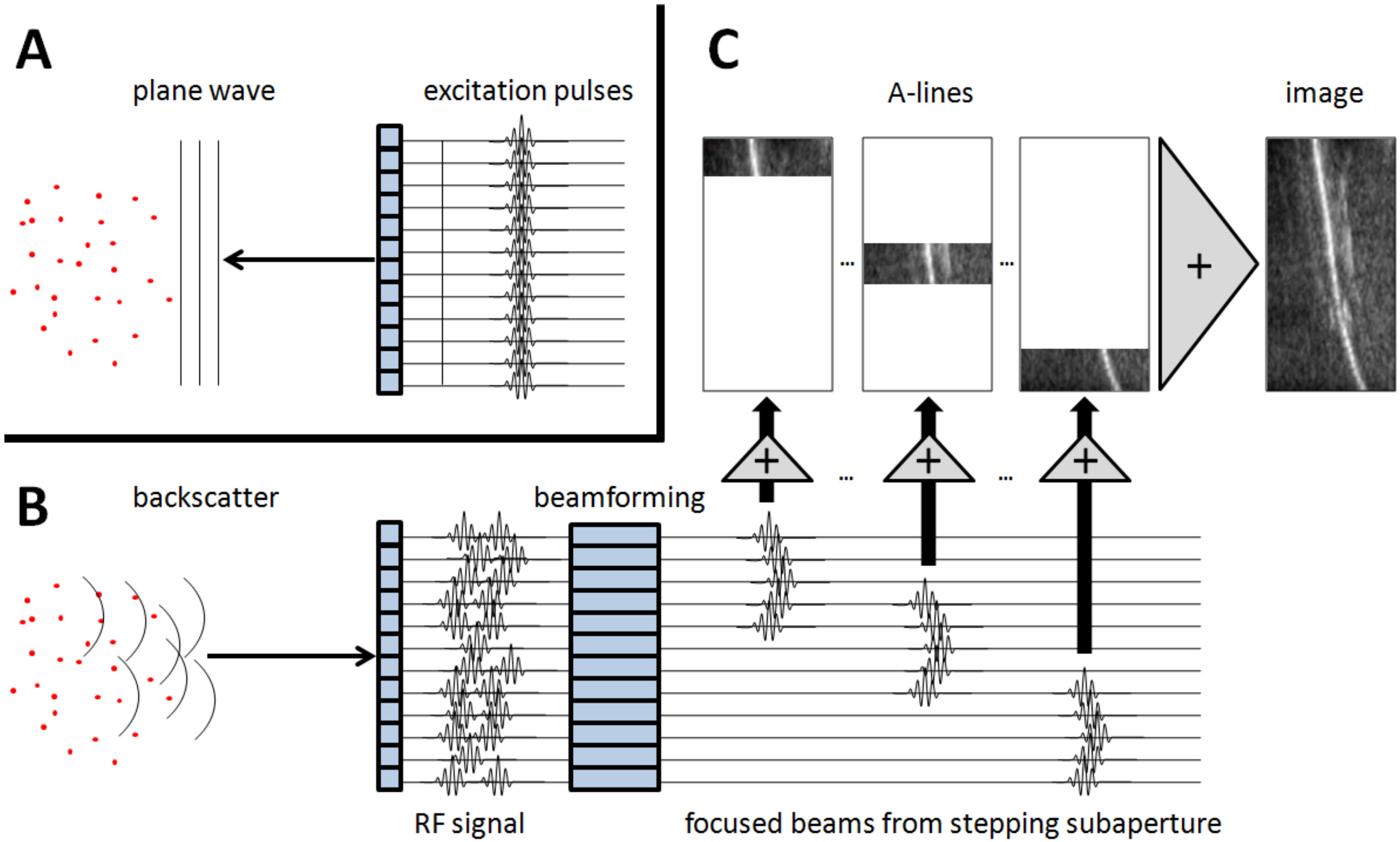
Rotational



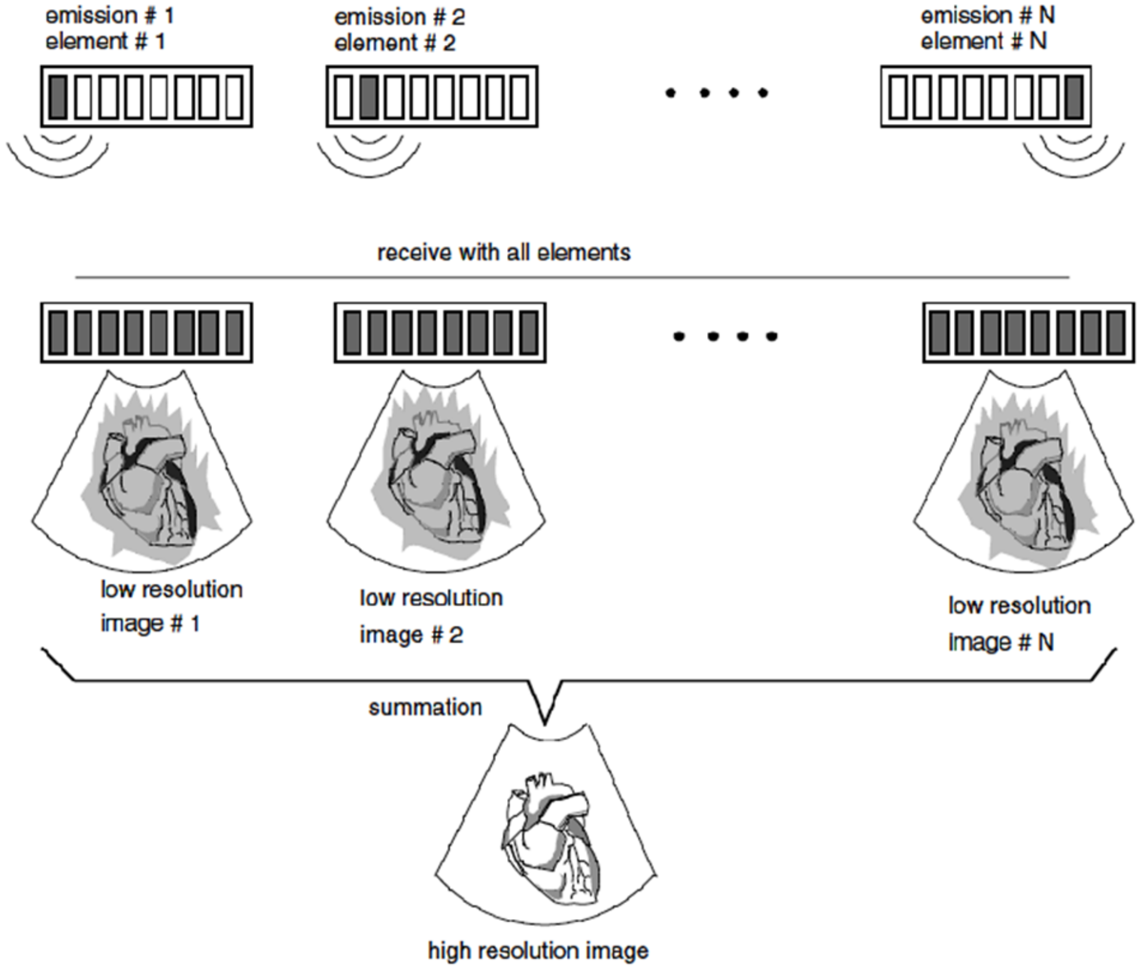
Frequency



Plane wave imaging



Synthetic aperture



Sequential firing by each element.

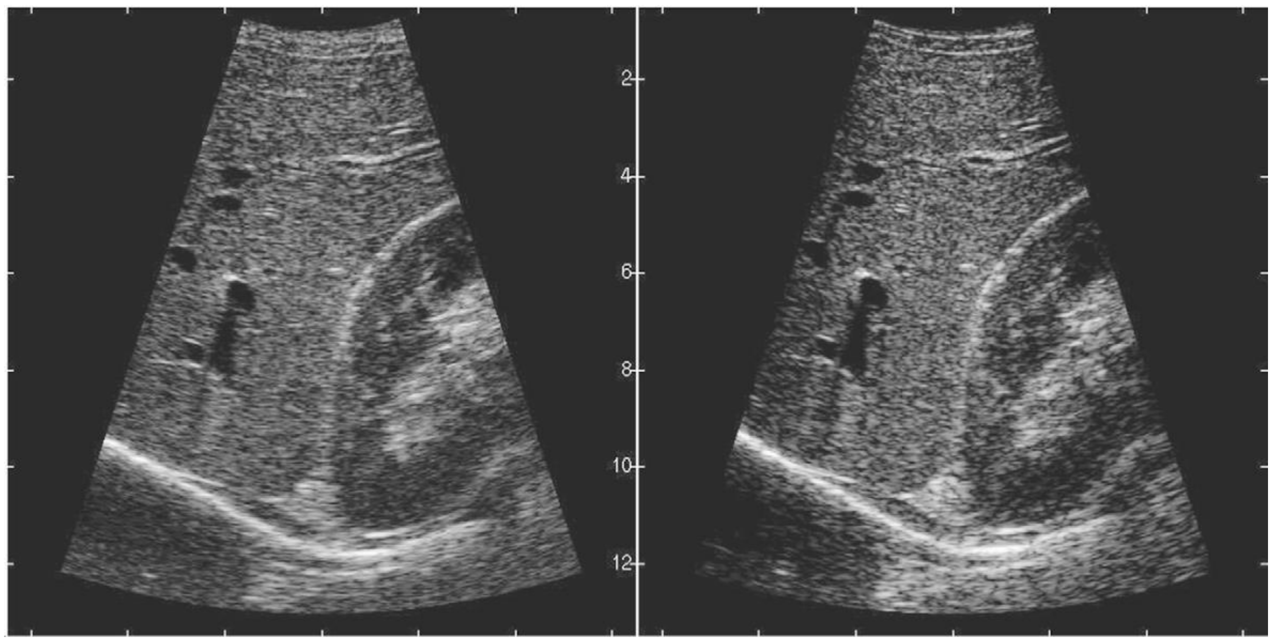
Reception by all elements generating low-resolution focused images.

Summation of low resolution images to form a high-resolution focused images.

Jensen JA, Nikolov SI, Gammelmark KL, et al. (2006) Synthetic aperture ultrasound imaging. Ultrasonics 44: e5–e15.

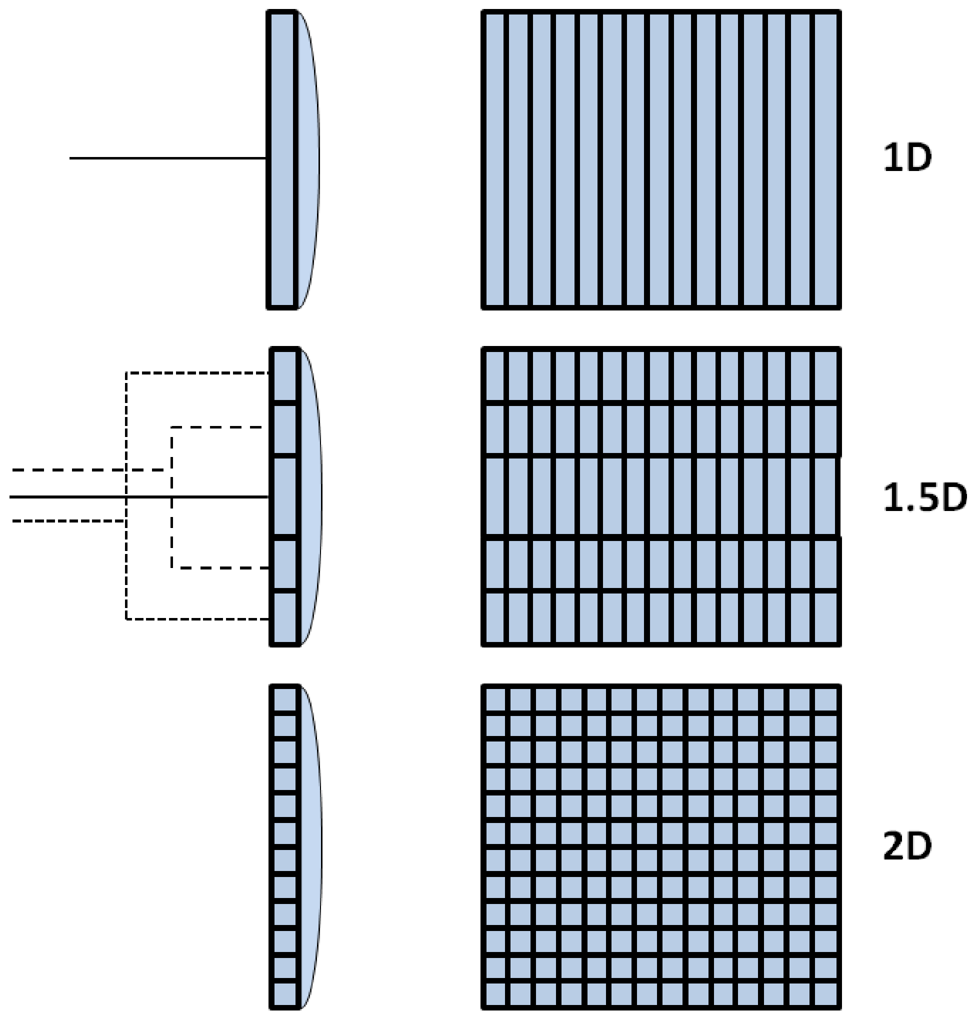
Synthetic aperture

Conventional Synthetic aperture



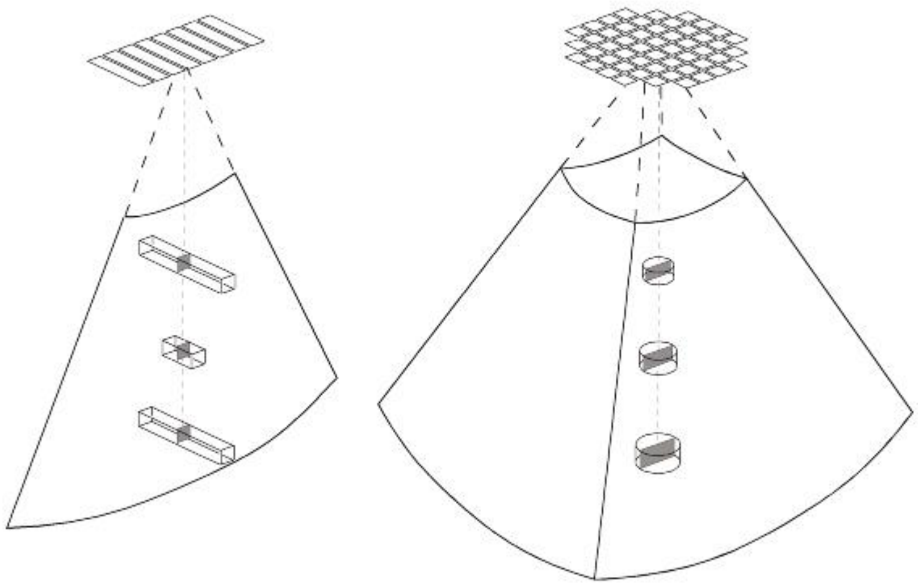
Jensen JA, Nikolov SI, Gammelmark KL, et al. (2006) Synthetic aperture ultrasound imaging. Ultrasonics 44: e5–e15.

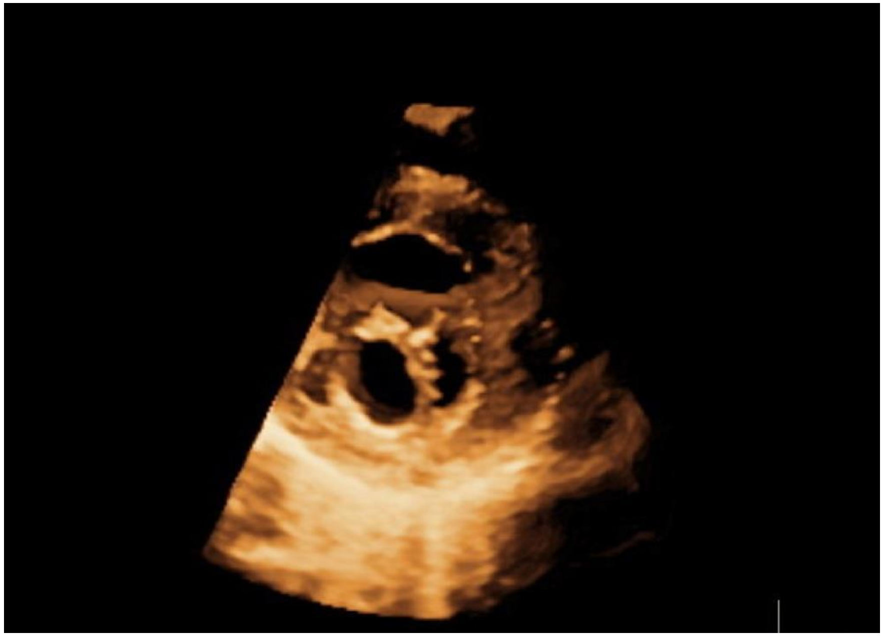
Array transducers'classification



3-D echography

1D array → 2D array
2D imaging → 3D imaging





3-D echocardiographic view of a double-orifice mitral valve

Thank you!

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